MATH 300 Winter 2016
Sets and Logic Dr. Brice

Exam 1

30 points. 110 minutes.

1. [2 points] What is an assertion?

2. [2 points] What is a deduction?

3. [2 points] What does it mean when you say that a particular deduction is valid?

4. [2 points] How can you show that a particular deduction is invalid?

5. [2 points] Is it possible for there to be a valid deducation that has a false conclusion? If not, say why. If so, provide an example of one.

6. For each categorical syllogism below, either state that the syllogism is valid (no proof necessary) or demonstrate that it is invalid by drawing a Venn diagram in which the hypotheses are true but the conlusion is false.

   (a) [3 points] Snakes are lizards, and all lizards are carnivores, so snakes must be carnivores, I guess.

   (b) [3 points] People who live life to the fullest are contented people, but some people who are contented are full of lethargy. It stands to reason that some people who live life to the fullest are full of lethargy.

   (c) [3 points] Not every poet is a scholar, but every scholar is a gentle person. From this, we can conclude that not every poet is a gentle person.

7. [3 points] Give a two-column proof of the following theorem: $A \Rightarrow (B \vee C), \neg B, \therefore A \Rightarrow C$.

8. [3 points] Prove algebraically that $(A \vee B) \Rightarrow C$ is logically equivalent to $(A \Rightarrow C) \& (B \Rightarrow C)$. (You do not have to give justifications.)

9. Propositional Logic can be used to solve any Knights/Knaves puzzle. For your symbolization key, use "A: Alice is a knight, B: Bob is a knight, C: Carol is a knight, D: Dave is a knight, E: ...". Then, every statement a participant makes provides you with *two* hypotheses: If Alice claims "Bob is a knight and Carol is a knave", then we translate her claim via our translation key and we have both

$$A \Rightarrow (B\&\neg C) \qquad \text{and} \qquad \neg A \Rightarrow \neg(B\&\neg C)$$

as hypotheses, because if Alice is a knight, then she's telling the truth, but if she's a knave, then her statement was a lie (it's like a two-for-one special). Once we translate, it's just a matter of using truth tables to figure out who's a knight and who's a knave by finding the one row in which all of the hypotheses are true.

Now, consider the following puzzle: *Alice claims that Bob is a knave, but Bob tells you that neither he nor Alice are knaves.*

(a) [3 points] Write out the four hypotheses (symbolized into propositional assertions) that their statements give you.

(b) [3 points] Use a truth table to determine who is who (it might help to have a few auxiliary columns, like for the two pieces of an implication, for instance).

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Exam 1

30 points. 110 minutes.

1. [2 points] What is an assertion?

   A statement that can be either true or false.

2. [2 points] What is a deduction?

   A few assertions, one of which is called the conclusion, the others of which are called the hypotheses.

3. [2 points] What does it mean when you say that a particular deduction is valid?

   It means that whenever the hypotheses are true, the conclusion must also be true.

4. [2 points] How can you show that a particular deduction is invalid?

   Find a situation in which the hypotheses are true but the conclusion is false.

   (This is called a counterexample. In Categorical Logic, a counterexample would be a particular Venn diagram. In Propositional Logic, a counterexample would be a particular valuation.)

5. [2 points] Is it possible for there to be a valid deducation that has a false conclusion? If not, say why. If so, provide an example of one.

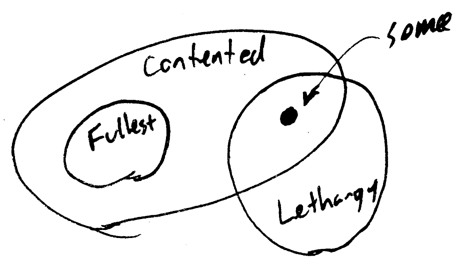
   It is possible. Here's one: *All spiders are from Mars, and Ziggy Stardust is a spider, so Ziggy Stardust is from Mars.*

6. For each categorical syllogism below, either state that the syllogism is valid (no proof necessary) or demonstrate that it is invalid by drawing a Venn diagram in which the hypotheses are true but the conlusion is false.

