

# 1st day: How to go ultracold

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# Overview

Why go for ultracold ?

see also the talk by Bill Philipps called  
,Einstein, time and the coolest stuff in the Universe‘

The first step: Getting ultracold by laser cooling

The second step: Get degeneracy by collisions

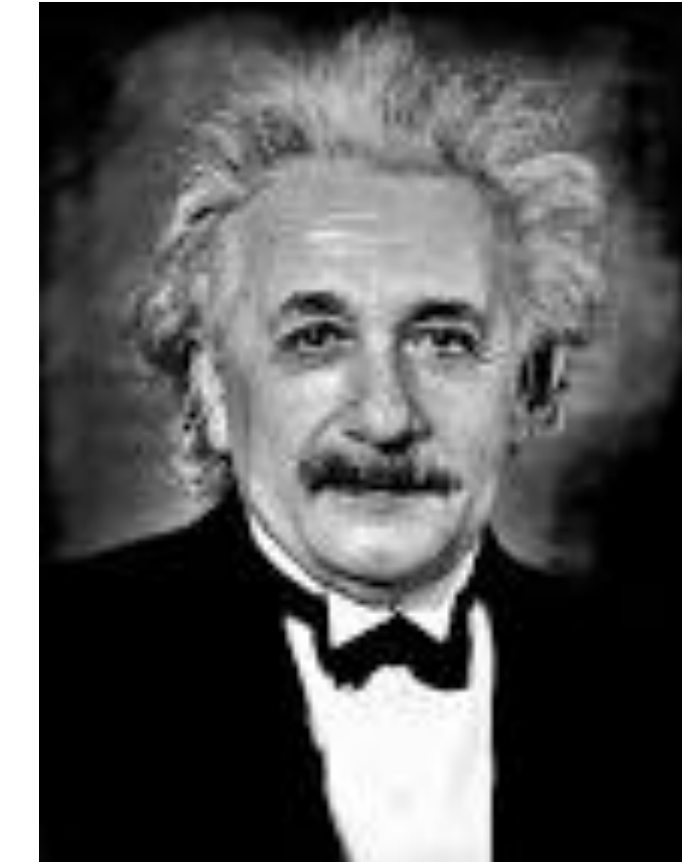
# Good reads

- **Lecture notes on AMO physics** by *Mikhail Lukin*
- **Making, probing understanding Bose-Einstein condensates** by *W. Ketterle*
- **Laser cooling and trapping** by *Metcalf and van der Straten*
- **Theory of Bose-Einstein condensation in trapped gases** by *Dalfovo et al.*
- **Bose-Einstein condensation in dilute gases** by *Pethick and Smith*

# What is time ?

Einsteins' special relativity:

Time is what a clock measures.



Experimentalists dilemma: What is a clock ?

Something that ,ticks', i.e. provides a regular series of events



# Traditional clocks



1 tick = 1 day



1 tick = few seconds



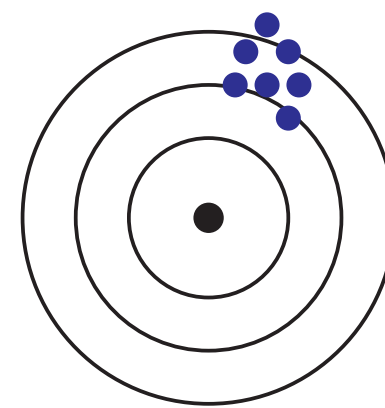
1 tick = 0.1 ms

## Problems:

- Not very stable
- Very slow ticking
- Reproducibility

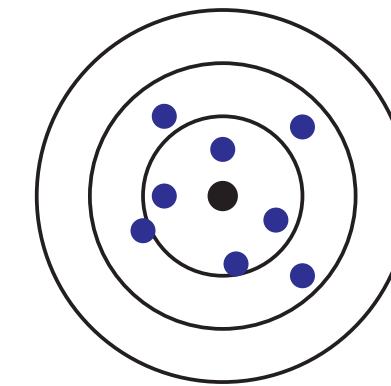
# What is a good clock ?

Stable



repeat with the same clock lots of measurements and get similar results

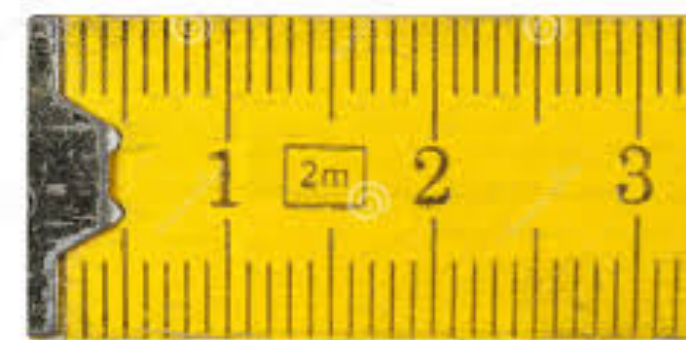
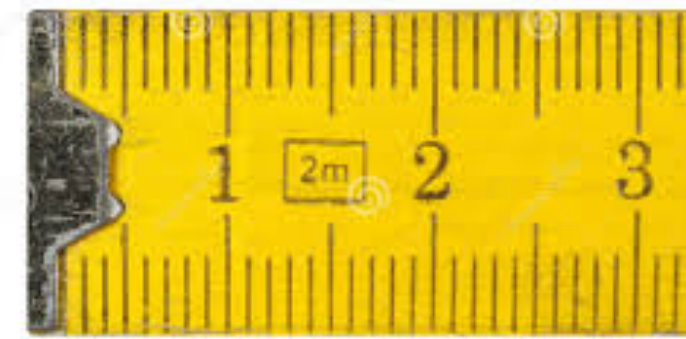
Precise



build several clocks and obtain same results

most of the time much, much harder to estimate

# Characterization of clocks



let them tick for a long time

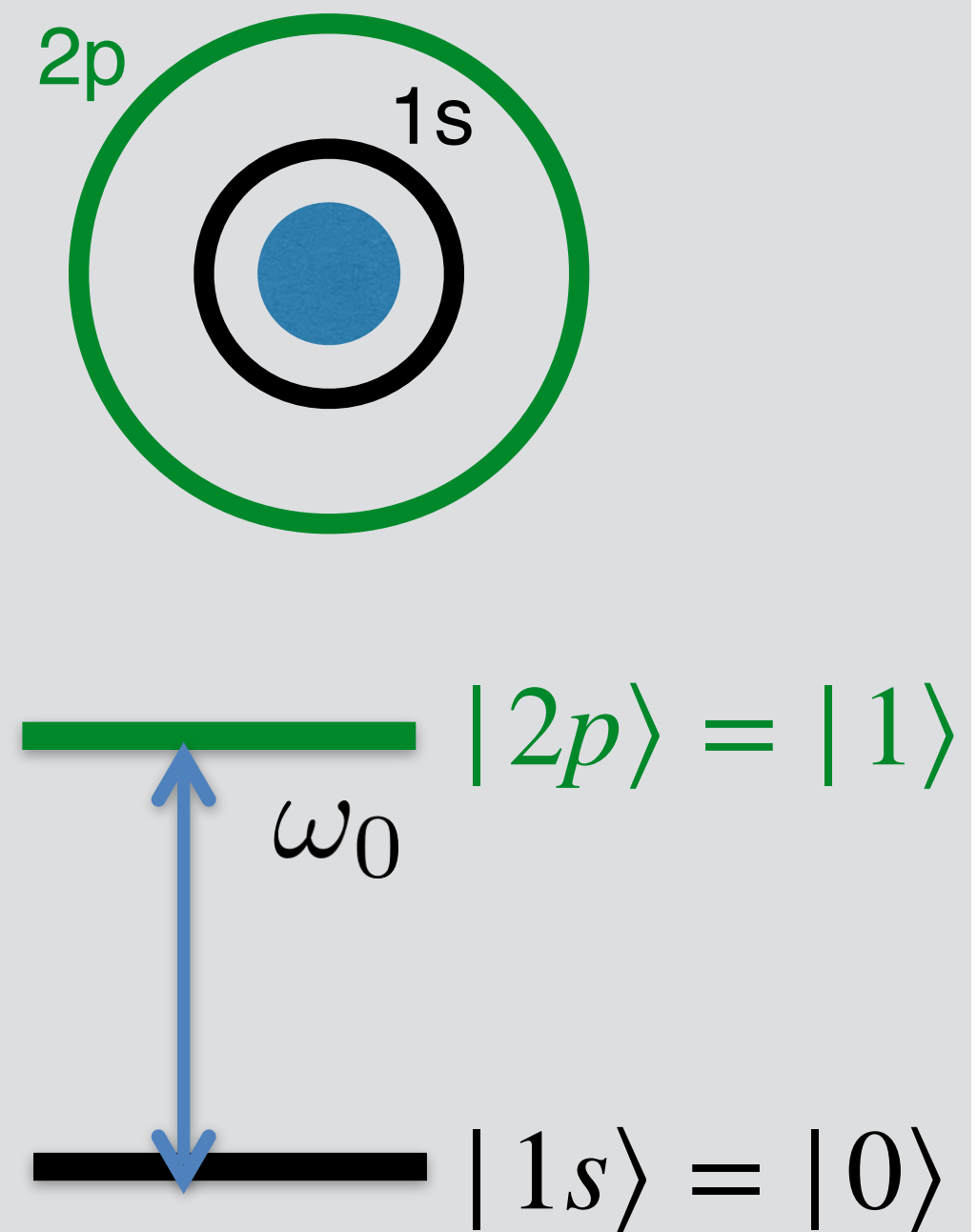
compare the result

What about precision?

