Introduction to Deep Learning

Bootcamp IID 2022

Frédérik Paradis

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2 mai 2022

Who Am I?

Frédérik Paradis

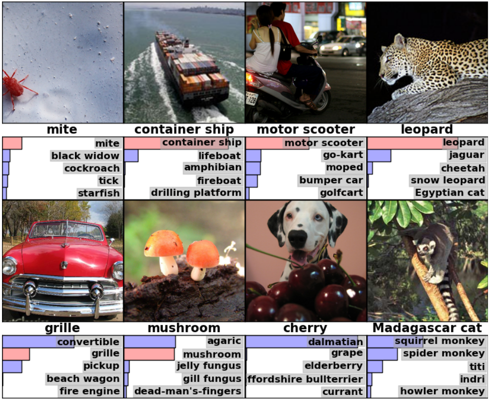
Lead developer of Poutyne

Ph.D. Student at Université Laval

Expert in AI at Baseline

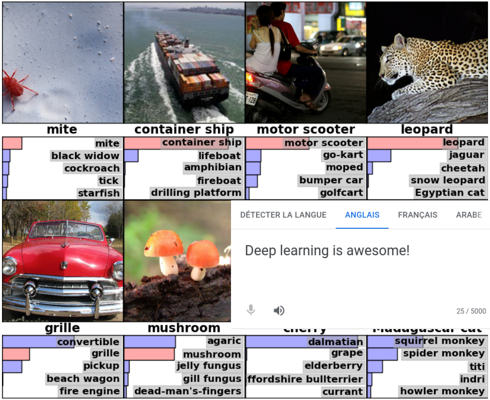
frederik-paradis

Deep Neural Networks



[Krizhevsky et al. 2012, "ImageNet Classification with Deep Convolutional Neural Networks"]

Deep Neural Networks



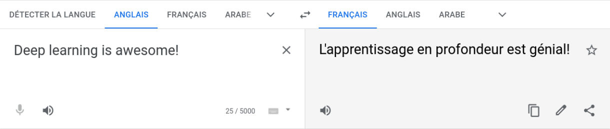


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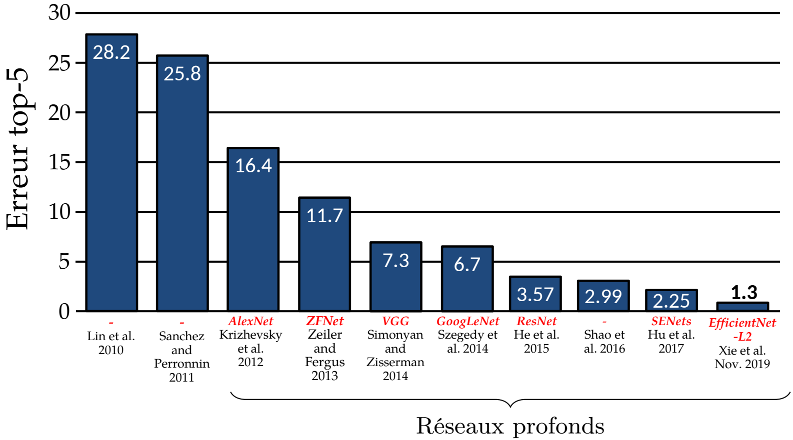
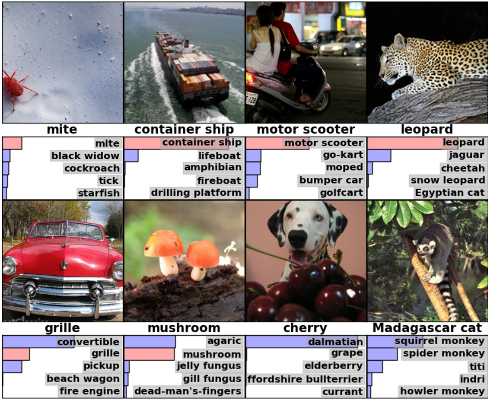
Image classification challenge on the 1,000 image classes of ImageNet



http://www2.ift.ulaval.ca/~pgiguere/cours/DeepLearning/2020/01-Introduction-2020.pdf

- Image classification



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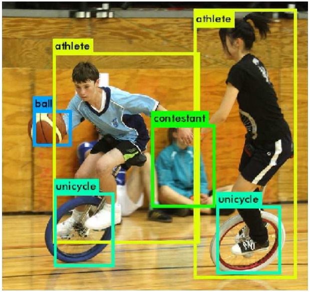
Computer Vision

- Image classification
- Object detection



[Redmon and Farhadi 2017, "YOLO9000: better, faster, stronger"]

