Gray code

Definition

Gray code numbering system is defined to be a non-negative integers, where codes of two adjacent numbers differ in exactly one bit.

For example, for numbers of length 3 bits have a sequence of Gray codes: 000, 001, 011, 010, 110, 111, 101, 100.Eg $G(4) = 6$.

This code was invented by Frank Gray (Frank Gray) in 1953.

Finding the Gray code

Consider the number of bits $n$and the bits of the number $G(n)$. Note that $i$the first bit $G(n)$equal to one only if $i$the first bit $n$equal to one, and $i + 1$the first bit is zero, or vice versa ($i$the first bit is zero, and $i + 1$equal to the first one). Thus, we have $G(n) = n \oplus (n >> 1)$:

```
int g (int n) {
      return n ^ (n >> 1);
}
```

Finding the inverse Gray code

Required by the Gray code $g$to restore the original number n.

We will go from older to younger bits (let LSB is 1, and the oldest - k). We obtain the following relations between the bits $n_i$of the number $n$bits and $g_i$number g:

$n_k = g_k$,

$n_{k-1} = g_{k-1} \oplus n_k = g_k \oplus g_{k-1}$,

$n_{k-2} = g_{k-2} \oplus n_{k-1} = g_k \oplus g_{k-1} \oplus g_{k-2}$,

$n_{k-3} = g_{k-3} \oplus n_{k-2} = g_k \oplus g_{k-1} \oplus g_{k-2} \oplus g_{k-3}$,

...

In the form of program code is written as the easiest:

```
int rev_g (int g) {
      int n = 0;
      for (; g; g>>=1)
```

```
        n ^= g;
    return n;
}
```

Applications

Gray codes have several applications in different areas, sometimes quite unexpected:

- nBit Gray code corresponds to a Hamiltonian cycle on n-cube.
- In the art, Gray codes are used to **minimize errors** when converting the analog signals into digital signals (e.g. in sensors). In particular, the Gray codes were discovered in connection with this application.
- Gray codes are used in solving the problem of **the Tower of Hanoi** .
- Let n- number of disks. Let's start with the Gray code length n, consisting of zeros (ie $G(0)$), and will move on Gray codes (from $G(i)$pass to $G(i+1)$).With every $i$bit of the current CB Gray code ith disk (and most significant bit corresponds to the smallest disk size and the oldest bat - the highest). Since at each step exactly one bit is changed, then we can understand the bit change $i$as moving i-disc. Note that for all drives except the smallest, at each step there is exactly one option course (except for the starting and final products). For the smallest disk always has two variations, however, there is the strategy of choice course, always leads to the answer: if $n$odd, the sequence of movements of the smallest drive has the form $f \to t \to r \to f \to t \to r \to \ldots$(where f- starting rod t- the final rod r- the remainder of the rod), and if $n$even, then $f \to r \to t \to f \to r \to t \to \ldots$.
- Gray codes are also finding application in the theory **of genetic algorithms** .

Problem in online judges

List of tasks that can be taken using the Gray code:

- SGU # 249 **"Matrix"** [Difficulty: Medium]