22 – S1 – Q1

Q : (a) DCT $\quad A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 \\ 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Solution $\quad N = 4$

$$\alpha(k) = \begin{cases} \frac{1}{2} & \text{for } k = 0 \\ \frac{\sqrt{2}}{2} & \text{for } k = 1, 2, 3 \end{cases}$$

$$S_{uv} = \alpha(u)\,\alpha(v) \sum_{i=0}^{3} \sum_{j=0}^{3} S_{ij} \cos\frac{(2i+1)u\pi}{8} \cos\frac{(2j+1)v\pi}{8}$$

$$S_{uv} = \alpha(u)\,\alpha(v) \Big[S_{11} \cos\frac{3u\pi}{8} \cos\frac{3v\pi}{8} + S_{12} \cos\frac{3u\pi}{8} \cos\frac{5v\pi}{8} + S_{21} \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} + S_{22} \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} \Big]$$

$$= 10\,\alpha(u)\,\alpha(v) \Big[\cos\frac{3u\pi}{8} \cos\frac{3v\pi}{8} + \cos\frac{3u\pi}{8} \cos\frac{5v\pi}{8} + \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} + \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} \Big]$$

$$S_{00} = 10 \times \frac{1}{2} \times \frac{1}{2} \left[1 + 1 + 1 + 1 \right] = 10$$

偷鸡，用T矩阵，$TAT^T = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$T(i,j) = \begin{cases} \sqrt{\frac{1}{N}} \\ \sqrt{\frac{2}{N}} \cos\frac{(2j+1)i\pi}{2N} \end{cases}$$ ，用i换u

simplely

$S_{00} = 10 \quad S_{01} = 0 \quad S_{02} = -10 \quad S_{03} = 0$

$S_{10} = 0 \quad S_{11} = 0 \quad S_{12} = 0 \quad S_{13} = 0$

$S_{20} = -10 \quad S_{21} = 0 \quad S_{22} = 10 \quad S_{23} = 0$

$S_{30} = 0 \quad S_{31} = 0 \quad S_{32} = 0 \quad S_{33} = 0$

$$S_{uv} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)

$$B = 20I_4 - \frac{1}{2}A$$

2-D DCT of $20I_4 = T\,20I_4\,T^T = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2-D DCT of $B = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \frac{1}{2} \times \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 75 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$