

Question 1.

$$\alpha = 0.05$$

t - statistic.

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

 $\bar{X}_1 = 5.1$ kg (sample mean Program B) $\bar{X}_2 = 4.3$ kg (sample mean Program A)

~~$\mu_1 - \mu_2 = \mu_1 - \mu_2 = 0$~~

 $s_1 =$ standard deviation Program B $= 1.3$ kg $s_2 =$ standard deviation Program A $= 1.1$ kg $n_1 = 28$ (size program B) $n_2 = 25$ (size program A)

$$t = \frac{(5.1 - 4.3) - 0}{\sqrt{0.060357 + 0.0484}}$$

$$t = \frac{0.8}{\sqrt{0.108757}}$$

$$t = \frac{0.8}{0.32978}$$

$$t = 2.426$$

Degrees of Freedom

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1}}$$

$$\frac{s_1^2}{n_1} = \frac{1.3^2}{28} = \frac{1.69}{28} = 0.060357$$

$$\frac{s_2^2}{n_2} = \frac{1.1^2}{25} = \frac{1.21}{25} = 0.0484$$

$$\begin{aligned} \left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2 &= (0.060357 + 0.0484)^2 \\ &= (0.108757)^2 \\ &= 0.011828 \end{aligned}$$

$$\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} = \frac{(0.060357)^2}{28 - 1} = \frac{0.003643}{27} = 0.0001349$$

$$\frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1} = \frac{(0.0484)^2}{25 - 1} = \frac{0.00234256}{24} = 0.0000976$$

$$0.0001349 + 0.0000976 = 0.0002325$$

$$df = \frac{0.011828}{0.0002325}$$

$$\approx 50.87$$

$$= 51$$

calculated t -value to critical value:

$$2.426 >$$

$$df = 51$$

$$\text{critical } t\text{-value} = 1.675$$

$$t = 2.426 > 1.675$$

To conclude mean weight loss for Program B is significantly greater than Program A.

Assignment 3

Machine A

$$s_1 = 2.15, n_1 = 21, n_1 - 1 = 20$$

Machine B

$$s_2 = 1.15, n_2 = 16, n_2 - 1 = 15$$

① $H_0: \sigma_1^2 = \sigma_2^2$ (no difference in variances)

$H_1: \sigma_1^2 \neq \sigma_2^2$ (variances are different)

② $F = \frac{s_1^2}{s_2^2} = \frac{2.1^2}{1.15^2} = \frac{4.41}{1.3225} \approx 1.96$

③ $df_1 = n_1 - 1 = 20$
 $df_2 = n_2 - 1 = 15$

④ Lower critical value = $F_{0.025}(20, 15)$
 Upper critical value = $F_{0.975}(20, 15)$

= $F_{0.025}(20, 15) \approx 0.40$
 $F_{0.975}(20, 15) \approx 2.54$

⑤ Reject H_0 if $F < 0.40$ or $F > 2.54$
 Test statistic $F = 1.96$ lies within the interval
 $(0.40, 2.54)$

No.

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Question 3

a) Calculate D , D^2 and their sums.

Patient	Before	After	$D(\text{Before} - \text{After})$	D^2
1	220	200	20	400
2	210	190	20	400
3	215	212	3	9
4	225	210	15	225
5	230	215	15	225
6	218	205	13	169
7	212	200	12	144
8	208	198	10	100
9	220	210	10	100
10	214	205	9	81

$$\sum D = 127 \quad \sum D^2 = 1853$$

b) Calculate \bar{D} and SD .

$$\bar{D} = \frac{\sum D}{n}$$

$$= \frac{127}{10} = 12.7$$

$$SD = \sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{1853 - \frac{127^2}{10}}{9}}$$

$$= \sqrt{\frac{1853 - 1612.9}{9}}$$

$$= \sqrt{\frac{240.1}{9}}$$

$$= \sqrt{26.68}$$

$$\approx 5.17$$

c) Hypothesis Test

1. Hypotheses:

$$H_0 : \mu_D = 0 \text{ (No effect)}$$

$$H_1 : \mu_D > 0 \text{ (cholesterol decreases)}$$

2. Test Statistics:

$$t = \frac{\bar{D} - 0}{s_D / \sqrt{n}}$$

$$= \frac{12.70}{5.17 / \sqrt{10}}$$

$$= \frac{12.70}{1.635}$$

$$= 7.77$$

3. Critical Value (Right-tailed test):

$$\text{Degree of freedom (df)} = n - 1 = 9.$$

$$t_{0.05, 9} = 1.833$$

4. Decision:

$$\text{Since } t_s = 7.77 > t_{\text{critical}} = 1.833, \text{ reject } H_0.$$

5. Conclusion:

There is significant evidence at the 0.05 level to conclude that the diet plan reduces cholesterol levels.