Computer Vision

- Image classification
- Object detection
- Image segmentation

[Cordts et al. 2016, "The cityscapes dataset for semantic urban scene understanding"]

Computer Vision

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- Object detection
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- Image matching (e.g. facial recognition)

[Jesorsky et al. 2001, "Robust face detection using the hausdorff distance"]

Computer Vision

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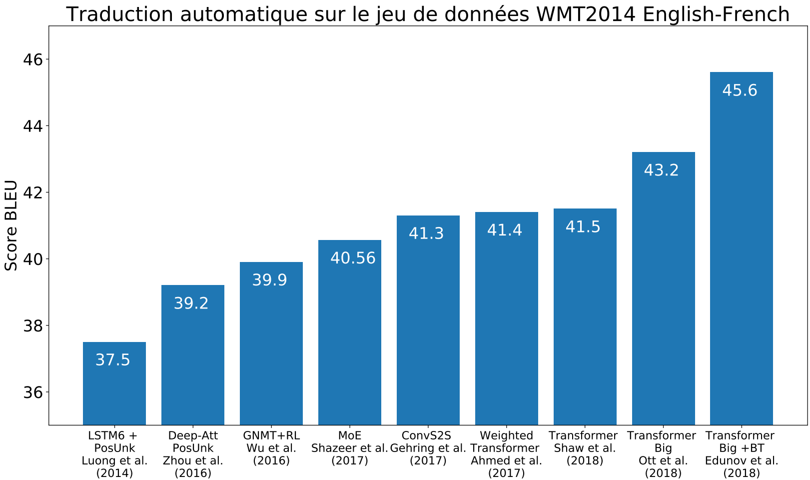
- etc.

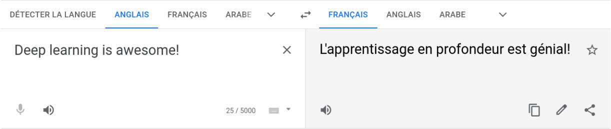
Natural Language Processing (NLP)



https://paperswithcode.com/sota/machine-translation-on-wmt2014-english-french

Natural Language Processing

- Translation



Natural Language Processing

- Translation
- Text classification (e.g. topic, sentiment)


John Doe

Awesome product

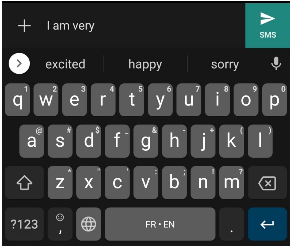
Reviewed in Canada on May 04, 2021

This is an awesome product. It couldn't be better!!!

16 people found this helpful

Natural Language Processing

- Translation
- Text classification (e.g. topic, sentiment)
- Autocorrect



Natural Language Processing

- Translation
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- etc.

Dataset $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$

Machine Learning

Dataset $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$

$$\downarrow$$

Learning algorithm

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Dataset $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$

$\downarrow$

Learning algorithm

$\downarrow$

Model

$$\begin{tikzcd}
 & \text{Dataset } \{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\} \arrow[d] \\
 & \text{Learning algorithm} \arrow[d] \\
\text{Input } x \arrow[r] & \text{Model}
\end{tikzcd}$$

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The Basis of Neural Networks: The Neuron

Let $x = (x_1, x_2, \ldots, x_n)$ be n features (or variables in statistics).

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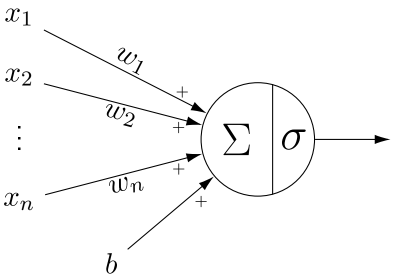
A neuron computes the following.



$$\sigma\left(\sum_{i=1}^{n} w_i x_i + b\right)$$

where w are called the weights of the neuron and $\sigma(\cdot)$ is a non-linear activation function.

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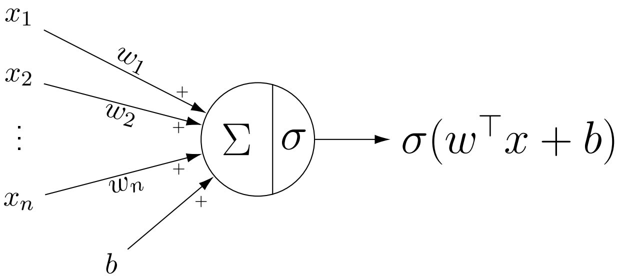

