20 – S2 – Q5

Q (a)(i) input → output $<$ vector ? scalar ?

Solution ① understand

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{98} \end{bmatrix}$$

$98\ \boxed{\ W_{ij}\ }^{100} \times \boxed{\ }^{100} + \boxed{\theta}^{98} = \boxed{Y}^{98}$

$1\ 2\ 3 \cdots i \cdots 98$
$1\ 2\ 3 \cdots j \cdots 100$

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1\,100} \\ W_{21} & W_{22} & \cdots & W_{2\,100} \\ \vdots & \vdots & \ddots & \vdots \\ W_{981} & W_{982} & \cdots & W_{98\,100} \end{bmatrix} \qquad \vec{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{98} \end{bmatrix}$$

② Vector form

$$\vec{y} = W\vec{x} + \vec{\theta}$$

③ Scalar form

$$y_i = \sum_{j=1}^{100} W_{ij}\, x_j + \theta_i$$

(ii) Q : parameters ?   mul ?  sum ?

Solution ① parameters

parameters = number of $W_{ij}$ + number of $\theta_i$

$$= 98 \times 100 + 98$$

$$= 9898$$

② multiplication

multiplication = number of $w \cdot x$

$$= 98 \times 100$$

$$= 9800$$

③ summations

summations = number of $w \cdot x$ add $wx$ + number of add $\theta$
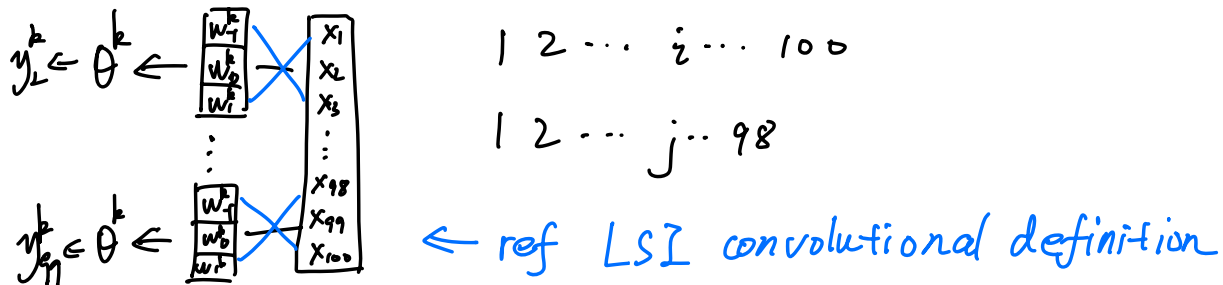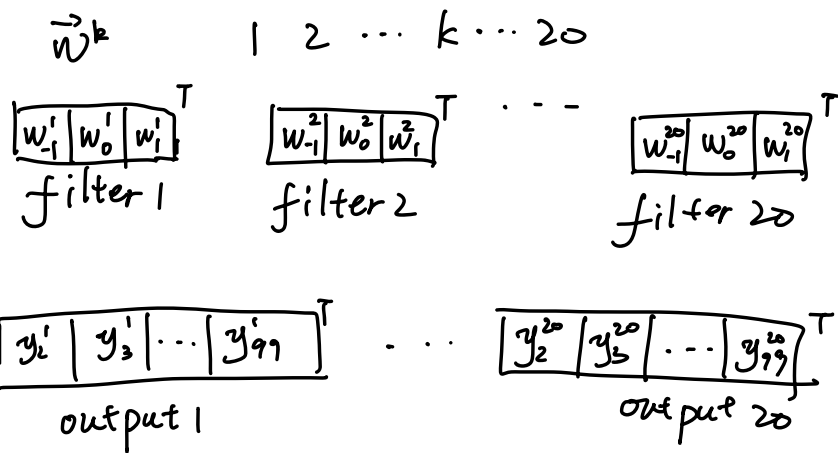
$$= 98 \times (99 + 1)$$

$$= 9800$$

(iii) ratio = ?

Solution

$$\text{ratio} = \frac{\text{number of outputs}}{\text{number of trainable parameters}}$$

$$= \frac{98}{9898}$$

$$= \frac{1}{101} = 0.009901$$

(b) (i) Q: Scalar form

Solution ① understand

$\vec{w}^k$     1 2 ⋯ k ⋯ 20

$[\,w^1_{-1}\,|\,w^1_0\,|\,w^1_1\,]^T$    $[\,w^2_{-1}\,|\,w^2_0\,|\,w^2_1\,]^T$  ⋯⋯  $[\,w^{20}_{-1}\,|\,w^{20}_0\,|\,w^{20}_1\,]^T$

   filter 1      filter 2       filter 20

$[\,y^1_2\,|\,y^1_3\,|\,\cdots\,|\,y^1_{99}\,]^T$  ⋯⋯  $[\,y^{20}_2\,|\,y^{20}_3\,|\,\cdots\,|\,y^{20}_{99}\,]^T$

   output 1         output 20

$y^k_2 \Leftarrow \theta^k \Leftarrow \begin{bmatrix} w^k_{-1} \\ w^k_0 \\ w^k_1 \end{bmatrix} \!\!\times\!\! \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix}$    1 2 ⋯ i ⋯ 100

$\vdots$          1 2 ⋯ j ⋯ 98

$y^k_{99} \Leftarrow \theta^k \Leftarrow \begin{bmatrix} w^k_{-1} \\ w^k_0 \\ w^k_1 \end{bmatrix} \!\!\times\!\! \begin{bmatrix} x_{98} \\ x_{99} \\ x_{100} \end{bmatrix}$   ← ref LSI convolutional definition

② Scalar form

$$y^k_j = w^k_{-1} x_{j+1} + w^k_0 x_j + w^k_1 x_{j-1} + \theta^k \qquad (2 \leq j \leq 99)$$

(ii) ① trainable parameters

trainable parameters

= number of filter × filter parameter + bias numbers

= 20 × 3 + 20 = 80

② mul

$$\text{multiplications} = \overset{\text{filter}}{\underset{\text{num}}{}} \times \overset{\text{filter}}{\underset{\substack{\text{parameter} \\ \text{num}}}{}} \times \overset{xy}{\underset{\text{num}}{}}$$

$$= 20 \times 3 \times 98$$

$$= 5880$$

③ add

$$\text{summation} = (2 \text{ sum between } wx + 1 \text{ bias}) \times y \text{ num} \times \text{filter num}$$

$$= (2+1) \times 98 \times 20$$

$$= 5880$$

(iii) $\text{ratio} = \dfrac{98 \times 20}{80} = \dfrac{1960}{80} = 24.5$