22 - S1 - Q1

Q: (a) DCT $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 \\ 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Solution $N = 4$

$$\alpha(k) = \begin{cases} \dfrac{1}{2} & \text{for } k = 0 \\ \dfrac{\sqrt{2}}{2} & \text{for } k = 1, 2, 3 \end{cases}$$

$$S_{uv} = \alpha(u)\,\alpha(v) \sum_{i=0}^{3} \sum_{j=0}^{3} S_{ij} \cos\frac{(2i+1)u\pi}{8} \cos\frac{(2j+1)v\pi}{8}$$

$$S_{uv} = \alpha(u)\,\alpha(v)\left[S_{11} \cos\frac{3u\pi}{8} \cos\frac{3v\pi}{8} \right.$$

$$+ S_{12} \cos\frac{3u\pi}{8} \cos\frac{5v\pi}{8}$$

$$+ S_{21} \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8}$$

$$\left. + S_{22} \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} \right]$$

$$= 10\,\alpha(u)\,\alpha(v)\left[\cos\frac{3u\pi}{8} \cos\frac{3v\pi}{8} \right.$$

$$+ \cos\frac{3u\pi}{8} \cos\frac{5v\pi}{8}$$

$$+ \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8}$$

$$\left. + \cos\frac{5u\pi}{8} \cos\frac{3v\pi}{8} \right]$$

$$S_{00} = 10 \times \frac{1}{2} \times \frac{1}{2}\left[1 + 1 + 1 + 1 \right] = 10$$

偷鸡，用T矩阵，$TAT^T = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$T(i,j) = \begin{cases} \sqrt{\frac{1}{N}} \\ \sqrt{\frac{2}{N}} \cos \frac{(2j+1)i\pi}{2N} \end{cases}$ ，用 i 换 u

simplely

$S_{00} = 10 \qquad S_{01} = 0 \qquad S_{02} = -10 \qquad S_{03} = 0$

$S_{10} = 0 \qquad S_{11} = 0 \qquad S_{12} = 0 \qquad S_{13} = 0$

$S_{20} = -10 \qquad S_{21} = 0 \qquad S_{22} = 10 \qquad S_{23} = 0$

$S_{30} = 0 \qquad S_{31} = 0 \qquad S_{32} = 0 \qquad S_{33} = 0$

$S_{uv} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(b)

$B = 20I_4 - \frac{1}{2}A$

2D DCT of $20I_4 = T\,20I_4\,T^T = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2D DCT of $B = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \frac{1}{2} \times \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 75 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
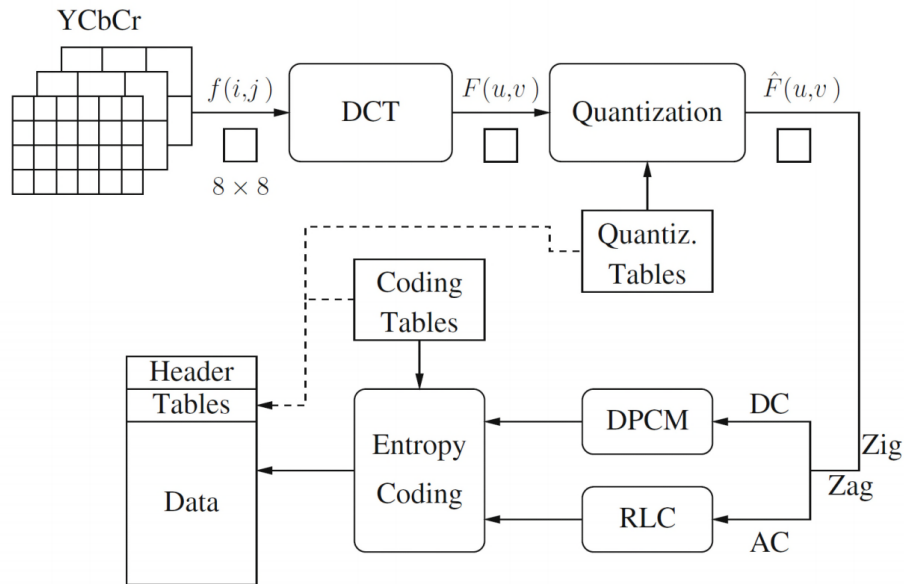
(c)   # JPEG Encoder

YCbCr

$f(i,j)$ → DCT → $F(u,v)$ → Quantization → $\hat{F}(u,v)$

8 × 8

Quantiz. Tables

Coding Tables

Header Tables

Data ← Entropy Coding ← DPCM ← DC

Entropy Coding ← RLC ← AC

Zig Zag

Fig. 9.1: Block diagram for JPEG encoder.

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(d) disagree

Justify ① DCT is effective at energy compaction primarily for smooth image regions, not textured ones.

② textured image patches contain rapid intensity variation and high-frequency content.
DCT will produce more non-zero quantizes coefficients

③ since energy is not as easily compacted, entropy encoding becomes less efficient