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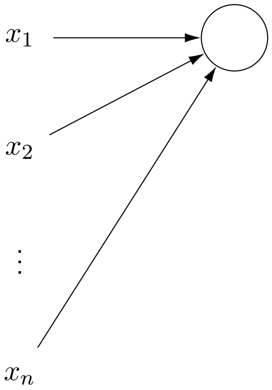
Neural Network



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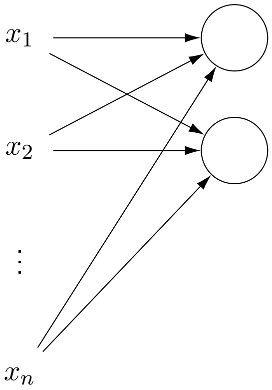

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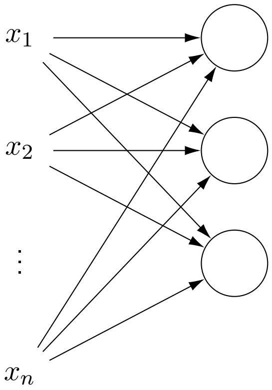

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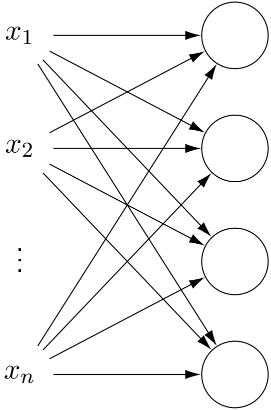

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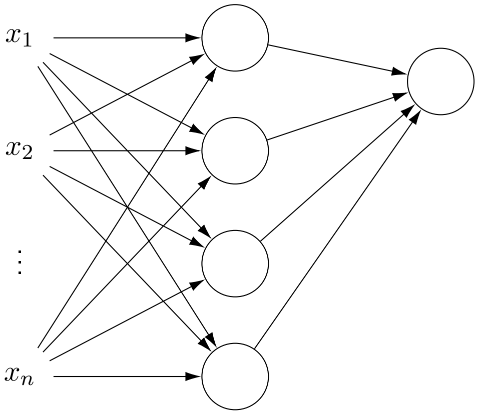

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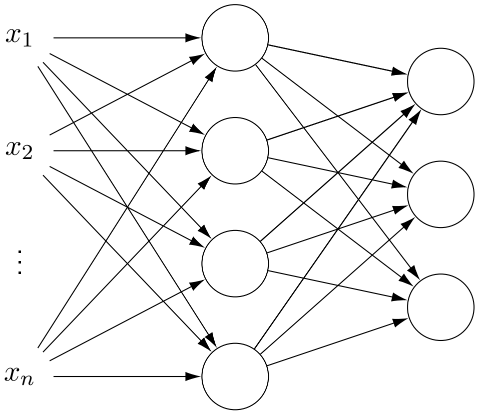

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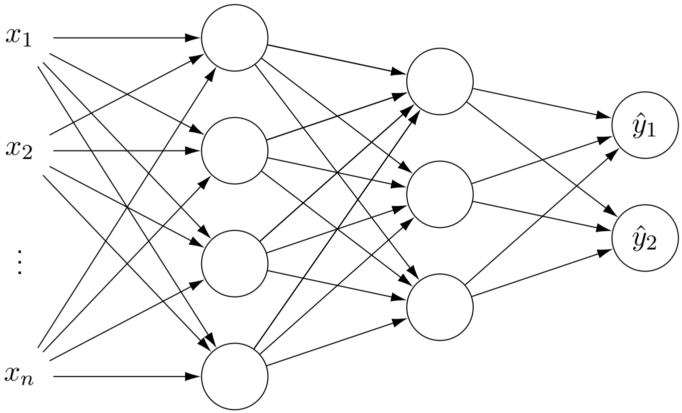

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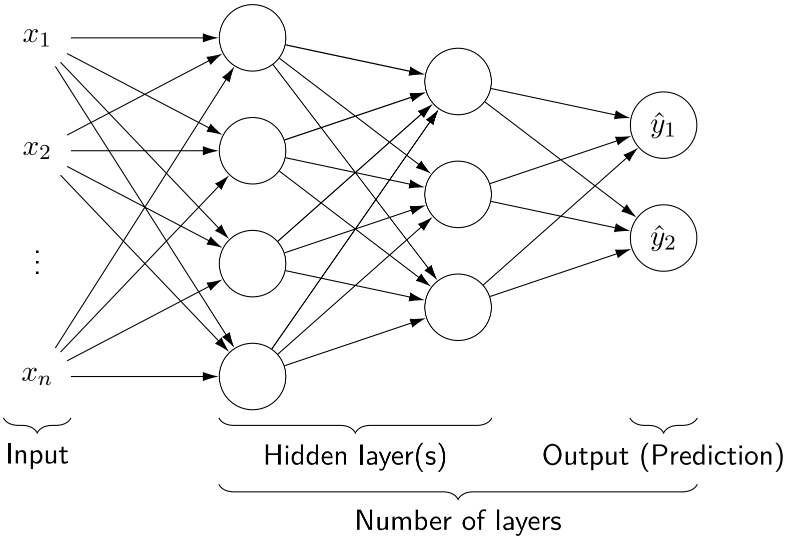

