

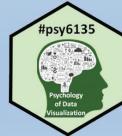
# Visualizing Uncertainty

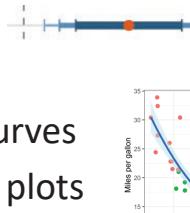


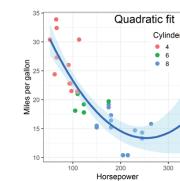
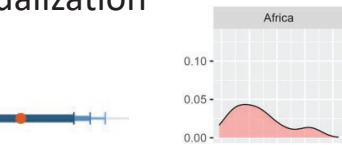
# Michael Friendly

# Psych 6135

<https://friendly.github.io/6135/>



- Uncertainty in statistics & visualization
  - Visualizing distributions
  - “Error bars”
  - Bayesian uncertainty
  - Uncertainty in fitted curves
  - Hypothetical outcome plots
  - Cartographic uncertainty

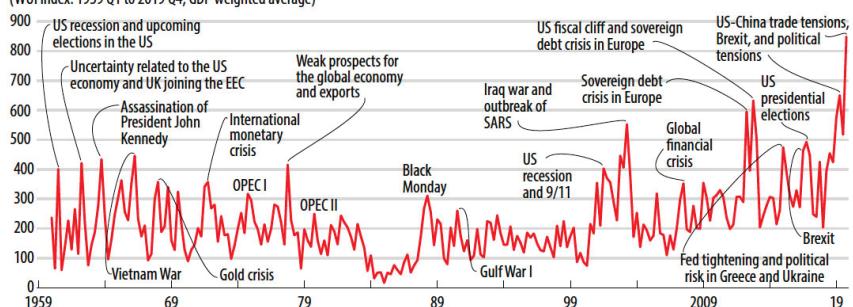


2

# What is uncertainty?

## **Uncertain times**

Global uncertainty has surged to a record high.  
(WUI index: 1959 01 to 2019 04. GDP weighted average)

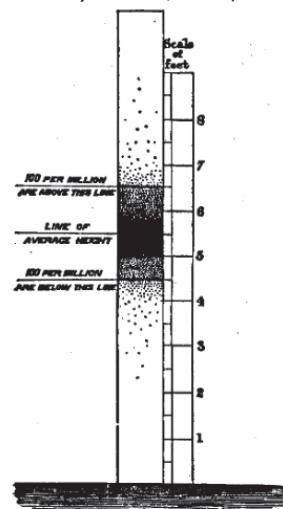


Sources: Abir, H., N. Bloom and D. Furceri (2018), World Uncertainty Index (WUII), mimeo.

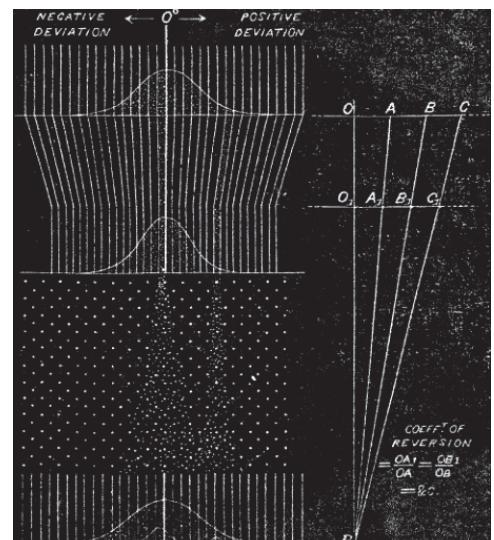
**Note:** The WUI is computed by counting the frequency of the word "uncertain" (or the variant) in Economist Intelligence Unit country reports. The WUI is then normalized by total number of words and rescaled by multiplying by 1,000. A higher number means higher uncertainty and vice versa. The aggregate and disaggregate data by country and regions are available at [www.worlduncertaintyindex.com](http://www.worlduncertaintyindex.com).

# Galton: Variation & Uncertainty

## Distribution of human height (*Heredity Genius*, 1867)



Quincunx: How many small effects → Normal



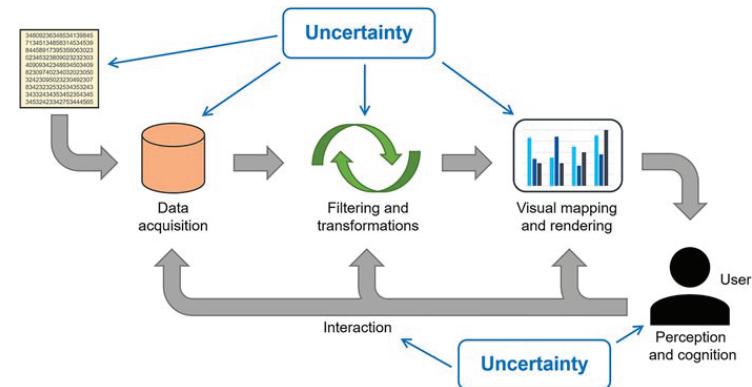
# Sources of uncertainty

- Where does the uncertainty in statistics come from? There are four main sources:
  - Data:** data can contain random error or have missing entries.
  - Assumptions:** model assumptions assess plausible values from distributions.
  - Models:** there is choice over the techniques and models we use.
    - Different analysts may choose different methods, yielding different estimates.
  - Replications:** Estimates of effects can vary from study to study. How to synthesize these?

See: [Uncertainty Toolkit, Ch 3](#) for other terms to understand uncertainty

5

# Where does uncertainty come from?



From: D. Weiskopf, <https://www.frontiersin.org/articles/10.3389/fbinf.2022.793819/full>

6

# Problems: data, models, graphics

- Uncertainty is fundamental to data analysis & models
  - data:** IQR, std dev., std error, ... (variation)
  - assumptions:** we assume some distribution for errors, e.g.,  $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ , independent with constant variance
  - models:**
    - classical: confidence intervals, p-values;
    - Bayesian: credible intervals, posterior distributions
- In data graphics,
  - Easy to show “fit” – means, regression estimates, ...
  - Harder to show the uncertainty in these numbers

7

# P-values, significance & uncertainty

ASA President's Task Force on Statistical Significance

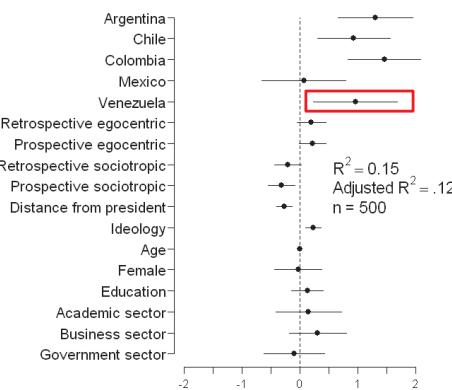
- “Much of the controversy surrounding **statistical significance** can be dispelled by better understanding of uncertainty, variability, multiplicity & replicability”
- “Different measures of uncertainty can **complement** each other; no single measure serves all purposes”
- “Controlling and accounting for uncertainty begins with the **design** of the study”
- “The **theoretical basis** of statistical science offers general strategies for dealing with uncertainty”
  - Frequentist** approach: p-values, confidence intervals & prediction intervals
  - Bayesian** approach: Bayes factors, posterior probability distributions, credible intervals

8

## Model fits as uncertainties

Variable	Coefficient (Standard Error)
Constant	.41 (.93)
<b>Countries</b>	
Argentina	1.31 (.33) *** B.M
Chile	.95 (.32) *** B.M
Colombia	1.46 (.32) *** B.M
Mexico	.07 (.32) ^CH.CO.V
Venezuela	.96 (.37) #P.B.M
<b>Threat</b>	
Retrospective egocentric economic perceptions	.20 (.13)
Prospective egocentric economic perceptions	.22 (.12) *
Retrospective sociotropic economic perceptions	-.21 (.12) *
Prospective sociotropic economic perceptions	-.32 (.12) **
Ideological Distance from president	
<b>Ideology</b>	
Ideology	.23 (.07) ***
<b>Individual Differences</b>	
Age	.00 (.01)
Female	-.03 (.21)
Education	.13 (.14)
Academic Sector	.15 (.29)
Business Sector	.31 (.25)
Government Sector	-.10 (.27)
R <sup>2</sup>	.15
Adjusted R <sup>2</sup>	.12
n	500
*** p < .01, ** p < .05, * p < .10 (two-tailed)	
A Coefficient is significantly different from Argentina's at p < .05;	
B Coefficient is significantly different from Brazil's at p < .05;	
CH Coefficient is significantly different from Chile's at p < .05;	
CO Coefficient is significantly different from Colombia's at p < .05;	
M Coefficient is significantly different from Mexico's at p < .05;	
V Coefficient is significantly different from Venezuela's at p < .05	

Coefficients \*\* & std errors express uncertainty  
Can we do better?

