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MEK4300

Mandatory assignment 2

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Flat plate integral analysis

We have the following velocity profile:

$$u = U \sin\left(\frac{\pi y}{2\delta}\right) \quad (1)$$

To check for how realistic this velocity profile is, we have to check which boundary conditions that will be fulfilled with this profile.

$$u(0) = U \sin(0) = 0 \quad (2)$$

$$u(\delta) = U \sin\left(\frac{\pi}{2}\right) = U \quad (3)$$

$$\left.\frac{du}{dy}\right|_{y=\delta} = \frac{\pi}{2\delta} U \cos\left(\frac{\pi}{2}\right) = 0 \quad (4)$$

$$\left.\frac{d^2u}{dy^2}\right|_{y=0} = -\frac{\pi^2}{4\delta^2} U \sin(0) = 0 \quad (5)$$

We can conclude that this sinus shaped velocity profile is a more realistic profile than the quadratic one ((4-11) in the book), since this velocity profile also satisfies the zero pressure-gradient condition (5), whereas the parabolic velocity profile only satisfies (2) - (4).

For the momentum thickness we have:

$$\begin{aligned} \theta &= \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 \frac{u}{U} \left(1 - \frac{u}{U}\right) d\eta \quad , \quad \text{where: } \eta = \frac{y}{\delta} \\ \theta &= \delta \int_0^1 \sin\left(\frac{\pi}{2}\eta\right) \left(1 - \sin\left(\frac{\pi}{2}\eta\right)\right) d\eta \\ &= \delta \int_0^1 \left(\sin\left(\frac{\pi}{2}\eta\right) - \sin^2\left(\frac{\pi}{2}\eta\right)\right) d\eta \\ &= \delta \left[-\frac{2}{\pi} \cos\left(\frac{\pi}{2}\eta\right) - \frac{\eta}{2} + \frac{1}{2\pi} \sin(\pi\eta) \right]_0^1 \\ &= \delta \left(-\frac{1}{2} + \frac{2}{\pi}\right) = \frac{4-\pi}{2\pi} \delta = 0.1366 \delta \end{aligned} \quad (6)$$

Displacement thickness:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \sin\left(\frac{\pi y}{2\delta}\right)\right) dy \quad (7)$$

$$\begin{aligned} y &= \eta\delta \quad , \quad dy = \delta d\eta \\ y &= 0 \Rightarrow \eta = 0 \\ y &= \delta \Rightarrow \eta = 1 \end{aligned}$$

$$\Rightarrow \delta^* = \delta \int_0^1 \left(1 - \sin\left(\frac{\pi}{2}\eta\right)\right) d\eta = \left[\eta + \frac{2}{\pi} \cos\left(\frac{\pi}{2}\right)\right]_0^1 = \delta \left(1 - \frac{2}{\pi}\right) = 0.3634 \delta \quad (8)$$

For the friction coefficient we have:

$$\begin{aligned} C_f &= \frac{t\tau_w}{\rho U^2} = 2 \frac{d\theta}{dx}, \quad \text{where: } \tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu \frac{\pi}{2\delta} U \cos(0) = \mu \frac{\pi U}{2\delta} \\ \Rightarrow C_f &= \frac{\mu\pi}{\rho U \delta} = 2 \frac{d}{dx} (0.1366\delta) = 2 \cdot 0.1366 \frac{d\delta}{dx} = \frac{\pi\mu}{\rho U \delta} \end{aligned} \quad (9)$$

We can then use (9) to find the boundary layer thickness δ :

$$\int \delta d\delta = \int \frac{1}{2 \cdot 0.1366} \frac{\mu\pi}{\rho U} dx \Rightarrow \delta = \sqrt{\frac{\pi\mu x}{0.1366\rho U}} = 4.796\sqrt{\frac{\mu x}{\rho U}} \quad (10)$$

$$\Rightarrow \frac{\delta}{x} = \sqrt{\frac{\pi}{0.1366}} \sqrt{\frac{\mu}{\rho U x}} = \frac{4.796}{\sqrt{Re_x}} \quad (11)$$

$$\text{where: } Re_x = \frac{\rho U x}{\mu}$$

We can then substitute the expression for the boundary layer thickness (10) into (9), which gives us:

$$C_f = \frac{\pi\mu}{\rho U \delta} = \frac{\pi\mu}{\rho U} \frac{1}{\sqrt{\frac{\pi\mu x}{0.1366\rho U}}} = \sqrt{\frac{0.1366\pi^2\mu^2\rho U}{\pi\mu\rho^2 U^2 x}} = \sqrt{\frac{0.1366\pi}{Re_x}} \quad (12)$$

We can use the expression for friction coefficient (9) to find the drag coefficient.