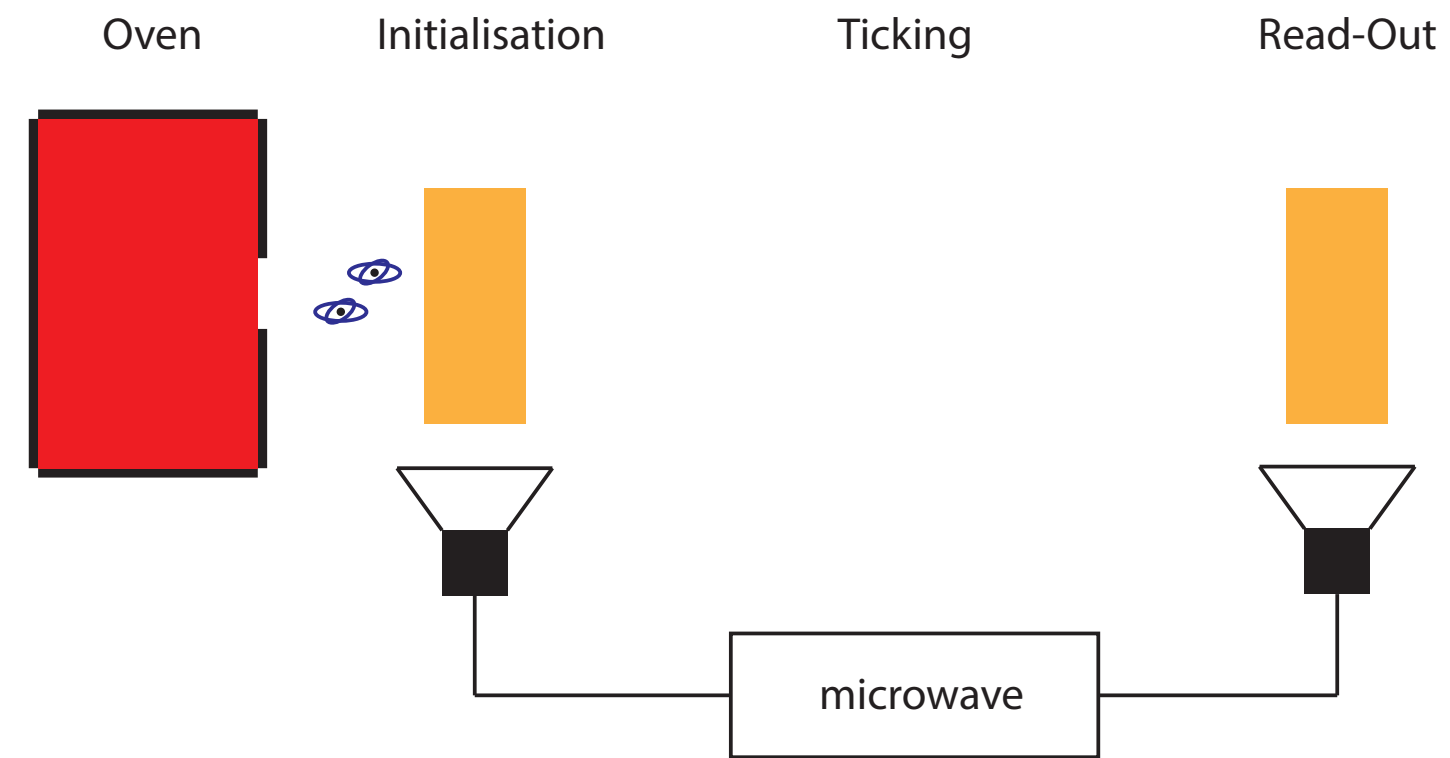
**We need a good standard and atoms give this**

