1.2 The Nambu-Goto Action

A particle sweeps out a worldline in Minkowski space. A string sweeps out a *worldsheet*. We'll parameterize this worldsheet by one timelike coordinate τ, and one spacelike coordinate σ. In this section we'll focus on closed strings and take σ to be periodic, with range

$$\sigma \in [0, 2\pi) \ . \tag{1.11}$$

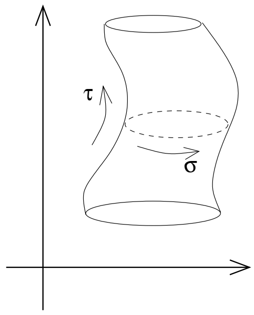

Figure 5:

We will sometimes package the two worldsheet coordinates together as $\sigma^\alpha = (\tau, \sigma)$, $\alpha = 0, 1$. Then the string sweeps out a surface in spacetime which defines a map from the worldsheet to Minkowski space, $X^\mu(\sigma, \tau)$ with $\mu = 0, \ldots, D-1$. For closed strings, we require

$$X^\mu(\sigma, \tau) = X^\mu(\sigma + 2\pi, \tau) \ .$$

In this context, spacetime is sometimes referred to as the *target space* to distinguish it from the worldsheet.

We need an action that describes the dynamics of this string. The key property that we will ask for is that nothing depends on the coordinates σ^α that we choose on the worldsheet. In other words, the string action should be reparameterization invariant. What kind of action does the trick? Well, for the point particle the action was proportional to the length of the worldline. The obvious generalization is that the action for the string should be proportional to the area, A, of the worldsheet. This is certainly a property that is characteristic of the worldsheet itself, rather than any choice of parameterization.

How do we find the area A in terms of the coordinates $X^\mu(\sigma, \tau)$? The worldsheet is a curved surface embedded in spacetime. The induced metric, $\gamma_{\alpha\beta}$, on this surface is the pull-back of the flat metric on Minkowski space,

$$\gamma_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} \eta_{\mu\nu} \ . \tag{1.12}$$

Then the action which is proportional to the area of the worldsheet is given by,

$$S = -T \int d^2\sigma \ \sqrt{-\det \gamma} \ . \tag{1.13}$$

Here T is a constant of proportionality. We will see shortly that it is the *tension* of the string, meaning the mass per unit length.