Finding factorial power divider

Given two numbers n and k. Required to calculate what degree divider k among $n!$ ie find the largest x such that $n!$ is divided into k^x.

Solution for the case of simple k

Consider first the case when k simple.

Let us write the expression for the factorial explicitly:

$$n! = 1\ 2\ 3\ \ldots\ (n-1)\ n$$

Note that each k member of the first of this work is divided into k, ie gives one to the answer, the number of members of such power $\lfloor n/k \rfloor$.

Furthermore, we note that every k^2 nth term of this series is divided into k^2, ie gives another one to answer (given that k in the first degree has already been considered before), the number of members equal $\lfloor n/k^2 \rfloor$.

And so on, each k^i member of the first series gives one to answer, and the number of members equal $\lfloor n/k^i \rfloor$.

Thus, the magnitude of response is:

$$\frac{n}{k} + \frac{n}{k^2} + \ldots + \frac{n}{k^i} + \ldots$$

This amount, of course, is not infinite, because only about the first $\log_k n$ members are nonzero. Consequently, the asymptotic behavior of the algorithm is $O(\log_k n)$.

Implementation:

```
int fact_pow (int n, int k) {
      int res = 0;
      while (n) {
            n /= k;
            res += n;
      }
      return res;
}
```

Solution for the case of composite k

The same idea is applied directly anymore.

But we can factorize k, solve the problem for each of its prime divisors, and then select a minimum number of responses.

More formally, let k_i- this is $i$the first factor of the number $k$belongs to him in power p_i. To solve the problem $k_i$with the above formula for $O(\log n)$, even though we got an answer Ans_i. Then answer for the composite $k$will be a minimum of values Ans_i / p_i.

Given that the factorization is performed in the simplest way $O(\sqrt{k})$, we obtain the asymptotic behavior of the final $O(\sqrt{k})$.