# Atomic clocks

## The Atom

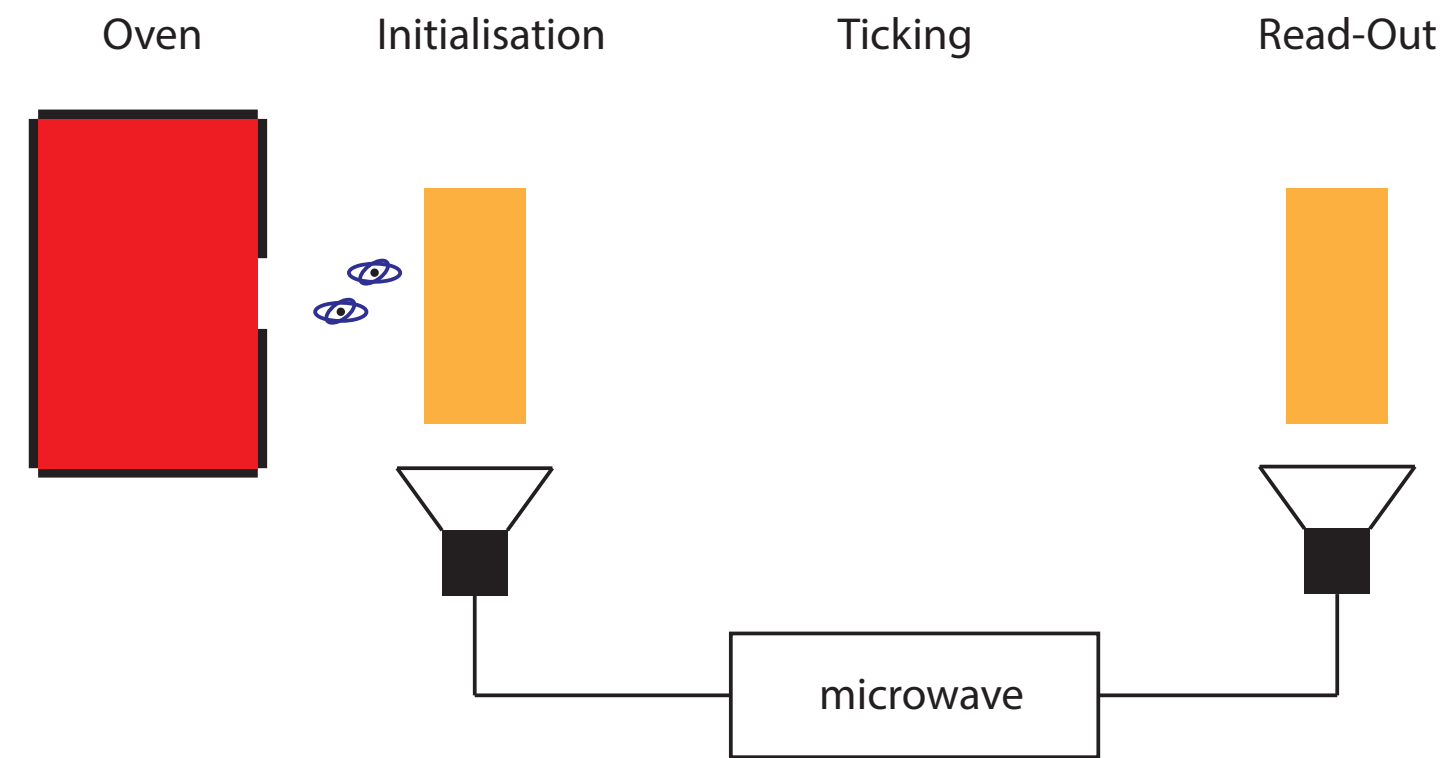


$$\mathcal{H} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

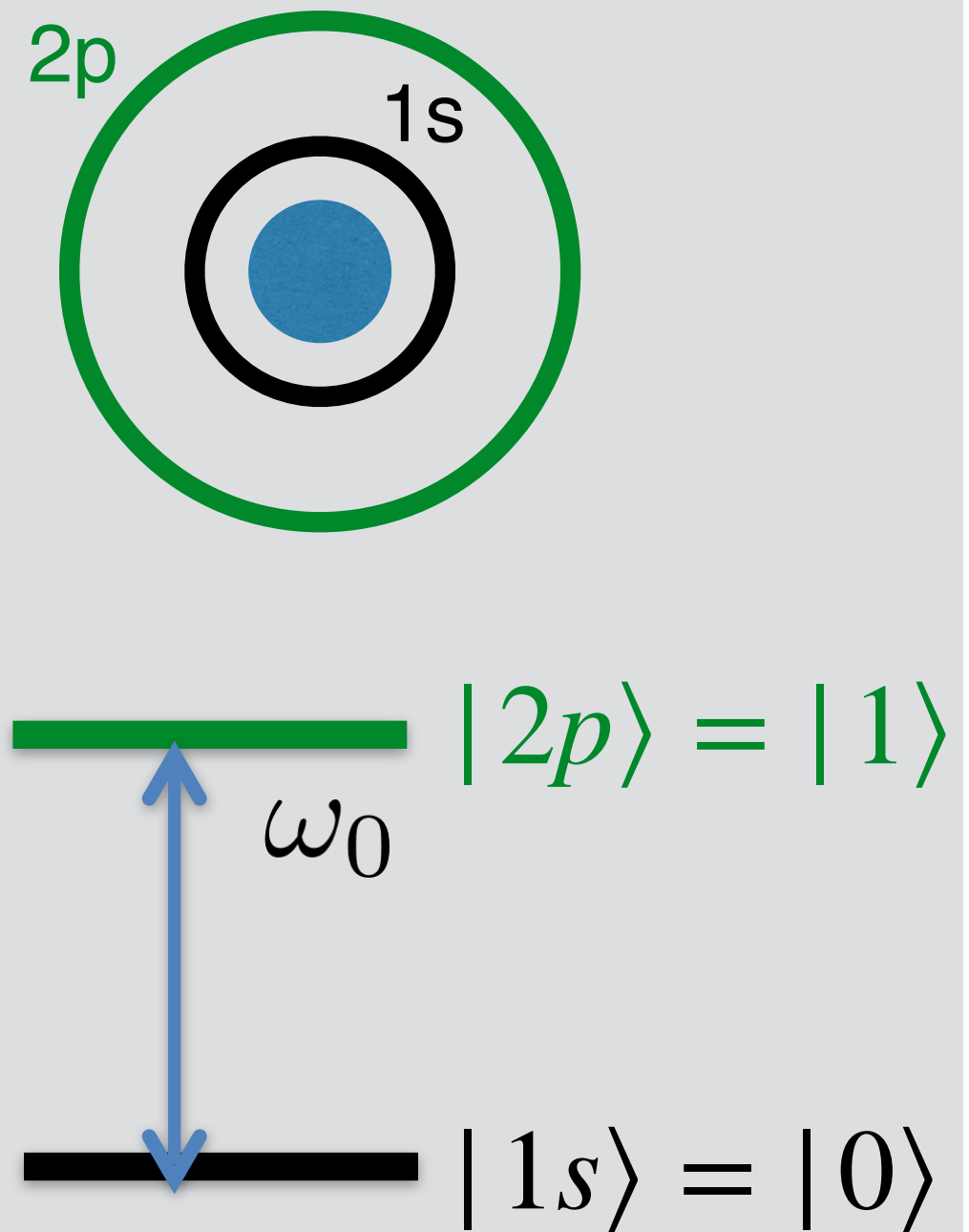




# Atomic clocks



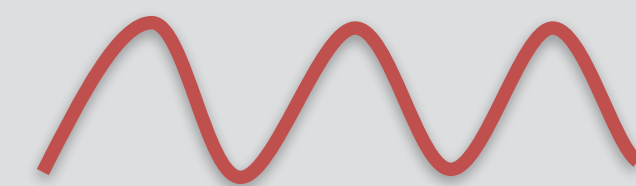
**The Atom**



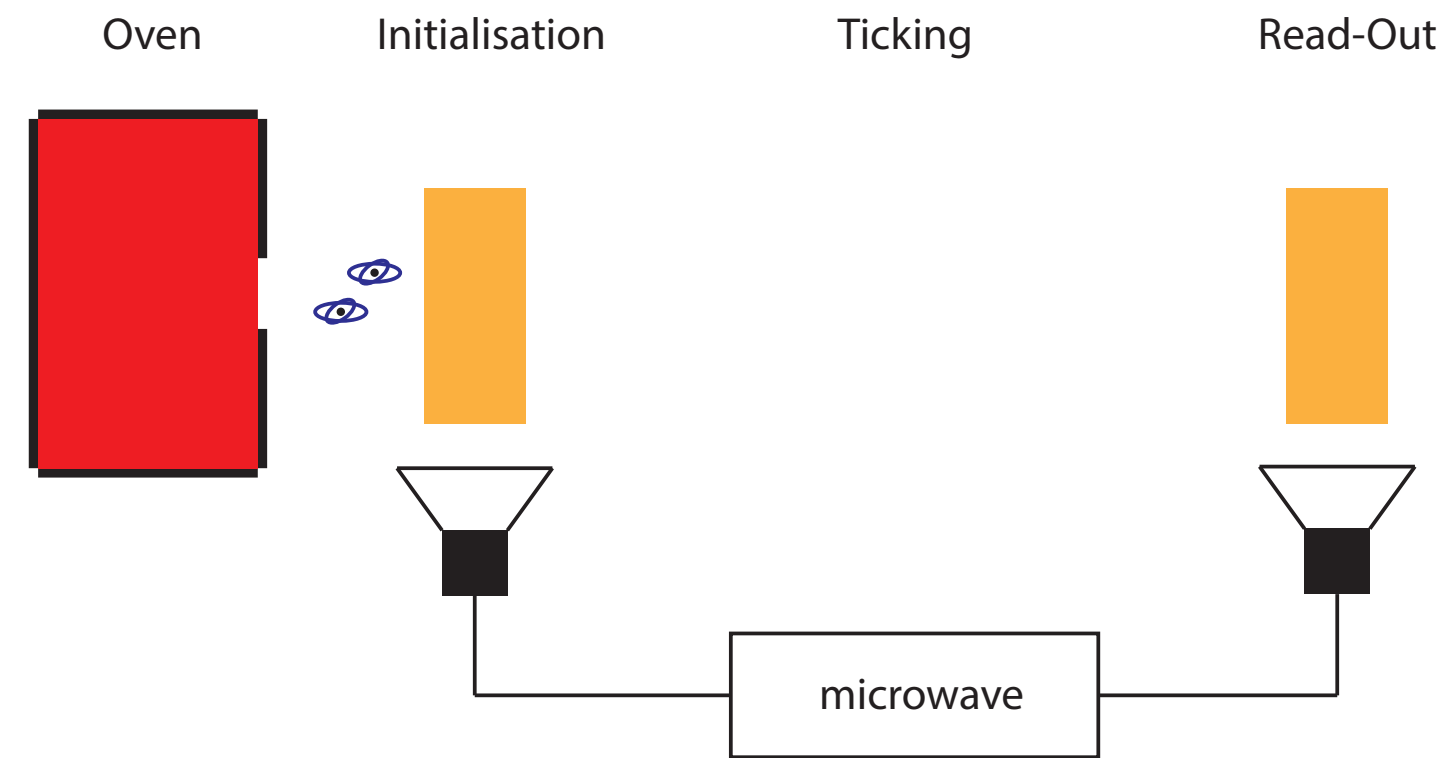
$$\mathcal{H} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