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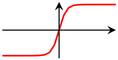
- $\tanh(z)$



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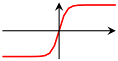
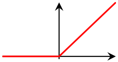


- The ReLU function:

$$\mathsf{ReLU}(z) = \max(0, z)$$



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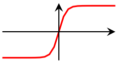

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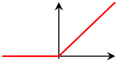
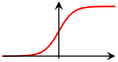


- The logistic function (also called sigmoid function):

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$



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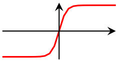

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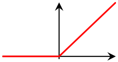

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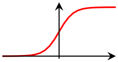

- The softmax function:

$$\text{softmax}(z)_i = \frac{\exp(z_i)}{\sum_{j=1}^{m} \exp(z_j)}$$

for $z = Wx + b \in \mathbb{R}^m$.

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- Cross-entropy loss for classification with C classes ($\mathcal{Y} = \{1, \ldots, C\}$)
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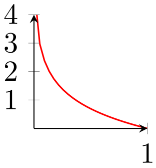

Regularization

To avoid overfitting (i.e. learning the training examples by heart), we may add a penalty term to the loss[1]:

$$\mathcal{L}(f(x), y) = \mathcal{L}_{\mathsf{MLE}}(f(x), y) + \lambda\Omega(f)$$

[1] MLE stands for Maximum Likehood Estimation.

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Goal: minimize the loss function for all examples (pairs (x, y))!

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$$J(\theta) = \mathop{\mathbb{E}}_{(x,y)\in\mathcal{D}} \mathcal{L}(f(x;\theta), y)$$

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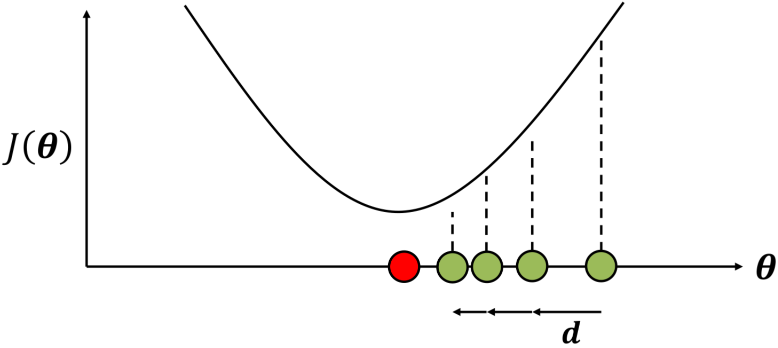
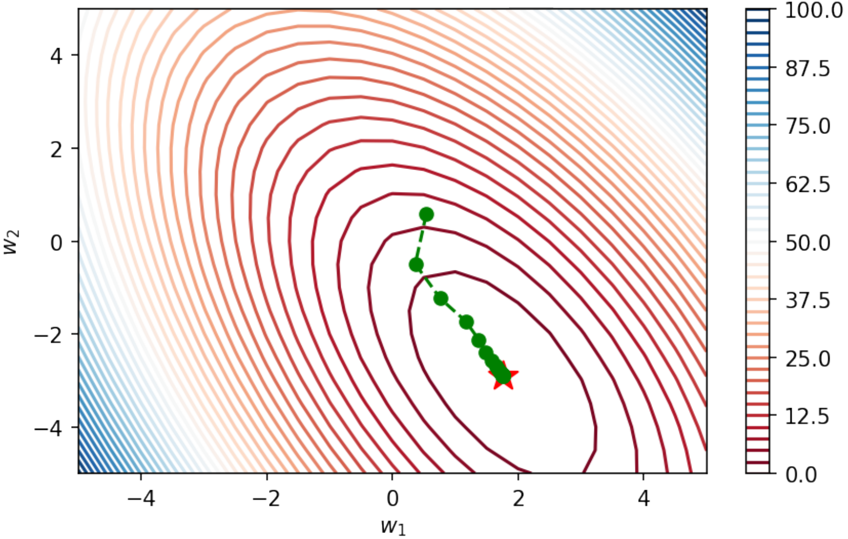
$$
\begin{aligned}
J(\theta) &= \mathop{\mathbb{E}}_{(x,y)\in\mathcal{D}} \mathcal{L}(f(x;\theta), y) \\
&\approx \frac{1}{N}\sum_{i=1}^{N} \mathcal{L}(f(x_i;\theta), y_i)
\end{aligned}
$$