   (a) [3 points] Snakes are lizards, and all lizards are carnivores, so snakes must be carnivores, I guess.

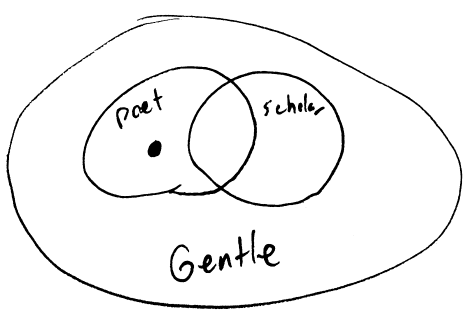
   It's valid.

   (b) [3 points] People who live life to the fullest are contented people, but some people who are contented are full of lethargy. It stands to reason that some people who live life to the fullest are full of lethargy.



   It's invalid.

   (c) [3 points] Not every poet is a scholar, but every scholar is a gentle person. From this, we can conclude that not every poet is a gentle person.



   It's invalid.

7. [3 points] Give a two-column proof of the following theorem: $A \Rightarrow (B \vee C), \neg B, \therefore A \Rightarrow C$.

1.	$A \Rightarrow (B \vee C)$	Hypothesis
2.	$\neg B$	Hypothesis
3.	$\quad A$	Assume
4.	$\quad B \vee C$	$\Rightarrow$-elim (lines 1, 3)
5.	$\quad C$	$\vee$-elim (lines 4, 2)
6.	$A \Rightarrow C$	$\Rightarrow$-intro (lines 3-5)

8. [3 points] Prove algebraically that $(A \vee B) \Rightarrow C$ is logically equivalent to $(A \Rightarrow C)\&(B \Rightarrow C)$. (You do not have to give justifications.)

$$\begin{aligned}(A \vee B) \Rightarrow C &\equiv \neg(A \vee B) \vee C \\ &\equiv (\neg A \& \neg B) \vee C \\ &\equiv (\neg A \vee C)\&(\neg B \vee C) \\ &\equiv (A \Rightarrow C)\&(B \Rightarrow C)\end{aligned}$$

9. Propositional Logic can be used to solve any Knights/Knaves puzzle. For your symbolization key, use "A: Alice is a knight, B: Bob is a knight, C: Carol is a knight, D: Dave is a knight, E: ...". Then, every statement a participant makes provides you with *two* hypotheses: If Alice claims "Bob is a knight and Carol is a knave", then we translate her claim via our translation key and we have both

$$A \Rightarrow (B \& \neg C) \qquad \text{and} \qquad \neg A \Rightarrow \neg(B \& \neg C)$$

as hypotheses, because if Alice is a knight, then she's telling the truth, but if she's a knave, then her statement was a lie (it's like a two-for-one special). Once we translate, it's just a matter of using truth tables to figure out who's a knight and who's a knave by finding the one row in which all of the hypotheses are true.

Now, consider the following puzzle: *Alice claims that Bob is a knave, but Bob tells you that neither he nor Alice are knaves.*

(a) [3 points] Write out the four hypotheses (symbolized into propositional assertions) that their statements give you.

$$A \Rightarrow \neg B$$
$$\neg A \Rightarrow B$$
$$B \Rightarrow (A \& B)$$
$$\neg B \Rightarrow \neg(A \& B)$$

(b) [3 points] Use a truth table to determine who is who (it might help to have a few auxiliary columns, like for the two pieces of an implication, for instance).

