

Name: Key

Hour: 11 AM, 1 PM MATH 107-12, 18, Fall 2014 (D. Brice)

## Exam 1

**Directions:** (1) Answer each problem completely, show your work (answers without work will not receive credit), and where appropriate box your final answer. (2) No notes, no textbooks, no communication devices, no discussion. (3) You may use an ACT-approved calculator.

100 points. 50 minutes.

1. Express  $x^3(x^2 + 2x + 1)$  as a polynomial in standard form.

- (a)  $x^6 + 2x^3 + 1$   
(b)  $x^6 + 2x^2 + 1$   
(c)  $x^3 + x^2 + 2x + 1$   
(d)  $x^5 + 2x^4 + x^3$   
(e)  $(x^2 + 2x + 1)^3$   
(f) None of these.

$$\begin{array}{c} x^3(x^2 + 2x + 1) \\ x^5 + 2x^4 + x^3 \end{array}$$

2. Factor  $1 - x^4$  completely. *factorable*

- (a)  $(1 - x)^4$   
(b)  $(1 - x)^3(1 + x)$   
(c)  $(1 + x)^2(1 - x)^2$   
(d)  $(1 + x^2)(1 - x^2)$   
(e)  $(1 + x^2)(1 + x)(1 - x)$   
(f) None of these.

$$\begin{array}{c} 1 - x^4 \\ (1 - x^2)(1 + x^2) \\ (1 - x)(1 + x)(1 + x^2) \end{array}$$

*irreducible*

3. Which of the following is equivalent to  $\sqrt[3]{x^2}$ ?

- (a)  $x^{3/2}$  (b)  $x^{-3}$  (c)  $x^{2/3}$  (d)  $x^5$  (e)  $x^6$  (f) None of these.

4. Simplify  $\frac{1}{x-3} + \frac{2x}{x+2}$ .

- (a)  $\frac{2x^2 - 5x + 2}{(x-3)(x+2)}$  (b)  $\frac{2x+1}{(x-3)(x+2)}$  (c)  $\frac{2x+1}{2x-1}$  (d) -1 (e) -2 (f) None of these.

$$\frac{1}{x-3} + \frac{2x}{x+2} = \frac{x+2}{(x-3)(x+2)} + \frac{2x(x-3)}{(x-3)(x+2)}$$

$$= \frac{x+2 + 2x(x-3)}{(x-3)(x+2)}$$

$$= \frac{x+2 + 2x^2 - 6x}{(x-3)(x+2)}$$

$$= \frac{2x^2 - 5x + 2}{(x-3)(x+2)}$$

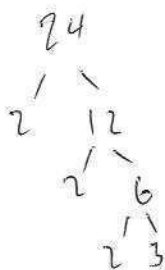
5. Simplify  $\frac{\frac{1}{x} + 6}{\frac{2}{x} - 3}$ .

- (a) -2 (b)  $\frac{-12}{x}$  (c)  $\frac{3}{x^2}$  (d)  $\frac{1+6x}{1-3x}$  (e)  $\frac{x-3}{x+6}$  (f) None of these.

$$\frac{\frac{1}{x} + 6}{\frac{2}{x} - 3} = \frac{\frac{1}{x} + \frac{6}{1}}{\frac{2}{x} - \frac{3}{1}} = \frac{\frac{1+6x}{x}}{\frac{2-3x}{x}} = \frac{1+6x}{x} \cdot \frac{x}{2-3x} = \frac{1+6x}{2-3x}$$

6. Simplify  $\frac{\sqrt{24}}{\sqrt{3}}$ .

- (a)  $\sqrt{21}$  (b)  $2\sqrt{2}$  (c) 8 (d)  $\sqrt[3]{24}$  (e) It can be simplified, but none of these. (f) It is already fully simplified.



$$\frac{\sqrt{24}}{\sqrt{3}} = \frac{\sqrt{2^3 \cdot 3}}{\sqrt{3}} = \frac{2\sqrt{2}\sqrt{3}}{\sqrt{3}} = 2\sqrt{2}$$

7. Simplify  $\sqrt{12} \cdot \sqrt{6}$ .

- (a)  $2\sqrt{6}$  (b)  $6\sqrt{2}$  (c) 36 (d)  $\sqrt{18}$  (e) It can be simplified, but none of these. (f) It is already fully simplified.



