Foundations2
Assignment 2020
Turing Machine multiplier

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Contents

1 Turing machine multiplication

1.1 Graph - Multiplication positive and negative numbers

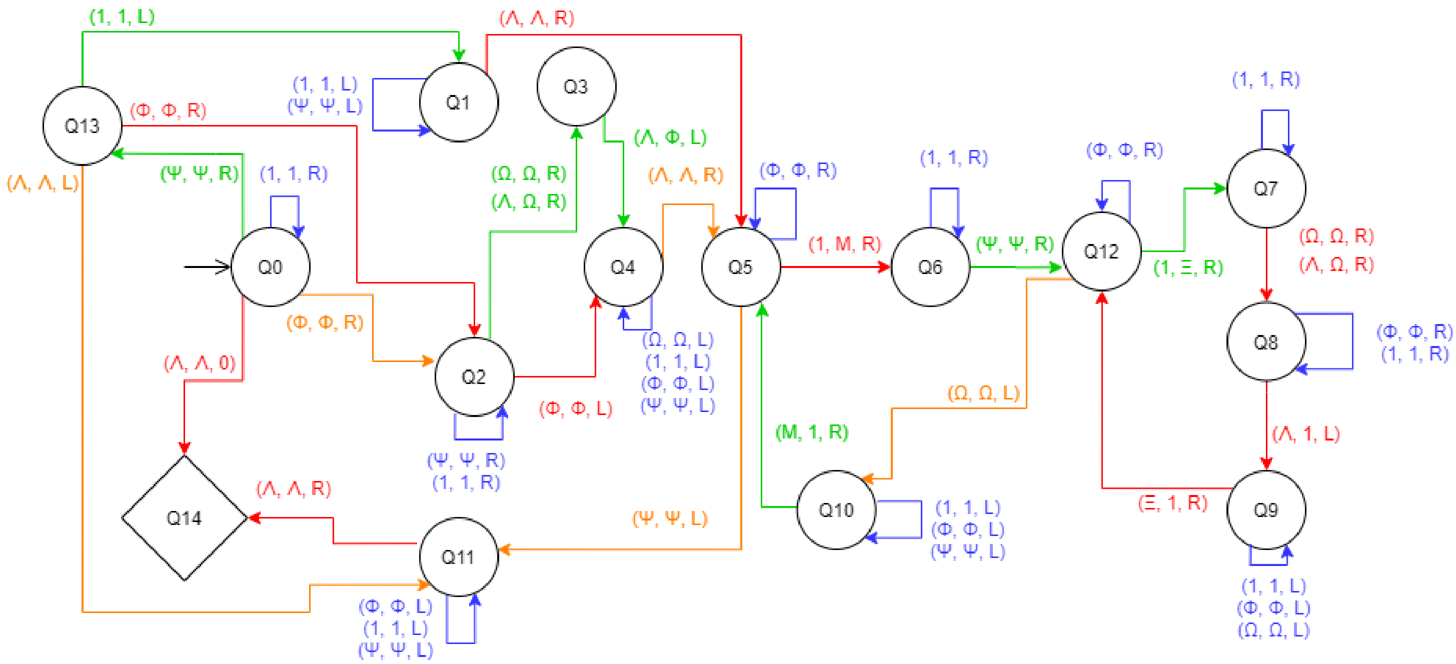

Figure 1: Turing machine to multiply, accepts negative input

1.2 Formal definition

The following is the formal definition of the multiplier machine with negative numbers.

