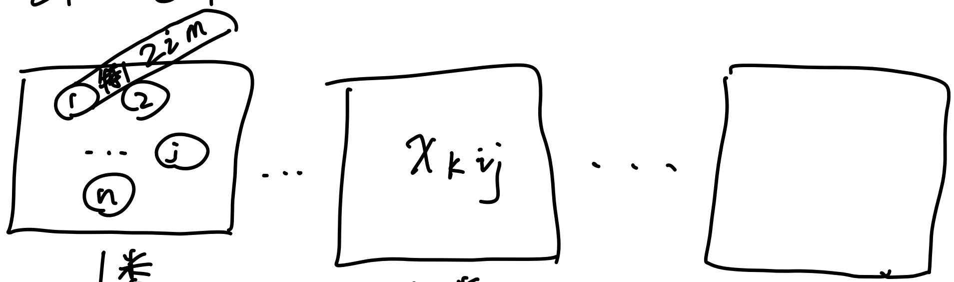
21-S1-Q4

Q



1类 k类 C类

Q(a) class-conditional mean μ_{ki} & covariance martix σ_{kpq}

Solution ① μ_{ki} 类内→卡死于k类，卡死于第i个特征的均值

对于 $1, 2 \cdots j \cdots n$

$$\mu_{ki}=\frac{1}{n}\sum_{j=1}^{n}x_{kij}$$

② covariance martix σ_{kpq}

→固定k类

→feature $1\ 2 \cdots \underbrace{p \cdots q}_{\text{之间的cov}} \cdots i \cdots m$

Solution

$$\sigma_{kpq}=\frac{1}{n}\sum_{j=1}^{n}(x_{kpj}-\mu_{kp})(x_{kqj}-\mu_{kq})$$

$$\mu_{kp}=\frac{1}{n}\sum_{j=1}^{n}x_{kpj} \qquad \mu_{kq}=\frac{1}{n}\sum_{j=1}^{n}x_{kqj}$$

(b) Q μ_k? Σ_k?

Solution

① $X_{kij} \rightarrow \vec{X}_{kj}$ i没了，用向量表示样本

$$\vec{X}_{kj} = \begin{bmatrix} x_{k1j} \\ x_{k2j} \\ \vdots \\ x_{kmj} \end{bmatrix}$$

$$\vec{\mu}_k = \frac{1}{n}\sum_{j=1}^{n} \vec{X}_{kj}$$

$$\Sigma_k = \frac{1}{n}\sum_{j=1}^{n} (\vec{X}_{kj} - \vec{\mu}_k)(\vec{X}_{kj} - \vec{\mu}_k)^T$$

(c) Q $\Sigma_k = ?$

Solution ① $\left.\begin{matrix}\vec{X}_{kj} \\ \vec{\mu}_k\end{matrix}\right\} \to X_k$ — mean feature $= 0$

$\hat{\vec{X}}_{kj} = \vec{x}_{kj} - \vec{\mu}_k$

$\downarrow$ compute

Σ_k

$[\hat{\vec{X}}_{k1} \ \hat{\vec{X}}_{k2} \cdots \hat{\vec{X}}_{kn}]$

$$\hat{\vec{X}}_{kj} = \vec{x}_{kj} - \vec{\mu}_k$$

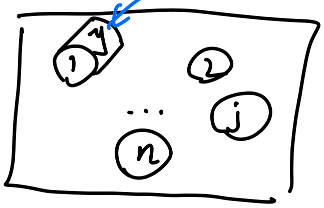
$$X_k = [\hat{\vec{X}}_{k1} \ \hat{\vec{X}}_{k2} \ \cdots \ \hat{\vec{X}}_{kn}]$$

$$\Sigma_k = \frac{1}{n} X_k X_k^T$$

(d) Q:

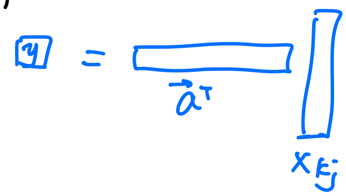
Solution ① understand

one-dimensional feature



k类

variance $V_k = \frac{1}{n}\sum(y-\mu)^2$



② $y_{kj} = \vec{a}^T \vec{X}_{kj}$

③ mean of y

$$\mu_{yk} = \frac{1}{n}\sum_{j=1}^{n} y_{kj} = \frac{1}{n}\sum_{j=1}^{n} \vec{a}^T \vec{x}_{kj} = \vec{a}^T \vec{\mu}_k$$

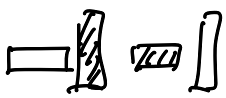
④ class-conditional variance V_k

$$V_k = \frac{1}{n}\sum_{j=1}^{n}(y_{kj} - \mu_{yk})^2$$

$$= \frac{1}{n}\sum_{j=1}^{n}(\vec{a}^T \vec{X}_{kj} - \vec{a}^T \vec{\mu}_k)^2$$

$$= \frac{1}{n}\sum_{j=1}^{n}\left[\vec{a}^T(\vec{X}_{kj} - \vec{\mu}_k)\right]^2$$

$$= \frac{1}{n}\sum_{j=1}^{n}(\vec{a}^T \hat{\vec{X}}_{kj})^2$$



点积可换

$$= \frac{1}{n}\sum_{j=1}^{n} \vec{a}^T \hat{\vec{X}}_{kj} \hat{\vec{X}}_{kj}^T \vec{a}$$

$$\Sigma_k = \frac{1}{n}\sum_{j=1}^{n} \vec{x}_{kj}\, \vec{x}_{kj}^{\,T}$$

$$V_k = \vec{a}^{\,T}\left(\frac{1}{n}\sum_{j=1}^{n} \vec{x}_{kj}\, \vec{x}_{kj}^{\,T}\right)\vec{a}$$

$$= \vec{a}^{\,T}\,\Sigma_k\,\vec{a}$$