(4a) Basic Lifting Scheme for Haar Wavelet Transform

(4a) Haar小波变换的基本提升方案

The Haar Wavelet Transform is one of the simplest wavelet transformations, which operates by splitting the input signal into approximation (average) and detail (difference) components.

Haar 小波变换是最简单的小波变换之一，它通过将输入信号分为近似（平均）和细节（差异）分量来进行操作。

The lifting scheme for the Haar Wavelet Transform consists of three main steps:

Haar 小波变换的提升方案包含三个主要步骤：

1. **Split Step**: Split the input sequence into two parts – even and odd samples.

   分割步骤：将输入序列分割为两部分 - 偶数样本和奇数样本。

2. **Predict Step**: Use the even part to predict the odd part. Subtract the prediction from the actual odd sample to obtain the detail coefficient.

   预测步骤：使用偶数部分来预测奇数部分。从实际奇数样本中减去预测值即可得到细节系数。

3. **Update Step**: Use the detail coefficient to update the even part, producing the average coefficient.**更新步骤**：使用细节系数更新偶数部分，产生平均系数。

Let's start with the input sequence $X = \{X_0, X_1\} = \{10, 14\}$.

让我们从输入序列开始 $X = \{X_0, X_1\} = \{10, 14\}$ 。

Step-by-Step Calculation逐步计算

1. **Split Step**:
   - Separate X into even and odd indices:分离 X 分为偶数和奇数索引：
     - Even (approximation): $X_{\text{even}} = X_0 = 10$偶数（近似值）： $X_{\text{even}} = X_0 = 10$
     - Odd (detail): $X_{\text{odd}} = X_1 = 14$
2. **Predict Step**:
   - Predict the odd element using the even element. For Haar, the prediction is simply the value of the even sample.

     使用偶数元素预测奇数元素。对于 Haar 来说，预测只是偶数样本的值。
   - $\text{Detail (d)} = X_{\text{odd}} - X_{\text{even}} = 14 - 10 = 4$
3. **Update Step**:
   - Update the even element with the average value.用平均值更新偶数元素。
   - $\text{Average (a)} = \frac{X_{\text{even}} + X_{\text{odd}}}{2} = \frac{10+14}{2} = 12$

Diagram

Here's a simple diagram to show the structure of the basic lifting scheme for the Haar Wavelet Transform:下面是一个简单的图表，显示了哈尔小波变换的基本提升方案的结构：

```mathematica
Input Sequence: X = {10, 14}

       | X₀ (10)           | X₁ (14)
       |                   |
      Split               Split
       |                   |
  X_even = 10         X_odd = 14
       |                   |
    Predict ----------------> (Detail) d = 4
       |
     Update
       |
     (Average) a = 12

Output: Average = 12, Detail = 4
```

So, the output of the basic lifting scheme for the Haar Wavelet Transform for input $X = \{10, 14\}$ is:因此，哈尔小波变换的基本提升方案的输出为输入 $X = \{10, 14\}$ 是：

- Approximation (average): 12近似值（平均）：12
- Detail: 4

(4b) Structure of 5/3 Lifting Discrete Wavelet Transform

(4b) 5/3提升离散小波变换的结构

The 5/3 Lifting Scheme is a widely used transform in image compression standards like JPEG 2000. The 5/3 scheme involves prediction and update steps using specific coefficients.

5/3 提升方案是 JPEG 2000 等图像压缩标准中广泛使用的变换。5/3 方案涉及使用特定系数的预测和更新步骤。

Diagram and Steps示意图和步骤

1. **Split Step**:
   - Separate the input signal into even and odd indexed samples.

     将输入信号分为偶数和奇数索引样本。
2. **Predict Step**:
   - Predict the odd samples using a linear combination of the neighboring even samples:使用相邻偶数样本的线性组合来预测奇数样本：
     - $d_i = x_{\text{odd}} - \frac{1}{2}(x_{\text{even-left}} + x_{\text{even-right}})$
3. **Update Step**:
   - Update the even samples using the predicted detail coefficients:

     使用预测的细节系数更新偶数样本：
     - $a_i = x_{\text{even}} + \frac{1}{4}(d_{\text{left}} + d_{\text{right}})$

This 5/3 Lifting Scheme structure results in approximation and detail coefficients for the wavelet transform.该 5/3 提升方案结构产生小波变换的近似系数和细节系数。

Here's a simplified diagram illustrating the 5/3 lifting scheme:

这是说明 5/3 提升方案的简化图：

```mathematica
Input Sequence: X = {X₀, X₁, X₂, X₃, ...}

    Split
     /    \
X_even    X_odd
     \    /
  Predict Step
     /    \
     d     a
     |     |
   Update Step
     |     |
   Output (a, d)
```

The 5/3 transform diagram shows the split, predict, and update steps, resulting in an approximation coefficient a and detail coefficient d for the signal.

5/3 变换图显示了分割、预测和更新步骤，从而产生近似系数 a 和细节系数 d 为信号。