$$States = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}\}$$

$$Symbols = \{\wedge, \Phi, \Psi, \Omega, 1, M, \Xi\}$$

$$M_{mult}(q_0, \wedge) = (q_{14}, \wedge, 0) \qquad M_{mult}(q_0, 1) = (q_0, 1, R)$$
$$M_{mult}(q_0, \Psi) = (q_{13}, \Psi, R) \qquad M_{mult}(q_0, \Phi) = (q_2, \Phi, R)$$
$$M_{mult}(q_1, 1) = (q_1, 1, L) \qquad M_{mult}(q_1, \Psi) = (q_1, \Psi, L)$$
$$M_{mult}(q_1, \wedge) = (q_5, \wedge, R) \qquad M_{mult}(q_{13}, \Phi) = (q_2, \Phi, R)$$
$$M_{mult}(q_2, 1) = (q_2, 1, R) \qquad M_{mult}(q_2, \Psi) = (q_2, \Psi, R)$$
$$M_{mult}(q_2, \wedge) = (q_3, \Omega, R) \qquad M_{mult}(q_2, \Omega) = (q_3, \Omega, R)$$
$$M_{mult}(q_2, \Phi) = (q_4, \Phi, L) \qquad M_{mult}(q_3, \wedge) = (q_4, \Phi, L)$$
$$M_{mult}(q_4, \Omega) = (q_4, \Omega, L) \qquad M_{mult}(q_4, 1) = (q_4, 1, L)$$
$$M_{mult}(q_4, \Phi) = (q_4, \Phi, L) \qquad M_{mult}(q_4, \Psi) = (q_4, \Psi, L)$$
$$M_{mult}(q_4, \wedge) = (q_5, \wedge, R) \qquad M_{mult}(q_5, \Phi) = (q_5, \Phi, R)$$
$$M_{mult}(q_5, 1) = (q_6, M, R) \qquad M_{mult}(q_5, \Psi) = (q_{11}, \Psi, L)$$
$$M_{mult}(q_6, 1) = (q_6, 1, R) \qquad M_{mult}(q_6, \Psi) = (q_{12}, \Psi, R)$$
$$M_{mult}(q_{12}, 1) = (q_7, \Xi, R) \qquad M_{mult}(q_{12}, \Omega) = (q_{10}, \Omega, L)$$
$$M_{mult}(q_7, \Omega) = (q_8, \Omega, R) \qquad M_{mult}(q_7, \wedge) = (q_8, \Omega, R)$$
$$M_{mult}(q_7, 1) = (q_7, 1, R) \qquad M_{mult}(q_8, \wedge) = (q_9, 1, L)$$
$$M_{mult}(q_8, 1) = (q_8, 1, R) \qquad M_{mult}(q_8, \Phi) = (q_8, \Phi, R)$$
$$M_{mult}(q_9, 1) = (q_9, 1, L) \qquad M_{mult}(q_9, \Phi) = (q_9, \Phi, L)$$
$$M_{mult}(q_9, \Omega) = (q_9, \Omega, L) \qquad M_{mult}(q_9, \Xi) = (q_{12}, 1, R)$$
$$M_{mult}(q_{10}, 1) = (q_{10}, 1, L) \qquad M_{mult}(q_{10}, \Phi) = (q_{10}, \Phi, L)$$
$$M_{mult}(q_{10}, \Psi) = (q_{10}, \Psi, L) \qquad M_{mult}(q_{10}, M) = (q_5, 1, R)$$
$$M_{mult}(q_{11}, 1) = (q_{11}, 1, L) \qquad M_{mult}(q_{11}, \Phi) = (q_{11}, \Phi, L)$$
$$M_{mult}(q_{11}, \wedge) = (q_{14}, \wedge, R) \qquad M_{mult}(q_{11}, \Psi) = (q_{11}, \Psi, L)$$
$$M_{mult}(q_{12}, \Phi) = (q_{12}, \Phi, R) \qquad M_{mult}(q_{13}, 1) = (q_1, 1, L)$$
$$M_{mult}(q_{13}, \wedge) = (q_{11}, \wedge, L)$$

2 Discussion of graph

2.1 Logic of graph

Initially the machine determines if either, or both, of the pair of numbers on the existing tape are negative. In the case that one (and only one) input number is negative it places the symbol representing negative numbers 'Φ' just after the symbol separating the result from the input 'Ω'. otherwise it leaves the result without such a symbol.

Following this the tape returns to the start index by moving left until it reaches the wedge symbol '$\wedge$', it then moves right until it reaches the first '1' which it changes to an 'M'. 'M' represents a pointer indicating how many times we copy the second number on the tape. Thus the machine moves along the tape until it reaches the 'Ψ' symbol, indicating it has reached the second number.

