



# Introduction to Database Systems

## IDBS – Spring 2024



# Functional Dependencies

**Readings**  
PDBM 6.2

**Eleni Tzirita Zacharatou**

Based on slides by Andy Pavlo

# Course Schedule (Illustrated)

Week	Lecture	Date	Lecture Topic	Readings	Exercises / Homeworks	Notes
15	1	Aug 28	Introduction, Course overview, Relational model	POBM: 1, 1.1, 2.1-2.2	Exercise 13	Lecturer: Eloni Tzorit Zacharatos
16	2	Sep 04	SQL DDL/DML, Basic SQL queries: Joins, Aggregations	POBM: 7.3	Exercise 12	Lecturer: Omar Shahbaz Khan
17	3	Sep 11	Complex SQL queries, Subqueries, Views	POBM: 7.3 - 7.4	Homework 1 (Sep 2 - 18)	Lecturer: Omar Shahbaz Khan
18	4	Sep 18	Triggers, Transactions, Using SQL from Python	POBM: 9.2, 14.1, 14.2.1, 14.5 + more (details below)	DB only Exercise 14B	Lecturer: Omar Shahbaz Khan
19	Sep 25		No lecture - only exercise session for MSc students		MSc only Exercise 14B	
40	5	Oct 02	ER modelling	POBM: 3.0-3.3, 6.3-6.4	Exercise 15	Lecturer: Omar Shahbaz Khan
41	6	Oct 09	Normalisation	POBM: 6.2-6.4	Homework 2 (Oct 1 - 16)	Lecturer: Eloni Tzorit Zacharatos
42	Oct 16		Semester Break			
43	7	Oct 23	Storage hierarchy, Physical database design, Indexing	POBM: 12.1, 12.2, 12.3.1 - 12.3.7 + more (details below)	Exercise 17	Lecturer: Eloni Tzorit Zacharatos
44	8	Oct 30	B-trees, Performance tuning, Access methods, Join implementation	POBM: 12.3.8, 13.1	Exercise 18	Lecturer: Omar Shahbaz Khan
45	9	Nov 06	Storage Models, Architecture of a DBMS, Main memory DBMS	POBM: 2 + more (details below)	Homework 3 (Oct 18 - Nov 13)	Lecturer: Eloni Tzorit Zacharatos
46	10	Nov 13	Transaction Management, Transactions in main memory DBMS	POBM: 14 + more (details below)	Old exercises/exams/homeworks	Lecturer: Omar Shahbaz Khan
47	11	Nov 20	Scaling-out, NoSQL, Eventual consistency, CAP theorem	POBM: 11 (+ optional papers)	Old exercises/exams/homeworks	Lecturer: Omar Shahbaz Khan
48	12	Nov 27	Big data analytics, Distributed computing Frameworks	POBM: 19.1-19.2, 19.4, 20.1-20.3	Homework 4 (Nov 18 - Dec 4)	Lecturer: Eloni Tzorit Zacharatos + guest Online
49	Dec 04		Final exam			
50	Dec 15		Exam			

Weeks 2 & 3



User

declaratively



Query



Developer

programmatically



Week 4



DBMS Interface

Weeks 7--9



Database Administrator  
(DBA)

Maintain  
& tune



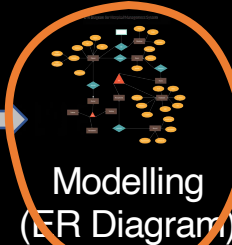
RDBMS

Week 6

Week 1



Raw Data



Modelling  
(ER Diagram)

Week 5

Advanced -- Weeks 10-12

# **DATABASE DESIGN**

How do we design a “good” database schema?

We want to ensure the integrity of the data.

We also want to get good performance.

# EXAMPLE DATABASE

**student(sid,cid,room,grade,name,address)**

sid	cid	room	grade	name	address
123	15-445	GHC 6115	A	Andy	Pittsburgh
456	15-721	GHC 8102	B	Tupac	Los Angeles
789	15-445	GHC 6115	A	Obama	Chicago
012	15-445	GHC 6115	C	Waka Flocka	Atlanta
789	15-721	GHC 8102	A	Obama	Chicago

# EXAMPLE DATABASE

**student(sid,cid,room,grade,name,address)**

sid	cid	room	grade	name	address
123	15-445	GHC 6115	A	Andy	Pittsburgh
456	15-721	GHC 8102	B	Tupac	Los Angeles
789	15-445	GHC 6115	A	Obama	Chicago
012	15-445	GHC 6115	C	Waka Flocka	Atlanta
789	15-721	GHC 8102	A	Obama	Chicago

# REDUNDANCY PROBLEMS

## Update Anomalies

- If the room number changes, we need to make sure that we change all students records.

