1. Our car is travelling at a constant velocity of 30 m/s when we spot a priceless painting on the road 100 m in front of us! We apply the brakes, giving our car an acceleration of -5 m/s^2.
   Putting our initial position as 0 m and the position of the painting as 100 m, find our position when the car comes to a stop (i.e., when velocity is 0). Did we save the painting?

$$\begin{aligned} a(t) &= -5 \\ v(t) &= -5t + v_0 \\ &= -5t + 30 \\ x(t) &= -2.5t^2 + 30t + x_0 \\ &= -2.5t^2 + 30t \end{aligned}$$

   The car comes to a stop when $v(t) = 0$.

$$\begin{aligned} 0 &= v(t) \\ 0 &= -5t + 30 \\ t &= 6 \end{aligned}$$

   We need to know if we hit the painting (which is at position 100). We calculate the car's position when it stops.

$$\begin{aligned} x(t) &= -2.5t^2 + 30t \\ x(6) &= 90 \end{aligned}$$

   We stopped at position 90, so we saved the painting!

2. We are designing a car, and we need to ensure that it has a stopping distance of 50 m when traveling at a speed of 25 m/s. How much acceleration must the brakes be designed to provide?

$$\begin{aligned} a(t) &= -A \\ v(t) &= -At + v_0 \\ &= -At + 25 \\ x(t) &= -\frac{A}{2}t^2 + 25t + x_0 \\ &= -\frac{A}{2}t^2 + 25t \end{aligned}$$

We need to find the time t when the velocity is zero.

$$\begin{aligned} 0 &= v(t) \\ 0 &= -At + 25 \\ t &= \frac{25}{A} \end{aligned}$$

Our car stops at time $t = \frac{25}{A}$. We need to see how far it goes.

$$\begin{aligned} x(t) &= -\frac{A}{2}t^2 + 25t \\ x\left(\frac{25}{A}\right) &= -\frac{A}{2}\left(\frac{25}{A}\right)^2 + 25\left(\frac{25}{A}\right) \\ &= \frac{625}{2A} \end{aligned}$$

So our stopping distance is $\frac{652}{2A}$. On the other hand, we need our stopping distance to be 50 m. Equate, and solve for A.

$$\begin{aligned} 50 &= \frac{625}{A} \\ A &= 6.25 \end{aligned}$$

We need our brakes to be able to apply -6.25 m/s^2 of acceleration.

3. Our car has brakes that can provide -6 m/s^2 acceleration. Find the stopping distance as a function of velocity. Find the velocity at which the stopping distance becomes 100 m, and find the velocity at which the stopping distance becomes 200 m (give your answers in m/s and correct to 2 decimal places).
   We know that our acceleration is -6. Our initial velocity is unknown, so we give it a symbol, say v_0.
   We need to find the time t^* when our car stops (*i.e.*, when the velocity equals 0) and the distance we travel before stopping (in terms of v_0).

$$\begin{aligned} a(t) &= -6 \\ v(t) &= -6t + v_0 \\ x(t) &= -3t^2 + v_0 t \end{aligned}$$

   Set $0 = v(t^*)$ and solve for t^*.

$$\begin{aligned} 0 &= v(t^*) \\ 0 &= -6t^* + v_0 \\ t^* &= \frac{v_0}{6} \end{aligned}$$

   So our car stops at time $t^* = \frac{v_0}{6}$. Our stopping distance is $x(t^*)$.

$$\begin{aligned} x(t^*) &= -3(t^*)^2 + v_0 t^* \\ &= \frac{1}{9}{v_0}^2 \end{aligned}$$

   So our stopping distance in terms of our velocity v_0 is $\frac{1}{9}{v_0}^2$.
   We need to find the velocities that result in stopping distances of 100 m and 200 m, respectively.

$$\begin{aligned} x(t^*) &= 100 && \Longrightarrow && v_0 = 30 \\ x(t^*) &= 200 && \Longrightarrow && v_0 = 42.42 \end{aligned}$$

   Notice that increasing our speed a relatively small amount doubles our stopping distance.