

### 3.4.6 Derivation of PCA

$$x_1 = [4, 6, 10] \quad x_2 = [3, 10, 13] \quad x_3 = [-2, -6, -8]$$

$$PC1 = ?$$

$$\text{Solution } X = \begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 29 & 66 & 95 \\ 66 & 172 & 238 \\ 95 & 238 & 333 \end{bmatrix}$$

$$|\lambda I - X^T X| = 0$$

$$-\lambda(\lambda^2 - 534\lambda + 1896) = 0$$

$$\lambda_1 = 530.425$$

$$\lambda_2 = 3.575$$

$$\lambda_3 = 0$$

$$(530.425I - X^T X)U = 0$$

$$\begin{bmatrix} -501.425 & 66 & 95 \\ 66 & -358.425 & 238 \\ 95 & 238 & -197.425 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{let } V_z = 1, \begin{cases} -501.425V_x + 66V_y + 95 = 0 \\ 66V_x + -358.425V_y + 238 = 0 \end{cases}$$

$$\begin{cases} V_x = 0.2837 \\ V_y = 0.7163 \\ V_z = 1 \end{cases}$$

$\Rightarrow$

$$\begin{cases} V_x = 0.2247 \\ V_y = 0.5674 \\ V_z = 0.7922 \end{cases}$$

$$V_1 = \begin{bmatrix} -0.22 \\ -0.57 \\ -0.79 \end{bmatrix}$$