## Insert Anomalies

- May not be possible to add a student unless they're enrolled in a course.

## Delete Anomalies

- If all the students enrolled in a course are deleted, then we lose the room number.

# EXAMPLE DATABASE

student(sid, name, address)

sid	name	address
123	Andy	Pittsburgh
456	Tupac	Los Angeles
789	Obama	Chicago
012	Waka Flocka	Atlanta

rooms(cid, room)

cid	room
15-415	GHC 6115
15-721	GHC 8102

courses(sid, cid, grade)

sid	cid	grade
123	15-415	A
456	15-721	B
789	15-415	A
012	15-415	C
789	15-721	A

*Why this decomposition is better and how to find it.*

# THIS VIDEO

Functional Dependencies: Constraints between attributes



# FUNCTIONAL DEPENDENCIES

A functional dependency (FD) is a form of a constraint.

Part of a relation's schema to define a valid instance.

Definition:  $X \rightarrow Y$

→ The value of  $X$  functionally defines the value of  $Y$ .

# FUNCTIONAL DEPENDENCIES

Formal Definition:

→  $X \rightarrow Y \Rightarrow (t_1[x]=t_2[x] \Rightarrow t_1[y]=t_2[y])$

If two tuples ( $t_1$   $t_2$ ) agree on the  $X$  attribute, then they must agree on the  $Y$  attribute too.

$R1(\underline{sid}, name, address)$

sid	name	address
123	Andy	Pittsburgh
456	Tupac	Los Angeles
789	Obama	Chicago
012	Waka Flocka	Atlanta

# FUNCTIONAL DEPENDENCIES

Formal Definition:

→  $X \rightarrow Y \Rightarrow (t_1[x]=t_2[x] \Rightarrow t_1[y]=t_2[y])$

If two tuples ( $t_1$ ,  $t_2$ ) agree on the  $X$  attribute, then they must agree on the  $Y$  attribute too.

$R1(\underline{sid}, name, address)$

sid	name	address
123	Andy	Pittsburgh
456	Tupac	Los Angeles
789	Obama	Chicago
012	Waka Flocka	Atlanta

$X$

$Y$

✓  $sid \rightarrow name$

# FUNCTIONAL DEPENDENCIES

FD is a constraint that allows instances for which the FD holds.

You can check if an FD is violated by an instance, but you cannot prove that an FD is part of the schema using an instance.

**R1(sid, name, address)**

sid	name	address
123	Andy	Pittsburgh
456	Tupac	Los Angeles
789	Obama	Chicago
012	Waka Flocka	Atlanta
555	Andy	Providence

??? name → address

# FUNCTIONAL DEPENDENCIES

Two FDs  $X \rightarrow Y$  and  $X \rightarrow Z$  can be written in shorthand as  $X \rightarrow YZ$ .

But  $XY \rightarrow Z$  is not the same as the two FDs  $X \rightarrow Z$  and  $Y \rightarrow Z$ .

# WHY SHOULD I CARE?

FDs seem important, but what can we actually do with them?

They allow us to decide whether a database design is correct.

→ Note that this is different than the question of whether it's a good idea for performance...

# IMPLIED DEPENDENCIES

**student(sid,cid, room, grade, name, address)**

sid	cid	room	grade	name	address
123	15-445	GHC 6115	A	Andy	Pittsburgh
456	15-721	GHC 8102	B	Tupac	Los Angeles
789	15-445	GHC 6115	A	Obama	Chicago
012	15-445	GHC 6115	A	Waka Flocka	Atlanta

Provided FDs

**sid → name, address**  
**sid, cid → grade**

Implied FDs

**sid, cid → grade**  
**sid, cid → sid**  
**sid, cid → cid**

# IMPLIED DEPENDENCIES

Given a set of FDs  $\{f_1, \dots, f_n\}$ , how  
do we decide whether FD  $g$  holds?

Compute the closure using  
Armstrong's Axioms  
→ This is the set of all implied FDs.



