

# An Introduction to String Theory

Mate Zoltan Farkas

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► Metric:

- $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$

# The Relativistic Point Particle – The Action

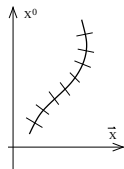
- ▶ Consider the action of a point particle (with fixed coordinates  $X_\mu = (t, \vec{x})$  in a given frame):

$$S = -m \int dt \sqrt{1 - \dot{\vec{x}}\dot{\vec{x}}}$$

→ not Lorentz-invariant, due to mixture of spacial and temporal coordinates under a Lorentz-transformation  $\Lambda$ .

- ▶ Consider instead for a generalized coordinate  $\tau$  along the line element:

$$S = -m \int d\tau \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} \eta_{\mu\nu}}$$



Ref: [1]

Remark:  $S$  is proportional to the integral over the worldline of the particle

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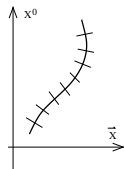
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# Dynamics of a Relativistic String – The Nambu-Goto Action

Action of the Relativistic String?

$$\text{particle} \quad \Leftrightarrow \quad \text{worldline} \quad \Leftrightarrow \quad S = -m \int d\tau \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} \eta_{\mu\nu}}$$

$$\text{closed string} \quad \Leftrightarrow \quad ? \quad \Leftrightarrow \quad ?$$

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closed string  $\Leftrightarrow$  worldsheet  $\Leftrightarrow$  Nambu-Goto/Dirac Action



# The Relativistic Point Particle – Action Symmetries

What are the symmetries of

$$S = -m \int d\tau \sqrt{-\frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} \eta_{\mu\nu}}$$

- ▶ Reparametrization invariance:

Let  $\tilde{\tau} = \tilde{\tau}(\tau)$ . Then:

$$S' = -m \int d\tilde{\tau} \sqrt{-\frac{dX^\mu}{d\tilde{\tau}} \frac{dX^\nu}{d\tilde{\tau}} \eta_{\mu\nu}} = S$$

→ gauge symmetry of the action → still D-1 dof!

- ▶ Poincaré invariance:

Let:

$$X'^\mu = \Lambda^\mu{}_\nu X^\nu + c^\mu$$

Then:

$$S' = S, \quad \text{as} \quad \Lambda^\mu{}_\rho \eta_{\mu\nu} \Lambda^\nu{}_\sigma = \eta_{\rho\sigma}$$

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# Never forget the Titles!

## ▶ first item

- subitem

- subsubitem

## ▶ second item

### 1. item 1

- 1.1 subitem 1

- 1.2 subitem 2

### 2. item 2

## ▶ third item

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# Covariant Quantization of the Solutions of the Nambu-Goto Action

$$S = \frac{1}{2\pi\alpha'} \int \sqrt{-\det g} \, \partial_\mu \quad (1)$$



# Quantization of $X^\mu$ in the Lightcone Gauge

- [1] David Tong. “Lectures on String Theory”. In: *arXiv:0908.0333 [hep-th]* (Feb. 2012). arXiv: 0908.0333. URL: <http://arxiv.org/abs/0908.0333> (visited on 07/13/2020).