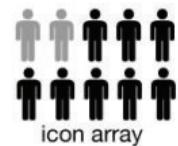
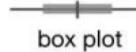
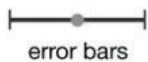


Source: [tables2graphs.com](https://tables2graphs.com)

9

## Graphical annotations for uncertainty

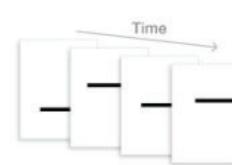
### Intervals and Ratios



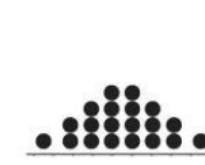
### Distributions



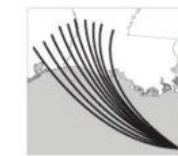
violin plot



hypothetical outcome plot



quantile dot plot



ensemble plot

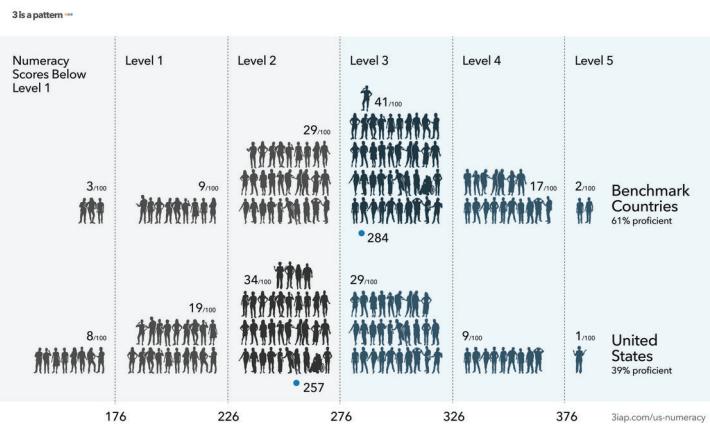
From: Padilla, Kay & Hullman (2021), *Uncertainty Visualization*, DOI: 10.1002/9781118445112.stat08296

10

## Putting the people back into charts about them:

### U.S. Numeracy Education Has Room for Improvement

Comparing PIAAC Numeracy Scores



From: <https://3iap.com/us-numeracy>

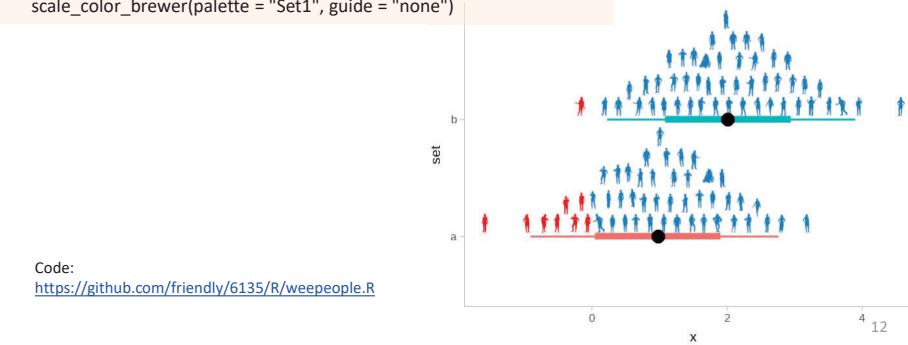
Weepeople font: <https://github.com/propublica/weepeople>

### Using weepeople font in R graphics

See: [https://github.com/mjskay/uncertainty-examples/blob/master/weepeople\\_dotplots.md](https://github.com/mjskay/uncertainty-examples/blob/master/weepeople_dotplots.md)

1. Download font from: <https://github.com/propublica/weepeople/>
2. Register font: systemfonts::register\_font(name = "weepeople")

```
df = data.frame(x = qnorm(ppoints(100), 1:2),
  set = c("a", "b"), icon = factor(sample(52, 100, replace = TRUE)))
people = c(letters, toupper(letters))
df |> ggplot(aes(x = x, y = set, group = set, shape = icon, color = x > 0)) +
  geom_dots(family = "weepeople", dotsize = 2.4, layout = "swarm") +
  scale_shape_manual(values = people, guide = "none") +
  scale_color_brewer(palette = "Set1", guide = "none")
```



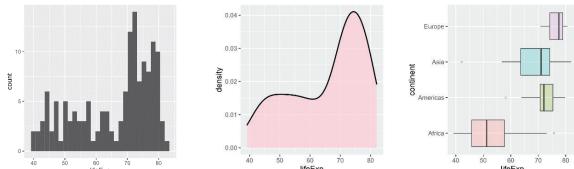
Code:  
<https://github.com/friendly/6135/R/weepeople.R>

11

# Visualizing distributions

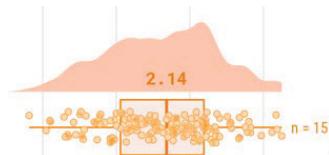
- The basics:

- Histograms
- Density plots
- Boxplots



- Doing better:

- violin plots
- rainclouds
- {ggdist}: data, distribution, interval



13

# Histograms

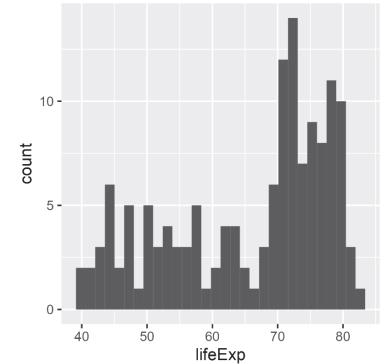
- Perhaps the simplest display

- divide the data into bins: [40-42), [42-44), ...
- bar plot of the frequencies: length ~ frequency in bin

```
library(gapminder)  
gapminder_2002 <- gapminder %>%  
  filter(year == 2002)
```

```
ggplot(gapminder_2002,  
       aes(x = lifeExp)) +  
  geom_histogram()
```

'stat\_bin()' using `bins = 30`. Pick better value with `binwidth`.