Assignment 3

Question 4.

- a. Calculate the difference between the observed (o) and expected count (e) as in the table below.

Species	Observed count	e_i	$(o - e_i)^2 / e$
Sparrows	42	40	$(42 - 40)^2 / 40 = 0.10$
Finches	33	30	$(33 - 30)^2 / 30 = 0.30$
Robins	15	20	$(15 - 20)^2 / 20 = 1.25$
Woodpeckers	10	10	$(10 - 10)^2 / 10 = 0$

- b. Calculate the χ^2 test.

$$\chi^2 = \frac{(o - e)^2}{e} = 0.10 + 0.30 + 1.25 + 0 = 1.65$$

- c. At the 0.05 significance level, test whether the observed distribution matches the expected proportions.

From part (b), the calculated chi-square test statistic is: $\chi^2 = 1.65$

Degree of freedom (df), $df = \text{number of categories} - 1 = 4 - 1 = 3$.

- The Critical value is.

$$\alpha = 0.05, df = 3$$

$$\chi^2 = 7.815$$

Since $1.65 < 7.815$ falls inside the acceptance region, we fail to reject the hypothesis.

Hence we assume that the observed data fits the proportions.

Question 5

A researcher wants to determine whether preferred mode of transportation is associated with a person's type of employment. A random sample of 120 individuals is surveyed, and the results are recorded in the following table:

	Public Transport	Private Car	Bicycle	Total
Office Workers	20	30	10	60
Freelancers	15	20	5	40
Students	10	5	5	20
Total	45	55	20	120

At the 0.05 significance level, test whether type of employment and preferred mode of transportation are independent.

Step 1 : State the hypotheses

- Null Hypothesis (H_0): Type of employment and preferred mode of transportation are independent.
- Alternative Hypothesis (H_1): Type of employment and preferred mode of transportation are associated.

Step 2: Calculate Expected Frequency.

$$E = \frac{(\text{Row Total}) \times (\text{Column Total})}{\text{Grand Total}}$$

	Public Transport	Private Car	Bicycle	Total
Office workers	$(60 \times 45) / 120 = 22.5$	$(60 \times 55) / 120 = 27.5$	$(60 \times 20) / 120 = 10$	60
Freelancers	$(40 \times 45) / 120 = 15$	$(40 \times 55) / 120 = 18.33$	$(40 \times 20) / 120 = 6.67$	40
Students	$(20 \times 45) / 120 = 7.5$	$(20 \times 55) / 120 = 9.17$	$(20 \times 20) / 120 = 3.33$	20
Total	45	55	20	120

Step 3 : Calculate Chi-Square Test Statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

- $(20 - 22.5)^2 / 22.5 = 0.278$
- $(30 - 27.5)^2 / 27.5 = 0.227$
- $(10 - 10)^2 / 10 = 0$
- $(15 - 15)^2 / 15 = 0$
- $(20 - 18.33)^2 / 18.33 = 0.153$
- $(5 - 6.67)^2 / 6.67 = 0.417$
- $(10 - 7.5)^2 / 7.5 = 0.833$
- $(5 - 9.17)^2 / 9.17 = 1.897$
- $(5 - 3.33)^2 / 3.33 = 0.833$

Total Chi Square

$$0.278 + 0.227 + 0 + 0 + 0.15 + 0.417 + 0.83 + 1.87 + 0.83 = 4.638$$

Step 4 : Determine the critical Value

- Degrees of freedom : 4
- Significance level (α) : 0.05
- Chi-Square critical Value for $df = 4$ and $\alpha = 0.05$: 9.488

$$4.638 < 9.488 \text{ fail to reject } H_0$$

= There is no significant association between type of employment and preferred mode of transportation at 0.05 significance level.