Gradient Descent



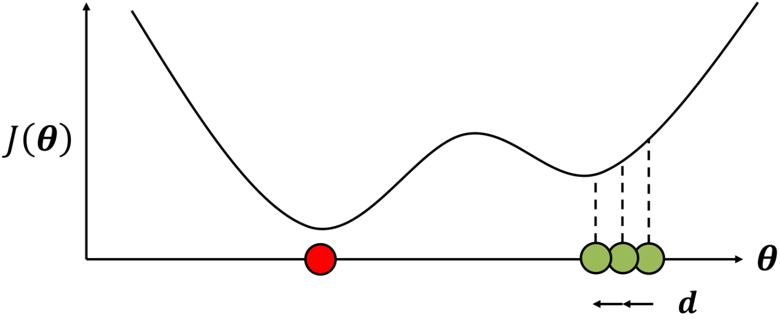
From Université Laval GLO-7030 by Ludovic Trottier

Gradient Descent



From Université Laval GLO-7030 by Pascal Germain

Gradient Descent For Deep Neural Networks



From Université Laval GLO-7030 by Ludovic Trottier

Gradient Descent For Deep Neural Networks

Adapted by Philippe Giguère for Université Laval GLO-7030 from Standford CS231N

Réalité : trouver le fond de la vallée embrumée, à tâtons

Adapted by Philippe Giguère for Université Laval GLO-7030 from Standford CS231N

Computation of Gradient

Using **"backpropagation"**, we compute the **partial derivative** of each parameter **with respect to the loss**. The vector containing all partial derivatives is called the **gradient**.

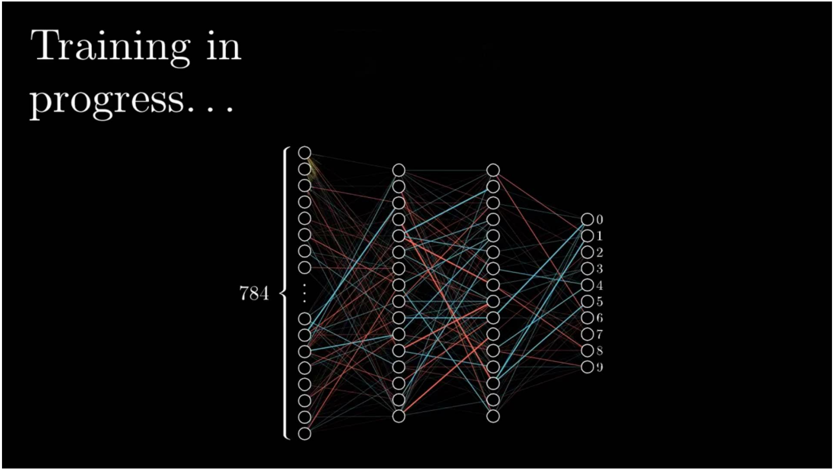
$$d = \nabla_\theta J(\theta) = \left[\frac{\partial J(\theta)}{\partial \theta_1}, \ldots, \frac{\partial J(\theta)}{\partial \theta_n}\right]$$

To update the parameters:

$$\theta \leftarrow \theta - \epsilon d$$

ϵ is called the learning rate.

Gradient



Extrait de `https://youtu.be/IHZwWFHWa-w` 3Blue1Brown

Optimization Algorithms

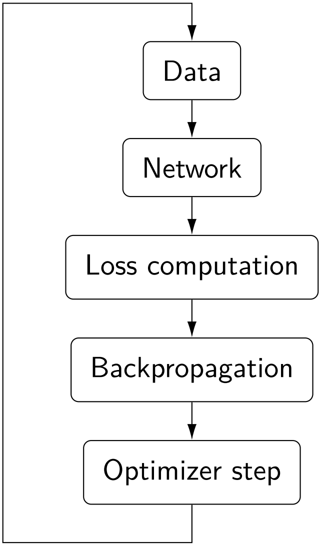
Simplest update rule:

$$\theta \leftarrow \theta - \epsilon d$$

Many types exist:

- **SGD**
- SGD with momentum
- SGD with momentum Nesterov
- Adagrad
- RMSprop
- **Adam**

Training Procedure



Training Procedure

procedure $\textsc{Train}(f(\cdot;\theta),\ S)$

  input: Neural network f parameterized by θ

  input: Dataset $S = \{(x_1, y_1), \ldots, (x_N, y_N)\}$

end procedure

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  for n epochs **do**

  end for

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for n epochs **do**

Let $S' = S$

while $S' \neq \emptyset$ **do**

Draw a batch $B \subseteq S'$ of b examples without replacement in S'.