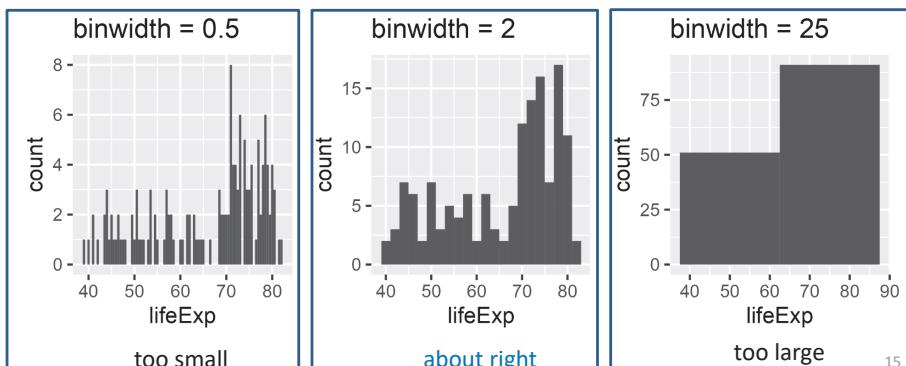


14

## Histograms: bin width

- Explicitly selecting the binwidth shows:
  - the “Goldilocks” principle: **just about right**
  - the default is often OK, but optimal “best” is harder to define

```
ggplot(gapminder_2002, aes(x = lifeExp)) + geom_histogram(binwidth = )
```



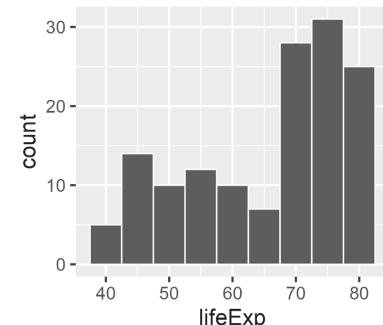
15

## Histograms: other properties

- Pay attention to graphic details
  - border color to make bars distinct
  - set bar boundaries: to edges? – it can make a difference

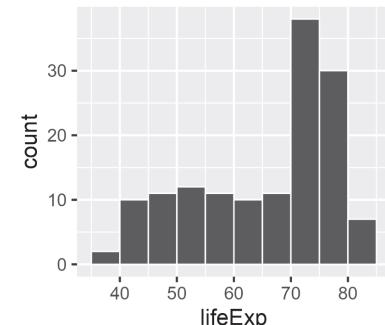
```
geom_histogram(..., color = "white")
```

color = "white"



```
geom_histogram(..., boundary=50)
```

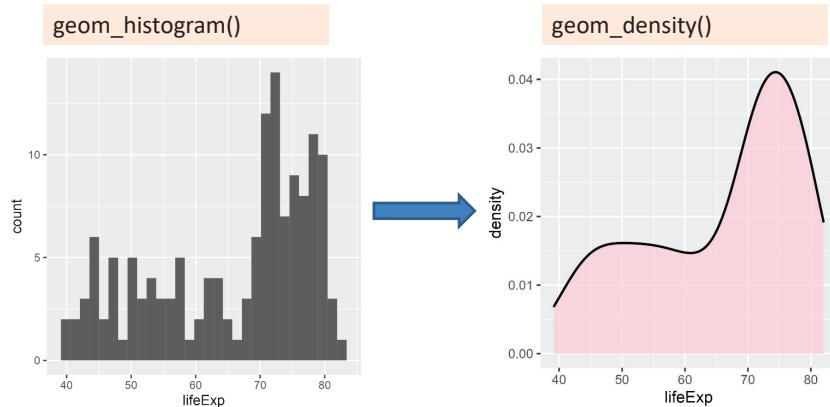
boundary = 50



16

# Density plots

- Basic idea: Smooth the distribution to avoid artifacts of discrete bins and bin centers
  - Uses a “kernel”, e.g. gaussian, averaged over a moving window



17

# Kernel density estimation

Imagine a distribution of potential density centered at each  $X_i$ , w/ sd = h (bandwidth)  
 $x \sim \mathcal{N}(\mu=X_i, \sigma=h)$

Five observations, each with a distribution

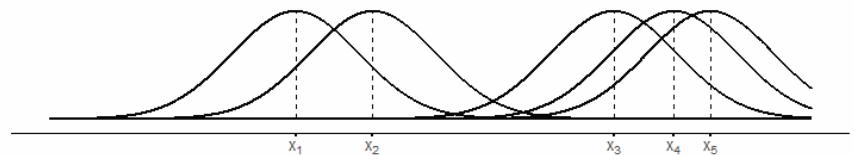
A moving window sweeps across, averaging the density for all observations

Kernel function

$$K(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$$

The Kernel Density Estimator is:

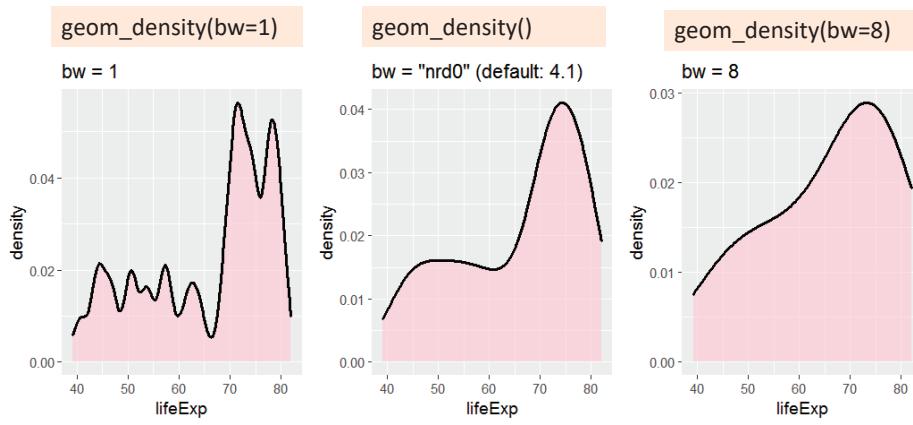
$$f(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right)$$



18

# Density plots: bandwidth

- The result depends on the width of the moving window – **bandwidth**
  - The default calculation is usually reasonable, but beware of weird data

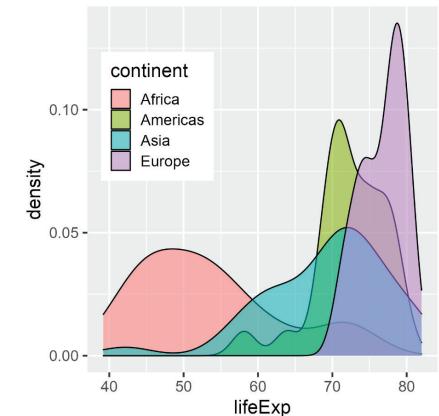


19

# Comparing groups

For multiple groups, using the fill aesthetic → overlaid curves -- is a decent start  
 But even with transparency it may be hard to see the separate curves

```
gap_2002c <-  
  gapminder_2002 %>%  
  filter(continent != "Oceania")  
  
ggplot(gap_2002c,  
       aes(x = lifeExp,  
            fill = continent)) +  
  geom_density(alpha = 0.5) +  
  theme(legend.position = c(.2, .7))
```



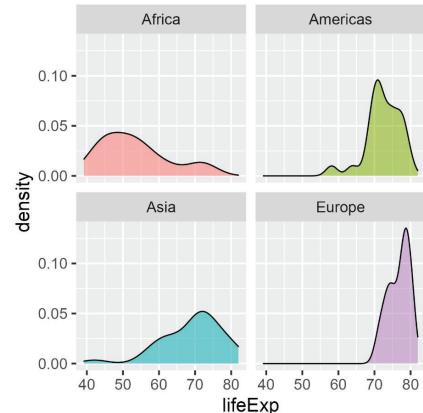
NB: ggplot picks a joint bandwidth, here: 2.52

20

## Comparing groups: Facets

Faceting solves the overlap problem, but the eye has to move from panel to panel to make comparisons.

