23-S1-Q1

Q (i) T = ?

Solution $N = 4$

$$T(i,j)\begin{cases} \frac{1}{2}, & \text{if } i=0 \\ \frac{\sqrt{2}}{2}\cos\frac{(2j+1)i\pi}{8} & \text{if } i>0 \end{cases}$$

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{2}\cos\frac{\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{3\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{5\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{7\pi}{8} \\ \frac{\sqrt{2}}{2}\cos\frac{2\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{6\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{10\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{14\pi}{8} \\ \frac{\sqrt{2}}{2}\cos\frac{3\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{9\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{15\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{21\pi}{8} \end{bmatrix}$$

$$= \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 & 0.5000 \\ 0.6534 & 0.2706 & -0.2706 & -0.6533 \\ 0.5000 & -0.5000 & -0.5000 & 0.5000 \\ 0.2706 & -0.6533 & 0.6533 & -0.2706 \end{bmatrix}$$

(ii) 2-D DCT

$$TAT^T = \begin{bmatrix} 20.000 & 18.480 & 0.000 & -7.654 \\ 18.480 & 17.076 & 0.000 & -7.072 \\ 0 & 0 & 0 & 0 \\ -7.654 & -7.072 & 0 & 2.929 \end{bmatrix}$$

(b)(i) similarity: both basic function have increasing horizontal, vertical and diagonal frequencies

Difference: the new scheme uses 4x4 DCT that consists of 16 different types of basis function where the baseline JPEG uses 8x8 DCT that consist of 64 different types of basis function

(ii) A suitable quantization table

$$Q = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 20 & 30 & 40 & 50 \\ 30 & 40 & 50 & 60 \\ 40 & 50 & 60 & 70 \end{bmatrix}$$

The quantization table should be 4x4 with increasing step sizes in the horizontal vertical and diagonal directions. This is to reduce quantization errors for the DC and low AC coefficients as their DCT values are larger and human ar more sensitive to error/distortion in them.

(iii)

Solution: B1 will experience more reconstruction error

Justify ① C_1 has larger high-frequency coefficients compared to C_2

② During Quantization, high-frequency DCT coefficients are divided by larger quantization steps, leading to greater rounding errors.

Since C_1 has more high-frequency coefficients, C_1 will loss more of these components compared to C_2

③ So B2 has less Reconstruction error than B_1