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To match each expression in Table 1 with the corresponding expression in Table 2, let's analyze each operation and its definition in binary morphological image processing.

Table 1:

1. $A \circ B$ - **Opening** of A by B.
2. $A \oplus B$ - **Dilation** of A by B.
3. $A \cdot B$ - **Hit-or-Miss** transform.
4. $A - B$ - **Set difference** between A and B.
5. $A \ominus B$ - **Erosion** of A by B.
6. $(B)_z$ - **Translation** of B by vector z.

Table 2: a. $\{w - z | w \in B\}$ - Translation of B by z. b. $(A \oplus B) \ominus B$ - **Closing** of A by B. c. $\cup\{(B)_z | (B)_z \subseteq A\}$ - The union of all translations of B that are subsets of A (related to **Opening**). d. $\{z | (\hat{B})_z \cap A \neq \emptyset\}$ - Definition of **Dilation**. e. $A \cap B^c$ - Set difference, as $A - B = A \cap B^c$. f. $\{z | (B)_z \subseteq A\}$ - Definition of **Erosion**.

Matching:

- **1.** $A \circ B$ matches with **c.** $\cup\{(B)_z | (B)_z \subseteq A\}$
- **2.** $A \oplus B$ matches with **d.** $\{z | (\hat{B})_z \cap A \neq \emptyset\}$
- **3.** $A \cdot B$ matches with **b.** $(A \oplus B) \ominus B$
- **4.** $A - B$ matches with **e.** $A \cap B^c$
- **5.** $A \ominus B$ matches with **f.** $\{z | (B)_z \subseteq A\}$
- **6.** $(B)_z$ matches with **a.** $\{w - z | w \in B\}$

Completed Answer Table:

1	2	3	4	5	6
c	d	b	e	f	a

So, the association is:

- 1: **c**
- 2: **d**
- 3: **b**
- 4: **e**
- 5: **f**
- 6: **a**

Answer Table:

1	2	3	4	5	6
c	d	b	e	f	a

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Problem (b):

We are given a binary image A (Figure 1) and two structuring elements B (Figures 2 and 3). We are to compute the set $C = A - (A \ominus B)$ for each structuring element and shade the elements of C in Figure 4. We will also analyze how the two results relate to image A.

(i) Using Structuring Element B from Figure 2:

Structuring Element B (Figure 2):

- A 3x3 matrix with all ones.
- Origin at the center (coordinates $(0, 0)$).

Interpretation:

- This structuring element considers all neighboring pixels in a 3x3 window.

Steps to Compute $C = A - (A \ominus B)$:

1. **Erosion ($A \ominus B$):**
   - For each pixel in A, check if the structuring element B fits entirely within A when centered at that pixel.
   - Pixels where B fits entirely are kept; others are eroded.
2. **Compute C:**
   - Subtract the eroded image from the original image: $C = A - (A \ominus B)$.
   - This results in the **boundary** of A corresponding to the thickness defined by B.

Shaded Elements of Set C:

Based on the erosion, the pixels in A that are not completely surrounded by other pixels (as per B) become part of C. The shaded elements in C are:

- **Boundary Pixels of A:**

Row	Column(s)
1	3
2	2, 3, 4
3	2, 4, 5
4	2, 5
5	2, 5, 6
6	1, 2, 5
7	2, 5
8	2, 4, 5
9	1, 2, 3, 4
10	3

Representation in Figure 4:

mathematica 复制代码

```
| 0 | 0 | 1 | 0 | 0 | 0 |  (Row 1)
| 0 | 1 | 1 | 1 | 0 | 0 |  (Row 2)
| 0 | 1 | 0 | 1 | 1 | 0 |  (Row 3)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 4)
| 0 | 1 | 0 | 0 | 1 | 1 |  (Row 5)
| 1 | 1 | 0 | 0 | 1 | 0 |  (Row 6)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 7)
| 0 | 1 | 0 | 1 | 1 | 0 |  (Row 8)
| 1 | 1 | 1 | 1 | 0 | 0 |  (Row 9)
| 0 | 0 | 1 | 0 | 0 | 0 |  (Row 10)
```

(ii) Using Structuring Element B from Figure 3:

Structuring Element B (Figure 3):

- A 3x3 matrix in a cross shape:
  - Ones at positions: center, up, down, left, right.
- Origin at the center (coordinates $(0, 0)$).

Interpretation:

- This structuring element considers only the immediate north, south, east, and west neighbors.

Steps to Compute $C = A - (A \ominus B)$:

1. **Erosion ($A \ominus B$):**
   - For each pixel in A, check if the cross-shaped structuring element B fits entirely within A when centered at that pixel.
   - Pixels where B fits entirely are kept; others are eroded.
2. **Compute C:**
   - Subtract the eroded image from the original image: $C = A - (A \ominus B)$.
   - This results in a thinner **boundary** of A compared to part (i).

Shaded Elements of Set C:

With a smaller structuring element, fewer pixels are eroded. The shaded elements in C are:

- **Thinner Boundary Pixels of A:**

Row	Column(s)
1	3
2	2, 3, 4
3	2, 5
4	2, 5
5	2, 6
6	1, 5
7	2, 5
8	2, 5
9	1, 4
10	3

Representation in Figure 4:

mathematica 复制代码

```
| 0 | 0 | 1 | 0 | 0 | 0 |  (Row 1)
| 0 | 1 | 1 | 1 | 0 | 0 |  (Row 2)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 3)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 4)
| 0 | 1 | 0 | 0 | 0 | 1 |  (Row 5)
| 1 | 0 | 0 | 0 | 1 | 0 |  (Row 6)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 7)
| 0 | 1 | 0 | 0 | 1 | 0 |  (Row 8)
| 1 | 1 | 0 | 1 | 0 | 0 |  (Row 9)
| 0 | 0 | 1 | 0 | 0 | 0 |  (Row 10)
```

Relation to Image A:

使用更大的结构元素（所有元素都在 3x3 矩阵中）会导致边界更厚。更多像素被侵蚀，因为结构元素要求所有周围像素都是 1。

- **Part (i):**
  - Using a larger structuring element (all ones in a 3x3 matrix) results in a thicker boundary. More pixels are eroded because the structuring element requires all surrounding pixels to be ones.
  - C represents the outer boundary of A, capturing more of the edge details.

C 表示的外边界 A1 ，捕捉更多边缘细节。

- **Part (ii):**
  - The cross-shaped structuring element is less restrictive, resulting in a thinner boundary. Fewer pixels are eroded since only the immediate neighbors in four directions are considered.
  - C represents a thinner version of the boundary of A, focusing on the most immediate edges.

十字形结构元素的限制较少，导致边界更薄。由于仅考虑四个方向上的直接邻居，因此被侵蚀的像素更少

C 表示边界的更薄版本 A ，关注最直接的边缘

Conclusion:

- The two results show how the choice of structuring element affects the erosion and, consequently, the boundary extraction.
- **Image A and the Results:**
  - Both C sets represent the boundaries of the original image A, but with different thicknesses.
  - The boundaries extracted in parts (i) and (ii) highlight the edges of A at different scales.

这两个结果显示了结构元素的选择如何影响侵蚀，从而影响边界提取

Note: The shading represents the elements of C after performing the morphological operation $C = A - (A \ominus B)$ using the respective structuring elements.