# ARMSTRONG'S AXIOMS

## Reflexivity:

$$\rightarrow X \supseteq Y \Rightarrow X \rightarrow Y$$

- This is a **trivial functional dependency**

## Augmentation:

$$\rightarrow X \rightarrow Y \Rightarrow XZ \rightarrow YZ$$

## Transitivity:

$$\rightarrow (X \rightarrow Y) \wedge (Y \rightarrow Z) \Rightarrow X \rightarrow Z$$

## Union:

$$\rightarrow (X \rightarrow Y) \wedge (X \rightarrow Z) \Rightarrow X \rightarrow YZ$$

## Decomposition:

$$\rightarrow X \rightarrow YZ \Rightarrow (X \rightarrow Y) \wedge (X \rightarrow Z)$$

## Pseudo-transitivity:

$$\rightarrow (X \rightarrow Y) \wedge (YW \rightarrow Z) \Rightarrow XW \rightarrow Z$$

# CLOSURES

Given a set **F** of FDs  $\{f_1, \dots, f_n\}$ , we define the closure **F+** is the set of all implied FDs.

**student**(sid, cid, room, grade, name, address)

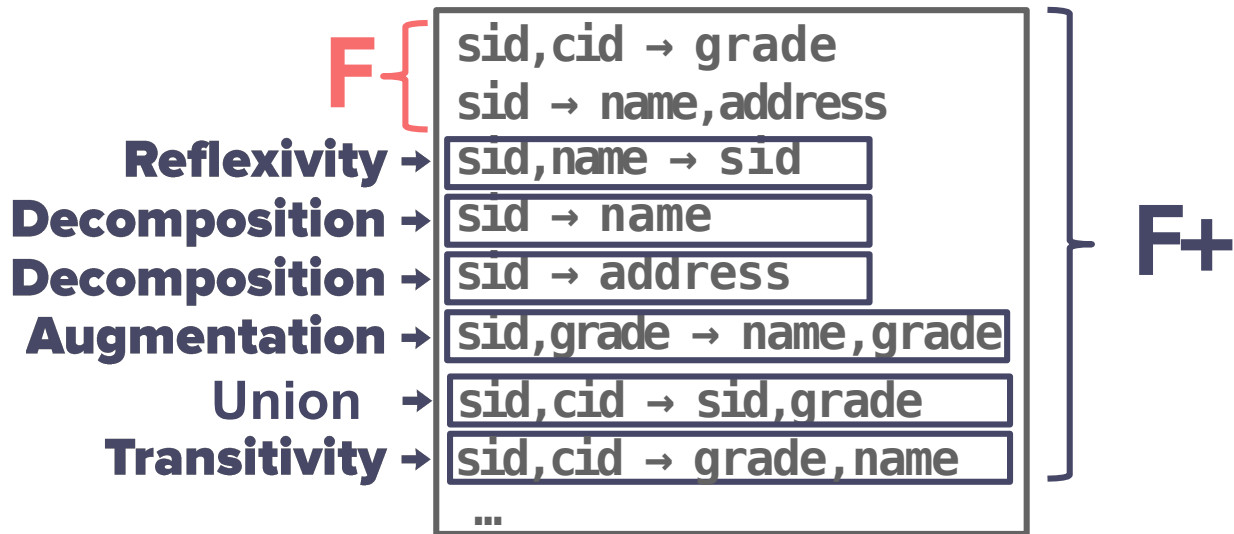
**F**{

sid, cid → grade  
sid → name, address

# CLOSURES

Given a set **F** of FDs  $\{f_1, \dots, f_n\}$ , we define the closure **F+** is the set of all implied FDs.

student(sid, cid, room, grade, name, address)



# UNAVOIDABLE AND REDUNDANT FDs

- FDs with a superkey on the left-hand side are **unavoidable**
  - If  $A$  is a candidate key for a relation, then clearly  $A \rightarrow B$  for any attribute  $B$
  - Similarly, if  $\{A_1, A_2\}$  forms a superkey, we have  $A_1 A_2 \rightarrow B$  for any  $B$
  - Etc
- FDs that can be computed from others are **redundant**
  - They do not require decomposition!
- Only decompose when not trivial, unavoidable, or redundant

# CONCLUSION

Functional dependencies are simple to understand.

They will allow us to reason about schema decompositions.

➤ **This week's lecture**