Exercise: 2D-DCT Using Matrix Implementation

Q (i) $T = ?$ (ii) $A \longrightarrow$ DCT

Solution

(i)
$$T(i,j) = \begin{cases} \dfrac{1}{\sqrt{N}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2} & \text{if } i=0 \\[3mm] \sqrt{\dfrac{2}{N}} \cos\dfrac{(2j+1)i\pi}{2N} = \dfrac{1}{\sqrt{2}} \cos\dfrac{(2j+1)i\pi}{8} & \text{if } i>0 \end{cases}$$

$$T = \begin{array}{c} i \backslash j \\ 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{bmatrix}
\dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{2} \\[3mm]
\dfrac{1}{\sqrt{2}}\cos\dfrac{\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{3\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{5\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{7\pi}{8} \\[3mm]
\dfrac{1}{\sqrt{2}}\cos\dfrac{2\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{6\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{10\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{14\pi}{8} \\[3mm]
\dfrac{1}{\sqrt{2}}\cos\dfrac{3\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{9\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{15\pi}{8} & \dfrac{1}{\sqrt{2}}\cos\dfrac{21\pi}{8}
\end{bmatrix}$$

$$= \begin{bmatrix}
0.5 & 0.5 & 0.5 & 0.5 \\
0.6533 & 0.2706 & -0.2706 & -0.6533 \\
0.5 & -0.5 & -0.5 & 0.5 \\
0.2706 & -0.6533 & 0.6533 & -0.2706
\end{bmatrix}$$

(ii) $F(u,v) = T \, f(i,j) \, T^{T}$

$$= \begin{bmatrix}
20 & 18.478 & 0 & -7.654 \\
18.478 & 17.072 & 0 & -7.072 \\
0 & 0 & 0 & 0 \\
-7.654 & -7.072 & 0 & 2.929
\end{bmatrix}$$

-7.071

答案为 -7.071
算出来是 Casio
-7.0715306