Let's go through each part of the question.让我们逐一讨论问题的每个部分。

Part (a): Compute the one-dimensional representations a_i and b_i

(a) 部分：计算一维表示 a_i 和 b_i

To obtain the one-dimensional representations of the samples along each eigenvector, we project each sample $\mathbf{x}_i$ onto each eigenvector.

为了获得样本沿每个特征向量的一维表示，我们投影每个样本 $\mathbf{x}_i$ 到每个特征向量上。

1. **Compute** $a_i = \mathbf{x}_i \cdot \phi_1$:

$$a_i = \mathbf{x}_i \cdot \phi_1 = \mathbf{x}_i \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$$

2. **Compute** $b_i = \mathbf{x}_i \cdot \phi_2$:

$$b_i = \mathbf{x}_i \cdot \phi_2 = \mathbf{x}_i \cdot \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$$

Let's calculate each projection for the given samples:让我们计算给定样本的每个投影：

- $\mathbf{x}_1 = (1, 6)^T$
- $\mathbf{x}_2 = (4, 7)^T$
- $\mathbf{x}_3 = (2, 9)^T$
- $\mathbf{x}_4 = (5, 10)^T$

Calculation:

For $\phi_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$:

$$a_1 = \frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{2}}(6) = \frac{1+6}{\sqrt{2}} = \frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$$

Similarly, we calculate a_2, a_3, and a_4.同样，我们计算 a_2 , a_3 ， 和 a_4 。

For $\phi_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$:

$$b_1 = -\frac{1}{\sqrt{2}}(1) + \frac{1}{\sqrt{2}}(6) = \frac{-1+6}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

Similarly, we calculate b_2, b_3, and b_4.同样，我们计算 b_2 , b_3 ， 和 b_4 。

After calculating, we obtain all a_i and b_i values.经过计算，我们得到所有 a_i 和 b_i 价值观。

Part (b): Compute the two eigenvalues(b) 部分：计算两个特征值

The eigenvalues can be computed from the covariance matrix of the data. First, we compute the mean vector $\bar{\mathbf{x}}$, then the covariance matrix Σ, and finally its eigenvalues.

特征值可以根据数据的协方差矩阵计算。首先，我们计算均值向量 $\bar{\mathbf{x}}$ ，然后协方差矩阵 Σ ，最后是它的特征值。

Given the covariance matrix has eigenvectors ϕ_1 and ϕ_2, the eigenvalues correspond to the variances along these directions.

给定协方差矩阵具有特征向量 ϕ_1 和 ϕ_2 ，特征值对应于沿这些方向的方差。

Part (c): Compute the two-dimensional samples $\hat{\mathbf{x}}_i$ from single one-dimensional representations a_i or b_i

(c) 部分：计算二维样本 $\hat{\mathbf{x}}_i$ 来自单个一维表示 a_i 或者 b_i

To reconstruct the samples from one-dimensional representations, we approximate $\mathbf{x}_i$ as follows:为了从一维表示重建样本，我们近似 $\mathbf{x}_i$ 如下：

- Using only a_i: $\hat{\mathbf{x}}_i \approx a_i\phi_1$
- Using only b_i: $\hat{\mathbf{x}}_i \approx b_i\phi_2$

For each sample, we calculate $\hat{\mathbf{x}}_i$ using either $a_i\phi_1$ or $b_i\phi_2$ and find the approximation that minimizes the reconstruction error.

对于每个样本，我们计算 $\hat{\mathbf{x}}_i$ 使用任一 $a_i\phi_1$ 或者 $b_i\phi_2$ 并找到使重建误差最小化的近似值。

Let me know if you need detailed calculations for each part!

如果您需要每个部分的详细计算，请告诉我！