Thought for 18 seconds

(a) Determining the Minimal and Maximal Grey Levels and the Number of Different Grey Levels

First, let's list the grey levels and their probabilities from the given histogram:

$$\begin{aligned} f = 0 \quad &\text{with probability} \quad P(f) = 0.1 \\ f = 0.7 \quad &\text{with probability} \quad P(f) = 0.2 \\ f = 0.8 \quad &\text{with probability} \quad P(f) = 0.3 \\ f = 0.9 \quad &\text{with probability} \quad P(f) = 0.2 \\ f = 1 \quad &\text{with probability} \quad P(f) = c \end{aligned}$$

Since the sum of all probabilities must equal 1, we can solve for c:

$$\begin{aligned} 0.1 + 0.2 + 0.3 + 0.2 + c &= 1 \\ 0.8 + c &= 1 \\ c &= 0.2 \end{aligned}$$

Now, the minimal grey level is $f_{\min} = 0$, and the maximal grey level is $f_{\max} = 1$. The image has **5 different grey levels**: 0, 0.7, 0.8, 0.9, and 1.

(b) Plotting the Histogram $P_{of}(f)$

The histogram can be represented by vertical lines (since it's composed of Dirac delta functions) at the specified grey levels with heights corresponding to their probabilities:

- At $f = 0$, height $= 0.1$
- At $f = 0.7$, height $= 0.2$
- At $f = 0.8$, height $= 0.3$
- At $f = 0.9$, height $= 0.2$
- At $f = 1$, height $= 0.2$

(c) Computing and Plotting the Histogram After Gamma Correction $g = f^2$

We apply the gamma correction to each grey level:

$$\begin{aligned} f = 0 \quad &\Rightarrow \quad g = (0)^2 = 0 \\ f = 0.7 \quad &\Rightarrow \quad g = (0.7)^2 = 0.49 \\ f = 0.8 \quad &\Rightarrow \quad g = (0.8)^2 = 0.64 \\ f = 0.9 \quad &\Rightarrow \quad g = (0.9)^2 = 0.81 \\ f = 1 \quad &\Rightarrow \quad g = (1)^2 = 1 \end{aligned}$$

The probabilities remain the same for the corresponding g values. The new histogram $P_{Yg}(g)$ is:

- At $g = 0$, height $= 0.1$
- At $g = 0.49$, height $= 0.2$
- At $g = 0.64$, height $= 0.3$
- At $g = 0.81$, height $= 0.2$
- At $g = 1$, height $= 0.2$

(d) Computing and Plotting the Histogram After Histogram Equalization

First, we compute the cumulative distribution function (CDF) of the original histogram:

$$\begin{aligned} \text{At } f = 0 \quad &\text{CDF}(0) = 0.1 \\ \text{At } f = 0.7 \quad &\text{CDF}(0.7) = 0.1 + 0.2 = 0.3 \\ \text{At } f = 0.8 \quad &\text{CDF}(0.8) = 0.3 + 0.3 = 0.6 \\ \text{At } f = 0.9 \quad &\text{CDF}(0.9) = 0.6 + 0.2 = 0.8 \\ \text{At } f = 1 \quad &\text{CDF}(1) = 0.8 + 0.2 = 1.0 \end{aligned}$$

Histogram equalization maps each grey level to a new grey level g using the CDF:

$$g = \text{CDF}(f)$$

Applying this mapping:

$$\begin{aligned} f = 0 \quad &\Rightarrow \quad g = 0.1 \\ f = 0.7 \quad &\Rightarrow \quad g = 0.3 \\ f = 0.8 \quad &\Rightarrow \quad g = 0.6 \\ f = 0.9 \quad &\Rightarrow \quad g = 0.8 \\ f = 1 \quad &\Rightarrow \quad g = 1.0 \end{aligned}$$

The probabilities remain unchanged. The new histogram $P_{\text{heg}}(g)$ is:

- At $g = 0.1$, height $= 0.1$
- At $g = 0.3$, height $= 0.2$
- At $g = 0.6$, height $= 0.3$
- At $g = 0.8$, height $= 0.2$
- At $g = 1.0$, height $= 0.2$

Summary:

- **(a)** The minimal grey level is $f_{\min} = 0$, the maximal grey level is $f_{\max} = 1$, and there are **5 different grey levels**.
- **(b)** The histogram $P_{of}(f)$ is plotted with grey levels at 0, 0.7, 0.8, 0.9, and 1, with corresponding probabilities.
- **(c)** After gamma correction, the new histogram $P_{Yg}(g)$ has grey levels at 0, 0.49, 0.64, 0.81, and 1.
- **(d)** After histogram equalization, the new histogram $P_{\text{heg}}(g)$ has grey levels at 0.1, 0.3, 0.6, 0.8, and 1, with probabilities unchanged.

Answer to Part (a):

- Minimal grey level: $f_{\min} = 0$
- Maximal grey level: $f_{\max} = 1$
- Number of different grey levels: **5**