From this point the machine essentially runs a modified version of copy machine found in lecture 7 (See Figure 2) where states q_{12} (Figure 1) corresponds to q_0 and q_{10} (Figure 1) corresponds to q_f. Once the copy finishes it goes back to the 'M' symbol and moves right 1 space and repeats until the second input number has been copied the same number of times as the value of the first number. Upon completion it returns to the '0th' index position and enters the final state.

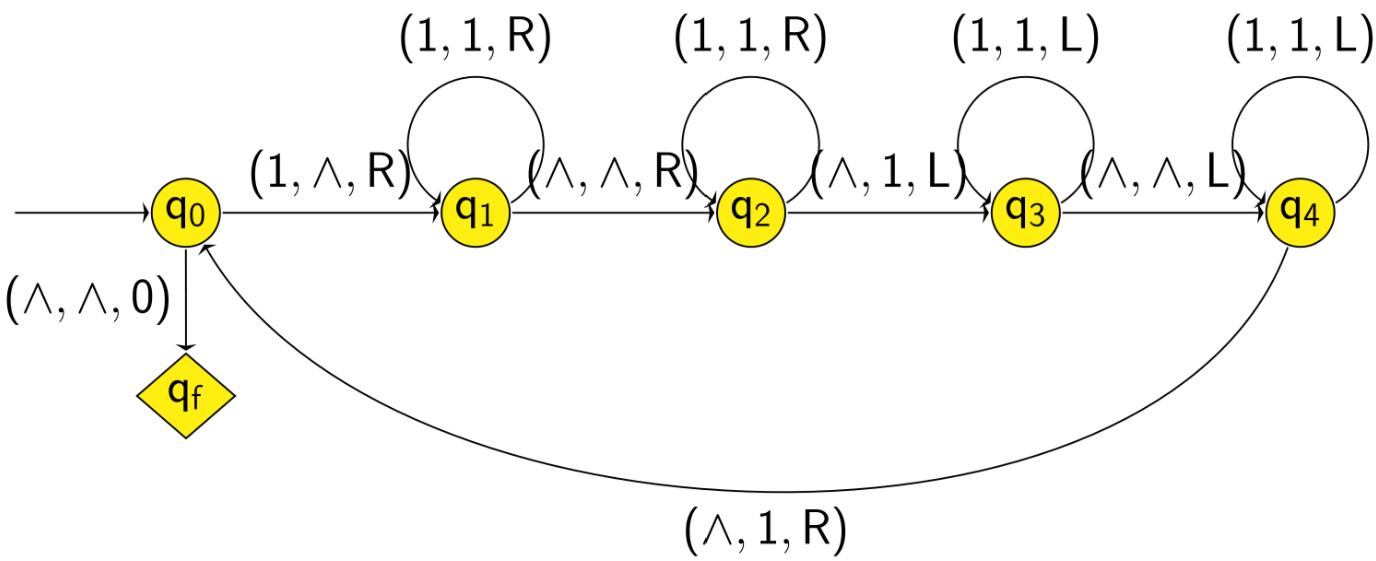

Figure 2: Copy machine from Lecture 7

2.2 States & Symbols

The states are mostly selected for their functionality. No 2 states have the same set of edges, and the total number of states has been minimized to some extent, such that where possible I have condensed what could be a singe step transition into a loop back to the same state rather than add an extra state, an example of this can be observed as a rule on state q_4 in Figure 1.

It may be possible to reduce the number of states in the machine from Figure 1 even further, a reduction in the number of states has already been achieved from the Figure 2 Copy Machine as can be observed by comparing the effective functionality of q_3 and q_4 in Figure 2 to q_9 in Figure 1.

Reducing the number of symbols is very possible, the machine in question (Figure 1) could be reduced to 3 symbols: '$Symbols = \{\wedge, \Phi, 1\}$', where the '$\wedge$ symbol replaces all spacer symbols. Readability would suffer as a consequence, the extra symbols have little or no effective performance overhead so a preference for human understanding of the tapes is more useful.

2.3 Clarity

The graph of the Turing machine with negative numbers does somewhat lack clarity, in part because some effort has been made to minimize the number of states; a version with some duplicate states might look prettier. Also adding negative number functionality and dealing with corner cases such as multiply by 0 required that some slightly difficult to route edges had to be introduced. A machine that simply performs multiplication on positive integers greater than 0 would be much clearer.

