

Exercise : 2D-DCT Using Matrix Implementation

Q (i) $T = ?$ (ii) $A \rightarrow \text{DCT}$

Solution

$$(i) T(i, j) = \begin{cases} \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{4}} = \frac{1}{2} & \text{if } i=0 \\ \frac{\sqrt{2}}{\sqrt{N}} \cos \frac{(2i+1)j\pi}{2N} = \frac{1}{\sqrt{2}} \cos \frac{(2i+1)j\pi}{8} & \text{if } i>0 \end{cases}$$

$$T = \begin{matrix} & \begin{matrix} ij & 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} \cos \frac{\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{3\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{5\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{7\pi}{8} \\ \frac{1}{\sqrt{2}} \cos \frac{2\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{6\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{10\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{14\pi}{8} \\ \frac{1}{\sqrt{2}} \cos \frac{3\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{9\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{15\pi}{8} & \frac{1}{\sqrt{2}} \cos \frac{21\pi}{8} \end{bmatrix} \end{matrix}$$

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.6533 & 0.2706 & -0.2706 & -0.6533 \\ 0.5 & -0.5 & -0.5 & -0.2706 \\ 0.2706 & -0.6533 & 0.6533 & -0.2706 \end{bmatrix}$$

(ii) $F(u, v) = T f(i, j) T^T$

$$= \begin{bmatrix} 20 & 18.478 & 0 & -7.654 \\ 18.478 & 17.072 & 0 & -7.072 \\ 0 & 0 & 0 & 0 \\ -7.654 & -7.072 & 0 & 2.929 \end{bmatrix}$$

答案为 -7.071
算出来是 casio
-7.0715306