A	B	$A \& B$	$A \Rightarrow \neg B$	$\neg A \Rightarrow B$	$B \Rightarrow (A \& B)$	$\neg B \Rightarrow \neg(A \& B)$
t	t	t	f	t	t	t
t	f	f	t	t	t	t
f	t	f	t	t	f	t
f	f	f	t	f	t	t

From the truth table, we see that the only row that make all of the hypotheses true is the second row, in which Alice is a knight and Bob is a knave.

Exam 2

30 points. 110 minutes.

1. For each assertion below, come up with a sympolization key and symbolize the assertion in First Order logic.

   (a) [2 points] For any real number, there is a natural number that is larger.

   (b) [2 points] If there is at least one perfect word in A, then every word in $A \cap B$ is perfectly cromulent.

   (c) [2 points] For all real numbers x, if there is a real number in A that domineers x, then there is also a real number in the compliment of A that x domineers.

2. For each assertion below, simplify so that the negations are on the most primitive assertions.

   (a) [3 points] $\neg\big(\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, \exists c \in \mathbb{R}, \varphi(a,c) \vee \varphi(b,c)\big)$

   (b) [3 points] $\neg\big((\exists x, x \in A \,\&\, x \in B \cup C) \Rightarrow (\forall y, y \in A \cup B \vee y \notin C)\big)$

3. Let A, B, and C be sets. For each deduction below, provide a counterexample that shows that the deduction is invalid (i.e. define sets A, B, and C for which the hypotheses are true but the conclusion is false).

   (a) [4 points] Assume A and B are disjoint. Then $A \cup C$ and $B \cup C$ are disjoint.

   (b) [4 points] $(A \cap C) \cup (B \cap C) \subset (A \cap B) \cup (A \cap C)$

4. [6 points] Let A, B, and C be sets, and let A and B be disjoint.
Show that $A \cap C$ and $B \cap C$ are disjoint.

5. [6 points] Recall that $U \Delta V = \{x | x \in U \cup V \,\&\, x \notin A \cap B\}$.

   Let A, B, and C be sets.
   Show that $A \cap (B \Delta C) \subset (A \cap B) \Delta (A \cap C)$

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Exam 2

30 points. 110 minutes.

1. For each assertion below, come up with a sympolization key and symbolize the assertion in First Order logic.

   (a) [2 points] For any real number, there is a natural number that is larger.

   $$\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, n > x$$

   (b) [2 points] If there is at least one perfect word in A, then every word in $A \cap B$ is perfectly cromulent.

   Let $P(x)$ mean "x is perfect," and let $C(x)$ mean "x is perfectly cromulent."

   $$\big(\exists w \in A, P(w)\big) \Rightarrow \big(\forall x \in A \cap B, C(x)\big)$$

   (c) [2 points] For all real numbers x, if there is a real number in A that domineers x, then there is also a real number in the compliment of A that x domineers.

   Let xDy mean "x domineers y."

   $$\forall x \in \mathbb{R}, \big((\exists y \in A, yDx) \Rightarrow (\exists z \in \bar{A}, xDz)\big)$$

2. For each assertion below, simplify so that the negations are on the most primitive assertions.

   (a) [3 points] $\neg\big(\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, \exists c \in \mathbb{R}, \varphi(a,c) \vee \varphi(b,c)\big)$

   $$\exists a \in \mathbb{R}, \exists b \in \mathbb{R}, \forall c \in \mathbb{R}, \neg\varphi(a,c) \,\&\, \neg\varphi(b,c)$$

   (b) [3 points] $\neg\big((\exists x, x \in A \,\&\, x \in B \cup C) \Rightarrow (\forall y, y \in A \cup B \vee y \notin C)\big)$

   $$\big(\exists x, x \in A \,\&\, x \in B \cup C\big) \,\&\, \big(\exists y, y \notin A \cup B \,\&\, y \in C\big)$$

3. Let A, B, and C be sets. For each deduction below, provide a counterexample that shows that the deduction is invalid (i.e. define sets A, B, and C for which the hypotheses are true but the conclusion is false).

