Exercise : 2D - DCT Using Matrix Implementation

Q (i) T = ? (ii) A → DCT

Solution

(i) $$T(i,j) = \begin{cases} \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}} = \frac{1}{2} & \text{if } i = 0 \\ \sqrt{\frac{2}{N}} \cos\frac{(2j+1)i\pi}{2N} = \frac{1}{\sqrt{2}} \cos\frac{(2j+1)i\pi}{8} & \text{if } i > 0 \end{cases}$$

$$T = \begin{array}{c} i\backslash j \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{3\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{5\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{7\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{2\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{6\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{10\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{14\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{3\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{9\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{15\pi}{8} & \frac{1}{\sqrt{2}}\cos\frac{21\pi}{8} \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.6533 & 0.2706 & -0.2706 & -0.6533 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.2706 & -0.6533 & 0.6533 & -0.2706 \end{bmatrix}$$

(ii) $F(u,v) = T\, f(i,j)\, T^T$

$$= \begin{bmatrix} 20 & 18.478 & 0 & -7.654 \\ 18.478 & 17.072 & 0 & -7.072 \\ 0 & 0 & 0 & 0 \\ -7.654 & -7.072 & 0 & 2.929 \end{bmatrix}$$

-7.071

答案为 -7.071

算出来是 casio -7.071530b

-7.071