Part (a):(一)部分：

We are asked to determine the type of fish with $x = 1.5$ and the corresponding probability of making a wrong decision using Bayes' theorem.

我们被要求确定鱼的类型 $x = 1.5$ 以及相应的利用贝叶斯定理做出错误决策的概率。

Step 1: Apply Bayes' Theorem第 1 步：应用贝叶斯定理

Bayes' theorem states:贝叶斯定理指出：

$$P(\omega_1|x) = \frac{P(x|\omega_1)P(\omega_1)}{P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)}$$

Where:在哪里：

- $P(\omega_1)$ is the prior probability for type ω_1 fish.$P(\omega_1)$ 是类型的先验概率 ω_1 鱼。
- $P(\omega_2)$ is the prior probability for type ω_2 fish.$P(\omega_2)$ 是类型的先验概率 ω_2 鱼。
- $P(x|\omega_1)$ is the likelihood of observing size x for type ω_1.
  $P(x|\omega_1)$ 是观察大小的可能性 x 对于类型 ω_1 。
- $P(x|\omega_2)$ is the likelihood of observing size x for type ω_2.
  $P(x|\omega_2)$ 是观察大小的可能性 x 对于类型 ω_2 。

We are given:我们得到：

- 40 fishes of type ω_1, and 60 fishes of type ω_2, so:
  40 类型的鱼 ω_1 ， 和 60 类型的鱼 ω_2 ， 所以：
  $$P(\omega_1) = \frac{40}{100} = 0.4 \quad \text{and} \quad P(\omega_2) = \frac{60}{100} = 0.6$$
- For $x = 1.5$, we can see from Figure 5:为了 $x = 1.5$ ， 由图5可知：
  $$P(x = 1.5|\omega_1) \approx 1.0 \quad \text{and} \quad P(x = 1.5|\omega_2) \approx 0.2$$

Step 2: Calculate Posterior Probability第 2 步：计算后验概率

Using Bayes' theorem, the probability that the fish is type ω_1 given $x = 1.5$:

使用贝叶斯定理，鱼是类型的概率 ω_1 给定 $x = 1.5$:

$$P(\omega_1|x = 1.5) = \frac{1.0 \times 0.4}{(1.0 \times 0.4) + (0.2 \times 0.6)} = \frac{0.4}{0.4 + 0.12} = \frac{0.4}{0.52} \approx 0.769$$

So, the probability that the fish is of type ω_1 is approximately 0.769.

所以，这条鱼属于 类型的概率 ω_1 大约是 0.769 。

Similarly, the probability that the fish is of type ω_2:同样，鱼属于类型的概率 ω_2 :

$$P(\omega_2|x = 1.5) = 1 - P(\omega_1|x = 1.5) = 1 - 0.769 = 0.231$$

Step 3: Wrong Decision Probability第三步：错误决策概率

If we classify the fish as type ω_1, the probability of making a wrong decision is the probability that the fish is actually of type ω_2, which is $P(\omega_2|x = 1.5) = 0.231$.

如果我们将鱼分类为类型 ω_1 ，做出错误决定的概率就是鱼实际上属于该类型的概率 ω_2 ，即 $P(\omega_2|x = 1.5) = 0.231$ 。

Thus, the probability of making a wrong decision is approximately 0.231.

因此，做出错误决定的概率大约为 0.231 。

Part (b):(b) 部分：

We are asked to generate a decision rule to determine the type of fish using Bayes' theorem and evaluate the probability of making a wrong decision using this decision rule.

我们被要求生成一个决策规则来使用贝叶斯定理来确定鱼的类型，并评估使用该决策规则做出错误决策的概率。

Step 1: Decision Rule Based on Maximum Posterior步骤1：基于最大后验的决策规则

The decision rule in Bayes' classification is to choose the class with the higher posterior probability. From part (a):贝叶斯分类的决策规则是选择后验概率较高的类别。来自（a）部分：

- If $P(\omega_1|x) > P(\omega_2|x)$, classify the fish as ω_1.
  如果 $P(\omega_1|x) > P(\omega_2|x)$ ，将鱼分类为 ω_1 。
- If $P(\omega_2|x) > P(\omega_1|x)$, classify the fish as ω_2.
  如果 $P(\omega_2|x) > P(\omega_1|x)$ ，将鱼分类为 ω_2 。

For $x = 1.5$:为了 $x = 1.5$:

- $P(\omega_1|x = 1.5) = 0.769$
- $P(\omega_2|x = 1.5) = 0.231$

Since $P(\omega_1|x = 1.5) > P(\omega_2|x = 1.5)$, we classify the fish as ω_1.

自从 $P(\omega_1|x = 1.5) > P(\omega_2|x = 1.5)$ ，我们将鱼分类为 ω_1 。

Step 2: Calculate Probability of Wrong Decision Using the Rule

第 2 步：使用规则计算错误决策的概率

The probability of making a wrong decision, if we classify the fish as type ω_1, is the probability that the fish is actually of type ω_2:

如果我们将鱼分类为类型，则做出错误决定的概率 ω_1 ，是该鱼实际上属于该类型的概率 ω_2 :

$$P(\text{Wrong Decision}) = P(\omega_2|x = 1.5) = 0.231$$

Therefore, using this decision rule, the probability of making a wrong decision is 0.231.

因此，使用该决策规则，做出错误决策的概率为 0.231 。

In summary:总之：

- For part (a), the fish is classified as type ω_1 with a probability of wrong decision ≈ 0.231.
  对于 (a) 部分，鱼被分类为类型 ω_1 有可能做出错误决定 ≈ 0.231 。
- For part (b), using the decision rule, the fish is classified as type ω_1, and the probability of making a wrong decision is also 0.231.
  对于 (b) 部分，使用决策规则，将鱼分类为类型 ω_1 ，做出错误决定的概率也是 0.231 。