

Name:

Key

Hour:

11 am

Exam 3

**Directions:** (1) Answer each problem completely, show your algebra (you need not show your arithmetic), and box your final answer. (2) No notes, no textbooks, no communication devices, no discussion. (3) You may use an ACT-approved calculator.

100 points. 50 minutes.

Math 107-12/18, Fall 2014 (Dr. Daniel Brice)

1. [5 points] A line parallel to  $y = 3x - 4$  has slope \_\_\_\_\_?

(a) 3 (b) -3 (c)  $\frac{1}{3}$  (d)  $-\frac{1}{3}$  (e) None of these.

2. [5 points] A line perpendicular to  $y = 3x - 4$  has slope \_\_\_\_\_?

(a) 3 (b) -3 (c)  $\frac{1}{3}$  (d)  $-\frac{1}{3}$  (e) None of these.

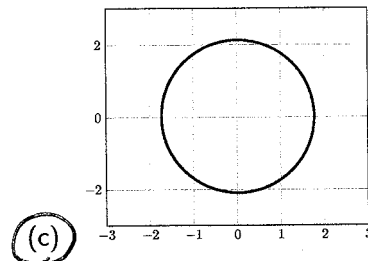
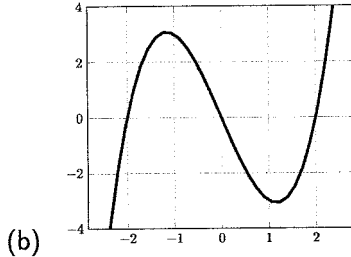
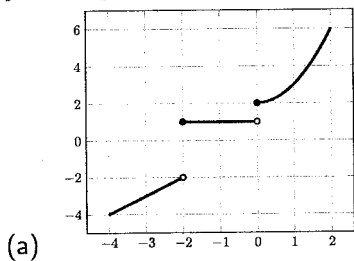
3. [5 points]  $f(x) = \sqrt{x-3}$ . Express the domain of  $f$  using interval notation.

(a)  $[3, \infty)$  (b)  $(3, \infty)$  (c)  $(-\infty, 3)$  (d)  $(-\infty, 3) \cup (3, \infty)$   
(e)  $(-\infty, 3]$  (f) None of these.

4. [5 points]  $f(x) = \frac{1}{x+2}$ . Express the domain of  $f$  using interval notation.

(a)  $[-2, \infty)$  (b)  $(-2, \infty)$  (c)  $(-\infty, -2)$  (d)  $(-\infty, -2) \cup (-2, \infty)$   
(e)  $(-\infty, -2]$  (f) None of these.

5. [5 points] Which of the following is not the graph of a function?



(d) a and b (e) a and c (f) b and c

6. [5 points] Find the slope of the line containing  $(-2, 5)$  and  $(3, 2)$ .

- (a)  $3/5$     ☒ (b)  $-3/5$     (c)  $5/3$     (d)  $-5/3$     (e) Undefined    (f) None of these.

7. [5 points] Write the equation of the line with slope 3 containing  $(2, -1)$ .

- (a) Undefined    (b)  $y = -3x + 7$     (c)  $y = 3x + 7$     (d)  $y = -3x - 7$     ☒ (e)  $y = 3x - 7$   
(f) None of these.

8. [5 points] Write the equation of the circle with center  $(1, -2)$  and radius 2.

- ☒ (a)  $(x - 1)^2 + (y + 2)^2 = 4$     (b)  $(x - 1)^2 + (y + 2)^2 = 2$     (c)  $(x + 1)^2 + (y - 2)^2 = 4$   
(d)  $(x + 1)^2 + (y - 2)^2 = 2$     (e)  $y = mx + p$     (f) None of these.

9. [5 points] Given  $f(x) = 3 + 2x - x^2$ , find  $f(2)$ .

- (a) 1    (b) 2    ☒ (c) 3    (d) 4    (e) 5    (f) None of these.

10. [5 points] Given  $f(x) = 3 + 2x - x^2$ , find  $f(x + h)$ .