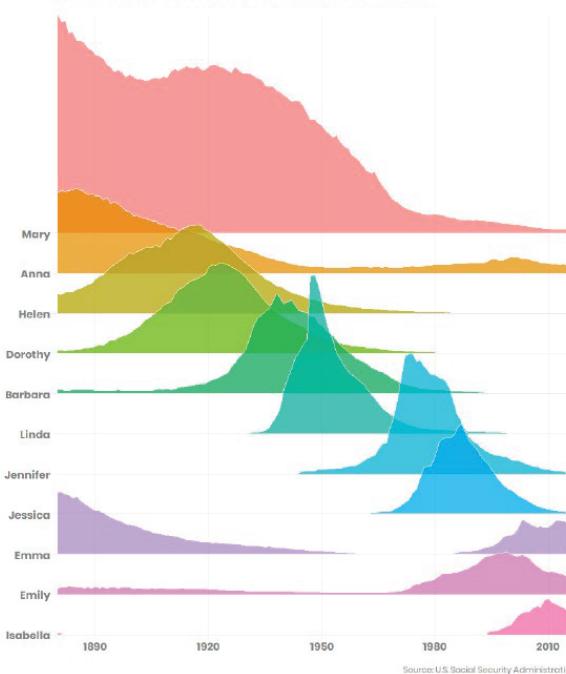
```
ggplot(gap_2002c,  
       aes(x = lifeExp,  
            fill = continent)) +  
  geom_density(alpha = 0.5) +  
  facet_wrap(~ continent) +  
  theme(legend.position = "none")
```



21

### Most popular girl names in the U.S.

Top 2 names with the highest mean and/or maximum per quarter are shown.



## Baby names

Ridgeline plots are particularly effective with more than a few categories, and when the distributions differ in [shape](#) as well as [central location](#)

Which names stand out from the rest?

What is the role of color here?

Note the subtle use of white to outline each distribution

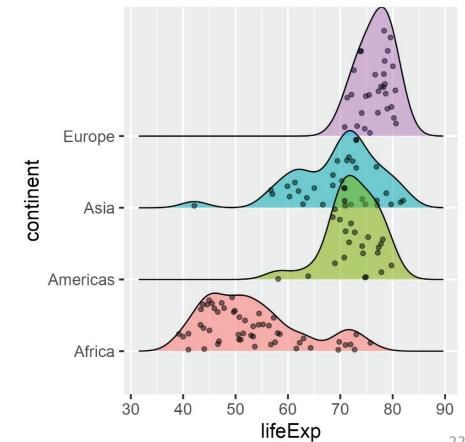
23

## {ggridges}: Ridgeline plots

Ridgeline plots are partially overlapping density plots, suggesting a mountain range.

- Useful for comparing distributions over time or circumstances
- Adding jittered points helps to show where the data are

```
library(ggridges)  
ggplot(gap_2002c,  
       aes(x=lifeExp,  
            y=continent,  
            fill=continent)) +  
  geom_density_ridges(  
    alpha = 0.5,  
    jittered_points=TRUE) +  
  theme(legend.position = "none")
```

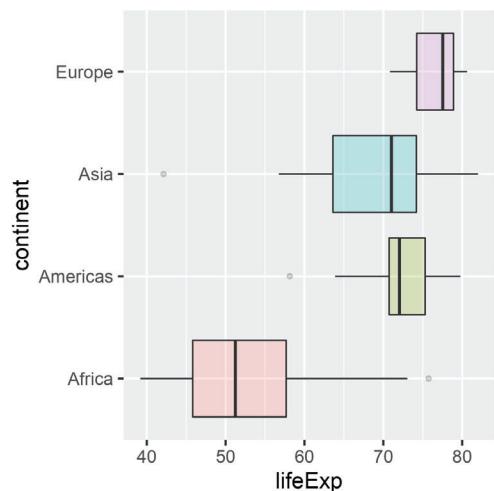


22

## Boxplots

Boxplots give a more schematic summary of a dataset— median, quartiles, whiskers & outliers

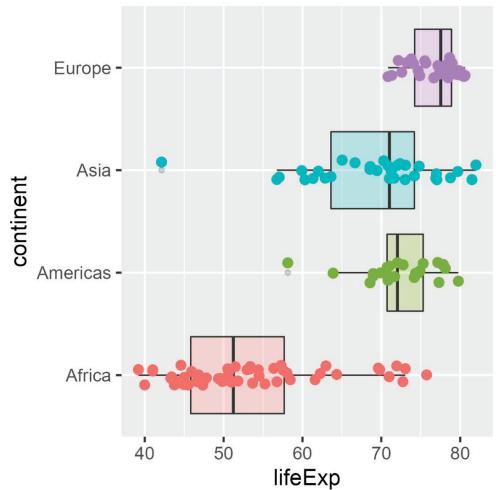
```
ggplot(gap_2002c,  
       aes(x=lifeExp,  
            y=continent,  
            fill=continent)) +  
  geom_boxplot(alpha = 0.2) +  
  theme(legend.position = "none")
```



# Boxplots

But perhaps too schematic— it sometimes helps to see the data as jittered points

```
ggplot(gap_2002c,
       aes(x=lifeExp,
           y=continent,
           fill=continent)) +
  geom_boxplot(alpha = 0.2) +
  geom_point(aes(color = continent),
             position =
               position_jitter(height=0.1)) +
  theme(legend.position = "none")
```

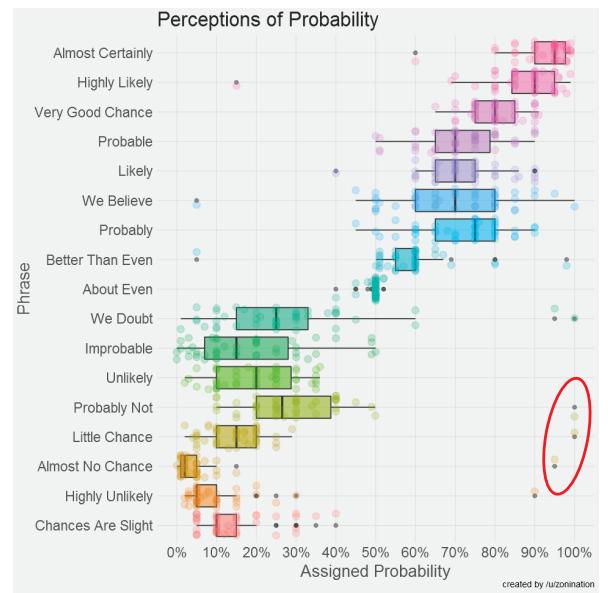


# How people view “probability”

What makes this graph successful?

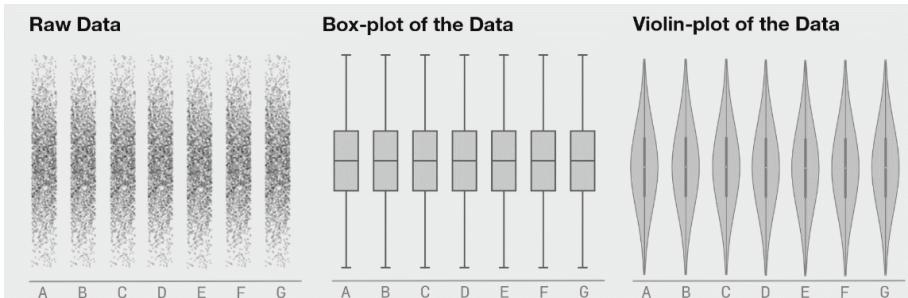
Note the wide range of variability (uncertainty) in the estimates:  
“about even” vs. “we believe”

Outliers: individuals who misunderstood instructions?



