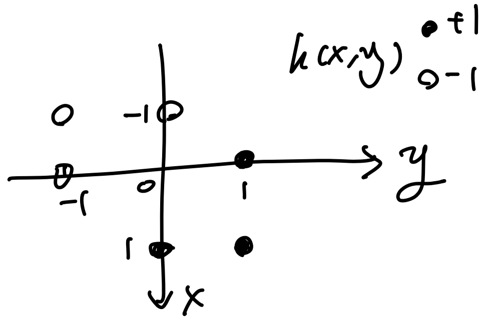
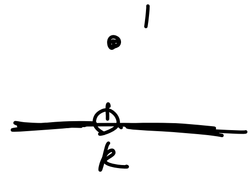
21-S2-Q1

Q(a)

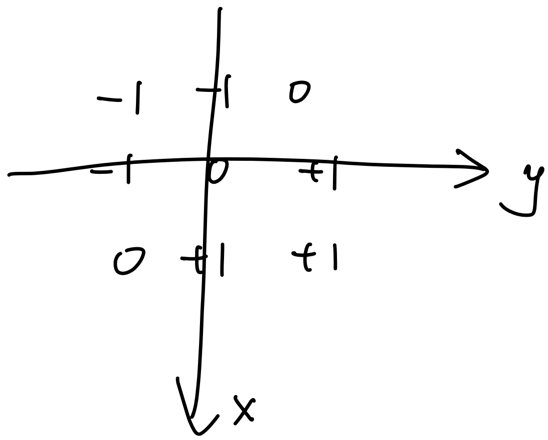


$\delta(n-k) =$ 

Solution

(a) filter mask

$$\begin{matrix} -1 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & +1 \end{matrix}$$



(b) Q: 输出 response

Solution

$$g(x,y) = f(x,y) * h(x,y)$$

$$= \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h(i,j)\, f(x-i, y-j)$$

(c) Q: Fourier transform of filter

Solution ① Fourier formula Discrete

$$H(u,v)=\frac{1}{mn}\sum_{x=0}^{m-1}\sum_{y=0}^{n-1}h(x,y)\,e^{-j2\pi\left(\frac{ux}{m}+\frac{vy}{n}\right)}$$

② $h(x,y)=\delta(x-1,y-1)+\delta(x-1,y)+\delta(x,y-1)$
$-\delta(x+1,y)-\delta(x,y+1)-\delta(x+1,y+1)$

③ $$H(u,v)=\frac{1}{3\times 3}\sum_{x=-1}^{1}\sum_{y=-1}^{1}h(x,y)\,e^{-j2\pi\left(\frac{ux}{3}+\frac{vy}{3}\right)}$$

$$=\frac{1}{9}\sum_{x=-1}^{1}\sum_{y=-1}^{1}h(x,y)\,e^{-j\frac{2\pi}{3}(ux+vy)}$$

$$=\frac{1}{9}\Big[h(1,1)e^{-j\frac{2\pi}{3}(u+v)}+h(1,0)e^{-j\frac{2\pi}{3}u}+h(0,1)e^{-j\frac{2\pi}{3}v}$$
$$+h(-1,-1)e^{j\frac{2\pi}{3}(u+v)}+h(-1,0)e^{j\frac{2\pi}{3}u}+h(0,-1)e^{j\frac{2\pi}{3}v}\Big]$$

$$=\frac{1}{9}\Big[e^{-j\frac{2\pi}{3}(u+v)}+e^{-j\frac{2\pi}{3}u}+e^{-j\frac{2\pi}{3}v}-e^{j\frac{2\pi}{3}(u+v)}-e^{j\frac{2\pi}{3}u}-e^{j\frac{2\pi}{3}v}\Big]$$

ref 欧拉公式

$$=\frac{1}{9}\Big[2j\sin\frac{2\pi}{3}(u+v)-2j\sin\frac{2\pi}{3}u-2j\sin\frac{2\pi}{3}v\Big]$$

$$= \frac{2j}{9}\left[\sin\frac{2\pi}{3}(u+v) - \sin\frac{2\pi}{3}u - \sin\frac{2\pi}{3}v\right]$$

(d) filter properties

Spatial Domain

① filter add the pixel values from southeast subtracts pixel values from northwest

② filter detect edges and derivative along diagonal line

Frequency domain

① Purely Imaginary indicate the filter is odd-symmetric in spatial domain

② when $u = v = 0$, $H(u, v) = 0$

so when the frequence is zero it will attenuates the DC component