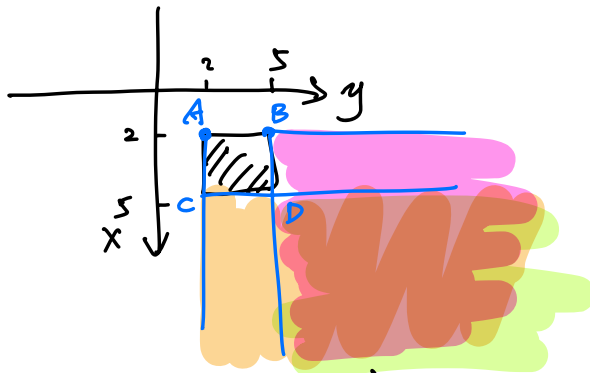21-S1-Q1

Q(a) $f(x,y)$ $h(x,y)$

Solution

(a) ① 2D unit step function
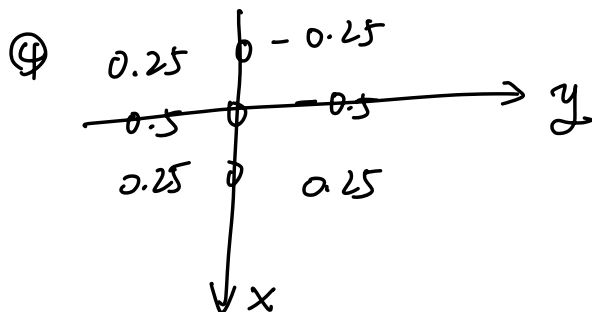
$$S(x,y) = \begin{cases} 1, & x \geq 0 \text{ and } y \geq 0 \\ 0, & \text{other wise} \end{cases}$$

②

③ $f(x,y) = S(x-2, y-2)$

$\quad - \quad S(x-6, y-2)$

$\quad - \quad S(x-2, y-6)$

$\quad + S(x-6, y-6)$

④

⑤ $h(x,y) = 0.25\,\delta(x-1,y-1) + 0.25\,\delta(x-1,y+1)$

$+ 0.5\,\delta(x,y+1) + 0.25\,\delta(x+1,y+1)$

$- 0.25\,\delta(x+1,y-1) - 0.5\,\delta(x,y-1)$

(b)

	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	0	1	1	1	1	0	0
3	0	1	1	1	1	0	0
4	0	1	1	1	1	0	0
5	0	1	1	1	1	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0

$0.25 \quad 0 \quad -0.25$

$0.5 \quad 0 \quad -0.5 \rightarrow y$

$0.25 \quad 0 \quad -0.25$

$g(1,1) = 0.25 \quad g(1,2) = 0.25 \quad g(1,3) = 0 \quad g(1,4) = 0 \quad \ldots$

$g(2,2) = 0.75 \quad g(2,3) = 0 \quad g(2,4) = 0 \quad g(2,5) = -0.75$

similarly

output image

	1	2	3	4	5	6	7
1	0.25	0.25	0	0	-0.25	-0.25	0
2	0.75	0.75	1	1	-0.75	-0.75	0
3	1	1	0	0	-1	-1	0
4	1	1	0	0	-1	-1	0
5	0.75	0.75	1	1	-0.75	-0.75	0
6	0.25	0.25	0	0	-0.25	-0.25	0
7	0	0	0	0	0	0	0

(c) $h(x,y)$ acts as horizontal gradient operator

② $h(x,y)$ computes the weighted difference between the pixels on the left and right of each position

③ $h(x,y)$ effectively detect vertical edge in the image

Verification

① Along the left edge of the rectangle in $f(x,y)$, where there is a transition from 0 to 1

② Along the right edge of the rectangle in $f(x,y)$, where there is a transition from 1 to -1

③ This behavior confirms that the filter effectively detects vertical edges