**The electric field**

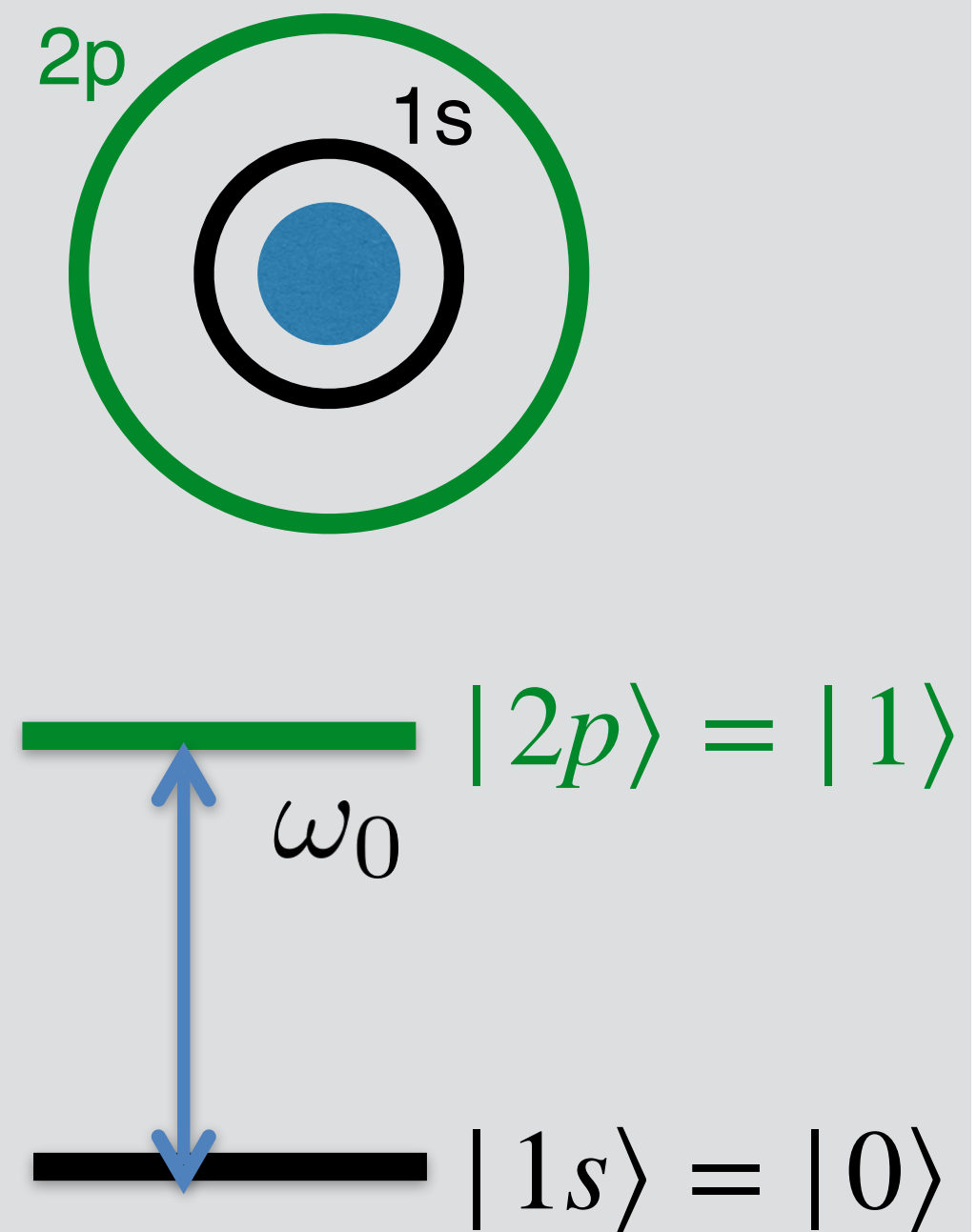
$$\mathbf{E} = E_0 \left( e^{i\omega_L t + i\varphi} + e^{-i\omega_L t - i\varphi} \right)$$



# Atomic clocks



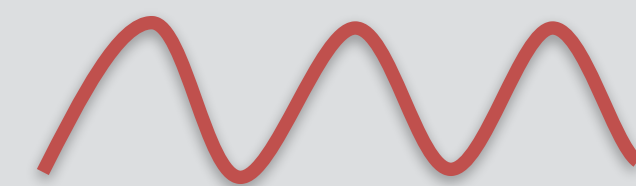
**The Atom**



$$\mathcal{H} = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1|$$

**The electric field**

$$\mathbf{E} = E_0 (e^{i\omega_L t + i\varphi} + e^{-i\omega_L t - i\varphi})$$



**Interaction via**

$$\mathcal{H} = -\mathbf{d} \cdot \mathbf{E}$$

$$\mathbf{d} = d (|0\rangle\langle 1| + |1\rangle\langle 0|)$$

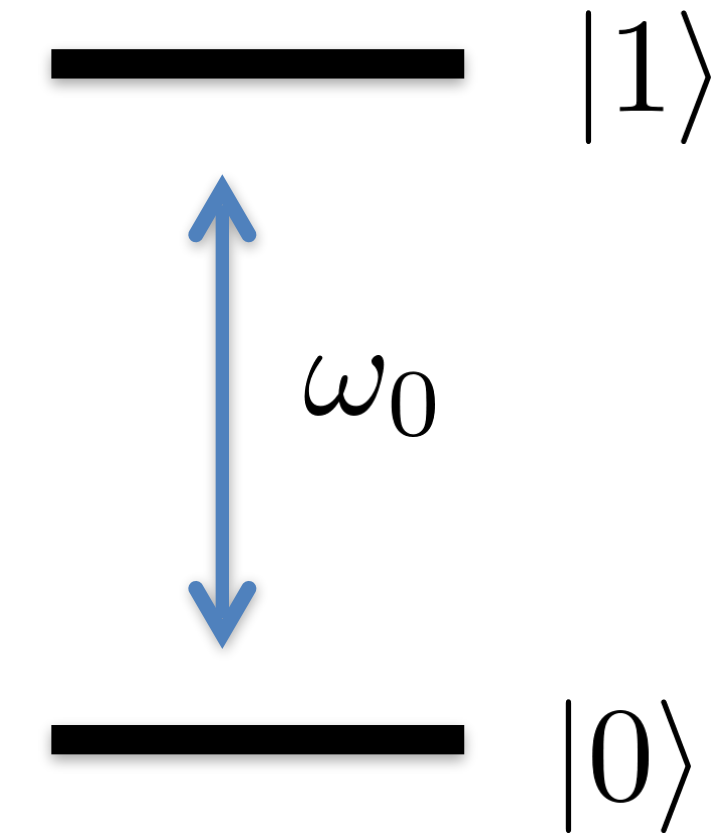
# The atom as a qubit

$$\mathcal{H} = E_0 |0\rangle \langle 0| + E_1 |1\rangle \langle 1|$$

$$\mathcal{H} = \frac{\hbar\omega_0}{2} |1\rangle \langle 1| - \frac{\hbar\omega_0}{2} |0\rangle \langle 0|$$

$$\mathcal{H} = \frac{\hbar\omega_0}{2} \sigma_z$$

→ write everything in terms of spins



$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# Interaction Hamiltonian

$$\mathcal{H} = -\mathbf{d} \cdot \mathbf{E}$$

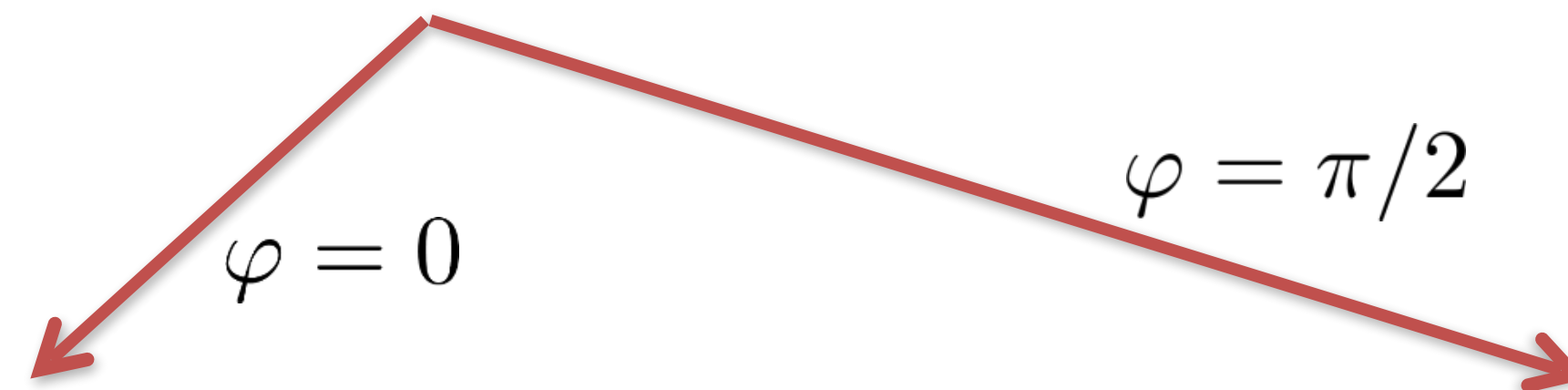
$$\mathbf{E} = E(e^{i\omega t + i\varphi} + e^{-i\omega t - i\varphi})$$

$$\mathbf{d} = d(\sigma_+ + \sigma_-)$$

**Rotating frame:**

$$\mathcal{H} = \frac{dE}{2}(\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi})$$

$$\mathcal{H} \sim \hbar\Omega(\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi})$$