- ☒ (a)  $3 + 2x + 2h - x^2 - 2xh - h^2$     (b)  $3 + 2x + 2h - x^2 - h^2$     (c)  $3 + 2x + 2h - x^2 + h^2$   
(d)  $3 + 2x - x^2 + h$     (e)  $3 + 2x - x^2 + 2h$     (f) None of these.

11. [10 points] For  $g(x) = x^2 - 3x$ , find the average rate of change as  $x$  goes from 1 to 3.

$$\begin{aligned} \frac{g(3) - g(1)}{3 - 1} &= \frac{[(3)^2 - 3(3)] - [(1)^2 - 3(1)]}{3 - 1} \\ &= \frac{0 - (-2)}{2} = \frac{2}{2} = \boxed{1} \end{aligned}$$

12. [10 points] Given  $f(x) = x^3 - 4x$ ,

(a) Compute  $f(-x)$ .

$$\begin{aligned} f(-x) &= (-x)^3 - 4(-x) \\ &= -x^3 + 4x \end{aligned}$$

(b) Is  $f$  even, odd, or neither?

odd

13. Given  $f(x) = \frac{1}{x}$ ,  $g(x) = \sqrt{x}$ , and  $k(x) = x - 2$ , compute and fully simplify:

(a) [5 points]  $(f + k)(x)$

$$\frac{1}{x} + x - 2 = \frac{1}{x} + \frac{x^2 - 2x}{x} = \boxed{\frac{x^2 - 2x + 1}{x}}$$

(b) [5 points]  $(g \circ k)(x)$

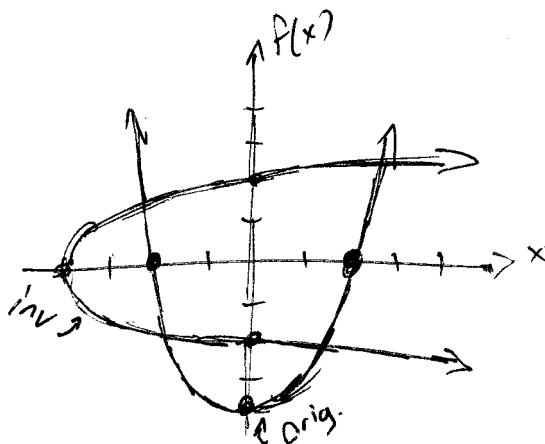
$$\sqrt{k(x)} = \boxed{\sqrt{x-2}}$$

(c) [5 points]  $\frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} &= \frac{\frac{x - x - h}{x(x+h)}}{h} = \frac{-h}{h \cdot x(x+h)} = \boxed{\frac{-1}{x(x+h)}} \end{aligned}$$

14. [10 points] Sketch the graph of  $f(x) = x^2 - 4$ , and sketch the inverse graph on the same coordinate plane. Is the inverse graph a function? If so, find the formula for  $f^{-1}(x)$ .

orig.	
x	f(x)
-2	0
-1	-3
0	-4
1	-3
2	0



inv	
x	y
0	-4
-3	-1
-4	0
-3	1
0	2

The inverse graph is not a function

15.  $f(x) = x^2 - 1$ .

- (a) [5 points] Write out and completely simplify the difference quotient  $\frac{f(x) - f(a)}{x - a}$  for  $a = 2$ .

$$\begin{aligned} \frac{x^2 - 1 - (a^2 - 1)}{x - a} &= \frac{x^2 - 1 - (2^2 - 1)}{x - 2} = \frac{x^2 - 1 - 3}{x - 2} \\ &= \frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{x-2} = \boxed{x+2} \end{aligned}$$

- (b) Use the difference quotient calculated above to find the average rate of change for each interval:

- i. [2 points] as  $x$  goes from 2 to 3.

$$x+2, \quad 3+2 = \boxed{5}$$

- ii. [2 points] as  $x$  goes from 2 to 2.1.

$$x+2, \quad 2.1+2 = \boxed{4.1}$$

- iii. [2 points] as  $x$  goes from 2 to 2.01.

$$x+2, \quad 2.01+2 = \boxed{4.01}$$