

To determine all possible values of the Hough transform $g(a, b)$ for the given binary image, we need to analyze how each point (x_i, y_i) in the image contributes to $g(a, b)$ through the model $y = ax + b$.

Step 1: Identify the Edge Points

The given binary image $f(x, y)$ is non-zero at the following points:

1. $\delta(x, y - 1) \Rightarrow x = 0, y = 1$
2. $\delta(x - 1, y - 2) \Rightarrow x = 1, y = 2$
3. $\delta(x + 1, y - 2) \Rightarrow x = -1, y = 2$
4. $\delta(x - 2, y - 3) \Rightarrow x = 2, y = 3$
5. $\delta(x + 2, y - 3) \Rightarrow x = -2, y = 3$

Step 2: Compute Hough Lines for Each Edge Point

For each point (x_i, y_i) , the Hough transform is the set of lines $b = y_i - ax_i$. This results in the following lines in (a, b) space:

1. Point $(0, 1)$: $b = 1$
2. Point $(1, 2)$: $b = 2 - a$
3. Point $(-1, 2)$: $b = 2 + a$
4. Point $(2, 3)$: $b = 3 - 2a$
5. Point $(-2, 3)$: $b = 3 + 2a$

Step 3: Identify All Possible Values of $g(a, b)$

The Hough transform $g(a, b)$ is non-zero at every point (a, b) lying on any of these lines. Therefore, all possible values of $g(a, b)$ occur along these lines in (a, b) space.

Answer:

(a) All possible values of the Hough transform $g(a, b)$ are at points (a, b) that satisfy any of the following equations:

- $b = 1$
- $b = 2 - a$
- $b = 2 + a$
- $b = 3 - 2a$
- $b = 3 + 2a$

That is, $g(a, b)$ is non-zero along these lines in (a, b) space.

Answer the question (b)

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To determine how many points in the Hough transform satisfy $g(a, b) > 1$, we need to find all (a, b) pairs where at least two of the lines obtained from the edge points intersect. At these intersection points, the value of $g(a, b)$ will be greater than 1 because multiple edge points contribute to the same (a, b) .

Step 1: List the Equations from Edge Points

From the given edge points, we have the following equations in (a, b) space:

1. Point $(0, 1)$: $b = 1$
2. Point $(1, 2)$: $b = 2 - a$
3. Point $(-1, 2)$: $b = 2 + a$
4. Point $(2, 3)$: $b = 3 - 2a$
5. Point $(-2, 3)$: $b = 3 + 2a$

Step 2: Find Intersection Points

We find the intersection points by solving the equations in pairs.

1. **Intersection at $a = 1, b = 1$:**
 - Lines $b = 1, b = 2 - a$, and $b = 3 - 2a$ intersect here.
 - **Contributing Lines:** Lines 1, 2, and 4
 - $g(a, b) = 3$
2. **Intersection at $a = -1, b = 1$:**
 - Lines $b = 1, b = 2 + a$, and $b = 3 + 2a$ intersect here.
 - **Contributing Lines:** Lines 1, 3, and 5
 - $g(a, b) = 3$
3. **Intersection at $a = 0, b = 2$:**
 - Lines $b = 2 - a$ and $b = 2 + a$ intersect here.
 - **Contributing Lines:** Lines 2 and 3
 - $g(a, b) = 2$
4. **Intersection at $a = -\frac{1}{3}, b = \frac{7}{3}$:**
 - Lines $b = 2 - a$ and $b = 3 + 2a$ intersect here.
 - **Contributing Lines:** Lines 2 and 5
 - $g(a, b) = 2$
5. **Intersection at $a = \frac{1}{3}, b = \frac{7}{3}$:**
 - Lines $b = 2 + a$ and $b = 3 - 2a$ intersect here.
 - **Contributing Lines:** Lines 3 and 4
 - $g(a, b) = 2$
6. **Intersection at $a = 0, b = 3$:**
 - Lines $b = 3 - 2a$ and $b = 3 + 2a$ intersect here.
 - **Contributing Lines:** Lines 4 and 5
 - $g(a, b) = 2$

Step 3: Count the Points Where $g(a, b) > 1$

There are **six unique intersection points** where $g(a, b) > 1$:

1. $a = 1, b = 1$ ($g(a, b) = 3$)
2. $a = -1, b = 1$ ($g(a, b) = 3$)
3. $a = 0, b = 2$ ($g(a, b) = 2$)
4. $a = -\frac{1}{3}, b = \frac{7}{3}$ ($g(a, b) = 2$)
5. $a = \frac{1}{3}, b = \frac{7}{3}$ ($g(a, b) = 2$)
6. $a = 0, b = 3$ ($g(a, b) = 2$)

Answer:

(b) There are 6 points in the Hough transform where $g(a, b) > 1$.

Answer the question (c)

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To determine the equation(s) of the line(s) $y = ax + b$ that have the highest number of pixels, we need to find the lines that pass through the maximum number of edge points in the given binary image $f(x, y)$.