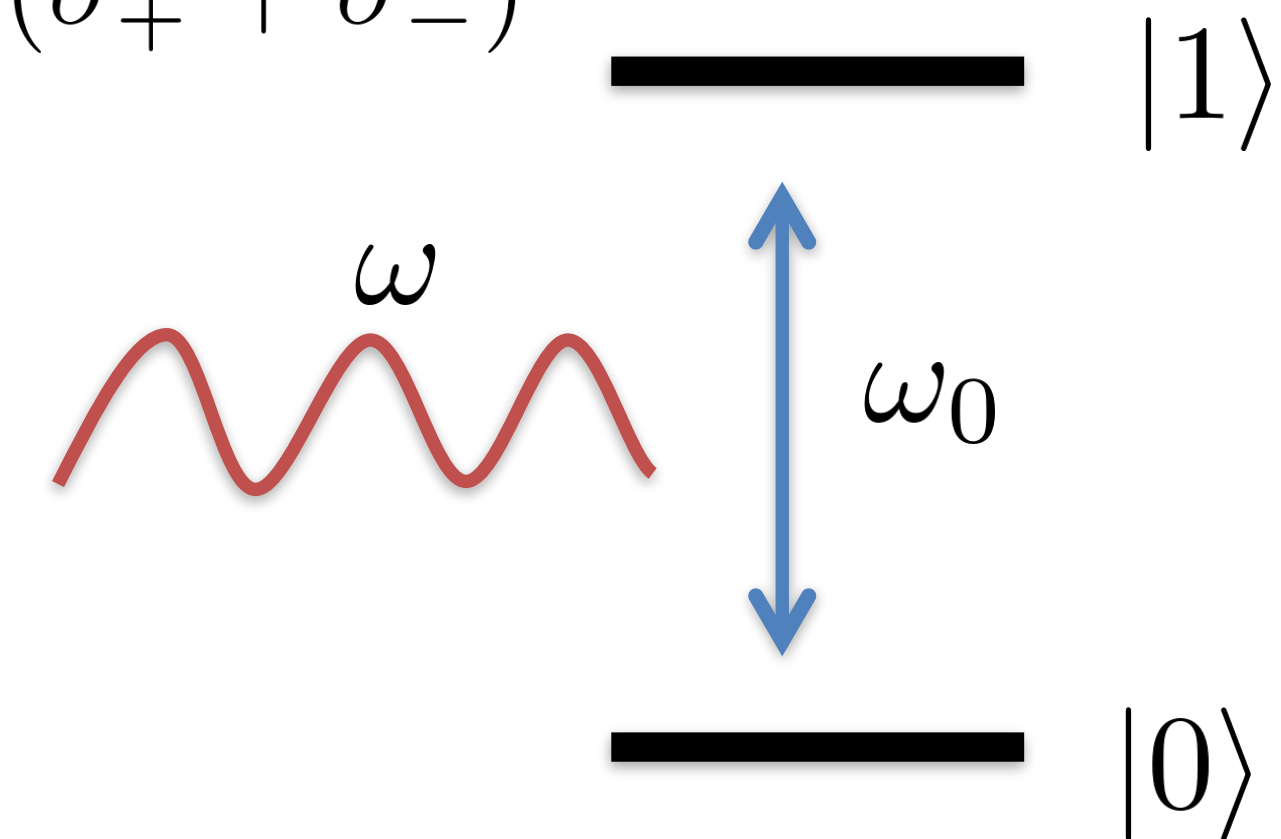


$$\mathcal{H} \sim \hbar\Omega(\sigma_+ + \sigma_-)$$

$$\mathcal{H} \sim \hbar\Omega\sigma_x$$

$$\mathcal{H} \sim \hbar\Omega(\sigma_+ - \sigma_-)$$

$$\mathcal{H} \sim \hbar\Omega\sigma_y$$



# Clocks as extremely precise qubits

Rotation about z-axis  
Detuning

$$\mathcal{H} = \hbar \Delta \hat{\sigma}_z$$

Rotation about x-axis  
Laser intensity

$$\mathcal{H} = \hbar \Omega_x \hat{\sigma}_x$$

Rotation about y-axis  
Laser intensity with phase  
adjusted

$$\mathcal{H} = \hbar \Omega_y \hat{\sigma}_y$$

$$U = e^{i\mathcal{H}t/\hbar}$$

$Z_{\pi/2}$

$$\Delta t = \frac{\pi}{2}$$