# Problems with boxplots revealed

Boxplots are fine for unimodal distributions – well summarized by Q1, Median, Q3  
They are insensitive to **multi-modal** data

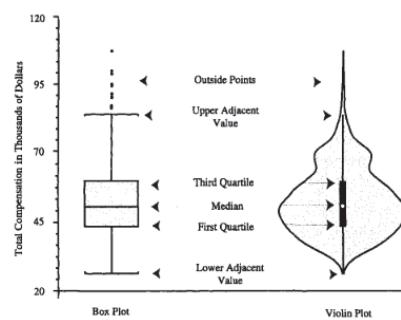


# Violin plots

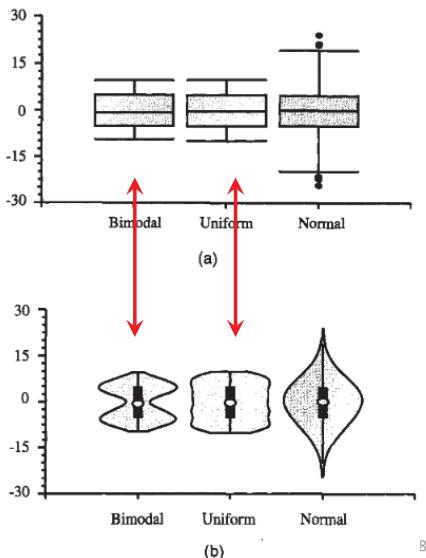
Boxplots are great for ~ normal data

- Shows center, spread, outliers

Violin plots add a (reflected) density curve to show the **shape** of the distribution



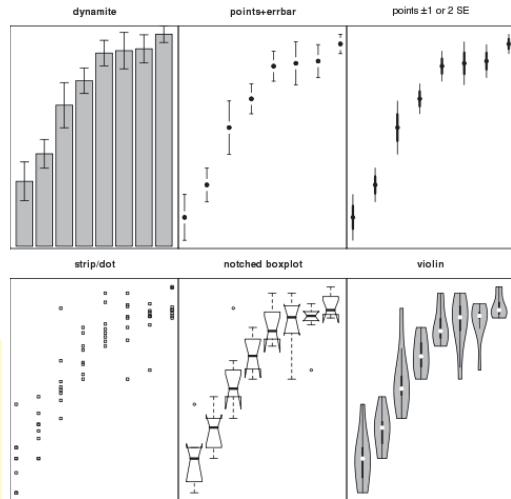
Hintze & Nelson (1998), *American Statistician*, 52:2, 181-184



## Comparing groups: Summary + Uncertainty

Six different graphs for comparing groups in a one-way design

- which group means differ?
- equal variability?
- distribution shape?
- what do error bars mean?
- unusual observations?



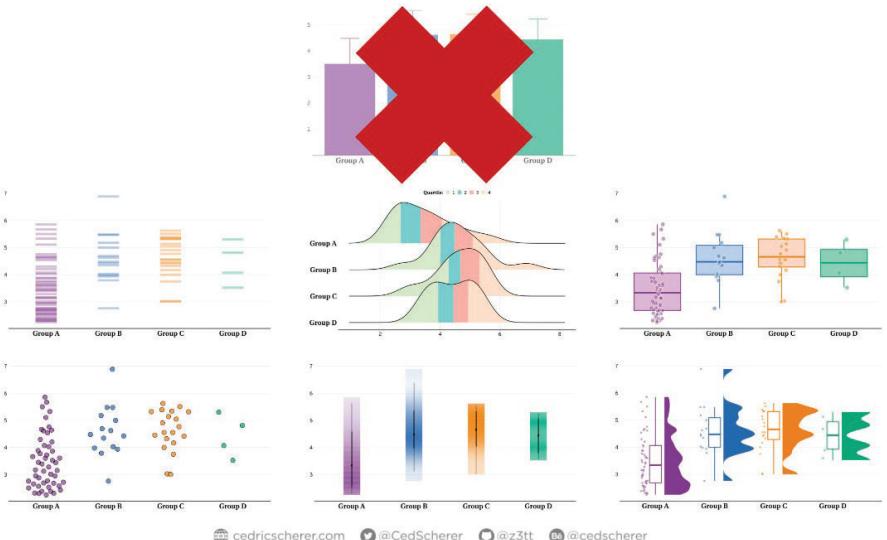
Never use dynamite plots

Always explain what error bars mean

Consider tradeoff between summarization & exposure

29

## Alternatives to dynamite plots



cedricscherer.com @CedScherer @z3tt @cedscherer

31

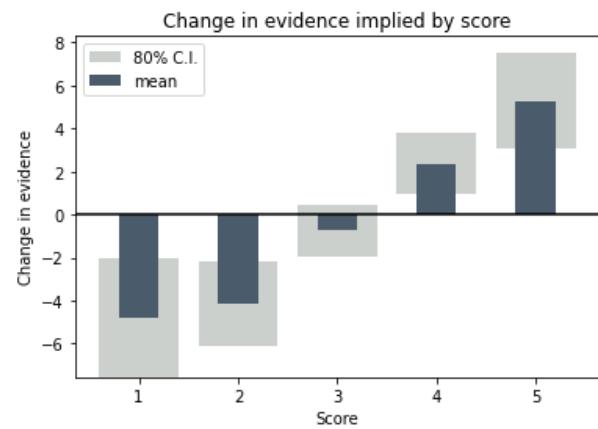
## De-fusing the barplot

If you insist on bars, use a better visual representation of uncertainty or CI



"Friends don't let friends make barplots" (video)

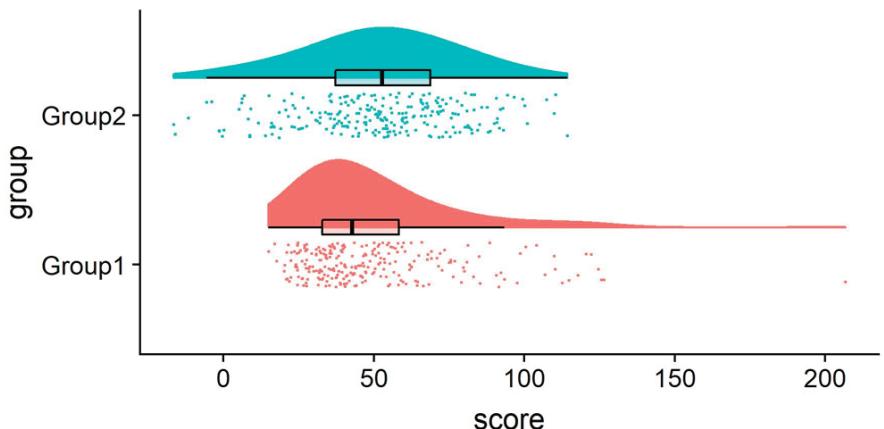
<https://barbarplots.github.io/>



32

## Raincloud plots

Raincloud plots combine **density curve** & **boxplot**, but also show the observations as jittered **points**

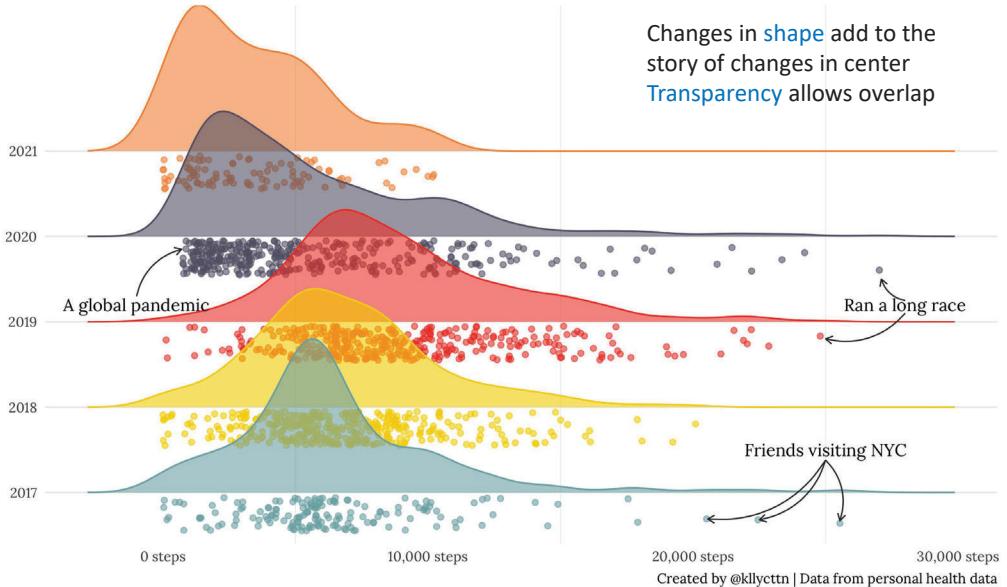


Allen M, Poggiali D, Whitaker K et al. Raincloud plots: a multi-platform tool for robust data visualization [version 2]. Wellcome Open Res 2021, 4:63 (doi: 10.12688/wellcomeopenres.15191.2)