   (a) [4 points] Assume A and B are disjoint. Then $A \cup C$ and $B \cup C$ are disjoint.

   If $A = \{1\}$, $B = \{2\}$, and $C = \{3\}$,
   then $A \cap B = \emptyset$, so the hypothesis is true,
   but $(A \cup C) \cap (B \cup C) = \{3\}$,
   so the conclusion is false.
   Thus, the deduction is invalid.

   (b) [4 points] $(A \cap C) \cup (B \cap C) \subset (A \cap B) \cup (A \cap C)$

   The deduction has only a conclusion, no hypotheses.
   We need three sets for which the statement is not true.

   Let $A = \{1, 2\}$, $B = \{2, 3\}$, and $C = \{1, 3\}$.
   Then $(A \cap C) \cup (B \cap C) = \{1, 3\}$,
   and $(A \cap B) \cup (A \cap C) = \{1, 2\}$.
   Since $3 \in (A \cap C) \cup (B \cap C)$
   but $3 \notin (A \cap B) \cup (A \cap C)$,
   $(A \cap C) \cup (B \cap C) \not\subset (A \cap B) \cup (A \cap C)$.

4. [6 points] Let A, B, and C be sets, and let A and B be disjoint.
Show that $A \cap C$ and $B \cap C$ are disjoint.

Let $A \cap B = \emptyset$.
Assume for contradiction that $(A \cap C) \cap (B \cap C) \neq \emptyset$,
so $\exists x, x \in (A \cap C) \cap (B \cap C)$.
Take one such x, and call it a, so $a \in (A \cap C) \cap (B \cap C)$.
Then $a \in A \cap C$ and $a \in B \cap C$.
Since $a \in A \cap C$, we get $a \in A$ and $a \in C$.
Since $a \in B \cap C$, we get $a \in B$ and $a \in C$.
In particular, we have $a \in A$ and $a \in B$,
so $\exists x, x \in A \cap B$, which contradicts $A \cap B = \emptyset$.
Thus, we know that our assumption must be wrong.
We conclude $(A \cap C) \cap (B \cap C) = \emptyset$.

5. [6 points] Recall that $U \Delta V = \{x | x \in U \cup V \,\&\, x \notin A \cap B\}$.

Let A, B, and C be sets.
Show that $A \cap (B \Delta C) \subset (A \cap B) \Delta (A \cap C)$

We want to show $A \cap (B \Delta C) \subset (A \cap B) \Delta (A \cap C)$,
that is, we want to show $\forall x, x \in A \cap (B \Delta C) \Rightarrow x \in (A \cap B) \Delta (A \cap C)$.
To that end, let $x \in A \cap (B \Delta C)$, and we want to show $x \in (A \cap B) \Delta (A \cap C)$.

Since $x \in A \cap (B \Delta C)$, we get $x \in A$ and $x \in B \Delta C$.
Since $x \in B \Delta C$, we know $x \in B$ or $x \in C$, but $x \notin B \cap C$.
We proceed in cases.

(Case 1) Assume $x \in B$, so $x \notin C$.
Then since $x \in A$, we get $x \in A \cap B$.
Also, since $x \notin C$, we get $x \notin A \cap C$.
Since both $x \in A \cap B$ and $x \notin A \cap C$,
we have $x \in (A \cap B) \Delta (A \cap C)$, as desired.

(Case 2) Assume $x \in C$, so $x \notin B$.
Then since $x \in A$, we get $x \in A \cap C$.
Also, since $x \notin B$, we get $x \notin A \cap B$.
Since both $x \in A \cap C$ and $x \notin A \cap B$,
we have $x \in (A \cap B) \Delta (A \cap C)$, as desired.