end while

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    Let $S' = S$
    while $S' \neq \emptyset$ **do**
      Draw a batch $B \subseteq S'$ of b examples without replacement in S'.
      $\ell = \frac{1}{b} \sum_{(x,y) \in B} \mathcal{L}(f(x;\theta), y)$
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$d = \nabla_\theta \ell$

Update θ with d using chosen optimizer (e.g. $\theta \leftarrow \theta - \epsilon d$).

end while

end for

end procedure

Deep Learning Libraries

TensorFlow

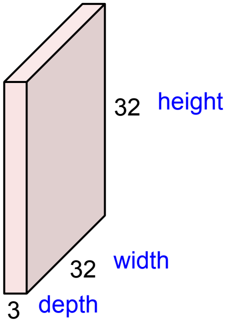
PyTorch

TensorFlow

PyTorch

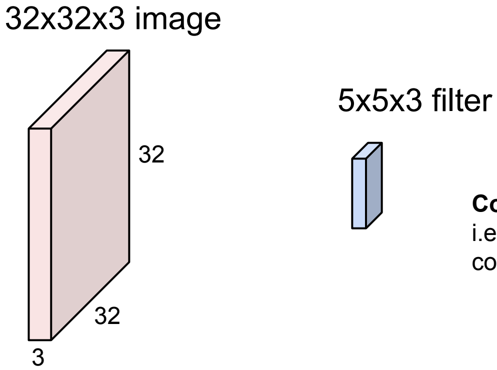
PyTorch Demo

Convolution Layer



Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer



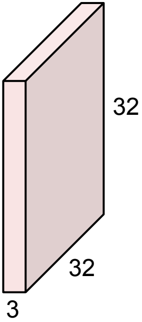
Convolve the filter with the image i.e. "slide over the image spatially, computing dot products"

Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

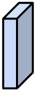
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image



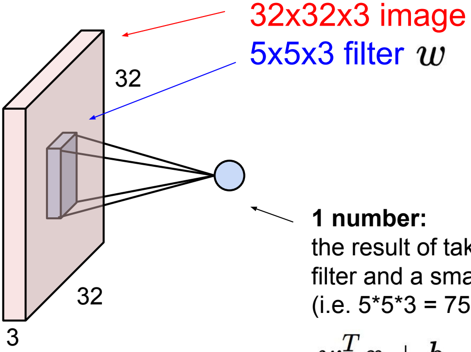
5x5x3 filter



Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer

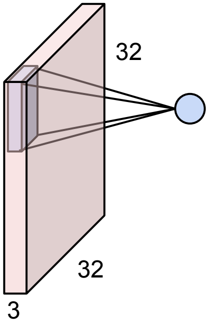


1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

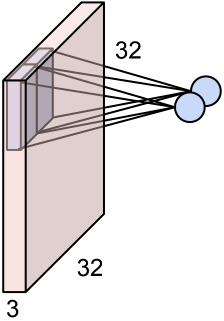
Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer



Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer



Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

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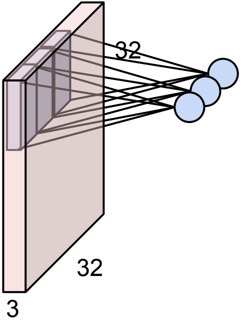
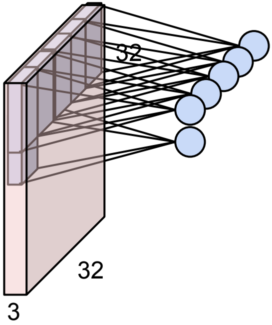


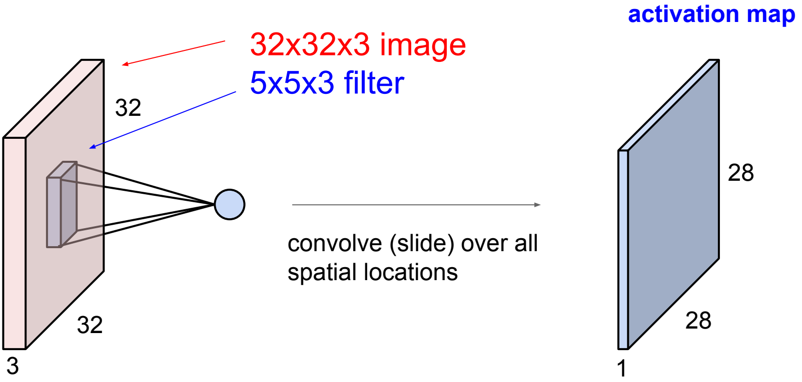
Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer

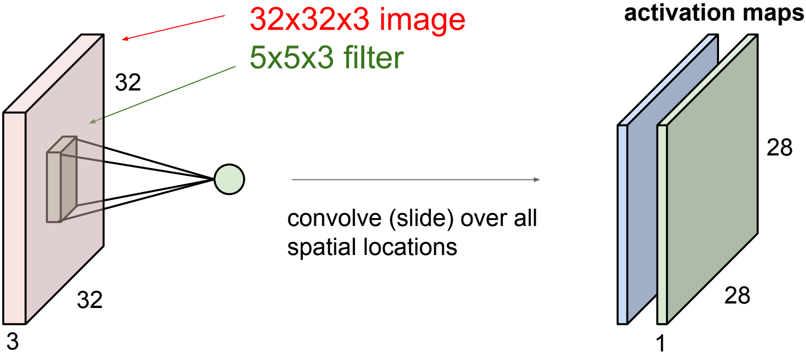


Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Convolution Layer



Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.



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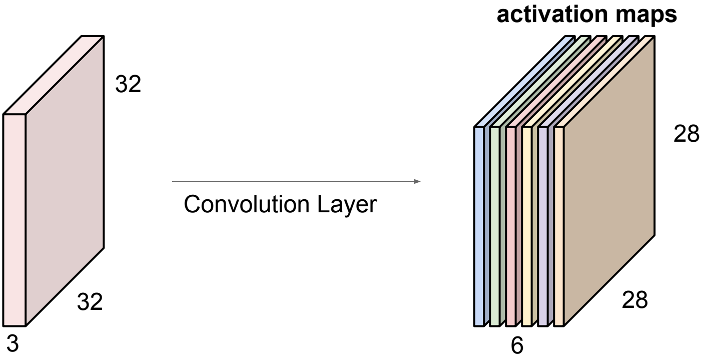

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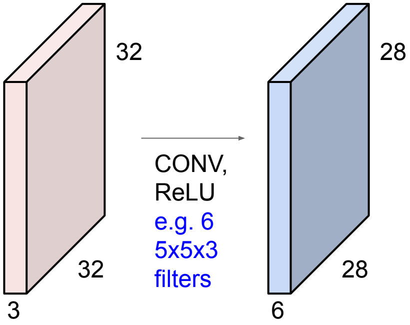
Convolution Layer



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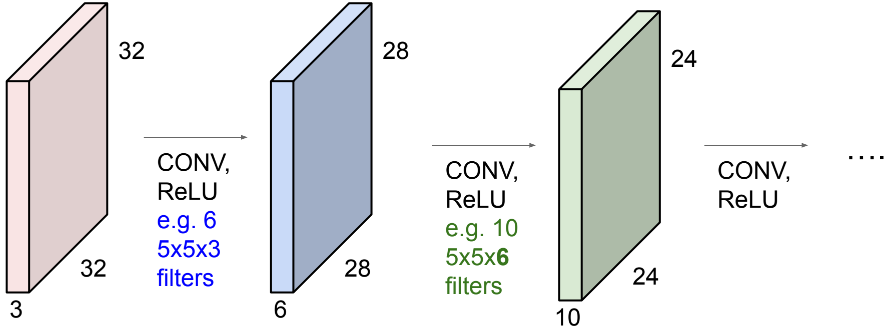
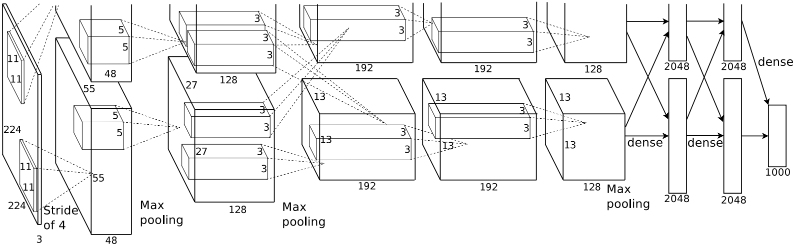


Slides of Fei-Fei Li, Ranjay Krishna, Danfei Xu; Standford CS231n.

Other Types of Layers

Many types of layers exist. Here is a few.