33

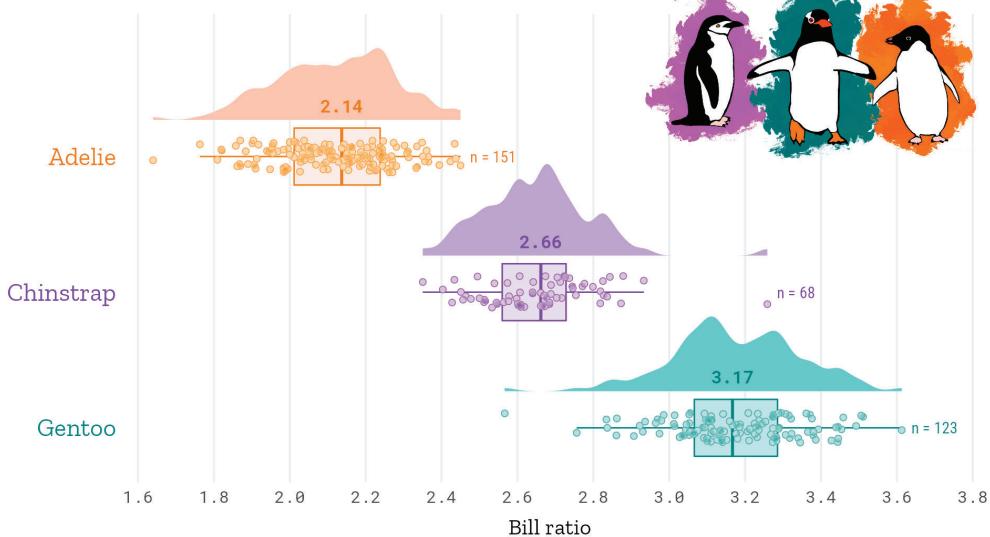
## How many steps have I taken since 2017?

Since July 2017, I have tracked the number of steps I've taken (almost) every day. In a little over 4 years, I have taken **9,232,798** steps. This includes days spent walking around New York with visiting friends, running a half-marathon, and a pandemic that dropped my step count to nearly 0.



## Bill Ratios of Brush-Tailed Penguins (*Pygoscelis spec.*)

Distribution of bill ratios, estimated as bill length divided by bill depth.



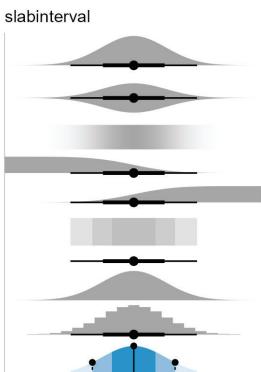
Gorman, Williams & Fraser (2014) PLoS ONE DOI: 10.1371/journal.pone.0090081  
Visualization: Cédric Scherer • Illustration: Allison Horst

## {ggdist} package

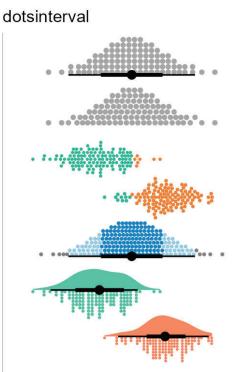


The {ggdist} package provides three families of geoms for visualizing uncertainty

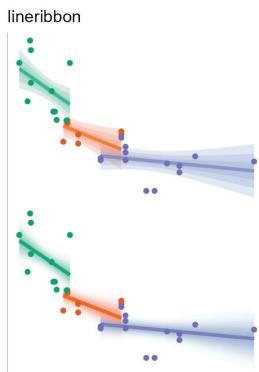
Point estimates + intervals



Dots + intervals



Fitted lines + uncertainty bands

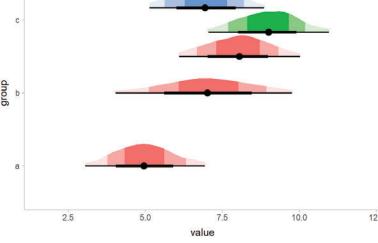


Some examples from the three main families of ggdist geometries

Design of {ggdist} makes it easy to combine two or more graphical representations –  
data + distribution + interval

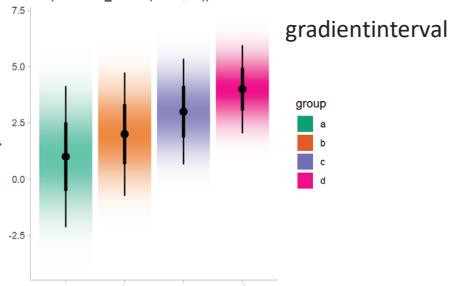
### halfeye, varying fill

`stat_halfeye(position = 'dodgejust')  
aes(fill = subgroup, fill_ramp = stat(cut_cdf_q(cdf)))`

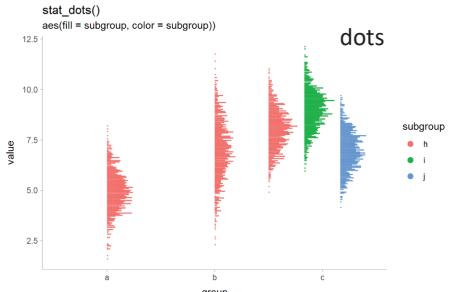


gradientinterval

`stat_dist_gradientinterval(fill_type = "gradient")  
aes(dist = dist_normal(mean, sd))`



dots



# One-way ANOVA example

```
set.seed(5)
n <- 10; ngrps <- 5
ABC <- tibble(
  grp = rep(c("A", "B", "C", "D", "E"), n),
  response = rnorm(n * ngrps, c(0, 1, 2, 1, -1), 0.5)
)
```

```
ABC.mod = lm(response ~ grp, data = ABC)
broom::tidy(ABC.mod)
```

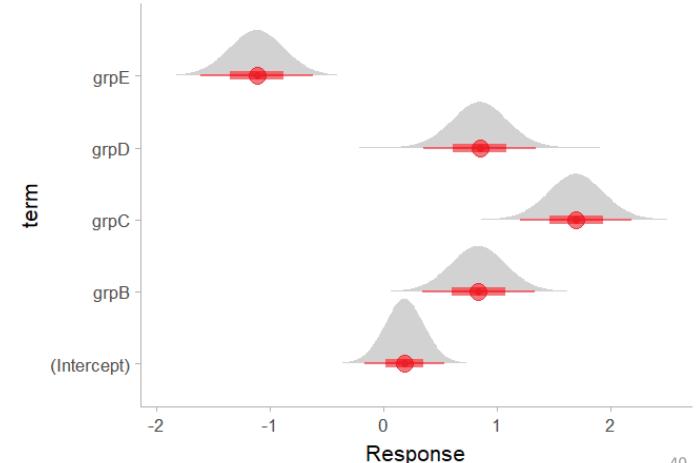
```
# A tibble: 5 × 5
  term      estimate std.error statistic   p.value
  <chr>     <dbl>    <dbl>     <dbl>    <dbl>
1 (Intercept) 0.182   0.173     1.05   0.300
2 grpB        0.833   0.245     3.40   0.00143
3 grpC        1.69    0.245     6.91   0.000000138
4 grpD        0.846   0.245     3.45   0.00122
5 grpE       -1.12    0.245    -4.56   0.0000394
```