$X_{\pi/2}$

$$\Omega_x t = \frac{\pi}{2}$$

$Y_{\pi/2}$

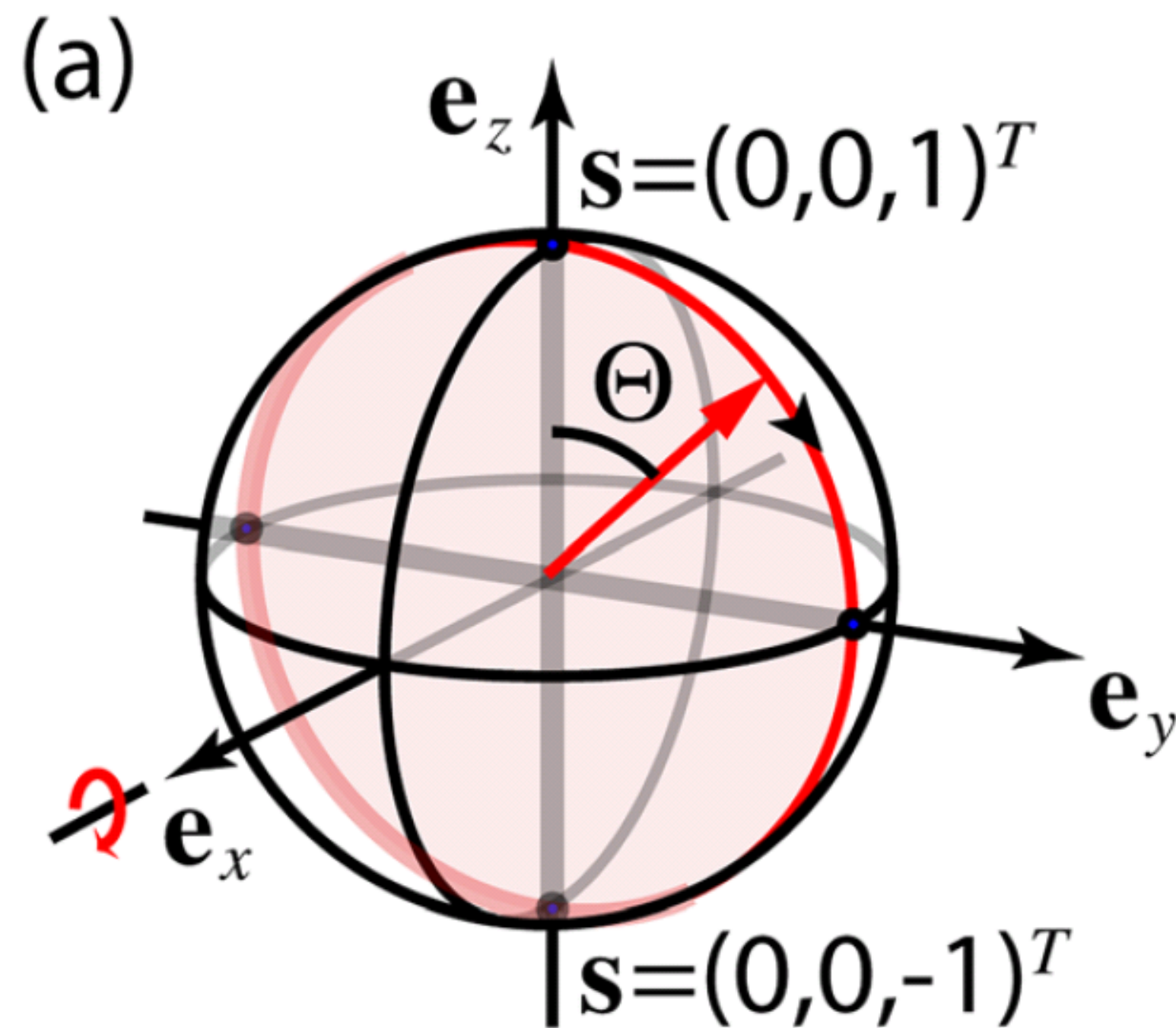
$$\Omega_y t = \frac{\pi}{2}$$

# Example: Rabi oscillations

$$\mathcal{H} = \hbar\Omega S_x$$

time evolution:  $e^{i\Omega\sigma_x t}$

Rotation about **x-axis** angle  $\Theta = \Omega t$

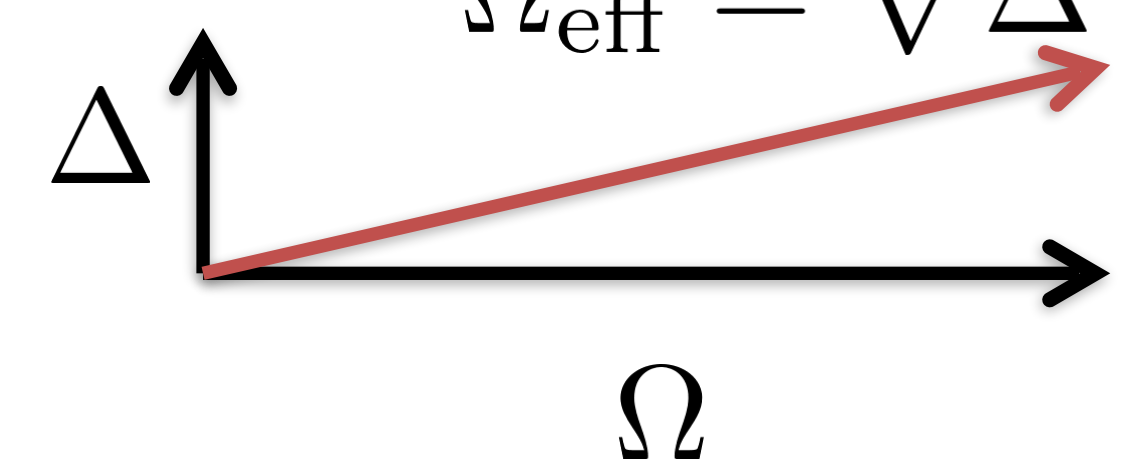


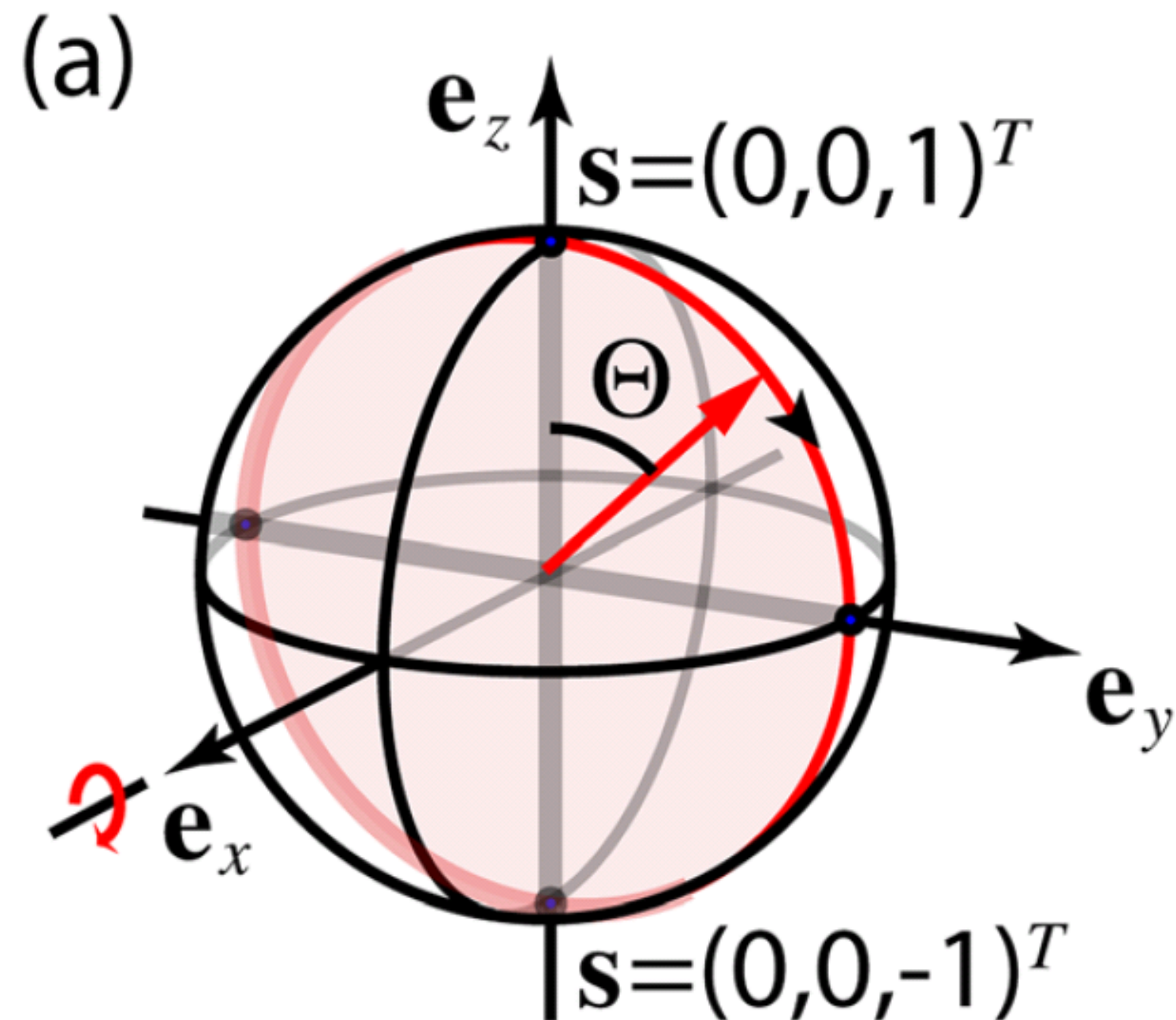
# Example: Offresonant Rabi oscillations

$$\mathcal{H} = \hbar\Omega S_x + \hbar\Delta S_z$$


**detuning**

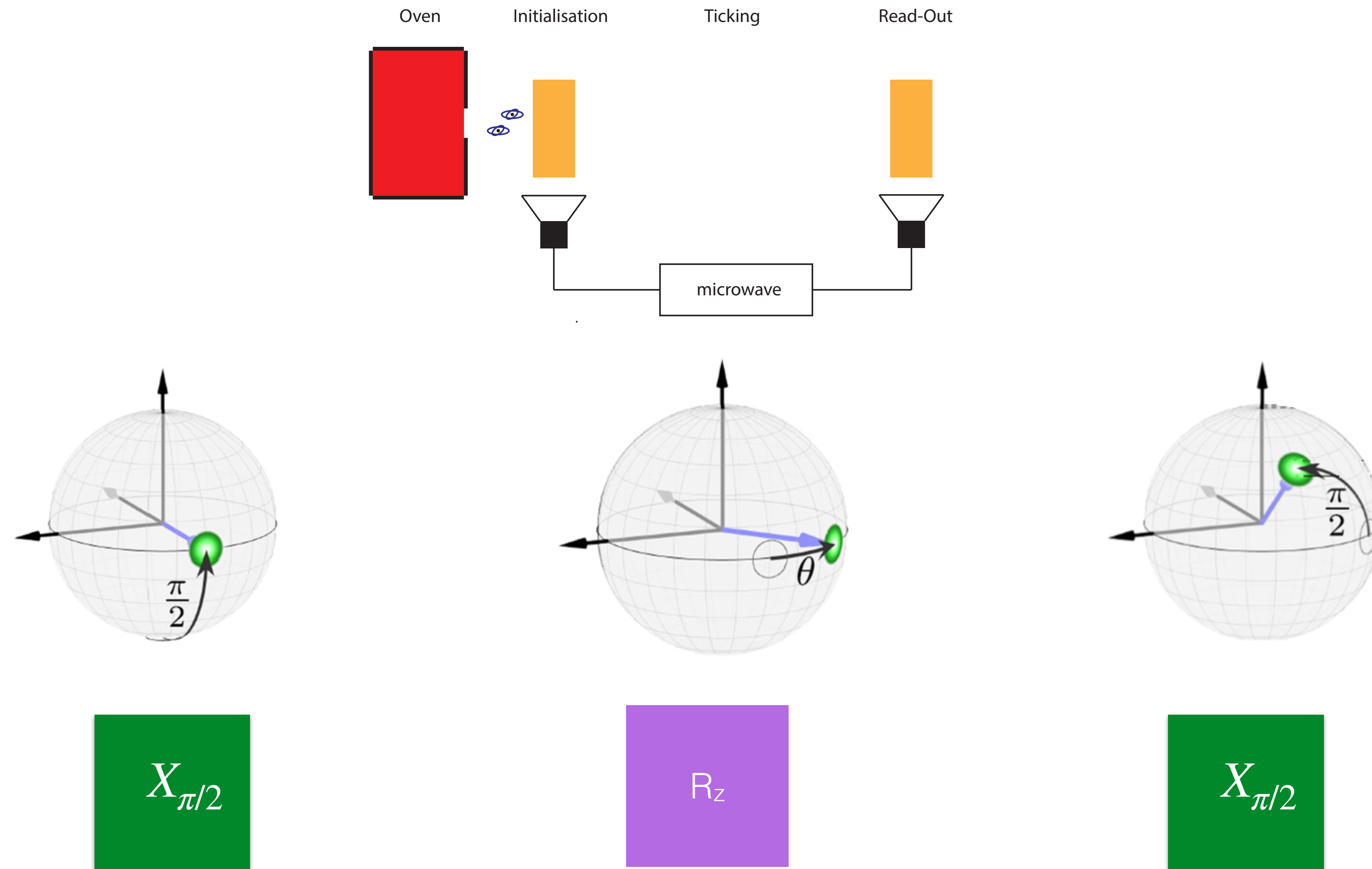
tilted rotation axis  $J = \begin{pmatrix} \Omega \\ 0 \\ \Delta \end{pmatrix}$