3 How it works

3.1 Definition of symbols

$$Symbols = \{\wedge, \Phi, \Psi, \Omega, 1, M, \Xi\}$$

$\wedge \rightarrow$ Represents an empty cell on the tape.
$\Phi \rightarrow$ Represents a minus character; the following number is negative.
$\Psi \rightarrow$ The divider between the 2 multiplicands.
$\Omega \rightarrow$ The divider between the data from the input tape and the additional data in the output tape.
$1 \rightarrow$ Represents the number '1', consecutive occurrences of this symbol indicates a number equal to the sum of the occurrences.
$M \rightarrow$ Effectively a pointer to the number being multiplied, this allows the Turing machine to track how far through the multiplication it is.
$\Xi \rightarrow$ Similar to 'M' but this indicates to the copy machine what symbol on the right hand side of the input is being copied.

3.2 Step through

An example tape of 2×3 would be input as in Figure 3 where the 'Ψ' symbol separates the multiplicands. As in Figure 3 the start state is specified as q_0, and the starting index on the tape will be '0'.

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$											
			$q0$											

Figure 3: Example Input tape for 2×3

Next the machine will parse the input to check for any negative inputs, if it finds only 1 'Φ' symbol between both inputs it will write a 'Φ' symbol at the start of the output and return to index '0' (See Figure 5), otherwise it will return to index '0' (See Figure 4). An example of 2 negative inputs can be found at Section 5.6

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$								
						$q13$								

Figure 4: Input parsed no negative, returning to 0

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q4$							

Figure 5: Input parse, negative found, written output and returning to 0

Now the multiplication part of the process begins. The first multiplicand's first '1' is converted to the 'M' symbol and the machine moves along to begin copying the second multiplicand (See Figure 6).

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q6$										

Figure 6: Begin multiplication

Once the machine reaches the first '1' on the second multiplicand, it begins the copy process by marking the 1 currently being copied with the 'Ξ' symbol (See Figure 7).

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q7$							

Figure 7: Begin Copy

The machine loops over the remaining '1' symbols until it reaches either the '$\wedge$' or 'Ω', it writes an 'Ω' and continues to search for the next occurence of '$\wedge$', once it finds it, it writes a '1' (See Figure 8) and returns to 'Ξ'.

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q8$					

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$						

Figure 8: 2 tapes: Copy first symbol and return

Now the program loops, until the last 'Ξ' is replaced by a '1', and the last 'M' is replaced by a '1'. At this point the machine returns to the 0th index and enters the final state: q_{14} (See Figure 9)

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...
			$\uparrow$ $q14$											

Figure 9: Copy complete, return to index 0

4 Implementation of Turing machine

The Language this Turing machine is implemented in is Scala. Scala is a fun language write code in, the object oriented style helps with separation of concerns, while the functional aspects; immutability and the structural sharing from data types such as Scala Vectors saves considerable memory when running tapes for (relatively) large multiplications such as 100*100. These features plus access to the easy to use Java printwriter to produce beautiful tapes as .tex files as an output of my implementation (see the Scala object at `/implementation/src/main/scala/Machine/TexTapePrinter.scala` in the source code) influenced my decision to use Scala.

5 Tests

Tests, including count of tapes printed, time to complete test

5.1 Quick summary

I have included a few tests on the following pages to demonstrate, basic functionality and corner cases.

Tests with tapes included in this document,

Test name	Time(ns)	Tapes
0×0	$300,300$	5
0×1	$185,300$	7
-1×2	$1,144,000$	45
2×2	$3,674,100$	65
-2×-2	$3,954,900$	73

5.2 0×0

Number of tapes: 5
Tape execution time: 0ms

	-2	-1	0	1	2	3	4	5	6	
...	$\wedge$	$\wedge$	Ψ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q0$							

Tape number: 0

	-2	-1	0	1	2	3	4	5	6	
...	$\wedge$	$\wedge$	Ψ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q13$						

Tape number: 1

	-2	-1	0	1	2	3	4	5	6	
...	$\wedge$	$\wedge$	Ψ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q11$							

Tape number: 2

	-2	-1	0	1	2	3	4	5	6	
...	$\wedge$	$\wedge$	Ψ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q11$								