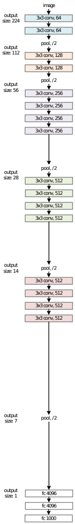
- Max pooling/averge pooling
- Batch normalization
- Dropout



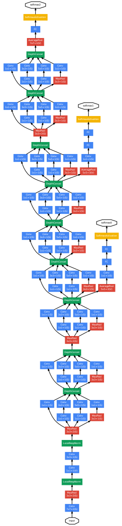
AlexNet

[Krizhevsky et al. 2012, "ImageNet Classification with Deep Convolutional Neural Networks"]

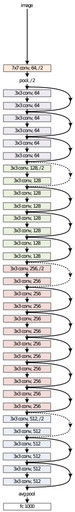
Deep Neural Network Architectures



VGG



GoogLeNet (or Inception)



ResNet

[Simonyan and Zisserman 2014, "Very deep convolutional networks for large-scale image recognition"]
[Szegedy et al. 2015, "Going deeper with convolutions"]
[He et al. 2016, "Deep residual learning for image recognition"]

Demo

Liens de référence

- Cours GLO-4030/7030 Apprentissage par réseaux de neurones profonds:
  - Slides: https://ulaval-damas.github.io/glo4030/
  - Laboratoire: https://github.com/ulaval-damas/glo4030-labs
- Vidéos du cours CS231n: https://www.youtube.com/watch?v=vT1JzLTH4G4&list=PLC1qU-LWwrF64f4QKQT-Vg5Wr4qEE1Zxk
- Tutoriels et documentation de PyTorch
  - https://pytorch.org/tutorials/ (pas tout le temps les meilleures pratiques)
  - https://pytorch.org/docs/stable/index.html
- Documentation de Poutyne: https://poutyne.org/

The End.

Questions?

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He, Kaiming et al. (2016). "Deep residual learning for image recognition". In: *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 770–778.

Jesorsky, Oliver, Klaus J Kirchberg, and Robert W Frischholz (2001). "Robust face detection using the hausdorff distance". In: *International conference on audio-and video-based biometric person authentication*. Springer, pp. 90–95.

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Redmon, Joseph and Ali Farhadi (2017). "YOLO9000: better, faster, stronger". In: *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 7263–7271.

Simonyan, Karen and Andrew Zisserman (2014). "Very deep convolutional networks for large-scale image recognition". In: *arXiv preprint arXiv:1409.1556*.

Szegedy, Christian et al. (2015). "Going deeper with convolutions". In: *Proceedings of the IEEE conference on computer vision and pattern recognition*, pp. 1–9.

Activation Functions

Given a non-activated output $z = w^\top x + b$, then $\sigma(z) = \dots$

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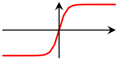
Given a non-activated output $z = w^\top x + b$, then $\sigma(z) = \ldots$

- $\tanh(z)$



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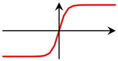
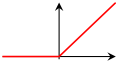


- The ReLU function:

$$\mathsf{ReLU}(z) = \max(0, z)$$



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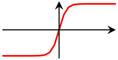

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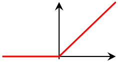
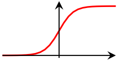


- The logistic function (also called sigmoid function):

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$



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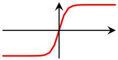

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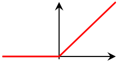

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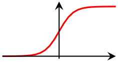

- The softmax function:

$$\mathsf{softmax}\,(z)_i = \frac{\exp(z_i)}{\sum_{j=1}^{m} \exp(z_j)}$$

for $z = Wx + b \in \mathbb{R}^m$.

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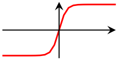

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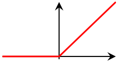

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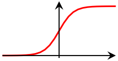

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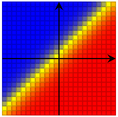

$$\boxed{\text{softmax}\,(z)_1 = \frac{\exp(x)}{\exp(x)+\exp(y)} \text{ for } z = [x, y]}$$

Loss Function

Measure of error between a prediction $\hat{y} = f(x)$ and a ground truth y.

Goal: minimize the loss function!

- Squared error (SE) for regression:

$$\mathcal{L}_{\mathsf{SE}}(\hat{y}, y) = (\hat{y} - y)^2$$

for $\hat{y}, y \in \mathbb{R}$.

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  - $\hat{y} = [\hat{y}_1, \ldots, \hat{y}_C] = \mathsf{softmax}\left([z_1, \ldots, z_C]\right)$

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  Based on the cross entropy:

$$H(q, p) = -\sum_{c \in \mathcal{Y}} p(c) \log q(c)$$

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where $p(c) = \mathbb{1}(y = c)$ and $q(c) = \hat{y}_c$.

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where $p(c) = \mathbb{1}(y = c)$ and $q(c) = \hat{y}_c$. This simplifies to:

$$\mathcal{L}_{\mathsf{CE}}(\hat{y}, y) = -\sum_{c=1}^{C} \mathbb{1}(y = c) \log \hat{y}_c = -\log \hat{y}_y$$