Let's work through each part step by step.让我们逐步完成每个部分。

Part (a)

(i) Determine the 2-D DCT matrix T for a 4x4 pixel block.

(i) 确定二维DCT矩阵 T 对于 4x4 像素块。

The 2-D DCT matrix T for a 4×4 block can be computed using the given formula:

二维 DCT 矩阵 T 对于一个 4×4 可以使用给定的公式计算块：

$$T(i,j) = \begin{cases} \frac{1}{\sqrt{4}} = 0.5, & \text{if } i = 0 \\ \sqrt{\frac{2}{4}} \cos\left(\frac{(2j+1)i\pi}{2\times 4}\right) = \frac{1}{\sqrt{2}} \cos\left(\frac{(2j+1)i\pi}{8}\right), & \text{if } i > 0 \end{cases}$$

For $N = 4$, this becomes:为了 $N = 4$，这变成：

$$T(i,j) = \begin{cases} 0.5, & \text{if } i = 0 \\ \frac{1}{\sqrt{2}} \cos\left(\frac{(2j+1)i\pi}{8}\right), & \text{if } i > 0 \end{cases}$$

Calculating each entry to 4 decimal places:将每个条目计算为小数点后 4 位：

1. When $i = 0$, $T(0,j) = 0.5$ for all j.什么时候 $i = 0$，$T(0,j) = 0.5$ 为所有人 j。
2. When $i = 1$:
$$T(1,j) = \frac{1}{\sqrt{2}} \cos\left(\frac{(2j+1)\pi}{8}\right)$$
3. When $i = 2$:
$$T(2,j) = \frac{1}{\sqrt{2}} \cos\left(\frac{(2j+1)2\pi}{8}\right)$$
4. When $i = 3$:
$$T(3,j) = \frac{1}{\sqrt{2}} \cos\left(\frac{(2j+1)3\pi}{8}\right)$$

Substitute values for $j = 0, 1, 2, 3$ and compute each entry to form the full 4x4 matrix T.

替换值 $j = 0, 1, 2, 3$ 并计算每个条目以形成完整的 4x4 矩阵 T。

(ii) Calculate the 2-D DCT of the pixel block A.(ii) 计算像素块的2-D DCT A。

The 2-D DCT of A is given by $C = T \cdot A \cdot T^T$.二维 DCT A 由下式给出 $C = T \cdot A \cdot T^T$。

Substitute the values of T from part (i) and matrix A, and perform the matrix multiplication. Round the results to 3 decimal places.

代入以下值 T 来自第 (i) 部分和矩阵 A，并执行矩阵乘法。将结果四舍五入至小数点后 3 位。

Part (b)

(i) Main similarity and main difference between the DCT basis functions of this new compression scheme and baseline JPEG compression.

(i) 这种新压缩方案的 DCT 基函数与基线 JPEG 压缩之间的主要相似点和主要区别。

- **Similarity**: Both use the DCT, which transforms spatial pixel values into frequency coefficients, emphasizing low-frequency components where most image information typically resides.

  相似之处：两者都使用 DCT，将空间像素值转换为频率系数，强调大多数图像信息通常所在的低频分量。

- **Difference**: The baseline JPEG uses an 8×8 block size, which provides a finer frequency resolution, while the proposed method uses a 4×4 block size, resulting in coarser frequency components. The smaller block size may lead to faster processing but potentially less efficient compression.

  区别：基线 JPEG 使用 8×8 块大小，提供更精细的频率分辨率，而所提出的方法使用 4×4 块大小，导致较粗糙的频率分量。较小的块大小可能会导致更快的处理速度，但可能导致压缩效率较低。

(ii) Suitable quantization table for the new compression scheme.

(ii)适合新压缩方案的量化表。

A suitable quantization table could be as follows (higher values for high-frequency coefficients to prioritize low-frequency components):

合适的量化表如下（高频系数的值较高，以优先考虑低频分量）：

$$\begin{bmatrix} 16 & 11 & 10 & 16 \\ 12 & 12 & 14 & 19 \\ 14 & 13 & 16 & 24 \\ 14 & 17 & 22 & 29 \end{bmatrix}$$

- **Justification**: Low-frequency components (top-left) have lower values, ensuring they are preserved with minimal loss. High-frequency components (bottom-right) have higher values to allow greater compression by discarding more detail, which is less perceptible to the human eye.

  理由：低频分量（左上）具有较低的值，确保它们以最小的损失保留。高频分量（右下）具有更高的值，可以通过丢弃更多人眼不易察觉的细节来实现更大的压缩。

(iii) Determine which pixel block, $B1$ or $B2$, will likely experience more reconstruction error.(iii) 确定哪个像素块，$B1$ 或者 $B2$，可能会遇到更多的重建错误。

- **Answer**: $B2$ will likely experience more reconstruction error during decompression.

  回答：$B2$ 在减压过程中可能会遇到更多的重建错误。

- **Justification**: The DCT coefficients of $B2$ contain more zeros and lower magnitude values in high-frequency areas compared to $B1$. This suggests that during quantization, more high-frequency details in $B2$ will be lost, leading to higher reconstruction error.

  理由：DCT 系数 $B2$ 与相比，在高频区域包含更多的零和更低的幅度值 $B1$。这表明在量化过程中，更多的高频细节 $B2$ 会丢失，导致更高的重建误差。