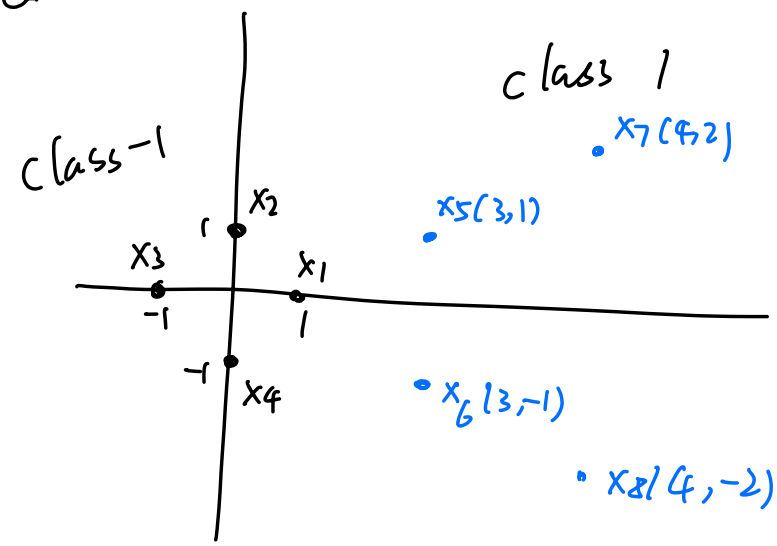
3.5 SVM Example

Q:



Solution : ① support vectors : x_1 x_5, x_6

$\lambda_1, \lambda_5, \lambda_6 \neq 0$

$y_1 = -1$, y_5 $y_6 = 1$

$$w = \sum_{i=1}^{N} \lambda_i y_i x_i$$

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = -\lambda_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda_5 \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \lambda_6 \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad (1)$$

$$\sum_{i=1}^{N} \lambda_i y_i = 0$$

$$-\lambda_1 + \lambda_5 + \lambda_6 = 0 \quad (5)$$

$$y_i (w \cdot x_i + b) \geq 1$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b = -1 \quad (2)$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}^T \begin{pmatrix} 3 \\ 1 \end{pmatrix} + b = 1 \quad (3)$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}^T \begin{pmatrix} 3 \\ -1 \end{pmatrix} + b = 1 \quad (4)$$

② From(1) $w_1 = -\lambda_1 + 3\lambda_5 + 3\lambda_6$ (10)

$w_2 = \lambda_5 - \lambda_6$ (11)

From(2) $w_1 + b = -1$ (8)

From(3) $3w_1 + w_2 + b = 1$ (6)

From(4) $3w_1 - w_2 + b = 1$ (7)

From(5) $-\lambda_1 + \lambda_5 + \lambda_6 = 0$ (12)

(6)+(7) $6w_1 + 2b = 2$

$3w_1 + b = 1$ (9)

(9)-(8) $2w_1 = 2$

$w_1 = 1$

From(9) $b = 1 - 3w_1$

$b = 1 - 3$

$b = -2$

From(7) $w_2 = 3w_1 + b - 1$

$= 3 - 2 - 1$

$= 0$

So $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $b = -2$

③ For on (10) (11) (12)

$$-\lambda_1 + 3\lambda_5 + 3\lambda_6 - 1 = 0$$

$$\lambda_5 - \lambda_6 = 0$$

$$-\lambda_1 + \lambda_5 + \lambda_6 = 0$$

$\lambda_1 = \frac{1}{2}$ $\lambda_5 = \frac{1}{4}$ $\lambda_6 = \frac{1}{4}$

④ Disctrimination function

$y = w^T x + b$ $y = (1,0)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 2$

Decision boundary

let $y = 0$

$(1,0)\begin{pmatrix} x \\ y \end{pmatrix} - 2 = 0$

$x - 2 = 0$

$\Rightarrow x = 2$