at will without fundamentally changing the system that we're talking about. Another way of saying this is that the particle has the option to move in space, but it doesn't have the option to move in time. It *has* to move in time. So we somehow need a way to promote time to a degree of freedom without it really being a true dynamical degree of freedom! How do we do this? The answer, as we will now show, is gauge symmetry.

Consider the action,

$$S = -m \int d\tau \sqrt{-\dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu}} \ , \tag{1.2}$$

where $\mu = 0, \ldots, D-1$ and $\dot{X}^\mu = dX^\mu/d\tau$. We've introduced a new parameter τ which labels the position along the worldline of the particle as shown by the dashed lines in the figure. This action has a simple interpretation: it is just the proper time $\int ds$ along the worldline.

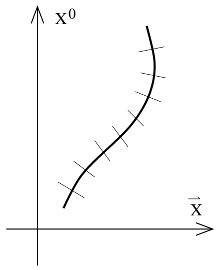

Figure 3:

Naively it looks as if we now have D physical degrees of freedom rather than $D-1$ because, as promised, the time direction $X^0 \equiv t$ is among our dynamical variables: $X^0 = X^0(\tau)$. However, this is an illusion. To see why, we need to note that the action (1.2) has a very important property: reparameterization invariance. This means that we can pick a different parameter $\tilde{\tau}$ on the worldline, related to τ by any monotonic function

$$\tilde{\tau} = \tilde{\tau}(\tau) \ .$$

Let's check that the action is invariant under transformations of this type. The integration measure in the action changes as $d\tau = d\tilde{\tau}\, |d\tau/d\tilde{\tau}|$. Meanwhile, the velocities change as $dX^\mu/d\tau = (dX^\mu/d\tilde{\tau})\,(d\tilde{\tau}/d\tau)$. Putting this together, we see that the action can just as well be written in the $\tilde{\tau}$ reparameterization,

$$S = -m \int d\tilde{\tau} \sqrt{-\frac{dX^\mu}{d\tilde{\tau}} \frac{dX^\nu}{d\tilde{\tau}} \eta_{\mu\nu}} \ .$$

The upshot of this is that not all D degrees of freedom X^μ are physical. For example, suppose you find a solution to this system, so that you know how X^0 changes with τ and how X^1 changes with τ, and so on. Not all of that information is meaningful because τ itself is not meaningful. In particular, we could use our reparameterization invariance to simply set

$$\tau = X^0(\tau) \equiv t \tag{1.3}$$