Frequentist theory

$$\tilde{\beta}_i \sim \text{student\_t}(\nu, \hat{\beta}_i, \sigma_{\hat{\beta}_i})$$

v: df.residual(mod)  
 $\hat{\beta}_i$ : estimate  
 $\sigma(\hat{\beta}_i)$ : std.error

```
ABC.mod |>
  tidy() |>
  ggplot(aes(y = term)) +
  stat_halfeye(
    aes(xdist = dist_student_t(df = df.residual(ABC.mod),
                                mu = estimate, sigma = std.error)),
    alpha = 0.5, interval_size_range = c(1, 3), color = "red") +
  xlab("Response")
```

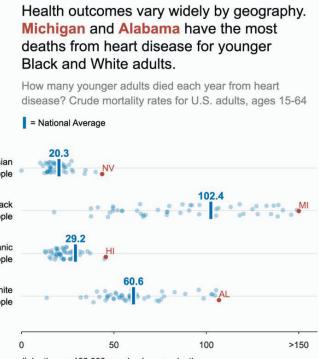
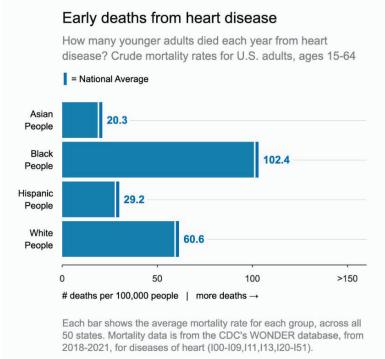


39

40

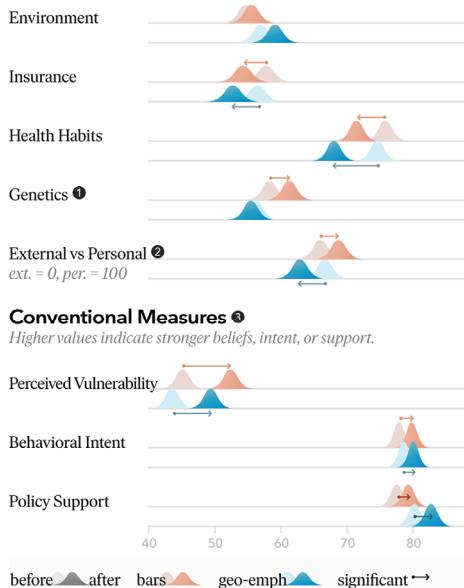
## What works?

Holder & Padilla (2024): Compare different graph forms for visualizing risk disparities between racial and other social groups



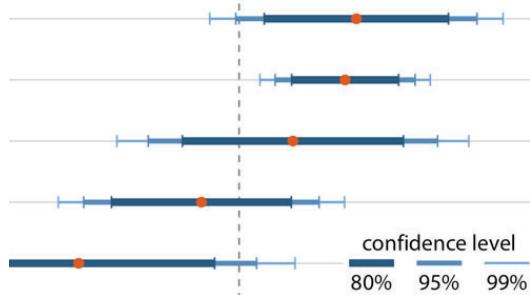
### Attribution Measures

Higher values indicate stronger beliefs.



41

42



Standard deviation vs. standard error

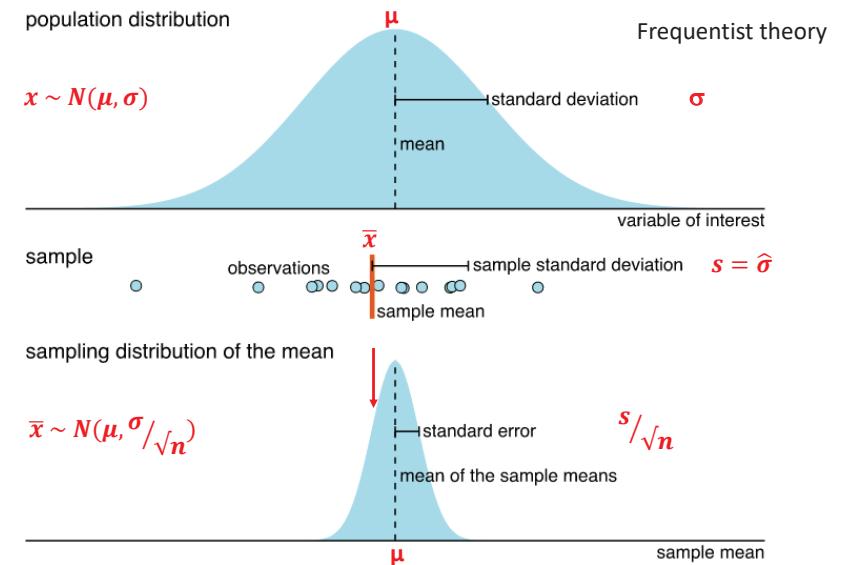
What is a “confidence interval”

Frequentist vs. Bayesian

## Error bars

43

## Key ideas of statistical sampling



From: Claus Wilke (2021), *Fundamentals of Data Visualization*, <https://clauswilke.com/dataviz/>, Ch 16

44

## What's a “confidence interval”?



**Steve Haroz** @steveharoz.com · 2h  
A 95% confidence interval could be

- 1.96 \* the standard error
- 1.96 \* the standard error of means
- 95% inner quantile of bootstrapped means
- 95% inner quantile of bootstrapped subject means
- 1.96 \* the standard deviation
- 95% inner quantile of values
- 95% of damn near anything

#stats



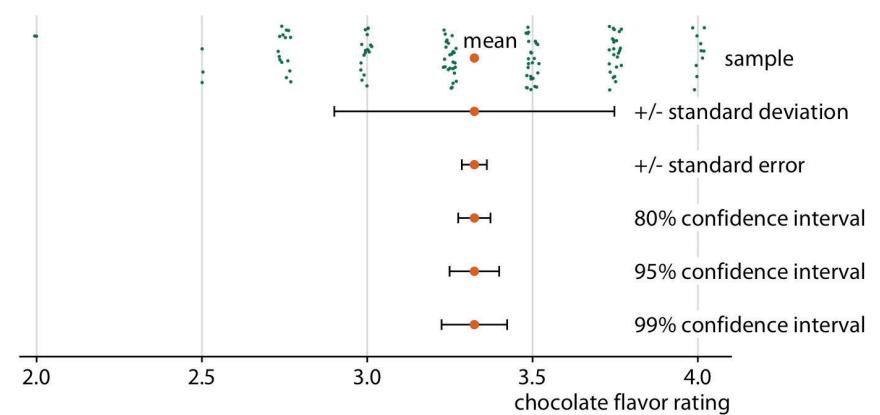
3

6

...

## Visualizing distributions: Error bars

There are many ways to show variability in a single sample



Expert ratings of 125 chocolate bars manufactured in Canada

<https://bsky.app/profile/steveharoz.com/post/3ko5xd7waa42m>

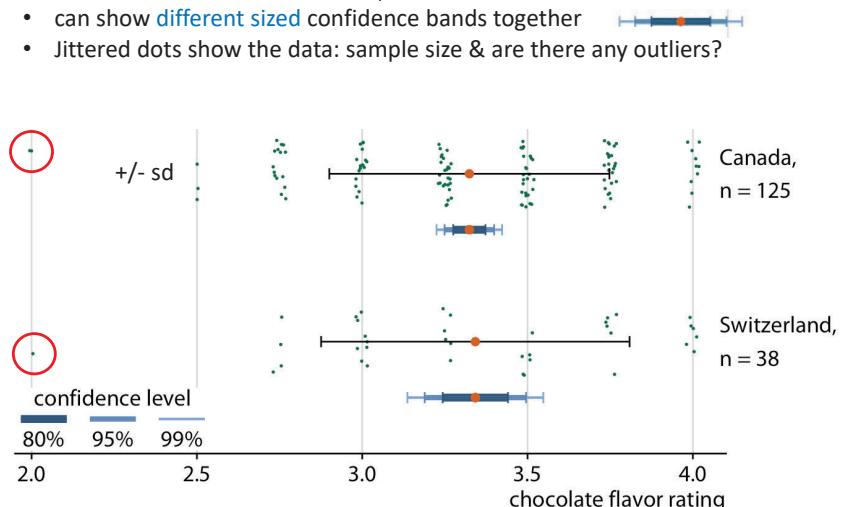
45

From: Claus Wilke (2021), *Fundamentals of Data Visualization*, <https://clauswilke.com/dataviz/>, Ch 16