$$\Omega_{\text{eff}} = \sqrt{\Delta^2 + \Omega^2}$$






# Back to our atomic clocks





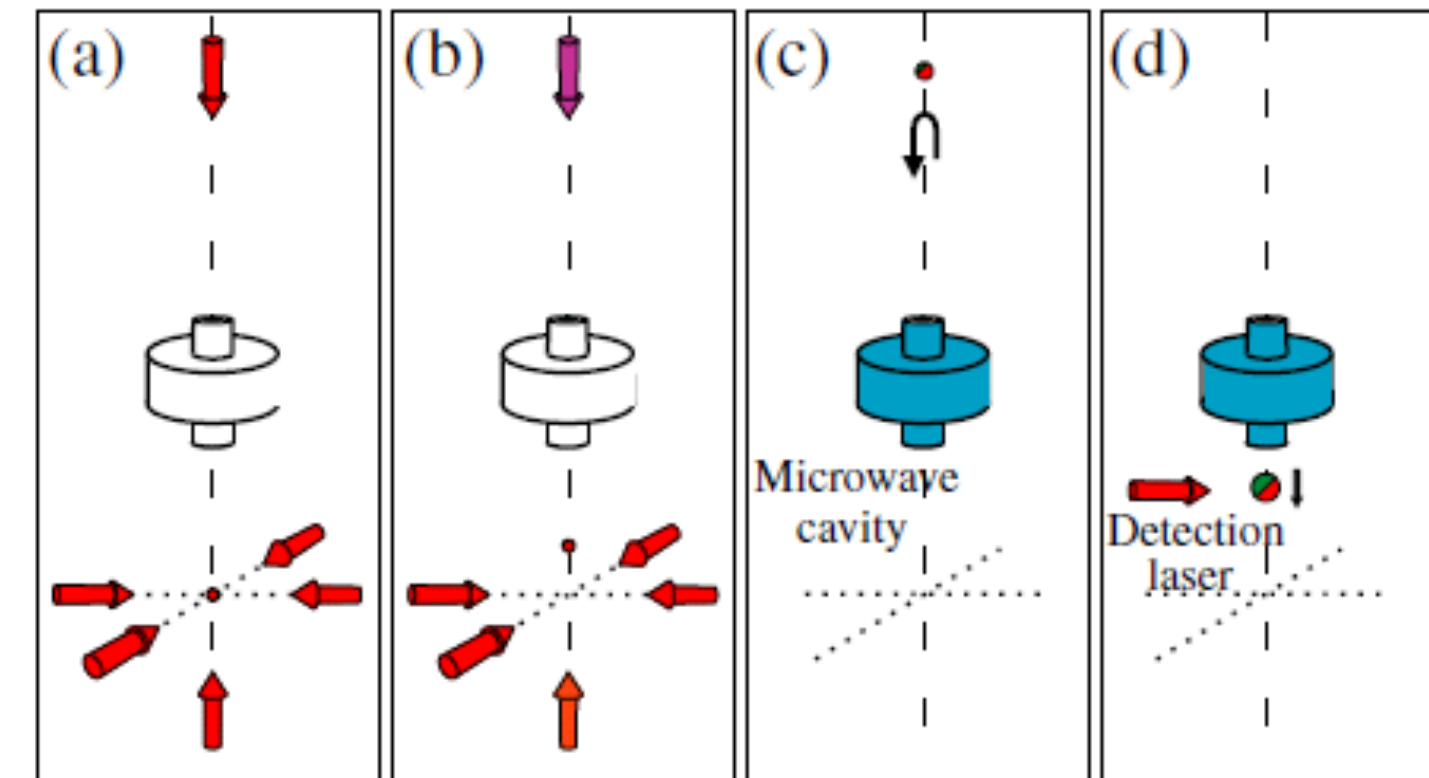
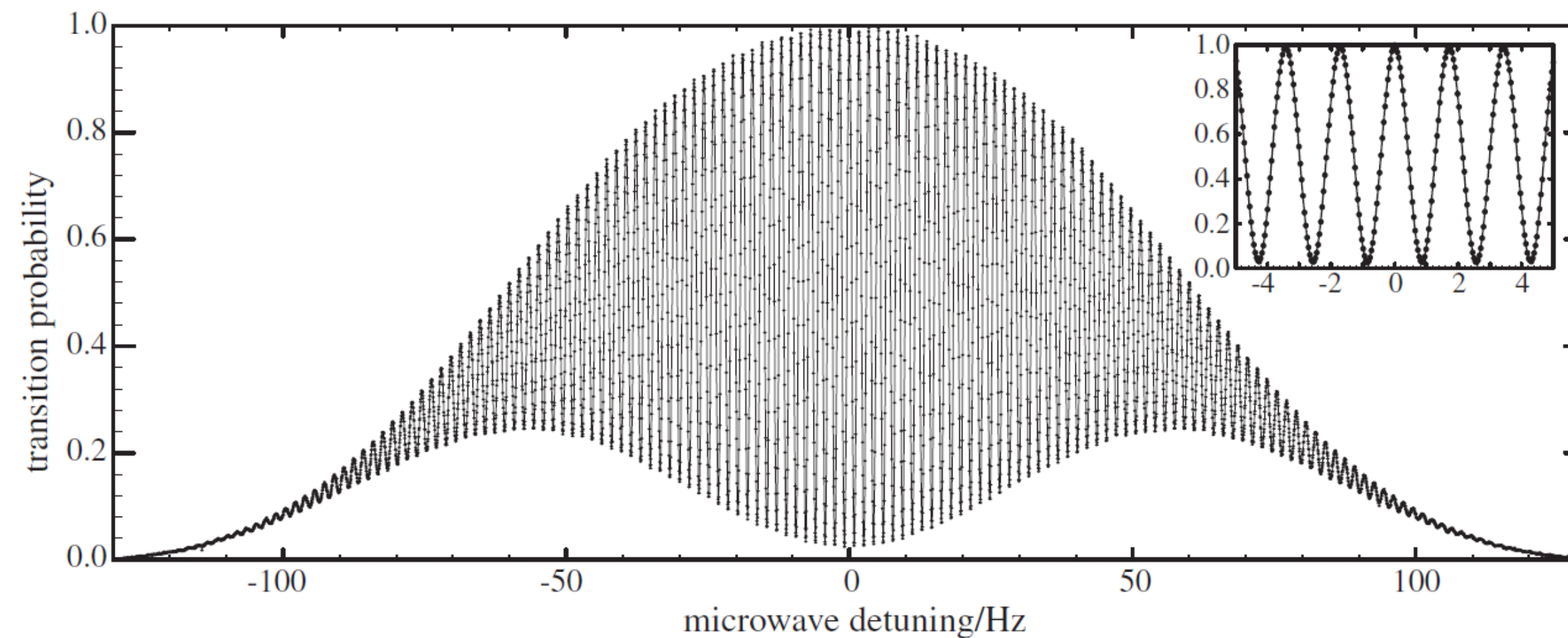
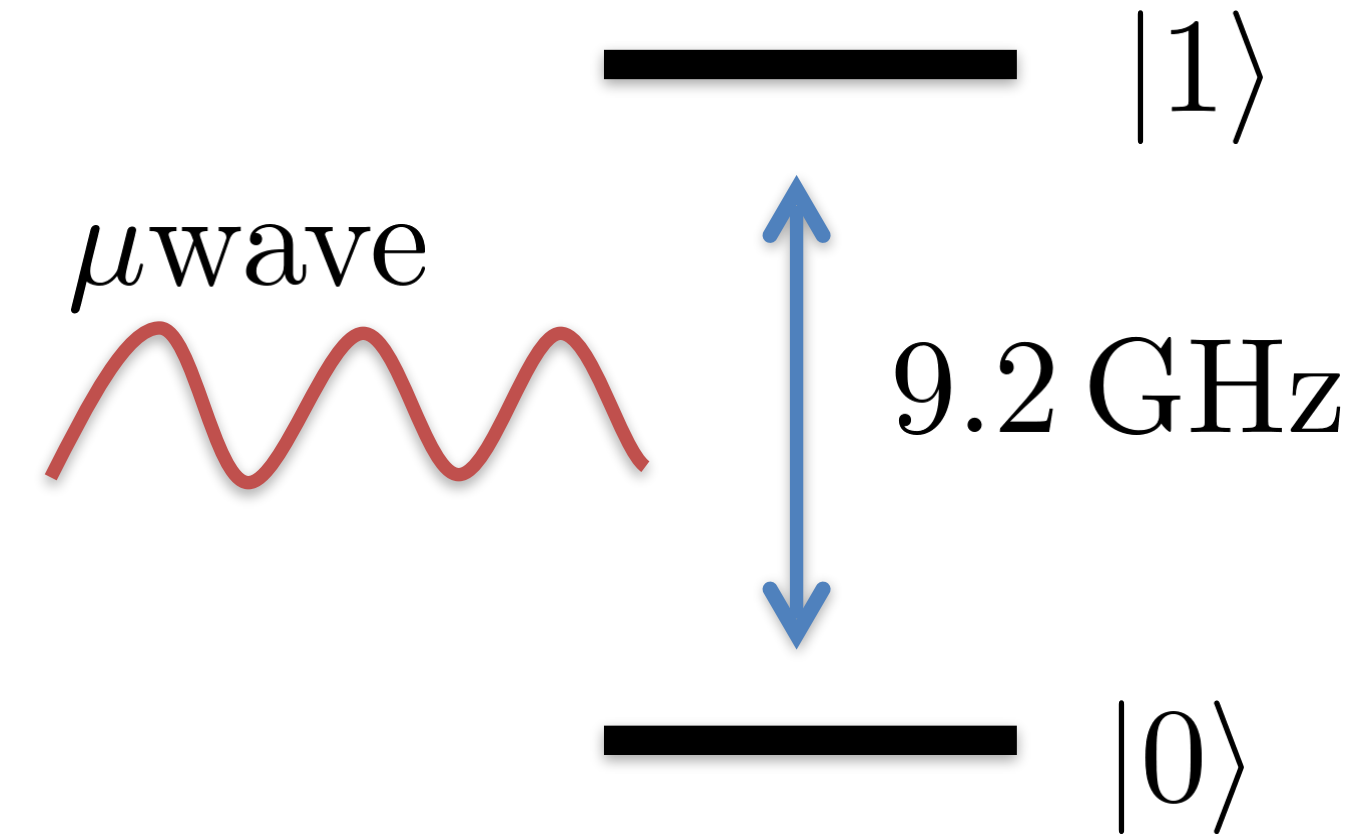
# Application: Time standard with Cesium fountain clock

