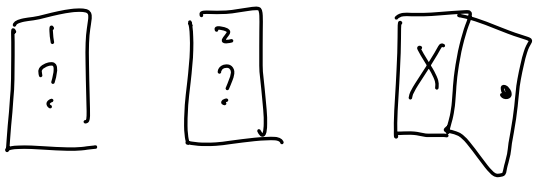
3.6.7.4 Quiz 4: Monty Hall Problem



Solution ① 1st Trail.

Case 1 : $p(win) = \frac{1}{3}$

$p(lose) = \frac{2}{3}$

Switch: case 1 and switch, $p(lose) = \frac{1}{3}$

case 2 and switch, $p(win) = \frac{2}{3}$

Not switch : case 1: $p(win) = \frac{1}{3}$

case 2: $p(lose) = \frac{2}{3}$

② Bayes

C_i denote price behind door i

$P(C1) = P(C2) = P(C3) = \frac{1}{3}$

play choose P1, the host opens D2

$$P(C1 \mid D2) = \frac{P(D2 \mid C1)\,P(C1)}{P(D2)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

NOT SWITCH

↓ random pick D2 or D3

$P(C2 \mid D2) = 0$ → player choose D1, host know D3 have prize so he only can open D2

$$P(C3 \mid D2) = \frac{P(D2 \mid C3)\,P(C3)}{P(D2)} = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$P(D2) = P(D2 \mid C1)P(C1) + P(D2 \mid C2)P(C2) + P(D2 \mid C3)P(C3)$$

$$= \frac{1}{2} \times \frac{1}{3} + 0 + \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{3}$$