Asked connected undirected graph of its adjacency matrix. Multiple edges in the graph are allowed. Required to count the number of different spanning trees of this graph.

The following formula belongs Kirchhoff (Kirchhoff), who proved it in 1847

Kirchhoff matrix theorem

Take the adjacency matrix of G, replace each element of this matrix is the opposite, and on a diagonal instead of the element $A_{I, I}$ put the degree of node i (if there are multiple edges, the vertex degree they are considered with their multiplicity). Then, according to Theorem Kirchhoff matrix, all the cofactors of this matrix are equal, and equal to the number of spanning trees of this graph. For example, you can delete the last row and last column of the matrix, and the modulus of its determinant is equal to the desired quantity.

Determinant of a matrix can be found in O (N^3) by using the Gauss method or the method of Kraut .

The proof of this theorem is rather complicated and is not presented here (see, for example, coming VB "dimer problem and the Kirchhoff theorem").

Communication with the laws of Kirchhoff circuit

Kirchhoff theorem between matrix and Kirchhoff laws for circuit has a surprising connection.

It can be shown (as a consequence of Ohm's law and Kirchhoff's first law), that the resistance R_{ij} between points i and j is equal to the electrical circuit:

$R_{ij} = | T^{(I, J)} | / | T^{J} |$

where the matrix T is obtained from the matrix A *reverse* resistance of the conductor (A_{ij} - inverse number of the resistance of the conductor between points i and j) transformation described in Theorem Kirchhoff matrix, and the notation $T^{(I)}$ denotes the deletion of the table with the number i, and $T^{(I, J)}$ - deletion of two rows and columns i and j.

Kirchhoff theorem gives this formula geometric meaning.