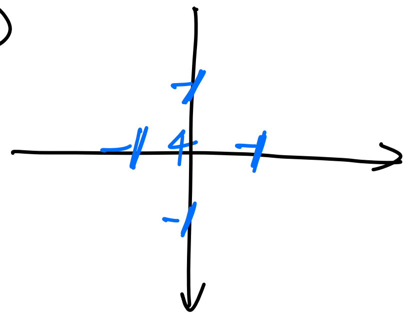
22-S1-Q1

(1) Q: $h(x,y) = ?$ 2 spatial properties?

Solution

① $g(x,y) = f(x,y) * h(x,y)$

②



③ $h(x,y) = 4\delta(x,y)$

$$- \delta(x-1,y) - \delta(x+1,y) - \delta(x,y-1) - \delta(x,y+1)$$

$$h(x,y) = \begin{cases} 4 & (x,y) = (0,0) \\ -1 & (x,y) = (\pm 1, 0) \text{ or } (0, \pm 1) \\ 0 & \text{otherwise} \end{cases}$$

④ spatial properties

1. High-Pass filtering: The filter emphasizes regions with high spatial frequency content (edges) by computing the difference between a pixel and its neighbors

2. Spatial Localization: The filter is localized affecting only a pixel and its immediate neighbors due to the non-zero coefficients being limited to the central pixel and its four direct neighbors

(b) $H(u,v) = \frac{1}{mn} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} h(x,y) e^{-j2\pi\left(\frac{ux}{m} + \frac{vy}{n}\right)}$

$$= \frac{1}{3\times 3} \sum_{x=-1}^{1} \sum_{y=-1}^{1} h(x,y) e^{-j\frac{2\pi}{3}(ux+vy)}$$

$$= \frac{1}{9}\Big[h(0,0)e^{0} + h(1,0)e^{-j\frac{2\pi}{3}u} + h(-1,0)e^{j\frac{2\pi}{3}u}$$

$$+ h(0,1)e^{-j\frac{2\pi}{3}v} + h(0,-1)e^{j\frac{2\pi}{3}v}\Big]$$

$$= \frac{1}{9}\left[4 - e^{-j\frac{2\pi}{3}u} - e^{j\frac{2\pi}{3}u} - e^{-j\frac{2\pi}{3}v} - e^{j\frac{2\pi}{3}v}\right]$$

$$= \frac{1}{9}\left[4 - \left(e^{-j\frac{2\pi}{3}u} + e^{j\frac{2\pi}{3}u}\right) - \left(e^{-j\frac{2\pi}{3}v} + e^{j\frac{2\pi}{3}v}\right)\right]$$

$$= \frac{1}{9}\left[4 - 2\cos\frac{2\pi}{3}u - 2\cos\frac{2\pi}{3}v\right]$$

(c)

0	0	0	0	0	0	0	0	0
0	0	$0 \to -1$	$0 \to -1$	$0 \to -1$	$0 \to -1$	$0 \to -1$	0	0
0	$0 \to -1$	$1 \to 2$	$1 \to 1$	$1 \to 1$	$1 \to 1$	$1 \to 2$	$0 \to -1$	0
0	$0 \to -1$	$1 \to 1$	$1 \to 0$	$1 \to 0$	$1 \to 0$	$1 \to 1$	$0 \to -1$	0
0	$0 \to -1$	$1 \to 1$	$1 \to 0$	$1 \to 0$	$1 \to 0$	$1 \to 1$	$0 \to -1$	0
0	$0 \to -1$	$1 \to 1$	$1 \to 0$	$1 \to 0$	$1 \to 0$	$1 \to 1$	$0 \to -1$	0
0	$0 \to -1$	$1 \to 2$	$1 \to 1$	$1 \to 1$	$1 \to 1$	$1 \to 2$	$0 \to -1$	0
0	0	$0 \to -1$	$0 \to -1$	$0 \to -1$	$0 \to -1$	$0 \to -1$	0	0
0	0	0	0	0	0	0	0	0

(d)

edge detection: as it suppresses uniform region and enhances areas with significant intensity changes.