$$\begin{aligned} C_D(L) &= \frac{1}{L} \int_0^L C_f(x) dx = \sqrt{\frac{0.1366\pi\mu}{L^2\rho U}} \int_0^L \frac{1}{\sqrt{x}} dx \\ \Rightarrow C_D(L) &= 0.655 \sqrt{\frac{\mu}{\rho U L^2}} \left[2\sqrt{x} \right]_0^L = 1.3101 \sqrt{\frac{\mu}{\rho U L}} = \frac{1.3101}{\sqrt{Re_L}} \end{aligned} \quad (13)$$

$$\text{where: } Re_L = \frac{\rho U L}{\mu}$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 L} \Rightarrow D = 0.655 \sqrt{\mu\rho U^3 L} \quad (14)$$

We have the following estimates for the boundary layer parameters:

$$\begin{aligned} \frac{\theta}{x} \sqrt{Re_x} &= 0.1366 \frac{\delta}{x} \sqrt{Re_x} = \sqrt{\frac{0.1366\pi\mu}{\rho U x}} \sqrt{Re_x} = 0.6550 \\ \frac{\delta^*}{x} \sqrt{Re_x} &= 0.3634 \sqrt{\frac{\pi\mu}{0.1366\rho U x}} \sqrt{Re_x} = 1.7427 \\ \frac{\delta}{x} \sqrt{Re_x} &= 4.796 \\ C_f \sqrt{Re_x} &= \sqrt{0.1366\pi} = 0.6550 \\ C_D \sqrt{Re_L} &= 1.3101 \end{aligned}$$

Comparison to Blasius:

Parameter	u from eq.(1)	Exact from Blasius	Error, %
$\frac{\theta}{x} \sqrt{Re_x}$	0.655	0.664	-1.36
$\frac{\delta^*}{x} \sqrt{Re_x}$	1.743	1.721	+1.3
$\frac{\delta}{x} \sqrt{Re_x}$	4.796	5.0	-4
$C_f \sqrt{Re_x}$	0.655	0.664	-1.36
$C_D \sqrt{Re_L}$	1.310	1.328	-1.35

Figure:(1) shows the drag coefficient of a flat plate as a function of the Reynold's number.

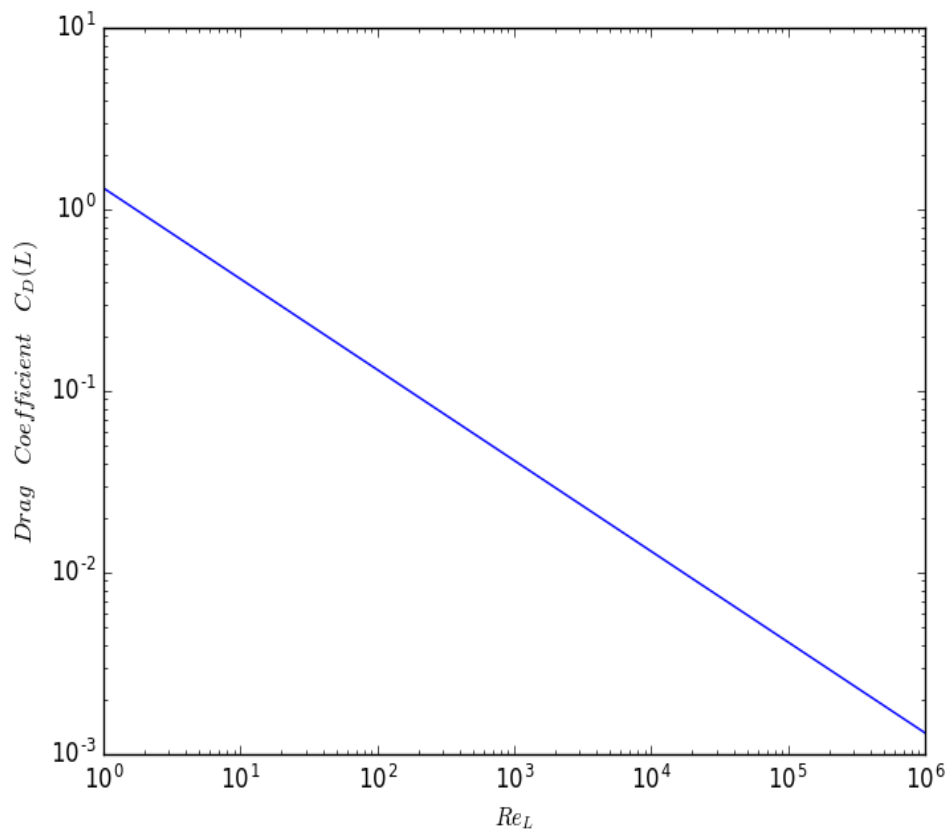


Figure 1: Drag of a plate