C A1 : 2D-DCT

$$S_{uv} = \alpha(u)\alpha(v) \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} S_{ij} \cos\frac{(2i+1)u\pi}{2N} \cos\frac{(2j+1)v\pi}{2N}$$

$$\alpha(k) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } k=0 \\ \sqrt{\frac{2}{N}} & \text{for } k=1,2,\cdots N-1 \end{cases}$$

$$A = \begin{bmatrix} 10 & 10 & 20 & 20 \\ 10 & 10 & 20 & 20 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution

$$S_{uv} = \alpha(u)\alpha(v) \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} S_{ij} \cos\frac{(2i+1)u\pi}{2N} \cos\frac{(2j+1)v\pi}{2N}$$

$$\overset{提u}{=} \alpha(u) \sum_{i=0}^{N-1} \cos\frac{(2i+1)u\pi}{2N} \left\{ \alpha(v) \sum_{j=0}^{N-1} S_{ij} \cos\frac{(2j+1)v\pi}{2N} \right\}$$

$$\overset{代換}{=} \alpha(u) \sum_{i=0}^{N-1} \underbrace{F_{iv}}_{F_{iv}} \cos\frac{(2i+1)u\pi}{2N}$$

First Stage: F_{iv} Second Stage: S_{uv}
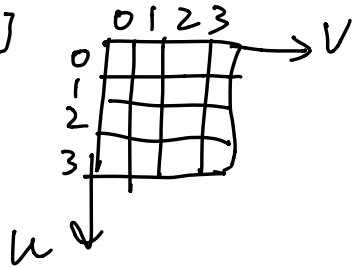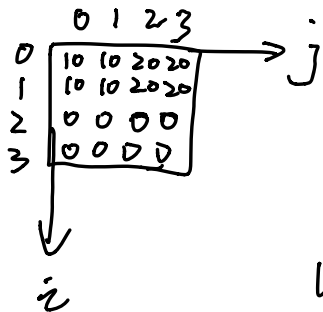
First Stage : $F_{iv} = \alpha(v) \sum_{j=0}^{N-1} S_{ij} \cos\frac{(2j+1)v\pi}{2N}$

$\because 4\times4$ 2D-DCT $\quad \therefore N=4$

$$F_{iv} = \alpha(v) \sum_{j=0}^{3} S_{ij} \cos\frac{(2j+1)v\pi}{8}$$

$$= \alpha(v)\left[S_{i0}\cos\frac{v\pi}{8} + S_{i1}\cos\frac{3v\pi}{8} \right.$$

$$\left. + S_{i2}\cos\frac{5v\pi}{8} + S_{i3}\cos\frac{7v\pi}{8} \right]$$

$$\alpha(0) = \frac{1}{2}$$

$$\alpha(k) = \frac{1}{\sqrt{2}}, \quad k=1,2\cdots N-1$$

let $i=0$, $S_{00}=10$, $S_{01}=10$, $S_{02}=20$, $S_{03}=20$

$$F_{0v} = \alpha(v)\left[10\cos\frac{v\pi}{8} + 10\cos\frac{3v\pi}{8} + 20\cos\frac{5v\pi}{8} + 20\cos\frac{7v\pi}{8} \right]$$

let $v=0$, $F_{00} = \frac{1}{2}[10+10+20+20] = 30$

$v=1$, $F_{01} = \frac{10}{\sqrt{2}}\left[\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + 2\cos\frac{5\pi}{8} + 2\cos\frac{7\pi}{8}\right]$

≈ -9.2388

$v=2$, $F_{02} = \frac{10}{\sqrt{2}}\left[\cos\frac{\pi}{4} + \cos\frac{3\pi}{4} + 2\cos\frac{5\pi}{4} + 2\cos\frac{7\pi}{4}\right]$