$^{133}\text{Cs}$ : Hyperfine splitting 9.2 GHz

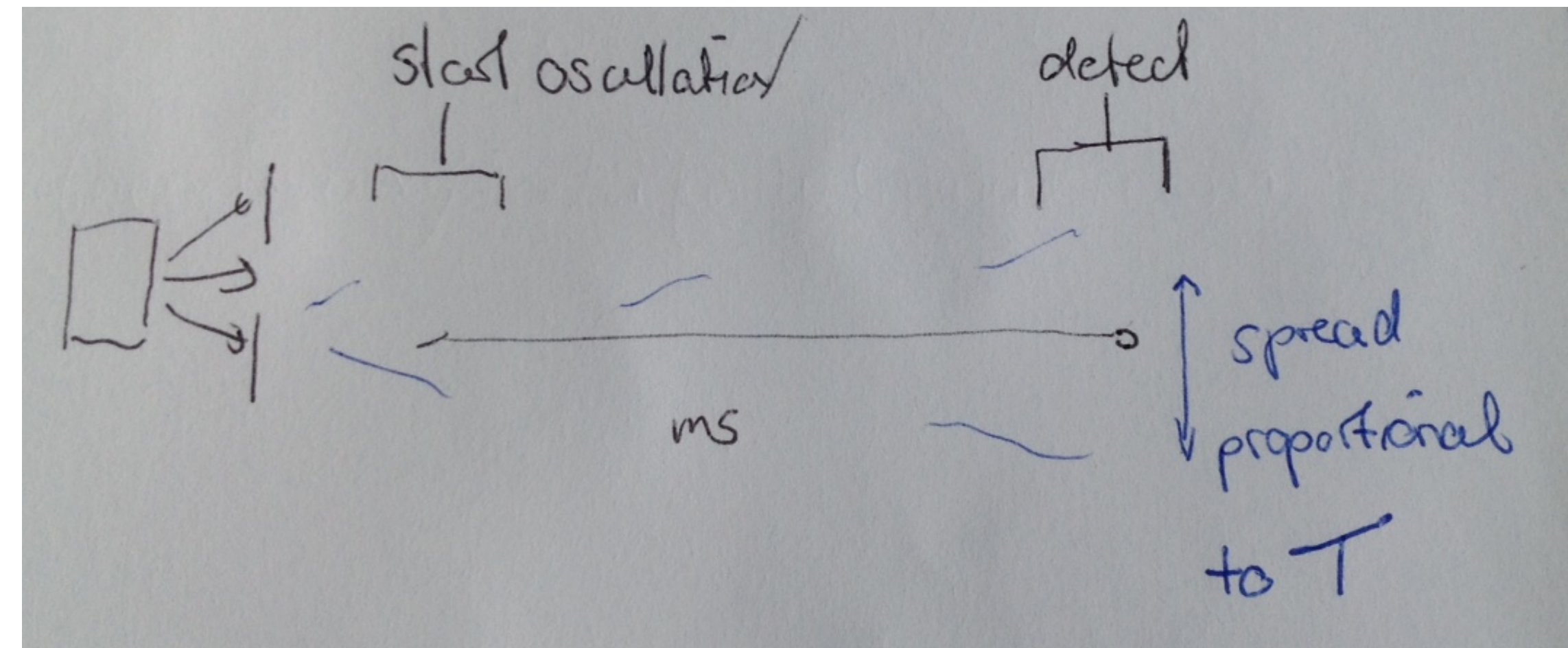
envelope  $\times \cos \Delta Et / \hbar$

$$\Delta\omega \times \Delta t \geq 1$$

$$\rightarrow \text{precision: } \frac{\Delta\omega}{\omega} \times \frac{1}{\sqrt{N}} \approx 10^{-13}$$



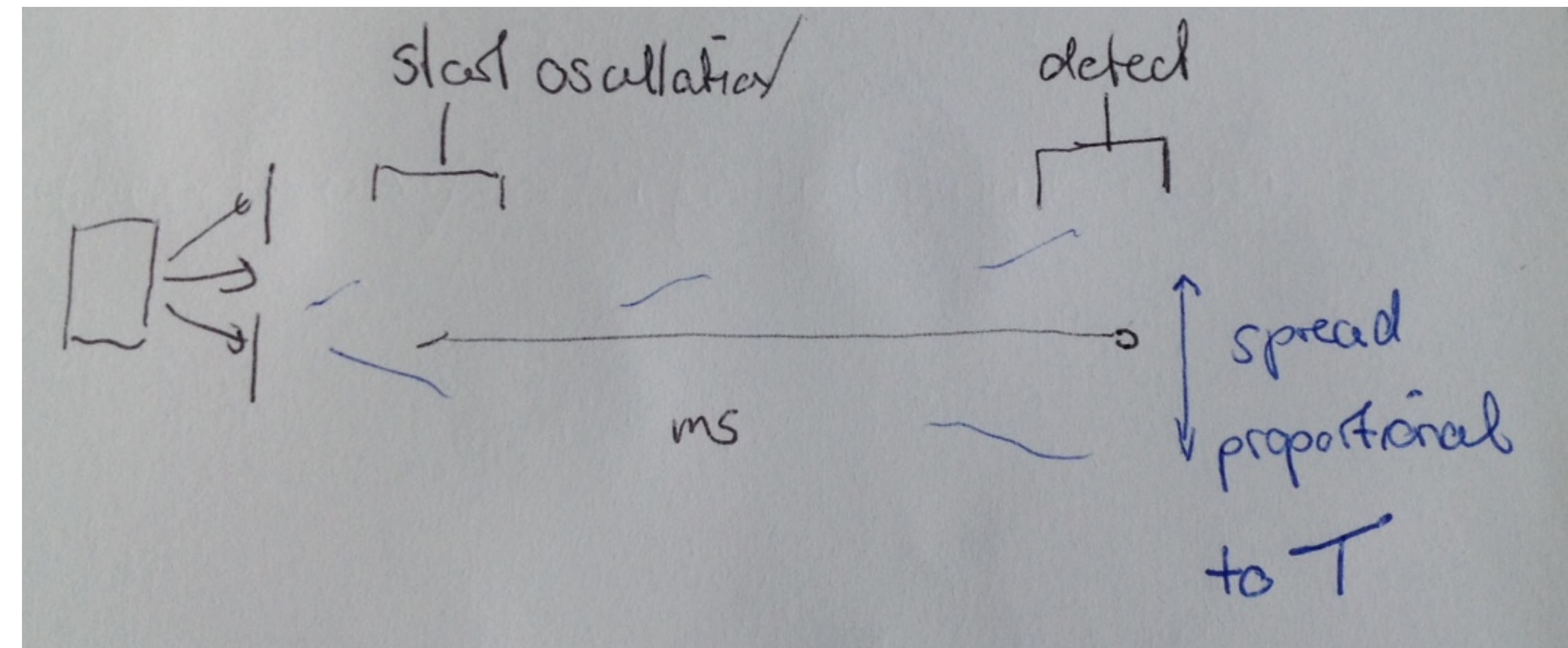
# Ramsey limitations



Detection better if atoms are slower



# Ramsey limitations



Detection better if atoms are slower

- We know how to perform qubit operation.
- How can we cool these atoms ?
- How can we trap them individually ?

