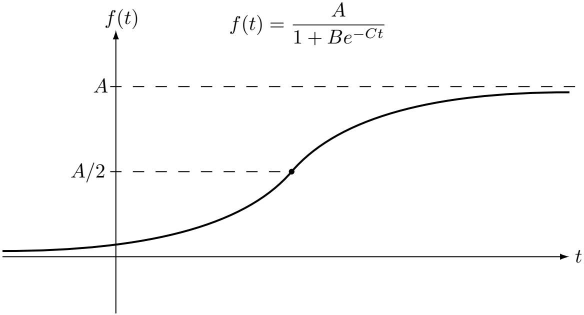
Repairing the Panda Population

September 4, 2016

Meet the Logistics Curve Family



Logistics Curve

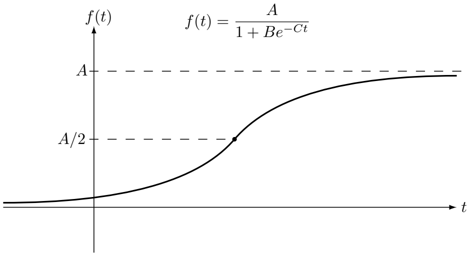
$$f(t) = \frac{A}{1 + Be^{-Ct}}$$

(A, B, C are constants)

Important Properties

- $f(t) \to A$ as $t \to \infty$
- $f(t) \to 0$ as $t \to -\infty$
- concave up when $f(t) < A/2$
- concave down when $f(t) > A/2$
- max growth rate when $f(t) = A/2$

Logistics Curves Model Population Growth



Ecology

- Subfield of Biology
- Studies population growth, especially as it relates to environment/ecosystem
- Logistics curves useful for modeling population growth

Logistics Curve as a Model

- $f(t)$, number of individuals living at time t
- A, the *carrying capacity* (the max number of individuals the environment can support)
- when $f(t) < A/2$, resources are plentiful, rate of population growth increases
- when $f(t) > A/2$, resources are scarce, competition drives rate of population growth down

Example: The Mountain Lions in Yosemite

Situation

A scientist is modeling the population of mountain lions in Yosemite National Park using a logistics curve with t measured in years and $t = 0$ corresponding to 2000.

$$f(t) = \frac{A}{1 + Be^{-Ct}}$$

Through observation and data analysis, she has determined that the constants $A = 4000$, $B = 10$, and $C = 0.12$ are a good fit for her observed data.

Questions

Use your calculator to plot the model from $t = 0$ to $t = 50$.
Now take a few minutes to answer the following questions about the model:

1. What is the carrying capacity for this population?
2. At what population level will the growth rate be the highest?
3. At what time will the growth rate be the highest?
4. What is the maximum growth rate (and what unit is that measured in)?

Solution: The Mountain Lions in Yosemite

1. A represents the carrying capacity, so the carrying capacity is 4000.
2. The logistics curve has it's fastest rate of growth when $f(t)$ is $A/2$, so the population will have the highest growth rate when there are 2000 mountain lions.
3. We need to find the value of t when $f(t) = 2000$.

$$\begin{aligned} 2000 &= \frac{4000}{1 + 10e^{-0.12t}} \\ 1 + 10e^{-0.12t} &= 2 \\ e^{-0.12t} &= 0.1 \\ -0.12t &= \ln(0.1) \\ t &= \frac{\ln(0.1)}{-0.12} \\ t &\approx 19.188 \end{aligned}$$

4. Since $f(t)$ tells us the population, the derivative $f'(t)$ will tell us the growth rate of the population.

$$\begin{aligned} f(t) &= \frac{4000}{1 + 10e^{-0.12t}} \\ f'(t) &= \frac{d}{dt} \frac{4000}{1 + 10e^{-0.12t}} \\ f'(t) &= \frac{0 - 4000(-1.2e^{-0.12t})}{(1 + 10e^{-0.12t})^2} \\ f'(t) &= \frac{4800e^{-0.12t}}{(1 + 10e^{-0.12t})^2} \\ f'(19.188...) &= 120 \end{aligned}$$

Since $f(t)$ is mountain lions in terms of time (in years), $f'(t)$ is mountain lions per year.