Since $x \in (A \cap B) \Delta (A \cap C)$ in both possible cases,
we conclude that it must be true that $x \in (A \cap B) \Delta (A \cap C)$.
Since x was a general element of $A \cap (B \Delta C)$,
we've shown $\forall x, x \in A \cap (B \Delta C) \Rightarrow x \in (A \cap B) \Delta (A \cap C)$,
or in other words, $A \cap (B \Delta C) \subset (A \cap B) \Delta (A \cap C)$, as desired.

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Final Exam

60 points. 150 minutes.

Directions: Work alone.

1. Given the following translation key, translate each assertion below into symbolic propositional logic. Your symbolization should translate the assertions exactly as they are stated. Order matters. Parentheses where needed. Do not simplify.

A : Alice is a logic student
B : Bob is a logic student
C : Alice is a math student
D : Bob is a math student

(a) [1 point] Either Bob is a logic student or neither Bob is a math student nor Alice is a logic student.

(b) [1 point] If Alice studies math, then both Alice studies logic and Bob studies logic.

(c) [1 point] If Alice and Bob both study math, then Alice studies logic.

(d) [1 point] If either Bob studies logic or math or Alice doesn't study math, then Alice studies logic.

(e) [1 point] Alice studies math and not logic implies that Bob either studies logic or doesn't study math.

(f) [1 point] Either Alice studies logic implies Bob studies logic, or Bob studies math implies Alice studies math.

(g) [1 point] If Bob studies neither math nor logic, then Alice studies logic or math.

(h) [1 point] If Alice's studying logic does not imply Bob studies logic, then either Alice or Bob doesn't study math.

2. Given the following translation key, translate each assertion below into symbolic first-order logic. Your symbolization should translate the assertions exactly as they are stated. Order matters. Parentheses where needed. Do not simplify.

$$x\,L\,y : x \text{ likes } y$$
$$F : \text{The set of Karl's friends}$$
$$N : \text{The set or Imre's neighbours}$$

(a) [1 point] $N \subset K$.

(b) [1 point] $K \not\subset N$.

(c) [1 point] $N \cap K = \emptyset$ (i.e. N and K are disjoint)

(d) [1 point] All of Imre's neighbours like all of Karl's friends.

(e) [1 point] At least one of Karl's friends likes at least one of Imre's neighbors.

(f) [1 point] There is one of Imre's neighbours, who is a friend of Karl and who likes all of Imre's neighbours.

3. Let A, B, and C be sets. Let $P(x)$, $Q(x)$, and xRy be predicates. For each string of symbols below, write either "well-formed" is the string is a grammatically-correct first-order assertion, or write "nonsense" if it is not.

(a) [1 point] $\forall x \in A, P(x) \,\&\, Q(x) \,\&\, x \in B$

(b) [1 point] $\exists x \in B, P(x) \,\&\, Q(x) \vee x \in B$

(c) [1 point] $\forall x \in B, \exists y \in A$

(d) [1 point] $\forall x \in A, x \in B \vee C$

(e) [1 point] $\exists x \in C, x \in A \cap x \in B$

(f) [1 point] $\forall x \in A, \exists y \in B, xRy \Rightarrow \exists z \in C, P(z)$

4. For each statement below, give the precise definition of that statement in symbolic first-order logic.

   (a) [1 point] $\sim$ is reflexive (where f is a binary relation on A).

   (b) [1 point] $\sim$ is symmetrix (where f is a binary relation on A).

   (c) [1 point] $\sim$ is transitive (where f is a binary relation on A).

   (d) [1 point] f is a function from A to B.

   (e) [1 point] f is 1-to-1 (where f is a function from A to B).

   (f) [1 point] f is onto (where f is a function from A to B).

5. Simplify each propositional assertion until the all the negations are attached to propositional variables and there are no double negations.