Tape number: 3

	-2	-1	0	1	2	3	4	5	6	
...	$\wedge$	$\wedge$	Ψ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q14$							

Tape number: 4

5.3 0×1

Number of tapes: 7
Tape execution time: 0ms

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q0$								

Tape number: 0

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q13$							

Tape number: 1

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q1$								

Tape number: 2

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q1$									

Tape number: 3

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q5$								

Tape number: 4

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q11$									

Tape number: 5

	-2	-1	0	1	2	3	4	5	6	7	
...	$\wedge$	$\wedge$	Ψ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q14$								

Tape number: 6

5.4 -1×2

Number of tapes: 45
Tape execution time: 0ms

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q0$											

Tape number: 0

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q2$										

Tape number: 1

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q2$									

Tape number: 2

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q2$								

Tape number: 3

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q2$							

Tape number: 4

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$						
								$q2$						

Tape number: 5

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$					
									$q3$					

Tape number: 6

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$						
								$q4$						

Tape number: 7

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$							
							$q4$							

Tape number: 8

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$								
						$q4$								

Tape number: 9

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$									
					$q4$									

Tape number: 10

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$										
				$q4$										

Tape number: 11

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q4$											

Tape number: 12

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q4$												

Tape number: 13

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q5$											

Tape number: 14

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q5$										

Tape number: 15

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q6$									

Tape number: 16

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q12$								

Tape number: 17

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q7$							

Tape number: 18

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q7$

Tape number: 19

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q8$

Tape number: 20

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q8$

Tape number: 21

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q9$

Tape number: 22

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q9$

Tape number: 23

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q9$

Tape number: 24

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	Ξ	1	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ $q9$

Tape number: 25

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q12$							

Tape number: 26

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q7$						

Tape number: 27

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q8$					

Tape number: 28

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q8$				

Tape number: 29

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$ $q8$			

Tape number: 30

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	1	$\wedge$	$\wedge$	...
										$\uparrow$ $q9$				

Tape number: 31

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	1	$\wedge$	$\wedge$	...
									$\uparrow$ $q9$					

Tape number: 32

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	1	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$						

Tape number: 33

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	Ξ	Ω	Φ	1	1	$\wedge$	$\wedge$	...
							$\uparrow$ $q9$							

Tape number: 34

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
								$\uparrow$ $q12$						

Tape number: 35

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
							$\uparrow$ $q10$							

Tape number: 36

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
						$\uparrow$ $q10$								

Tape number: 37

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
					$\uparrow$ $q10$									

Tape number: 38

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	M	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
				$\uparrow$ $q10$										

Tape number: 39

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
					$\uparrow$ $q5$									

Tape number: 40

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
				$\uparrow$ $q11$										

Tape number: 41

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
			$\uparrow$ $q11$											

Tape number: 42

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
		$\uparrow$ $q11$												

Tape number: 43

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	Φ	1	Ψ	1	1	Ω	Φ	1	1	$\wedge$	$\wedge$	...
			$\uparrow$ $q14$											

Tape number: 44

5.5 2×2

Number of tapes: 65
Tape execution time: 3ms

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 0)
$q0$

Tape number: 0

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 1)
$q0$

Tape number: 1

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 2)
$q0$

Tape number: 2

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 3)
$q13$

Tape number: 3

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 2)
$q1$

Tape number: 4

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...

$\uparrow$ (at 1)
$q1$

Tape number: 5

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q1$											

Tape number: 6

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q1$												

Tape number: 7

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q5$											

Tape number: 8

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q6$										

Tape number: 9

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q6$									

Tape number: 10

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q12$								

Tape number: 11

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$							
							$q7$							

Tape number: 12

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$						
								$q7$						

Tape number: 13

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$					
									$q8$					

Tape number: 14

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$						
								$q9$						

Tape number: 15

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$							
							$q9$							

Tape number: 16

-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$								
						$q9$								

Tape number: 17

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q12$							

Tape number: 18

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q7$						

Tape number: 19

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q8$					

Tape number: 20

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q8$				

Tape number: 21

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q9$					

Tape number: 22

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$						

Tape number: 23

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q9$							

Tape number: 24

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q12$						

Tape number: 25

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q10$							

Tape number: 26

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q10$								

Tape number: 27

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q10$									

Tape number: 28

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q10$										

Tape number: 29

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	M	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q10$											

