Hawking Radiation as Seen by Observers

Bachelor thesis

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1. QFT in curved spacetime
   - Scalar field
   - Unruh detector
   - Minkowski space
2. Static spacetime
   - Properties of the Wightman function
   - Observers
     - Static observer
     - General observer
     - Thermal

Massless scalar field in curved spacetime

- Spacetime metric: $g_{\mu\nu}$
- Lagrangian: $\mathcal{L} = -\frac{1}{2}\sqrt{|g|}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$
- Klein-Gordon equation: $\nabla_\mu\nabla^\mu\phi = \frac{1}{\sqrt{|g|}}\partial_\mu\left(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi\right) = 0$
- Scalar product: $(\phi|\psi) := i\int_\Sigma \mathrm{d}S^\mu\,\phi^*\nabla_\mu\psi - \psi\nabla_\mu\phi^*$
- Orthonormal basis: $(u_i|u_j) = \delta_{ij},\ (u_i|u_j^*) = 0,\ (u_i^*|u_j^*) = -\delta_{ij}$
- Quantisation: $\phi(\mathbf{x}) = \sum_i u_i a_i + u_i^* a_i^\dagger$

State of the QFT

- Vacuum: $a_i \left|0\right\rangle = 0$
- Problem: $u_i \to v_j, a_i \to b_j$: $b_i \left|0\right\rangle \neq 0$

  $\to$ Need to guess state!
- Static spacetime: choose vacuum w.r.t. positive frequency modes: $u_i \sim e^{-i\omega t}$
- What does an observer see?

Greens functions

- Vacuum:
  - Wightman function $D^+(\mathbf{x},\mathbf{x}') := \langle 0|\,\phi(\mathbf{x})\phi(\mathbf{x}')\,|0\rangle$
  - $iD(\mathbf{x},\mathbf{x}') := [\phi(\mathbf{x}),\phi(\mathbf{x}')] = 2i\,\mathrm{Im}\,D^+(\mathbf{x},\mathbf{x}')$
  - $D^{(1)}(\mathbf{x},\mathbf{x}') := \langle 0|\,\{\phi(\mathbf{x}),\phi(\mathbf{x}')\}\,|0\rangle = 2\,\mathrm{Re}\,D^+(\mathbf{x},\mathbf{x}')$
- Thermal:
  - replace $\langle 0|\ldots|0\rangle$ by $\frac{1}{Z}\mathrm{Tr}\,e^{-\beta H}\ldots$
  - D is c-number: $D_\beta = D$
  - $D^{(1)}_\beta(t,\vec{x};t',\vec{x}') = \sum_n D^{(1)}(t-i\beta n,\vec{x};t',\vec{x}')$

Unruh detector

- Detector model: $c \cdot m(\tau)\phi(\mathbf{x}(\tau)),\ c \ll 1$
- Transition amplitude: $Q_{|0,0\rangle \to |E,\psi\rangle}(\tau) \sim \int_{-\infty}^{\tau} e^{iE\tau'} \langle\psi|\,\phi(\mathbf{x}(\tau'))\,|0\rangle \,\mathrm{d}\tau'$
- Transition rate: $\frac{\mathrm{d}F_E}{\mathrm{d}\tau} = 2\mathrm{Re} \int_{-\infty}^{0} \mathrm{d}\tau' \, e^{-iE\tau'} D^{+}(\mathbf{x}(\tau+\tau'), \mathbf{x}(\tau))$
- For constant rate: $\frac{\mathrm{d}F_E}{\mathrm{d}\tau} = \int_{-\infty}^{\infty} \mathrm{d}\tau \, e^{-iE\tau} D^{+}(\mathbf{x}(\tau), \mathbf{x}(0))$
- Interpretation: F_E is particle population for observer

Minkowski space

- Wightman function: $D^+(\mathbf{x}, \mathbf{x}') = -\frac{1}{4\pi^2} \frac{1}{(t-t'-i\varepsilon)^2 - |\vec{x}-\vec{x}'|^2}$
- Static observer: $t(\tau) = \tau, \vec{x}(\tau) = \mathrm{const}$
  - $D^+(\mathbf{x}(\tau), \mathbf{x}(0)) = -\frac{1}{4\pi^2} \frac{1}{(\tau - i\varepsilon)^2}$
  - Fourier transform: $\frac{\mathrm{d}F_E}{\mathrm{d}\tau} = 0$

$\rightarrow$ Inertial observer: vacuum contains no particles

image

Unruh effect

- Accelerating observer: $t(\tau) = 1/\alpha \sinh \alpha\tau$, $x(\tau) = 1/\alpha \cosh \alpha\tau$
  - $D^+(\mathbf{x}(\tau), \mathbf{x}(\tau')) = -\frac{\alpha^2}{16\pi^2} \frac{1}{\sinh^2 \frac{\alpha(\tau-\tau')}{2}}$
- Thermal state: $D^+_\beta(\mathbf{x}, \mathbf{x}') = -\frac{1}{4\beta^2} \frac{1}{\sinh^2\left(\frac{\pi}{\beta}\sqrt{(t-t'-i\varepsilon)^2 - |\vec{x}-\vec{x}'|^2}\right)}$
  - Static observer: $D^+_\beta(\mathbf{x}(\tau), \mathbf{x}(\tau')) = -\frac{1}{4\beta^2} \frac{1}{\sinh^2\left(\frac{\pi}{\beta}(\tau-\tau')\right)}$
- Set $\beta = 2\pi/\alpha$
- Accelerating observer: vacuum is a thermal state

