(a) One-level Haar Wavelet Transform Decomposition of a 4×4 Image Block(a) 4×4 图像块的一级 Haar 小波变换分解

To perform a one-level Haar Wavelet Transform, we will decompose the image block into four sub-bands: **LL (approximation), LH (horizontal), HL (vertical), and HH (diagonal)**. Here are the steps:

为了执行一级 Haar 小波变换，我们将图像块分解为四个子带：**LL（近似）、LH（水平）、HL（垂直）和 HH（对角线）**。步骤如下：

1. **Separate Rows:** For each row, calculate the average and difference for pairs of pixels. The average goes into the LL (approximation) and LH (horizontal) sub-bands, while the differences go into HL and HH.

   单独的行：对于每一行，计算像素对的平均值和差值。平均值进入 LL（近似）和 LH（水平）子带，而差值进入 HL 和 HH。

2. **Separate Columns:** Perform the same operation on the results of step 1 by averaging and differencing pairs of rows to decompose the block fully.

   分离列：通过对行对进行平均和差分来对步骤 1 的结果执行相同的操作，以完全分解块。

Step-by-Step Calculation逐步计算

Given image block:给定图像块：

$$\begin{bmatrix} 61 & 57 & 16 & 24 \\ 59 & 59 & 18 & 18 \\ 13 & 9 & 13 & 17 \\ 11 & 11 & 13 & 17 \end{bmatrix}$$

Step 1: Horizontal Transformation (Row-wise)第 1 步：水平转换（按行）

- Calculate the average and difference for each pair of columns in each row.

  计算每行中每对列的平均值和差值。

$$\text{Row 1: } \begin{cases} \frac{61+57}{2} = 59 & \frac{61-57}{2} = 2 \\ \frac{16+24}{2} = 20 & \frac{16-24}{2} = -4 \end{cases}$$

Result: Row 1 becomes: $[59, 20, 2, -4]$

Continue this process for all rows:对所有行继续此过程：

1. Row 2: $[59, 18, 0, 0]$
2. Row 3: $[11, 15, 2, -2]$
3. Row 4: $[11, 15, 0, -2]$

So, after row-wise transformation, the matrix is:因此，经过行变换后，矩阵为：

$$\begin{bmatrix} 59 & 20 & 2 & -4 \\ 59 & 18 & 0 & 0 \\ 11 & 15 & 2 & -2 \\ 11 & 15 & 0 & -2 \end{bmatrix}$$

Step 2: Vertical Transformation (Column-wise)第 2 步：垂直转换（按列）

Now, apply the Haar transform on the columns (same process as before):

现在，对列应用 Haar 变换（与之前的过程相同）：

1. **Column 1 (59, 59, 11, 11):**

$$\frac{59+59}{2} = 59, \frac{59-59}{2} = 0, \frac{11+11}{2} = 11, \frac{11-11}{2} = 0$$

   Result: Column 1 becomes: $[59, 11, 0, 0]$结果：第 1 列变为：$[59, 11, 0, 0]$

Following similar calculations, we get the transformed 4×4 matrix:

经过类似的计算，我们得到变换后的4×4矩阵：

$$\begin{bmatrix} 59 & 17 & 1 & -2 \\ 11 & 15 & 0 & -1 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Block-Based Gradient Descent Search Algorithm in Block Motion Estimation(b) 块运动估计中基于块的梯度下降搜索算法

The **block-based gradient descent search algorithm** is used in block motion estimation to find the best match for a block in the current frame within a search window in the reference frame. The algorithm minimizes the **sum of absolute differences (SAD)** or another error metric.

基于块的梯度下降搜索算法用于块运动估计，以在参考帧的搜索窗口内找到当前帧中的块的最佳匹配。该算法最小化**绝对差之和 (SAD)**或其他误差度量。

Steps:

1. **Start at Initial Position:** Choose the center point of the search window in the reference frame.**从初始位置开始：**选择参考系中搜索窗口的中心点。

2. **Evaluate Neighboring Blocks:** Calculate the SAD for neighboring blocks within the search window.**评估相邻块：**计算搜索窗口内相邻块的 SAD。

3. **Move to Minimum Error Position:** Move to the position with the lowest SAD and repeat the process from this new location.

   移动到最小误差位置：移动到 SAD 最低的位置，并从这个新位置重复该过程。

4. **Convergence:** Continue this until the lowest SAD position is the same as the previous position, indicating a local minimum.

   收敛：继续此操作，直到最低 SAD 位置与前一个位置相同，表示局部最小值。

Advantages:优点：

- **Efficiency:** It requires fewer calculations than exhaustive search since it moves towards the optimal match instead of searching all positions.

  效率：它比穷举搜索需要更少的计算，因为它朝着最佳匹配方向移动，而不是搜索所有位置。

- **Good Accuracy:** It often provides close-to-optimal results with significantly lower computational cost.**良好的准确性：**它通常可以提供接近最佳的结果，并且计算成本显着降低。