

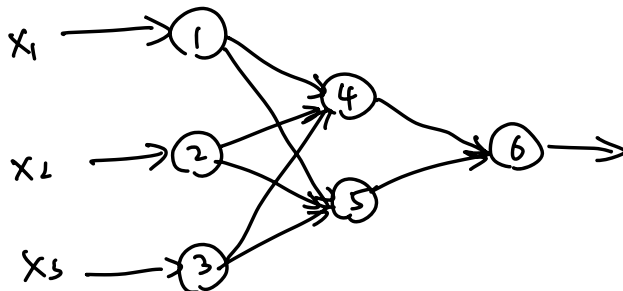
4.3.5.2.1 Backpropagation Example

Q: $x = (1, 0, 1)$ with label 1, learning rate
initial weights and biases $\eta_w = \eta_b = 0.9$

x_1 x_2 x_3 b_4 b_5 b_6

1 0 1 -0.4 0.2 0.1

w_{41}	w_{51}	w_{42}	w_{52}	w_{43}	w_{53}	w_{64}	w_{65}
0.2	-0.3	0.4	0.1	-0.5	0.2	-0.3	-0.2



Solution ① Unit 4

Net input net_j

$$\begin{aligned} net_4 &= x_1 w_{41} + x_2 w_{42} + x_3 w_{43} + b_4 \\ &= 1 \times 0.2 + 0 \times 0.4 + 1 \times (-0.5) - 0.4 \\ &= 0.2 - 0.5 - 0.4 \\ &= -0.7 \end{aligned}$$

$$\text{Output } O_j = \sigma(net_j) = \frac{1}{1 + e^{-net_j}}$$

$$O_4 = \frac{1}{1 + e^{-(-0.7)}} = 0.3318$$

② Unit 5

$$\begin{aligned} net_5 &= w_{51} x_1 + w_{52} x_2 + w_{53} x_3 + b_5 \\ &= -0.3 \times 1 + 0.1 \times 0 + 0.2 \times 1 + 0.2 \\ &= -0.3 + 0.2 + 0.2 \\ &= 0.1 \end{aligned}$$

$$O_5 = \frac{1}{1 + e^{-0.1}} = 0.5250$$

③ Unit 6

$$\begin{aligned} net_6 &= w_{64} O_4 + w_{65} O_5 + b_6 \\ &= -0.3 \times 0.3318 - 0.2 \times 0.525 + 0.1 \end{aligned}$$

$$= -0.1045$$

$$O_6 = \frac{1}{1 + e^{0.1045}} = 0.4739$$

④ Unit 6

$$\delta_6 = \sigma'(\text{net}_6)(t_6 - O_6)$$

$$= \sigma(\text{net}_6)(1 - \sigma(\text{net}_6))(t_6 - O_6)$$

$$\sigma(\text{net}_6) = \frac{1}{1 + e^{0.1045}} = 0.4739$$

$$\delta_6 = 0.4739 \times (1 - 0.4739) \times (1 - 0.4739)$$

$$= 0.1312$$

⑤ Unit 4

$$\delta_j = \sigma'(\text{net}_j) \sum_k \delta_k W_{kj}$$

$$\delta_4 = \sigma'(\text{net}_4) \delta_6 W_{64}$$

$$\sigma(\text{net}_4) = \frac{1}{1 + e^{0.7}} = 0.3318$$

$$\delta_4 = 0.3318 \times (1 - 0.3318) \times 0.1312 \times (-0.3)$$

$$= -0.008726$$

⑥ Unit 5

$$\delta_5 = \sigma'(\text{net}_5) \delta_6 W_{65}$$

$$\sigma'(\text{net}_5) = \frac{1}{1 + e^{-0.1}} = 0.5250$$

$$\begin{aligned}\delta_5 &= 0.525 \times (1 - 0.525) \times 0.1312 \times (-0.2) \\ &= -0.006544\end{aligned}$$

$$\textcircled{7} W_{64}$$

$$\begin{aligned}\Delta W_{kj} &= \eta_w \delta_k O_j \\ &= 0.9 \times \delta_6 \times O_4 \\ &= 0.9 \times 0.1312 \times 0.3318 \\ &= 0.03918\end{aligned}$$

$$\begin{aligned}W_{64} &= -0.3 + 0.03918 \\ &= -0.2608\end{aligned}$$

$$\begin{aligned}\textcircled{8} W_{65} &= -0.2 \times 0.9 \times 0.1312 \times 0.525 \\ &= -0.1380\end{aligned}$$

$$\begin{aligned}\textcircled{9} W_{41} &= 0.2 + 0.9 \times (-0.008726) \times 1 \\ &= 0.1921\end{aligned}$$

$$\begin{aligned}\textcircled{10} W_{51} &= -0.3 + 0.9 \times (-0.006544) \times 1 \\ &= -0.3059\end{aligned}$$

$$\textcircled{11} W_{42} = 0.4 + 0.9 \times (-0.008726) \times 0 \\ = 0.4$$

$$\textcircled{12} W_{52} = 0.1$$

$$\textcircled{13} W_{43} = -0.5 + 0.9 \times (-0.008726) \times 1 \\ = -0.5079$$

$$\textcircled{14} W_{53} = 0.2 + 0.9 \times (-0.006544) \times 1 \\ = 0.1941$$

$$\textcircled{15} \Delta b_j = \eta_b \delta_j$$

$$b_6 = 0.1 + 0.9 \times 0.1312 = 0.2181$$

$$\textcircled{16} b_5 = 0.2 + 0.9 \times (-0.006544) \\ = 0.1941$$

$$\textcircled{17} b_4 = -0.4 + 0.9 \times (-0.008726) \\ = -0.4078$$