Static spactimes

- Metric: $\mathrm{d}s^2 = -\beta(\vec{x})\,\mathrm{d}t^2 + g_{ij}(\vec{x})\,\mathrm{d}x^i\,\mathrm{d}x^j$
- Positive frequency solutions: $u_i(t,\vec{x}) = \frac{1}{\sqrt{2\omega_i}} e^{-i\omega_i t} A_i(\vec{x})$
- State of QFT – vacuum: $a_i\,|0\rangle = 0$
- Normal coordinates ($a = \beta(0)$):

$$D^+(\mathbf{x},0) = -\frac{1}{4\pi^2}\frac{1}{a(t-i\varepsilon)^2 - |\vec{x}|^2} + \mathcal{O}\left(x^2\right)$$

Pole at $\mathbf{x} = \mathbf{x}'$

- Consider trajectory $\mathbf{x}(\tau)$, $\mathbf{x}(0) = 0$
- D^+ has second order pole at $\tau = 0$: $\frac{1}{a(t(\tau)-i\varepsilon)^2 - |\vec{x}(\tau)|^2}$
- ε shift pole to upper half:
  - $a(t(\tau_\varepsilon) - i\varepsilon)^2 - |\vec{x}(\tau_\varepsilon)|^2 = 0$
  - $\delta\tau = \frac{\mathrm{d}\tau_\varepsilon}{\mathrm{d}\varepsilon}\Big|_{\varepsilon=0} = ia\dot{t}(0) \pm \sqrt{-a^2\dot{t}(0)^2 + 1}$
    $\rightarrow \delta\tau$ has positive imaginary part
- Only poles in the lower half contribute

  $\rightarrow$ drop pole at $\tau = 0$

Other singularities of D^+

- $D^+(\mathbf{x},\mathbf{x}') = \langle 0|\, \phi(\mathbf{x})\phi(\mathbf{x}')\, |0\rangle$ satisfies $\nabla_\mu \nabla^\mu D^+(\mathbf{x},\mathbf{x}') = 0$
- Define $A = 1/D^+ \Rightarrow A\nabla_\mu\nabla^\mu A = 2\nabla_\mu A \nabla^\mu A$
- $D^+ = \infty \Rightarrow A = 0 \Rightarrow \nabla_\mu A \nabla^\mu A = 0$

  $\rightarrow D^+ = \infty$ is a lightlike hypersurface
- D^+ is singular on the lightcone
- Assume no more singularities

  image

  $\rightarrow$ no singularities on timelike trajectories
- $D = [\phi(\mathbf{x}), \phi(\mathbf{x}')]$ is only non zero on lightcone

  $\rightarrow D^+$ is real for all trajectories

Static observers

Lemma:

box

In a static spacetime a static observer does not observe any particles

- $t(\tau) = \frac{\tau}{\sqrt{a}}$
- $u_i \; = \; \frac{1}{\sqrt{2\omega_i}} e^{-i\omega_i t} A_i(\vec{x})$
- $D^+(\mathbf{x}(\tau), \mathbf{x}(0)) = \sum_i \frac{1}{2\omega_i} e^{-i\omega_i \tau/\sqrt{a}} A_i(\vec{x}_0) A_i^*(\vec{x}_0)$

$$\begin{aligned}\frac{\mathrm{d}F_E}{\mathrm{d}\tau} &= \sum_i \frac{1}{2\omega_i} A_i(\vec{x}_0) A_i^*(\vec{x}_0) \left(\int_{-\infty}^{\infty} \mathrm{d}\tau \, e^{-iE\tau} e^{-i\omega\tau/\sqrt{a}} \right) \\ &= \sum_i \frac{1}{2\omega_i} A_i(\vec{x}_0) A_i^*(\vec{x}_0) \delta\big(E + \omega/\sqrt{a}\big) = 0\end{aligned}$$

General observers

- Transition rate not constant:
  $\frac{\mathrm{d}F_E}{\mathrm{d}\tau} = 2\mathrm{Re}\int_{-\infty}^{0} \mathrm{d}\tau'\, e^{-iE\tau'} D^{+}(\mathbf{x}(\tau+\tau'), \mathbf{x}(\tau))$
  $\rightarrow$ can not use residuum theorem
- How to handle pole at $\tau' = 0$?
- Expansion: $D^{+}(\mathbf{x}(\tau'), 0) = -\frac{1}{4\pi^2\tau'^2} + W(\tau')$
- $\frac{1}{\tau'^2}$ term does not contribute
- $W(\tau')$ is non singular

Thermal case

- Thermal Wightman function:

$$D^+_\beta(t(\tau'),\vec{x}(\tau');0) = \sum_{n=-\infty}^{\infty} D^+(t(\tau') - i\beta n, \vec{x}(\tau');0)$$
$$= \sum_{n=-\infty}^{\infty} -\frac{1}{4\pi^2(\tau' - i\beta\sqrt{a}n)^2} + \sum_{n=-\infty}^{\infty} W(\tau'(t - i\beta n))$$
$$= -\frac{1}{4\beta^2 a}\frac{1}{\sinh^2\left(\frac{\pi}{\beta\sqrt{a}}\tau'\right)} + W_\beta(\tau')$$

- Static observers: $T = \frac{T_0}{\sqrt{a}}$ (Tolman effect)
- General: Corrections from $W_\beta(\tau')$
  - shift temperature?
  - non thermal?

Quellen

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