$= 0$

$$V = 3 \quad F_{03} = \frac{10}{\sqrt{2}}\left[\cos\frac{3\pi}{8} + \cos\frac{7\pi}{8} + 2\cos\frac{15\pi}{8} + 2\cos\frac{21\pi}{8}\right]$$

$$\approx 3.8268$$

$$F_{0V} = \begin{bmatrix} 30 & -9.2388 & 0 & 3.8268 \end{bmatrix}$$

similarly $F_{1V} = \begin{bmatrix} 30 & -9.2388 & 0 & 3.8268 \end{bmatrix}$

$$F_{2V} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \qquad F_{3V} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, $F_{iV} = \begin{bmatrix} 30 & -9.2388 & 0 & 3.8268 \\ 30 & -9.2388 & 0 & 3.8268 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$S_{uV} = \alpha(u) \sum_{i=0}^{N-1} F_{iV} \cos\frac{(2i+1)u\pi}{2N}$$

$$= \alpha(u) \sum_{i=0}^{3} F_{iV} \cos\frac{(2i+1)u\pi}{8}$$

$$= \alpha(u)\left[F_{0V} \cos\frac{u\pi}{8} + F_{1V} \cos\frac{3u\pi}{8} \right.$$

$$\left. + F_{2V} \cos\frac{5u\pi}{8} + F_{3V} \cos\frac{7u\pi}{8} \right]$$

let $v = 0$, $F_{00} = 30$ $F_{10} = 30$ $F_{20} = 0$ $F_{30} = 0$

$$S_{u0} = \alpha(u)\left[30 \cos\frac{u\pi}{8} + 30 \cos\frac{3u\pi}{8}\right]$$

① let $u=0$, $S_{00} = \frac{1}{2}[30+30] = 30$

$u=1$, $S_{10} = \frac{30}{\sqrt{2}}\left[\cos\frac{\pi}{8} + \cos\frac{3\pi}{8}\right] \approx 27.7164$

$u=2$, $S_{20} = \frac{30}{\sqrt{2}}\left[\cos\frac{\pi}{4} + \cos\frac{3\pi}{4}\right] = 0$

$u=3$, $S_{30} = \frac{30}{\sqrt{2}}\left[\cos\frac{3\pi}{8} + \cos\frac{9\pi}{8}\right] \approx -11.4805$

$S_{u0} = \begin{bmatrix} 30 & 27.7164 & 0 & -11.4805 \end{bmatrix}^T$

② let $V=1$, $S_{u1} = \alpha(u)\left[-9.2388\cos\frac{u\pi}{8} - 9.2388\cos\frac{3u\pi}{8}\right]$

let $u=0$, $S_{01} = \frac{1}{2}[-9.2388 - 9.2388] = -9.2388$

$u=1$, $S_{11} = \frac{-9.2388}{\sqrt{2}}\left[\cos\frac{\pi}{8} + \cos\frac{3\pi}{8}\right] \approx -8.5355$

$u=2$, $S_{21} = \frac{-9.2388}{\sqrt{2}}\left[\cos\frac{2\pi}{8} + \cos\frac{6\pi}{8}\right] = 0$

$u=3$, $S_{31} = \frac{-9.2388}{\sqrt{2}}\left[\cos\frac{3\pi}{8} + \cos\frac{9\pi}{8}\right] \approx 3.5355$

$S_{u1} = \begin{bmatrix} -9.2388 & -8.5355 & 0 & 3.5355 \end{bmatrix}^T$

③ let $V=2$, $S_{u2} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

④ let $V=3$, $S_{u3} = \alpha(u)\left[3.8268\cos\frac{u\pi}{8} + 3.8268\cos\frac{3u\pi}{8}\right]$

let $u=0$, $S_{03} = \frac{1}{2}[3.8268 + 3.8268] = 3.8268$

$u=1$, $S_{13} = \frac{3.8268}{\sqrt{2}}\left[\cos\frac{\pi}{8} + \cos\frac{3\pi}{8}\right] \approx 3.5355$

$u=2$, $S_{23} = \frac{3.8268}{\sqrt{2}}\left[\cos\frac{2\pi}{8} + \cos\frac{6\pi}{8}\right] = 0$

$$k=3, \quad S_{33} = \frac{3.8268}{\sqrt{2}}\left[\cos\frac{3\pi}{8} + \cos\frac{9\pi}{8}\right] \approx -1.4645$$

$$S_{u3} = \begin{bmatrix} 3.8268 & 3.5355 & 0 & -1.4645 \end{bmatrix}^T$$

$$S_{uv} = \begin{bmatrix} 30 & -9.2388 & 0 & 3.8268 \\ 27.7164 & -8.5355 & 0 & 3.5355 \\ 0 & 0 & 0 & 0 \\ -11.4805 & 3.5355 & 0 & -1.4645 \end{bmatrix}$$