Let's go through each part step-by-step.让我们逐步了解每个部分。

Part (a): Fully-Connected Layer(a)部分：全连接层

(i) Express the outputs in terms of inputs in vector-matrix form and scalar form

(i) 用向量矩阵形式和标量形式的输入来表达输出

In a fully-connected layer, the outputs $\mathbf{y} = [y_1, y_2, \ldots, y_{98}]^T$ are given by:

在全连接层中，输出 $\mathbf{y} = [y_1, y_2, \ldots, y_{98}]^T$ 由下式给出：

$$\mathbf{y} = \mathbf{W}\mathbf{x} + \theta$$

where:

- $\mathbf{W}$ is a 98×100 weight matrix,
- $\mathbf{x} = [x_1, x_2, \ldots, x_{100}]^T$ is the input vector,$\mathbf{x} = [x_1, x_2, \ldots, x_{100}]^T$ 是输入向量，
- $\theta = [\theta_1, \theta_2, \ldots, \theta_{98}]^T$ is the bias vector.$\theta = [\theta_1, \theta_2, \ldots, \theta_{98}]^T$ 是偏置向量。

In scalar form, each output y_i for $i = 1, 2, \ldots, 98$ is given by:

以标量形式，每个输出 y_i 为了 $i = 1, 2, \ldots, 98$ 由下式给出：

$$y_i = \sum_{j=1}^{100} w_{ij} x_j + \theta_i$$

(ii) Compute the number of trainable parameters and the number of multiplications and summations required in this layer to compute the outputs from the inputs.

(ii) 计算该层中可训练参数的数量以及计算输入输出所需的乘法和求和的数量。

- **Number of trainable parameters**:**可训练参数的数量**:
  - Each weight in $\mathbf{W}$ is trainable, so we have $98 \times 100 = 9800$ weights.

    每个重量在 $\mathbf{W}$ 是可训练的，所以我们有 $98 \times 100 = 9800$ 重量。
  - Each bias term in θ is trainable, so there are 98 biases.

    中的每个偏置项 θ 是可训练的，所以有 98 个偏差。
  - Total trainable parameters = $9800 + 98 = 9898$.

    总可训练参数 = $9800 + 98 = 9898$ 。
- **Number of multiplications and summations**:**乘法和求和的次数**:
  - For each output y_i, we need to perform 100 multiplications (for each term $w_{ij}x_j$) and 99 summations (to sum 100 terms).

    对于每个输出 y_i ，我们需要执行 100 次乘法（对于每一项 $w_{ij}x_j$ ）和 99 次求和（总计 100 项）。
  - There are 98 outputs, so the total number of multiplications is $98 \times 100 = 9800$.

    有 98 个输出，因此乘法总数为 $98 \times 100 = 9800$ 。
  - The total number of summations is $98 \times 99 = 9702$.总和次数为 $98 \times 99 = 9702$ 。

(iii) What is the ratio of the number of outputs to the number of trainable parameters?

(iii) 输出数量与可训练参数数量的比率是多少?

The number of outputs is 98, and the number of trainable parameters is 9898. Thus, the ratio is:输出数量为 98，可训练参数数量为 9898。因此，比率为：

$$\text{Ratio} = \frac{98}{9898} \approx 0.0099$$

Part (b): Convolutional Neural Network Layer(b)部分：卷积神经网络层

(i) Express the outputs in terms of inputs in scalar form(i) 用标量形式的输入来表达输出

For a convolutional layer with 20 filters, each of size 3, the output of the k-th filter, y_i^k, for $i = 2, 3, \ldots, 99$, is computed as:

对于具有 20 个滤波器（每个滤波器大小为 3）的卷积层， k -th 过滤器， y_i^k ， 为了 $i = 2, 3, \ldots, 99$ ，计算如下：

$$y_i^k = w_{-1}^k x_{i-1} + w_0^k x_i + w_1^k x_{i+1} + \theta^k$$

where:

- w_{-1}^k, w_0^k, and w_1^k are the weights of the k-th filter,

  w_{-1}^k , w_0^k ， 和 w_1^k 是的权重 k 第-个过滤器，
- θ^k is the bias for the k-th filter.θ^k 是偏置 k -th 过滤器。

(ii) Compute the number of trainable parameters and the number of multiplications and summations required in this layer to compute the outputs from the inputs.

(ii) 计算该层中可训练参数的数量以及计算输入输出所需的乘法和求和的数量。

- **Number of trainable parameters**:**可训练参数的数量**:
  - Each filter has 3 weights, and there are 20 filters, so there are $3 \times 20 = 60$ weight parameters.每个filter有3个权重，有20个filter，所以有 $3 \times 20 = 60$ 重量参数。
  - Each filter has one bias term, so there are 20 biases.

    每个滤波器有一个偏置项，因此有 20 个偏置。
  - Total trainable parameters = $60 + 20 = 80$.总可训练参数 = $60 + 20 = 80$ 。
- **Number of multiplications and summations**:**乘法和求和的次数**:
  - For each output y_i^k, we need 3 multiplications (one for each weight) and 2 summations (to sum 3 terms).

    对于每个输出 y_i^k ，我们需要 3 次乘法（每个权重一次）和 2 次求和（对 3 项求和）。
  - There are 98 outputs per filter (from y_2 to y_{99}) and 20 filters.

    每个过滤器有 98 个输出（从 y_2 到 y_{99} ）和 20 个过滤器。
  - Total number of multiplications = $98 \times 3 \times 20 = 5880$.

    乘法总数 = $98 \times 3 \times 20 = 5880$ 。
  - Total number of summations = $98 \times 2 \times 20 = 3920$.

    求和总数 = $98 \times 2 \times 20 = 3920$ 。

(iii) What is the ratio of the number of outputs to the number of trainable parameters?

(iii) 输出数量与可训练参数数量的比率是多少?

The number of outputs is $98 \times 20 = 1960$, and the number of trainable parameters is 80. Thus, the ratio is:输出的数量是 $98 \times 20 = 1960$ ，可训练参数的数量为 80。因此，比率为：

$$\text{Ratio} = \frac{1960}{80} = 24.5$$

These calculations complete the solution. Let me know if you need further clarification on any part!这些计算完成了解决方案。如果您需要对任何部分进行进一步说明，请告诉我！