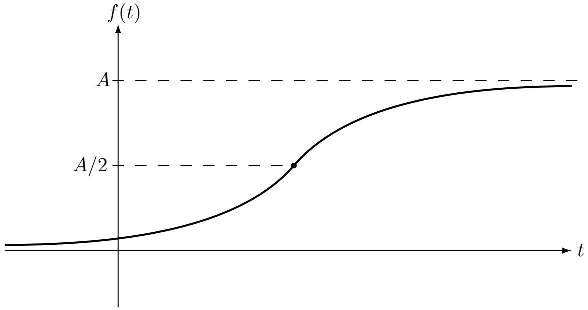
Panda Population Growth

September 3, 2016

Meet the Logistics Curve Family



A *logistics curve* is a function

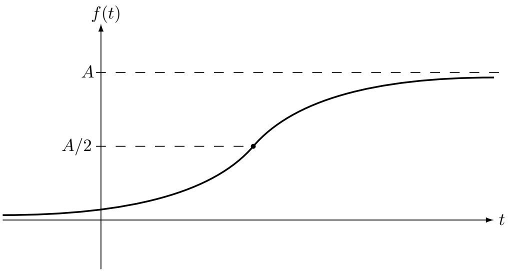
$$f(t) = \frac{A}{1 + Be^{-Ct}}$$

where A, B, and C are constants.

The important properties are:

- $f(t) \to A$ as $t \to \infty$
- $f(t) \to 0$ as $t \to -\infty$
- concave up when $f(t) < A/2$
- concave down when $f(t) > A/2$
- max growth rate when $f(t) = A/2$

Logistics Curves Model Population Growth



Population growth is an important topic of study in the subfield of Biology called *Ecology*.

$$f(t) = \frac{A}{1 + Be^{-Ct}}$$

Logistics curves are useful for modeling population growth.

- $f(t)$ represents the number of individuals living at time t.
- A represents the *carrying capacity* (the maximum number of individuals that the environment can support).
- When $f(t) < A/2$, resources are plentiful, and the population growth rate increases.
- When $f(t) > A/2$, resources are scarce, and competition for resources drives the population growth rate down.

Example: The Mountain Lions in Yosemite

A scientist is modeling the population of mountain lions in Yosemite National Park using a logistics curve with t measured in years and $t = 0$ corresponding to 2000.

$$f(t) = \frac{A}{1 + Be^{-Ct}}$$

Through observation and data analysis, she has determined that the constants $A = 4000$, $B = 10$, and $C = 0.12$ are a good fit for her observed data.

Use your calculator to plot the model from $t = 0$ to $t = 50$.
Now take a few minutes to answer the following:

1. What is the carrying capacity for this population?
2. At what population level will the growth rate be the highest?
3. At what time will the growth rate be the highest?
4. What is the maximum growth rate (and what unit is that in)?

Solution: The Mountain Lions in Yosemite

1. A represents the carrying capacity, so the carrying capacity is 4000.
2. The logistics curve has it's fastest rate of growth when $f(t)$ is $A/2$, so the population will have the highest growth rate when there are 2000 mountain lions.
3. We need to find the value of t when $f(t) = 2000$.

$$\begin{aligned}
2000 &= \frac{4000}{1 + 10e^{-0.12t}} \\
1 + 10e^{-0.12t} &= 2 \\
e^{-0.12t} &= 0.1 \\
-0.12t &= \ln(0.1) \\
t &= \frac{\ln(0.1)}{-0.12} \\
t &\approx 19.188
\end{aligned}$$

4. Since $f(t)$ tells us the population, the derivative $f'(t)$ will tell us the growth rate of the population.

$$\begin{aligned}
f(t) &= \frac{4000}{1 + 10e^{-0.12t}} \\
f'(t) &= \frac{d}{dt}\frac{4000}{1 + 10e^{-0.12t}} \\
f'(t) &= \frac{0 - 4000(-1.2e^{-0.12t})}{(1 + 10e^{-0.12t})^2} \\
f'(t) &= \frac{4800e^{-0.12t}}{(1 + 10e^{-0.12t})^2} \\
f'(19.188...) &= 120
\end{aligned}$$

Since $f(t)$ is mountain lions in terms of time (in years), $f'(t)$ is mountain lions per year.