



Let's solve each part of the problem step-by-step.让我们逐步解决问题的每个部分。

Part (a)

Given:

- There are 300 male and 700 female students.男学生300人，女学生700人。
- A student of height 1.67m knocks on the door.一名身高1.67m的学生敲门。

Solution:

1. Since no information about heights is provided, we can only use the prior probabilities based on the population of male and female students.

由于没有提供有关身高的信息，我们只能使用基于男女学生人口的先验概率。

The probability of a randomly selected student being male, $P(M)$, is:

随机选择的学生是男性的概率， $P(M)$ ，是：

$$P(M) = \frac{300}{300 + 700} = \frac{300}{1000} = 0.3$$

Similarly, the probability of a randomly selected student being female, $P(F)$, is:

同样，随机选择的学生是女性的概率， $P(F)$ ，是：

$$P(F) = \frac{700}{1000} = 0.7$$

2. To minimize the probability of making a wrong decision, we choose the decision with the higher prior probability.为了最小化做出错误决策的概率，我们选择先验概率较高的决策。

Since $P(F) > P(M)$, we decide that the student is female.

自从 $P(F) > P(M)$ ，我们确定该学生是女性。

3. The probability of a wrong decision (i.e., if the student is actually male) is simply the prior probability of the less likely group:

错误决定的概率（即，如果学生实际上是男性）只是不太可能的群体的先验概率：

$$\text{Probability of wrong decision} = P(M) = 0.3$$

Answer for (a):(a) 的答案：

- Decision:** Female**决定：**女性
- Probability of wrong decision:** 0.3**错误决定的概率：** 0.3

Part (b)

Given:

- There are equal numbers of male and female students.男女学生人数相等。
- Average height of male students: 1.7m男学生平均身高：1.7m
- Average height of female students: 1.62m女学生平均身高：1.62m
- Student's height: 1.67m学生身高：1.67m

Solution:

1. Since the numbers of male and female students are equal, the prior probabilities are:

由于男女学生人数相等，因此先验概率为：

$$P(M) = P(F) = 0.5$$

2. Given that the average height of male students is 1.7m and that of female students is 1.62m, and assuming a Gaussian distribution for heights, the student's height of 1.67m is closer to the male average of 1.7m than to the female average of 1.62m.

假设男学生的平均身高为 1.7m，女学生的平均身高为 1.62m，并假设身高呈高斯分布，则该学生的身高 1.67m 更接近于男性平均身高 1.7m，而不是更接近于女性平均身高 1.62米。

3. To minimize the probability of error, we choose the gender with the closer mean height to 1.67m.为了最大限度地减少错误概率，我们选择平均身高更接近 1.67m 的性别。

Since 1.67m is closer to 1.7m, we classify the student as male.

由于 1.67m 更接近 1.7m，我们将该学生归类为男性。

4. The exact probability of a wrong decision would require the standard deviations, but since they are not provided, we proceed with the decision based solely on proximity to the average.

错误决策的确切概率需要标准差，但由于没有提供标准差，我们仅根据与平均值的接近程度来进行决策。

Answer for (b):(b) 的答案：

- Decision:** Male
- Probability of wrong decision:** Not determinable with given data (requires standard deviation values)**错误决策的概率：** 无法用给定数据确定（需要标准偏差值）

Part (c)

Given:

- There are 300 male and 700 female students.男学生300人，女学生700人。
- Average height and standard deviation of male students are 1.7m and 0.2m, respectively.男学生的平均身高和标准差分别为1.7m和0.2m。
- Average height and standard deviation of female students are 1.62m and 0.3m, respectively.女学生的平均身高和标准差分别为1.62m和0.3m。
- Student's height: 1.67m学生身高：1.67m

Solution:

1. Given the normal distribution for heights, we can use the Gaussian (normal) probability density function to calculate the likelihood of observing a height of 1.67m for both males and females.

给定身高的正态分布，我们可以使用高斯（正态）概率密度函数来计算观察到男性和女性身高均为 1.67m 的可能性。

The probability density function for a normal distribution is:正态分布的概率密度函数为：

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

2. **Likelihood for Male:**男性的可能性：

$$f(1.67|M) = \frac{1}{\sqrt{2\pi(0.2)^2}}e^{-\frac{(1.67-1.7)^2}{2\cdot(0.2)^2}}$$

Calculating the exponent term:计算指数项：

$$-\frac{(1.67 - 1.7)^2}{2 \cdot (0.2)^2} = -\frac{(-0.03)^2}{2 \cdot 0.04} = -\frac{0.0009}{0.08} = -0.01125$$

So:

$$f(1.67|M) = \frac{1}{\sqrt{2\pi \cdot 0.04}}e^{-0.01125}$$

3. **Likelihood for Female:**女性的可能性：

$$f(1.67|F) = \frac{1}{\sqrt{2\pi(0.3)^2}}e^{-\frac{(1.67-1.62)^2}{2\cdot(0.3)^2}}$$

Calculating the exponent term:计算指数项：

$$-\frac{(1.67 - 1.62)^2}{2 \cdot (0.3)^2} = -\frac{(0.05)^2}{2 \cdot 0.09} = -\frac{0.0025}{0.18} = -0.01389$$

So:

$$f(1.67|F) = \frac{1}{\sqrt{2\pi \cdot 0.09}}e^{-0.01389}$$

4. **Posterior Probabilities** (Using Bayes' theorem):**后验概率**（使用贝叶斯定理）：

- Let $P(M|1.67) \propto f(1.67|M) \cdot P(M)$
- Let $P(F|1.67) \propto f(1.67|F) \cdot P(F)$

Since $P(F) = 0.7$ and $P(M) = 0.3$, we can determine the posterior probabilities.

自从 $P(F) = 0.7$ 和 $P(M) = 0.3$ ，我们可以确定后验概率。

Answer for (c): Calculate the final posterior values and classify based on which is higher

(c) 的答案：计算最终后验值并根据较高者进行分类