Tape number: 30

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q5$										

Tape number: 31

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q6$									

Tape number: 32

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q12$								

Tape number: 33

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q7$							

Tape number: 34

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q7$						

Tape number: 35

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q8$					

Tape number: 36

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q8$				

Tape number: 37

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$ $q8$			

Tape number: 38

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
										$\uparrow$ $q9$				

Tape number: 39

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
									$\uparrow$ $q9$					

Tape number: 40

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$						

Tape number: 41

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
							$\uparrow$ $q9$							

Tape number: 42

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
						$\uparrow$ $q9$								

Tape number: 43

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	$\wedge$	$\wedge$	...
							$\uparrow$ $q12$							

Tape number: 44

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
								$\uparrow$ $q7$						

Tape number: 45

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
									$\uparrow$ $q8$					

Tape number: 46

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
										$\uparrow$ $q8$				

Tape number: 47

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
											$\uparrow$			
											$q8$			

Tape number: 48

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
												$\uparrow$		
												$q8$		

Tape number: 49

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
											$\uparrow$			
											$q9$			

Tape number: 50

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
										$\uparrow$				
										$q9$				

Tape number: 51

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
									$\uparrow$					
									$q9$					

Tape number: 52

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
								$\uparrow$						
								$q9$						

Tape number: 53

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	Ξ	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q9$ (under position 4)

Tape number: 54

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q12$ (under position 5)

Tape number: 55

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q10$ (under position 4)

Tape number: 56

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q10$ (under position 3)

Tape number: 57

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q10$ (under position 2)

Tape number: 58

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	M	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ $q10$ (under position 1)

Tape number: 59

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ (2)

$q5$

Tape number: 60

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ (1)

$q11$

Tape number: 61

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ (0)

$q11$

Tape number: 62

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ (-1)

$q11$

Tape number: 63

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	
...	$\wedge$	$\wedge$	1	1	Ψ	1	1	Ω	1	1	1	1	$\wedge$	...

$\uparrow$ (0)

$q14$

Tape number: 64

5.6 -2 × -2

Number of tapes: 73
Tape execution time: 2ms

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			↑													
			$q0$													

Tape number: 0

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				↑												
				$q2$												

Tape number: 1

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					↑											
					$q2$											

Tape number: 2

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						↑										
						$q2$										

Tape number: 3

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							↑									
							$q2$									

Tape number: 4

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						↑										
						$q4$										

Tape number: 5

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q4$											

Tape number: 6

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q4$												

Tape number: 7

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q4$													

Tape number: 8

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
		$\uparrow$ $q4$														

Tape number: 9

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
			$\uparrow$ $q5$													

Tape number: 10

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q5$												

Tape number: 11

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$											
					$q6$											

Tape number: 12

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$										
						$q6$										

Tape number: 13

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$									
							$q12$									

Tape number: 14

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$								
								$q12$								

Tape number: 15

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$							
									$q7$							

Tape number: 16

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$						
										$q7$						

Tape number: 17

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	Ω	$\wedge$	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$ $q8$					

Tape number: 18

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q9$						

Tape number: 19

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q9$							

Tape number: 20

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	Ξ	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$								

Tape number: 21

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q12$							

Tape number: 22

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q7$						

Tape number: 23

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$					
											$q8$					

Tape number: 24

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	$\wedge$	$\wedge$	$\wedge$	$\wedge$	...
												$\uparrow$				
												$q8$				

Tape number: 25

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$					
											$q9$					

Tape number: 26

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$						
										$q9$						

Tape number: 27

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	Ξ	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$							
									$q9$							

Tape number: 28

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$						
										$q12$						

Tape number: 29

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q10$							

Tape number: 30

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q10$								

Tape number: 31

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q10$									

Tape number: 32

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q10$										

Tape number: 33

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q10$											

Tape number: 34

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	M	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
				$\uparrow$ $q10$												