Step 1: List the Edge Points

From the given $f(x, y)$, the edge points are:

1. $(0, 1)$
2. $(1, 2)$
3. $(-1, 2)$
4. $(2, 3)$
5. $(-2, 3)$

Step 2: Test Lines for Maximum Edge Points

We need to find lines $y = ax + b$ that pass through as many of these points as possible.

Line 1: $y = x + 1$

Test each edge point:

- $(0, 1)$: $1 = 0 + 1$ ✓
- $(1, 2)$: $2 = 1 + 1$ ✓
- $(2, 3)$: $3 = 2 + 1$ ✓
- $(-1, 2)$: $2 \neq -1 + 1$
- $(-2, 3)$: $3 \neq -2 + 1$

Total Edge Points on Line 1: 3

Line 2: $y = -x + 1$

Test each edge point:

- $(0, 1)$: $1 = -0 + 1$ ✓
- $(-1, 2)$: $2 = -(-1) + 1$ ✓
- $(-2, 3)$: $3 = -(-2) + 1$ ✓
- $(1, 2)$: $2 \neq -1 + 1$
- $(2, 3)$: $3 \neq -2 + 1$

Total Edge Points on Line 2: 3

Other Possible Lines:

Testing other lines like $y = 2, y = 3$, or $y = 0$ results in fewer than 3 edge points lying on them.

Conclusion:

The lines $y = x + 1$ and $y = -x + 1$ each pass through **3 edge points**, which is the maximum possible with the given set of points.

Answer:

(c) The lines with the highest number of pixels are $y = x + 1$ and $y = -x + 1$.

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Answer the question (d)

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To determine $G(a, b)$, we need to compute the Hough transform $g(a, b)$ and then apply the condition:

$$\begin{cases} g(a, b), & \text{if } g(a, b) > 1 \\ 0, & \text{otherwise} \end{cases}$$

Step 1: Recall the Hough Transform Lines

From the given edge points in $f(x, y)$, we have the following lines in (a, b) space:

1. $b = 1$ (from $(0, 1)$)
2. $b = 2 - a$ (from $(1, 2)$)
3. $b = 2 + a$ (from $(-1, 2)$)
4. $b = 3 - 2a$ (from $(2, 3)$)
5. $b = 3 + 2a$ (from $(-2, 3)$)

Step 2: Identify Points Where $g(a, b) > 1$

From part (b), we found six intersection points where $g(a, b) > 1$:

1. **At $(a, b) = (1, 1)$:**
 - Lines $b = 1, b = 2 - a$, and $b = 3 - 2a$ intersect.
 - $g(1, 1) = 3$
2. **At $(a, b) = (-1, 1)$:**
 - Lines $b = 1, b = 2 + a$, and $b = 3 + 2a$ intersect.
 - $g(-1, 1) = 3$
3. **At $(a, b) = (0, 2)$:**
 - Lines $b = 2 - a$ and $b = 2 + a$ intersect.
 - $g(0, 2) = 2$
4. **At $(a, b) = \left(-\frac{1}{3}, \frac{7}{3}\right)$:**
 - Lines $b = 2 - a$ and $b = 3 + 2a$ intersect.
 - $g\left(-\frac{1}{3}, \frac{7}{3}\right) = 2$
5. **At $(a, b) = \left(\frac{1}{3}, \frac{7}{3}\right)$:**
 - Lines $b = 2 + a$ and $b = 3 - 2a$ intersect.
 - $g\left(\frac{1}{3}, \frac{7}{3}\right) = 2$
6. **At $(a, b) = (0, 3)$:**
 - Lines $b = 3 - 2a$ and $b = 3 + 2a$ intersect.
 - $g(0, 3) = 2$

Step 3: Compute $G(a, b)$

Using the given definition, $G(a, b)$ is equal to $g(a, b)$ at these six points and zero elsewhere.

Thus, $G(a, b)$ is:

1. **At $(a, b) = (1, 1)$:**
 $G(1, 1) = g(1, 1) = 3$
2. **At $(a, b) = (-1, 1)$:**
 $G(-1, 1) = g(-1, 1) = 3$
3. **At $(a, b) = (0, 2)$:**
 $G(0, 2) = g(0, 2) = 2$
4. **At $(a, b) = \left(-\frac{1}{3}, \frac{7}{3}\right)$:**
 $G\left(-\frac{1}{3}, \frac{7}{3}\right) = g\left(-\frac{1}{3}, \frac{7}{3}\right) = 2$
5. **At $(a, b) = \left(\frac{1}{3}, \frac{7}{3}\right)$:**
 $G\left(\frac{1}{3}, \frac{7}{3}\right) = g\left(\frac{1}{3}, \frac{7}{3}\right) = 2$
6. **At $(a, b) = (0, 3)$:**
 $G(0, 3) = g(0, 3) = 2$

Everywhere else, $G(a, b) = 0$.

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