

21-S1-Q1

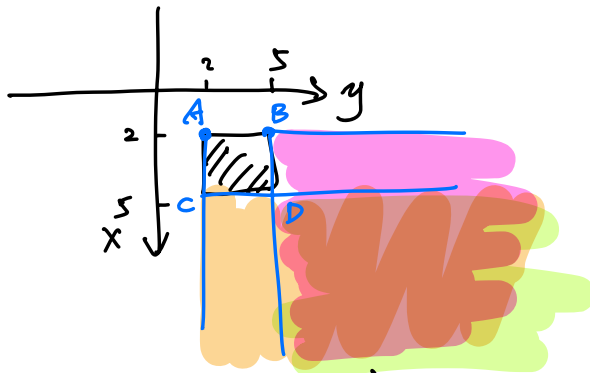
Q(a) $f(x,y)$ $h(x,y)$

Solution

(a) ① 2D unit step function

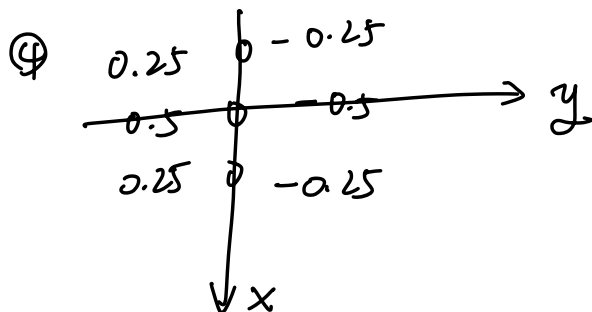
$$s(x,y) = \begin{cases} 1, & x \geq 0 \text{ and } y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

②



| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | 1 | 1 | 0 |
| 3 | 0 | 1 | 1 | 1 | 1 | 0 |
| 4 | 0 | 1 | 1 | 1 | 1 | 0 |
| 5 | 0 | 1 | 1 | 1 | 1 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 |

$$\begin{aligned} \textcircled{3} f(x,y) &= s(x-2, y-2) \\ &- s(x-6, y-2) \\ &- s(x-2, y-6) \\ &+ s(x-6, y-6) \end{aligned}$$



$$\textcircled{5} h(x, y) = 0.25 \delta(x-1, y+1) + 0.5 \delta(x, y+1) \\ + 0.25 \delta(x+1, y+1) - 0.25 \delta(x+1, y-1) \\ - 0.5 \delta(x, y-1) - 0.25 \delta(x-1, y-1)$$

(b)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|---|---|---|---|---|---|---|
| x \ y | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|------|---|-------|---|---|---|---|---|
| x \ y | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.5 | 0 | -0.5 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0.25 | 0 | -0.25 | 0 | 0 | 0 | 0 | 0 |

$$g(1,1) = 0.25 \quad g(1,2) = 0.25 \quad g(1,3) = 0 \quad g(1,4) = 0 \quad \dots$$

$$g(2,2) = 0.75 \quad g(2,3) = 0 \quad g(2,4) = 0 \quad g(2,5) = -0.75$$

similarly

output image

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|------|------|---|---|-------|-------|---|---|
| x \ y | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0.25 | 0.25 | 0 | 0 | -0.25 | -0.25 | 0 | 0 |
| 2 | 0.75 | 0.75 | 0 | 0 | -0.75 | -0.75 | 0 | 0 |
| 3 | 1 | 1 | 0 | 0 | -1 | -1 | 0 | 0 |
| 4 | 1 | 1 | 0 | 0 | -1 | -1 | 0 | 0 |
| 5 | 0.75 | 0.75 | 0 | 0 | -0.75 | -0.75 | 0 | 0 |
| 6 | 0.25 | 0.25 | 0 | 0 | -0.25 | -0.25 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

- (c) ① $h(x, y)$ acts as horizontal gradient operator
- ② $h(x, y)$ computes the weighted difference between the pixels on the left and right of each position
- ③ $h(x, y)$ effectively detect vertical edge in the image

Verification

- ① Along the left edge of the rectangle in $f(x, y)$, where there is a transition from 0 to 1
- ② Along the right edge of the rectangle in $f(x, y)$, where there is a transition from 1 to -1
- ③ This behavior confirms that the filter effectively detects vertical edges