   (a) [4 points] $\neg[P \vee (Q \,\&\, R)] \,\&\, \neg(\neg S \,\&\, T)$

   (b) [4 points] $\neg[(P \vee \neg Q) \Rightarrow ((S \vee \neg T) \,\&\, \neg(A \,\&\, B))]$

6. Write a two-column proof of each of the following valid deductions.

   (a) [4 points] $(P \Rightarrow Q\,;\ \neg P \Rightarrow R\,;\ \therefore Q \vee R)$

   (b) [4 points] $((P \vee Q) \Rightarrow \neg R\,;\ \neg R \Rightarrow \neg P\,;\ \therefore \neg P)$

7. [6 points] Let A, B, and C be sets. Assume $B \subset C$. Prove that $A \cap B \subset A \cap C$.

8. [6 points] Let A and B be sets. Show that $A \setminus B$ and $B \setminus A$ are disjoint.

9. [6 points] Let $f : X \to Y$ be a bijection. Let $\approx$ be an equivalence relation on X.

   Define a binary relation $\doteq$ on B by setting $y_1 \doteq y_2$ iff $f^{-1}(y_1) \approx f^{-1}(y_2)$ for $y_1, y_2 \in Y$.

   Prove that $\doteq$ is an equivalence relation on Y.

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Final Exam

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A : Alice is a logic student
B : Bob is a logic student
C : Alice is a math student
D : Bob is a math student

(a) [1 point] Either Bob is a logic student or neither Bob is a math student nor Alice is a logic student.

$B \vee (\neg D \,\&\, \neg A)$

(b) [1 point] If Alice studies math, then both Alice studies logic and Bob studies logic.

$C \Rightarrow (A \,\&\, B)$

(c) [1 point] If Alice and Bob both study math, then Alice studies logic.

$(C \,\&\, D) \Rightarrow A$

(d) [1 point] If either Bob studies logic or math or Alice doesn't study math, then Alice studies logic.

$(B \vee D \vee \neg C) \Rightarrow A$

(e) [1 point] Alice studies math and not logic implies that Bob either studies logic or doesn't study math.

$(C \,\&\, \neg A) \Rightarrow (B \vee \neg D)$

(f) [1 point] Either Alice studies logic implies Bob studies logic, or Bob studies math implies Alice studies math.

$(A \Rightarrow B) \vee (D \Rightarrow C)$

(g) [1 point] If Bob studies neither math nor logic, then Alice studies logic or math.

$(\neg D \,\&\, \neg B) \Rightarrow (A \vee C)$

(h) [1 point] If Alice's studying logic does not imply Bob studies logic, then either Alice or Bob doesn't study math.

$\neg(A \Rightarrow B) \Rightarrow (\neg C \vee \neg D)$

2. Given the following translation key, translate each assertion below into symbolic first-order logic. Your symbolization should translate the assertions exactly as they are stated. Order matters. Parentheses where needed. Do not simplify.

$$x\,L\,y : x \text{ likes } y$$
$$F : \text{The set of Karl's friends}$$
$$N : \text{The set or Imre's neighbours}$$

(a) [1 point] $N \subset K$.

$\forall x \in N, x \in K$

(b) [1 point] $K \not\subset N$.

$\exists x \in K, x \notin N$

(c) [1 point] $N \cap K = \emptyset$ (i.e. N and K are disjoint)

$\neg \exists x, x \in N \cap K$

(d) [1 point] All of Imre's neighbours like all of Karl's friends.

$\forall x \in N, \forall y \in F, x\,L\,y$

(e) [1 point] At least one of Karl's friends likes at least one of Imre's neighbors.

$\exists x \in K, \exists y \in N, x\,L\,y$

(f) [1 point] There is one of Imre's neighbours, who is a friend of Karl and who likes all of Imre's neighbours.

$\exists x \in N, x \in K \,\&\, \forall y \in N, x\,L\,y$

3. Let A, B, and C be sets. Let $P(x)$, $Q(x)$, and xRy be predicates. For each string of symbols below, write either "well-formed" is the string is a grammatically-correct first-order assertion, or write "nonsense" if it is not.