Tape number: 35

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
					$\uparrow$ $q5$											

Tape number: 36

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
						$\uparrow$ $q6$										

Tape number: 37

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
							$\uparrow$ $q12$									

Tape number: 38

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
								$\uparrow$ $q12$								

Tape number: 39

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
									$\uparrow$ $q7$							

Tape number: 40

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
										$\uparrow$ $q7$						

Tape number: 41

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
											$\uparrow$					
											$q8$					

Tape number: 42

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
												$\uparrow$				
												$q8$				

Tape number: 43

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	$\wedge$	$\wedge$	$\wedge$	...
													$\uparrow$			
													$q8$			

Tape number: 44

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
												$\uparrow$				
												$q9$				

Tape number: 45

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
											$\uparrow$					
											$q9$					

Tape number: 46

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
										$\uparrow$						
										$q9$						

Tape number: 47

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
									$\uparrow$ $q9$							

Tape number: 48

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	Ξ	1	Ω	1	1	1	$\wedge$	$\wedge$	...
								$\uparrow$ $q9$								

Tape number: 49

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	$\wedge$	$\wedge$	...
									$\uparrow$ $q12$							

Tape number: 50

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
										$\uparrow$ $q7$						

Tape number: 51

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
											$\uparrow$ $q8$					

Tape number: 52

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
												$\uparrow$ $q8$				

Tape number: 53

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
													$\uparrow$ $q8$			

Tape number: 54

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	$\wedge$	$\wedge$	...
														$\uparrow$ $q8$		

Tape number: 55

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
													$\uparrow$ $q9$			

Tape number: 56

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
												$\uparrow$ $q9$				

Tape number: 57

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
											$\uparrow$ $q9$					

Tape number: 58

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
										$\uparrow$ $q9$						

Tape number: 59

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	Ξ	Ω	1	1	1	1	$\wedge$	...
									$\uparrow$ $q9$							

Tape number: 60

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
										$\uparrow$ $q12$						

Tape number: 61

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
									$\uparrow$ $q10$							

Tape number: 62

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
								$\uparrow$ $q10$								

Tape number: 63

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
							$\uparrow$ $q10$									

Tape number: 64

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
						$\uparrow$ $q10$										

Tape number: 65

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	M	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
					$\uparrow$ $q10$											

Tape number: 66

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
						$\uparrow$ $q5$										

Tape number: 67

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
					$\uparrow$ $q11$											

Tape number: 68

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
				$\uparrow$ $q11$												

Tape number: 69

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
			$\uparrow$ $q11$													

Tape number: 70

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
		$\uparrow$ $q11$														

Tape number: 71

	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	
...	$\wedge$	$\wedge$	Φ	1	1	Ψ	Φ	1	1	Ω	1	1	1	1	$\wedge$	...
			$\uparrow$ $q14$													

Tape number: 72

5.7 Extended Test summary

The following tests can be generated from the source code.

To gain some perspective I have produced a second machine that multiplies in units of 2 instead of 1 called "FasterMult" (See Figure 11). I have run a couple of tests on each machine to demonstrate the difference in performance between them.

Tests with tapes not included in this document

Machine	Test name	Time(ms)	Tapes
With Negative Numbers	20×28	13753.4356	332067
Faster Mult	20×28	6751.9592	166323
With Negative Numbers	22×23	10793.5741	270343
Faster Mult	22×23	5297.7857	135435

