23-S1-Q1

Q (i) T = ?

Solution $N = 4$

$$T(i,j)\begin{cases} \frac{1}{2}, & \text{if } i=0 \\ \frac{\sqrt{2}}{2}\cos\frac{(2j+1)i\pi}{8} & \text{if } i>0 \end{cases}$$

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{\sqrt{2}}{2}\cos\frac{\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{3\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{5\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{7\pi}{8} \\ \frac{\sqrt{2}}{2}\cos\frac{2\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{6\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{10\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{14\pi}{8} \\ \frac{\sqrt{2}}{2}\cos\frac{3\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{9\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{15\pi}{8} & \frac{\sqrt{2}}{2}\cos\frac{21\pi}{8} \end{bmatrix}$$

$$= \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 & 0.5000 \\ 0.6534 & 0.2706 & -0.2706 & -0.6533 \\ 0.5000 & -0.5000 & -0.5000 & 0.5000 \\ 0.2706 & -0.6533 & 0.6533 & -0.2706 \end{bmatrix}$$

(ii) 2-D DCT

$$TAT^T = \begin{bmatrix} 20.000 & 18.480 & 0.000 & -7.654 \\ 18.480 & 17.076 & 0.000 & -7.072 \\ 0 & 0 & 0 & 0 \\ -7.654 & -7.072 & 0 & 2.929 \end{bmatrix}$$

(b) similarity: both basic function have increasing horizontal, vertical and diagonal frequencies

Difference: the new scheme uses 4x4 DCT that consists of 16 different types of basis function where the baseline JPEG uses 8x8 DCT that consist of 64 different types of basis function

(c) A suitable quantization table

$$Q = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 20 & 30 & 40 & 50 \\ 30 & 40 & 50 & 60 \\ 40 & 50 & 60 & 70 \end{bmatrix}$$

The quantization table should be 4x4 with increasing step sizes in the horizontal vertical and diagonal directions. This is to reduce quantization errors for the DC and low AC coefficients as their DCT values are larger and human ar more sensitive to error/distortion in them.