(a) [1 point] $\forall x \in A, P(x) \,\&\, Q(x) \,\&\, x \in B$

well-formed

(b) [1 point] $\exists x \in B, P(x) \,\&\, Q(x) \vee x \in B$

nonsense

(c) [1 point] $\forall x \in B, \exists y \in A$

nonsense

(d) [1 point] $\forall x \in A, x \in B \vee C$

nonsense

(e) [1 point] $\exists x \in C, x \in A \cap x \in B$

nonsense

(f) [1 point] $\forall x \in A, \exists y \in B, xRy \Rightarrow \exists z \in C, P(z)$

well-formed

4. For each statement below, give the precise definition of that statement in symbolic first-order logic.

(a) [1 point] $\sim$ is reflexive (where f is a binary relation on A).

$\forall x \in A, x \sim x$

(b) [1 point] $\sim$ is symmetrix (where f is a binary relation on A).

$\forall x, y \in A, x \sim y \Rightarrow y \sim x$

(c) [1 point] $\sim$ is transitive (where f is a binary relation on A).

$\forall x, y, z \in A, (x \sim y \,\&\, y \sim z) \Rightarrow x \sim z$

(d) [1 point] f is a function from A to B.

$f \subset A \times B$ and $\forall x \in A, \exists! y \in B, y = f(x)$

(e) [1 point] f is 1-to-1 (where f is a function from A to B).

$\forall x_1, x_2 \in A, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

(f) [1 point] f is onto (where f is a function from A to B).

$\forall y \in B, \exists x \in A, y = f(x)$

5. Simplify each propositional assertion until the all the negations are attached to propositional variables and there are no double negations.

   (a) [4 points] $\neg[P \vee (Q \,\&\, R)] \,\&\, \neg(\neg S \,\&\, T)$

$$\begin{aligned}
&\neg[P \vee (Q \,\&\, R)] \,\&\, \neg(\neg S \,\&\, T) \\
&\equiv [\neg P \,\&\, \neg(Q \,\&\, R)] \,\&\, (S \vee \neg T) \\
&\equiv \neg P \,\&\, (\neg Q \vee \neg R) \,\&\, (S \vee \neg T)
\end{aligned}$$

   (b) [4 points] $\neg[(P \vee \neg Q) \Rightarrow ((S \vee \neg T) \,\&\, \neg(A \,\&\, B))]$

$$\begin{aligned}
&\neg[(P \vee \neg Q) \Rightarrow ((S \vee \neg T) \,\&\, \neg(A \,\&\, B))] \\
&\equiv (P \vee \neg Q) \,\&\, \neg((S \vee \neg T) \,\&\, \neg(A \,\&\, B)) \\
&\equiv (P \vee \neg Q) \,\&\, (\neg(S \vee \neg T) \vee (A \,\&\, B)) \\
&\equiv (P \vee \neg Q) \,\&\, ((\neg S \,\&\, T) \vee (A \,\&\, B))
\end{aligned}$$

6. Write a two-column proof of each of the following valid deductions.

   (a) [4 points] $(P \Rightarrow Q\,;\ \neg P \Rightarrow R\,;\ \therefore Q \vee R)$

   (b) [4 points] $((P \vee Q) \Rightarrow \neg R\,;\ \neg R \Rightarrow \neg P\,;\ \therefore \neg P)$

7. [6 points] Let A, B, and C be sets. Assume $B \subset C$. Prove that $A \cap B \subset A \cap C$.

8. [6 points] Let A and B be sets. Show that $A \setminus B$ and $B \setminus A$ are disjoint.

9. [6 points] Let $f : X \to Y$ be a bijection. Let $\approx$ be an equivalence relation on X.

Define a binary relation $\doteq$ on B by setting $y_1 \doteq y_2$ iff $f^{-1}(y_1) \approx f^{-1}(y_2)$ for $y_1, y_2 \in Y$.

Prove that $\doteq$ is an equivalence relation on Y.