20 — S2 — Q5

Q (a)(i) input → output < vector ? scalar ?

Solution ① understand

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{100} \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{98} \end{bmatrix}$$

$$98 \begin{array}{c} 100 \\ \boxed{W_{ij}} \end{array} \times \boxed{}_{100} + \boxed{\theta}_{98} = \boxed{y}_{98}$$

1 2 3 ... i ·· — 98
1 2 3 ··· j ··· — 100

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1\,100} \\ W_{21} & W_{22} & \cdots & W_{2\,100} \\ \vdots & \vdots & \ddots & \vdots \\ W_{981} & W_{982} & \cdots & W_{98\,100} \end{bmatrix} \qquad \vec{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{98} \end{bmatrix}$$

② Vector form

$$\vec{y} = W\vec{x} + \vec{\theta}$$

③ Scalar form

$$y_i = \sum_{j=1}^{100} W_{ij}\, x_j + \theta_i$$

(ii) Q : parameters ?   mul ?   sum ?

Solution ① parameters

parameters = number of W_{ij} + number of θ_i

$$= 98 \times 100 + 98$$

$$= 9898$$

② multiplication

$$\text{multiplication} = \text{number of } w \cdot x$$

$$= 98 \times 100$$

$$= 9800$$

③ summations

$$\text{summations} = \text{number of } w \cdot x \text{ add } w x + \text{number of add } 0$$
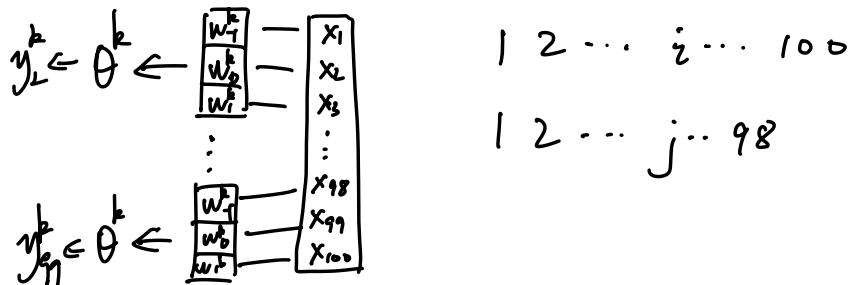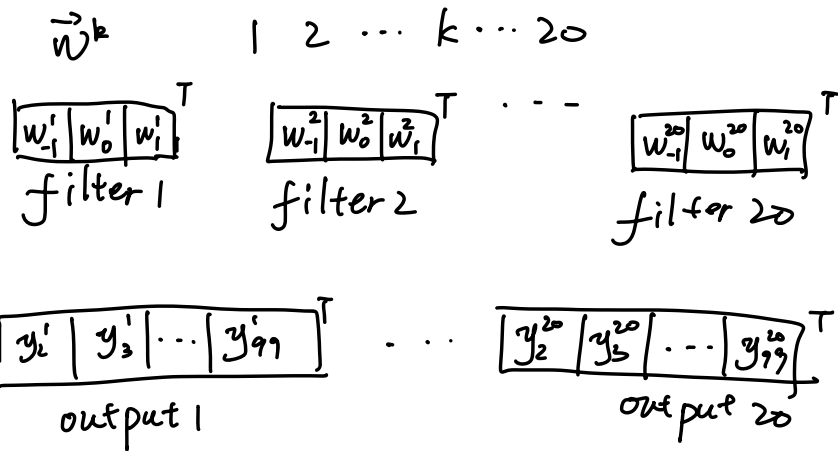
$$= 98 \times (99 + 1)$$

$$= 9800$$

(iii) ratio = ?

Solution

$$\text{ratio} = \frac{\text{number of outputs}}{\text{number of trainable parameters}}$$

$$= \frac{98}{9898}$$

$$= \frac{1}{101} = 0.009901$$

(b)(i) Q: Scalar form

Solution ① understand

$\vec{w}^k$ 1   2   ...   k ... 20

$\boxed{w_{-1}^1 | w_0^1 | w_1^1}^T$ $\boxed{w_{-1}^2 | w_0^2 | w_1^2}^T$ --- $\boxed{w_{-1}^{20} | w_0^{20} | w_1^{20}}^T$

filter 1   filter 2   filter 20

$\boxed{y_2^1 | y_3^1 | \cdots | y_{99}^1}^T$ ... $\boxed{y_2^{20} | y_3^{20} | \cdots | y_{99}^{20}}^T$

output 1   output 20

$y_2^k \Leftarrow \theta^k \Leftarrow \boxed{\begin{matrix} w_{-1}^k \\ w_0^k \\ w_1^k \end{matrix}} - \boxed{\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}}$ 1 2 ... i ... 100

$\vdots$ $\vdots$ 1 2 ... j ... 98

$y_{99}^k \Leftarrow \theta^k \Leftarrow \boxed{\begin{matrix} w_{-1}^k \\ w_0^k \\ w_1^k \end{matrix}} \boxed{\begin{matrix} x_{98} \\ x_{99} \\ x_{100} \end{matrix}}$

② Scalar form

$$y_j^k = w_{-1}^k x_{j-1} + w_0^k x_j + w_1^k x_{j+1} + \theta^k \qquad (2 \le j \le 99)$$

(ii) ① trainable parameters

trainable parameters

= number of filter $\times$ filter parameter + bias numbers

= 20 $\times$ 3 + 20 = 80

② mul

$$\text{multiplications} = \text{filter}_\text{num} \times \text{filter parameter}_\text{num} \times y_\text{num}$$

$$= 20 \times 3 \times 98$$

$$= 5880$$

③ add

$$\text{summation} = (2 \text{ sum between } wx + 1 \text{ bias}) \times y_\text{num} \times \text{filter}_\text{num}$$

$$= (2+1) \times 98 \times 20$$

$$= 5880$$

(iii) $\text{ratio} = \dfrac{98 \times 20}{80} = \dfrac{1960}{80} = 24.5$