FYS3150 - Project 1

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I. INTRODUCTION

Reference code in github repository. Problem to solve

$$-\frac{\mathrm{d}}{\mathrm{d}x}u(x) = f(x), \tag{1}$$

with

$$x \in (0,1) \quad \text{and} \quad u(0) = u(1) = 0. \tag{2}$$

II. THEORY

Approximation of second derivative of a general discretized function g_i:

$$g_i'' = \frac{g_{i+1} + g_{i-1} - 2g_i}{h^2} + \mathcal{O}(h^2). \tag{3}$$

approx by removing order.

Discretized approximation $u \to u_i$ and $x \to x_i = ih$, where the step length h is defined as

$$h = \frac{1}{n+1}, \tag{4}$$

where n are the total number of points on the interval $[x_0 = 0,\ x_{n+1} = 1]$. The boundary conditions are then discretized as $u_0 = u_{n+1} = 0$.

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Thus our equation, equation 1, can be rewritten as

$$-\frac{u_{i+1} + u_{i-1} - 2u_i}{h^2} = f_i.$$

By introducing $\tilde{b}_i = h^2 f_i$ and inserting for i we get

$$\begin{pmatrix} -u_0 + 2u_1 - u_2 \\ -u_1 + 2u_2 - u3 \\ \dots \\ -u_{n-1} + 2u_n - 2u_{n+1} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}.$$

Inserting we get

$$\begin{pmatrix} 2 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & \dots \\ 0 & -1 & 2 & -1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \tilde{b}_4 \\ \tilde{b}_5 \end{pmatrix}. \tag{5}$$

So we have

$$\mathbf{A}\mathbf{u} = \tilde{\mathbf{b}}, \tag{6}$$

with the matrix $\mathbf{A}$ given by equation 5.

III. METHOD

IV. RESULTS

V. DISCUSSION

VI. CONCLUSION