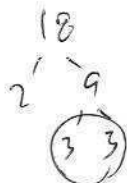
$$\sqrt{12} \cdot \sqrt{6} = \sqrt{2^2 \cdot 3} \cdot \sqrt{2 \cdot 3} = \sqrt{2^3 \cdot 3^2} = 2 \cdot 3 \cdot \sqrt{2} = 6\sqrt{2}$$



8. Simplify  $\sqrt{8} + \sqrt{18}$ .

- (a)  $\sqrt{26}$  (b)  $7\sqrt{2}$  (c) 12 (d) 13 (e) It can be simplified, but none of these. (f) It is already fully simplified.

$$\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$



9. Express  $(x+1)(2x-3)$  as a polynomial in standard form.

$$2x^2 - 3x + 2x - 3$$

$$\boxed{2x^2 - x - 3}$$

10. Express  $(x^4 + 2x^2 - 4) - (x^3 + x - 1)$  as a polynomial in standard form.

$$x^4 + 2x^2 - 4 - x^3 - x + 1$$

$$~~x^4 + 2x^2~~$$

$$\boxed{x^4 - x^3 + 2x^2 - x - 3}$$

11. Factor  $3y^3 + 6y^2$  completely.

$$3y^3 + 6y^2$$

$$\boxed{3y^2(y+2)}$$

12. Factor  $x^2 - x - 2$  completely.

$$(x-2)(x+1)$$

13. Factor  $2x^2 - 5x + 2$  completely.

$$\begin{array}{r} -2 \quad +4 \\ \hline -4 \\ +2x \\ +x \\ \hline -5 \end{array}$$

$$(x-2)(2x-1)$$

factors		sum
-1	-4	-5

14. Factor  $3x^2 - 12x + 9$  completely.

$$3x^2 - 12x + 9$$

$$= 3(x^2 - 4x + 3)$$

$$= 3(x-1)(x-3)$$

15. Multiply and simplify as much as possible  $\frac{(x-1)(x+4)}{x} \cdot \frac{3x^2}{(x-1)^2}$

$$\frac{3x^2(x+4)}{x(x-1)}$$

$$\frac{3x(x+4)}{x-1}$$

16. Add or subtract as indicated and simplify as much as possible  $\frac{3}{x+2} - \frac{x}{x+2}$

$$\frac{3-x}{x+2}$$

17. Add or subtract as indicated and simplify as much as possible  $\frac{x-1}{x+1} + \frac{3}{x}$

$$\frac{x(x-1)}{x(x+1)} + \frac{3(x+1)}{x(x+1)} = \frac{x(x-1) + 3(x+1)}{x(x+1)}$$

$$= \frac{x^2 - x + 3x + 3}{x(x+1)} = \frac{x^2 + 2x + 3}{x(x+1)}$$

18. Evaluate  $\sqrt[3]{-27}$ .

$$\boxed{-3}$$

19. Rationalize the denominator and simplify  $\frac{3}{2-\sqrt{7}}$ .

$$\frac{3}{2-\sqrt{7}} \cdot \frac{2+\sqrt{7}}{2+\sqrt{7}} = \frac{3(2+\sqrt{7})}{4-7} = \frac{\cancel{3}(2+\sqrt{7})}{-3} = -(2+\sqrt{7})$$
$$\boxed{= -2-\sqrt{7}}$$

20. Rationalize the numerator and simplify  $\frac{\sqrt{25-x^2}-4}{x-3}$ .

$$\frac{\sqrt{25-x^2}-4}{x-3} \cdot \frac{\sqrt{25-x^2}+4}{\sqrt{25-x^2}+4} = \frac{25-x^2-16}{(x-3)(\sqrt{25-x^2}+4)}$$
$$= \frac{9-x^2}{(x-3)(\sqrt{25-x^2}+4)}$$
$$= \frac{-(x-3)(x+3)}{(x-3)(\sqrt{25-x^2}+4)} = \boxed{\frac{-x-3}{\sqrt{25-x^2}+4}}$$