



Solution:

Part (a)

In the Hough transform, each pixel from a set of n pixels in the image contributes votes to possible lines that could pass through it. Peaks in the accumulator cells represent possible lines with several collinear points.

在霍夫变换中，一组中的每个像素 n 图像中的像素为可能穿过它的线贡献投票。累加器单元中的峰值代表具有多个共线点的可能线。

Given $n = 5$, we have five detected pixels that can form various possible lines:

给定 $n = 5$, 我们有五个检测到的像素可以形成各种可能的线条：

1. Case 1: All 5 pixels are collinear情况 1：所有 5 个像素共线

- **Number of peaks:** 1 peak in the accumulator.峰值数量：累加器中有 1 个峰值。
- **Peak value:** 5 (as all five points lie on the same line).
峰值：5（因为所有五个点都位于同一条线上）。
- **Line type:** This peak represents a single line with 5 collinear pixels.
线型：此峰值表示具有 5 个共线像素的单线。

2. Case 2: 4 pixels are collinear, with 1 outlier情况 2：4 个像素共线，有 1 个异常值

- **Number of peaks:** 2 peaks.峰数：2 个峰。
- **Peak values:** One peak with a value of 4 (for the 4 collinear pixels) and one peak with a value of 1 (for the outlier pixel, assuming it does not align with any other).
峰值：一个值为 4 的峰值（对于 4 个共线像素）和一个值为 1 的峰值（对于离群像素，假设它不与任何其他像素对齐）。
- **Line type:** One line with 4 pixels, and no significant line for the outlier pixel.
线类型：一条线有 4 个像素，离群像素没有显著线。

3. Case 3: 3 pixels are collinear, with 2 other outlier pixels

情况 3：3 个像素共线，还有 2 个异常像素

- **Number of peaks:** 3 peaks.峰数：3 个峰。
- **Peak values:** One peak with a value of 3 (for the 3 collinear pixels) and two additional peaks with values of 1 (each for the individual outlier pixels).
峰值：一个值为 3 的峰值（对于 3 个共线像素）和两个值为 1 的附加峰值（每个峰值对于各个离群值像素）。
- **Line type:** One line with 3 pixels, and no significant lines for the two outliers.
线型：一条线有 3 个像素，两个异常值没有明显的线。

4. Case 4: 2 pixels are collinear, and the other 3 pixels do not align with them or each other情况 4：2 个像素共线，另外 3 个像素不对齐或彼此不对齐

- **Number of peaks:** 4 peaks.峰数：4 个峰。
- **Peak values:** One peak with a value of 2 (for the 2 collinear pixels) and three additional peaks with values of 1 (each for the individual outlier pixels).
峰值：一个值为 2 的峰值（对于 2 个共线像素）和三个值为 1 的附加峰值（每个峰值对于各个离群值像素）。
- **Line type:** One line with 2 pixels, and no significant lines for the other three individual pixels.线型：一条线有 2 个像素，其他 3 个单独的像素没有明显的线。

5. Case 5: All 5 pixels are non-collinear (no three points align)

情况 5：所有 5 个像素都不共线（没有三点对齐）

- **Number of peaks:** 5 peaks.峰数：5 个峰。
- **Peak values:** Each peak has a value of 1.峰值：每个峰值的值为 1。
- **Line type:** No lines with more than 1 pixel are detected; each point represents an individual peak.线型：不检测超过1个像素的线；每个点代表一个单独的峰。

Summary for Part (a):(a) 部分摘要:

- Possible numbers of peaks: 1, 2, 3, 4, or 5.可能的峰数：1、2、3、4 或 5。
- Each configuration corresponds to a different arrangement of collinear points, and peak values decrease as fewer pixels align.
每种配置对应于共线点的不同排列，并且峰值随着对齐的像素的减少而减小。

Part (b)

Let $s_1 = a + ja, s_2 = b, s_3 = -a - ja, s_4 = -jb$.

Define:

- $c = s_1 + s_2 + s_3 + s_4$
- $d = s_1^2 + s_2^2 + s_3^2 + s_4^2$

Step-by-Step Calculations:逐步计算:

1. Compute $c = s_1 + s_2 + s_3 + s_4$:

Substitute s_1, s_2, s_3 , and s_4 :代替 s_1, s_2, s_3 , 和 s_4 :

$$c = (a + ja) + b + (-a - ja) + (-jb)$$

Simplify c :

$$c = a - a + ja - ja + b - jb = b(1 - j)$$

2. Angle of c :

$$c = b(1 - j).$$

- Compute the angle of c :

$$\text{Angle of } c = \arg(b(1 - j)) = \arg(1 - j)$$

- Since $1 - j$ lies in the fourth quadrant, its angle is $-\frac{\pi}{4}$.

自从 $1 - j$ 位于第四象限，其角度为 $-\frac{\pi}{4}$ 。

Comparison:

- The angles of s_1, s_2, s_3 , and s_4 are:的角度为 s_1, s_2, s_3 , 和 s_4 是:
 - $s_1 = a + ja \Rightarrow \text{angle} = \frac{\pi}{4}$
 - $s_2 = b \Rightarrow \text{angle} = 0$
 - $s_3 = -a - ja \Rightarrow \text{angle} = -\frac{3\pi}{4}$
 - $s_4 = -jb \Rightarrow \text{angle} = -\frac{\pi}{2}$
- The angle of c ($-\frac{\pi}{4}$) does not match any of the individual angles of s_1, s_2, s_3 , or s_4 .
的角度为 c ($-\frac{\pi}{4}$) 不匹配任何单独的角度 s_1, s_2, s_3 , 或者 s_4 。

3. Compute $d = s_1^2 + s_2^2 + s_3^2 + s_4^2$:

Calculate each square:计算每个平方:

- $s_1^2 = (a + ja)^2 = a^2(1 + j)^2 = a^2(1 + 2j + j^2) = a^2(-1 + 2j) = -a^2 + 2a^2j$
- $s_2^2 = b^2$
- $s_3^2 = (-a - ja)^2 = a^2(-1 - 2j) = -a^2 - 2a^2j$
- $s_4^2 = (-jb)^2 = -b^2$

Summing these:总结这些:

$$d = (-a^2 + 2a^2j) + b^2 + (-a^2 - 2a^2j) - b^2 = -2a^2 + b^2 + b^2 = 0$$

4. Angle of d :

Since $d = 0$, the angle is undefined.自从 $d = 0$, 角度未定义。

Comparison:

- The angle of d cannot be compared to the angles of s_1, s_2, s_3 , and s_4 because $d = 0$, making its angle undefined.
的角度为 d 无法与角度相比 s_1, s_2, s_3 , 和 s_4 因为 $d = 0$, 使其角度未定义。

Summary for Part (b):(b) 部分摘要:

- (i) The angle of c is $-\frac{\pi}{4}$, which does not match the angles of s_1, s_2, s_3 , or s_4 .
(i)角度 c 是 $-\frac{\pi}{4}$, 这与 的角度不匹配 s_1, s_2, s_3 , 或者 s_4 。
- (ii) The angle of d is undefined since $d = 0$.(ii)角度 d 未定义, 因为 $d = 0$ 。