Figure 10: Extended tests

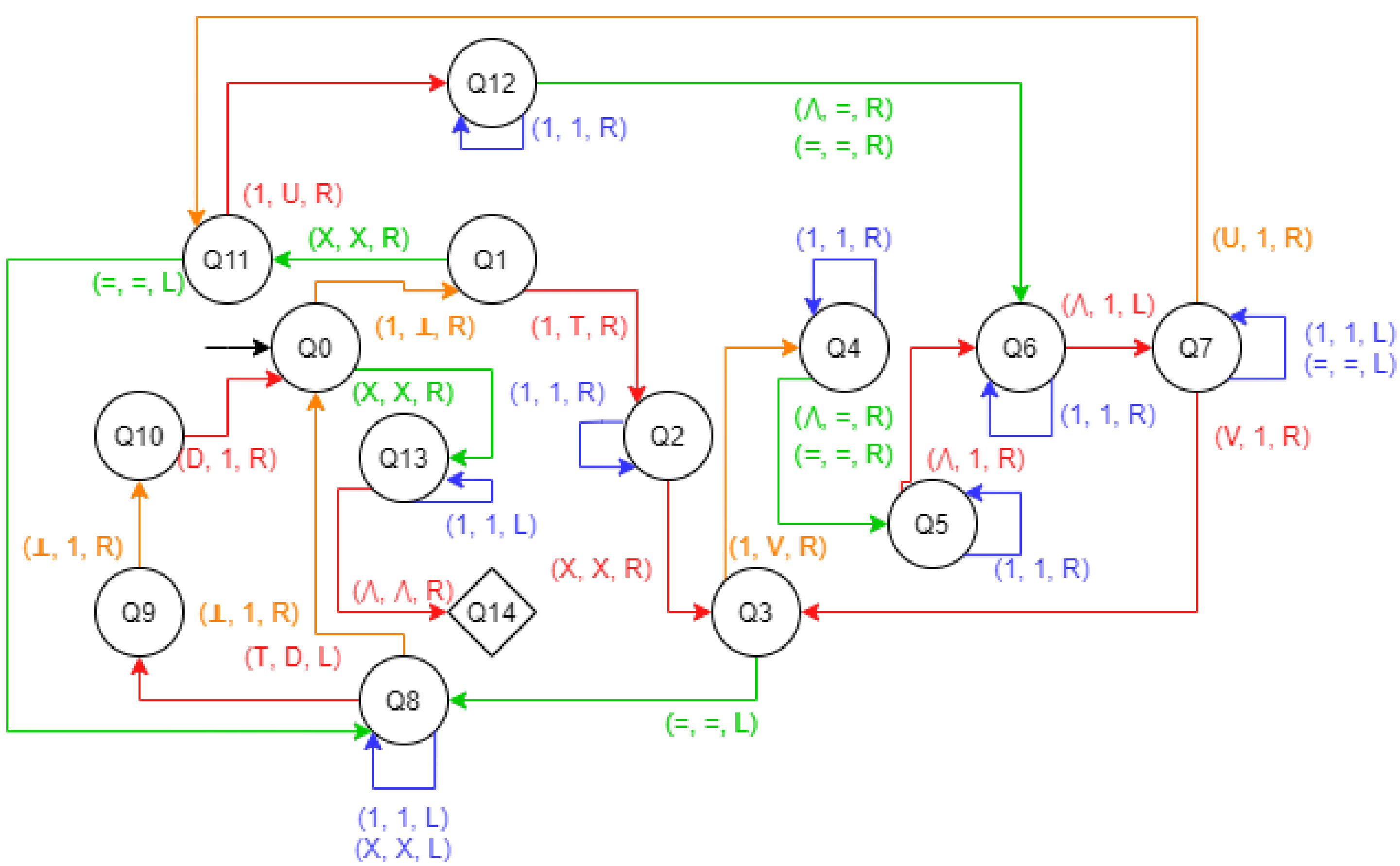

Figure 11: Turing machine to multiply faster

6 Efficiency of program

From the tests in Figure 10 can see that a performance suffers significantly on larger multiplications because the turing machine must iterate through every cell on the tape between it and the place it will write. This gets less efficient as the tape writes bigger and bigger numbers, the time to complete 100x100 for example would take more than 48 hours with this (faster) machine, because the closer to the end of the multiplication it gets the slower it becomes. Although the faster machine performs at nearly twice the speed of the slower one, it still suffers from the same problem.

This method of calculation is extremely inefficient, the speed can be significantly increased by increasing the number of symbols copied in a single pass along the tape, but that can only produce a linear improvement, and vastly increases the complexity, while larger multiplications take exponentially longer and use exponentially more tapes.

7 Power of 3 machine

I would build a Turing machine called $Mres()$, which results in a tape holding only the rightmost value from the the result of $Mmult()$ (everything to the right of Ω).
To extend a multiplier machine to a power of 3 machine, you could use the machine to calculate the value of n^2 first: $Mres(Mmult(n, n))$, and then multiply the result of n^2 by n to get n^3
In other words: $Mmult(res(Mmult(n, n)), n)$