46

## Comparing distributions: Sample size

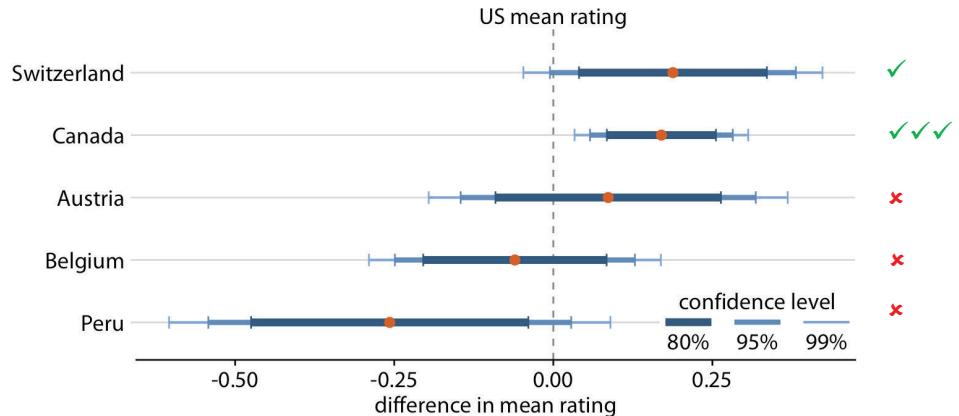
- means and standard deviations are similar for Canada & Switzerland
- confidence interval widths  $\sim 1/\sqrt{n}$
- can show different sized confidence bands together
- Jittered dots show the data: sample size & are there any outliers?



47

## Comparing distributions: Contrasts

- For comparison of one group to all others, plot the difference directly
- Easy to see which differences exclude 0, at what confidence level



48

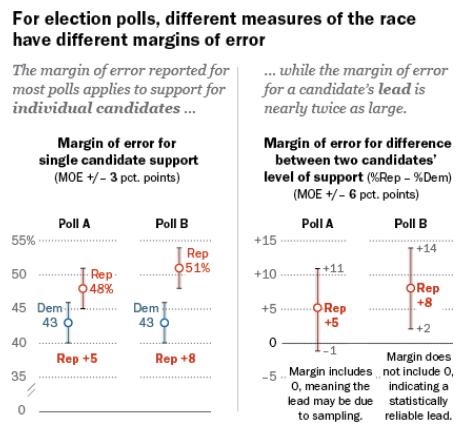
## Intervals: Direct vs. Differences

The standard error for the difference between two means is always larger than the standard error of either mean

$$SE(\bar{x}) = \sqrt{s^2 / n}$$

$$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{s_1^2 / n_1 + s_2^2 / n_2}$$

When separate intervals are shown, the visual inference is that groups differ significantly if intervals do not overlap.



49

## What kind of intervals?



### Frequentist

- Confidence interval
- Scope: repeated (hypothetical) samples
- Center: parameter estimate
  - $\mu \rightarrow \bar{x}; \beta \rightarrow \hat{\beta}$
- Width:  $\sim \text{std. error} = \hat{\sigma}/\sqrt{n}$
- Interpretation: true parameter w/in this interval  $1-\alpha$  % (in repeated samples)

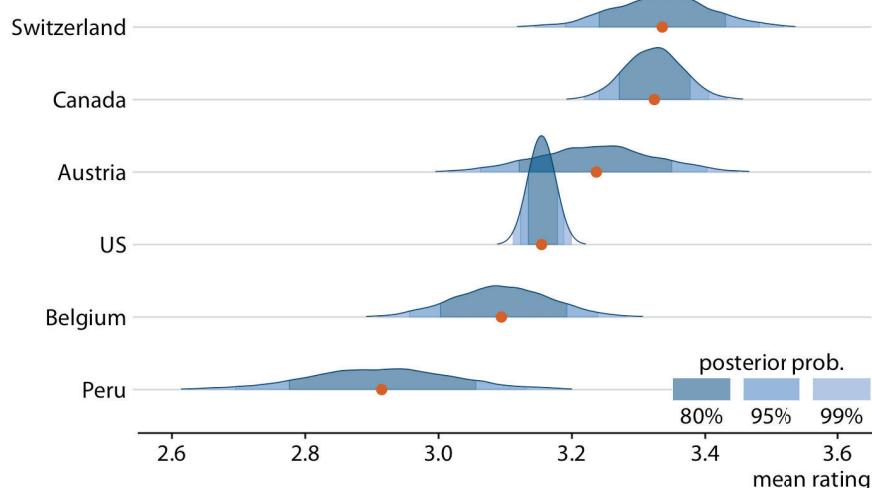
### Bayesian

- Credibility interval
- Scope: repeated draws from the posterior distribution
- Center: median of posterior distribution
- Width: MAD sd of posterior
- Interpretation: Given prior, expect parameter w/in this interval  $1-\alpha$  % of draws

50

# Bayesian intervals

Distribution of repeated draws from posterior distribution



From: Claus Wilke (2021), *Fundamentals of Data Visualization*, <https://clauswilke.com/dataviz/>, Ch 16

51



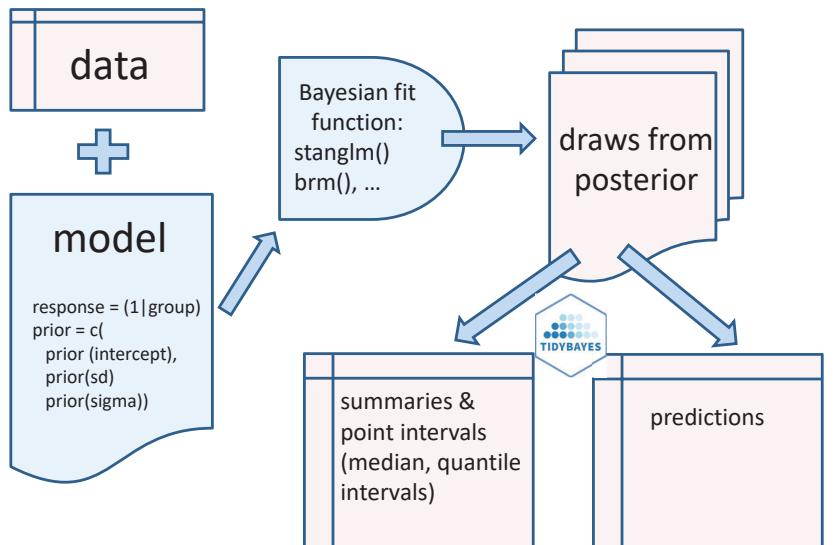
# tidybayes: Bayesian analysis + tidy data + geoms

- The {tidybayes} package makes it easier to combine Bayesian analysis with insightful ggplot visualization
  - Bayesian packages: JAGS, Stan (rstanarm), brms
  - Inputs: data, model specifications aren't tidy
    - Need to translate data into forms these packages expect
  - Outputs: Posterior draws, distributions aren't tidy
    - Need to translate these into form suitable for summaries & plotting
      - Extract tidy fits and predictions from models
      - Summarize posterior distributions
      - Visualize priors and posteriors

Docs: <http://mjskay.github.io/tidybayes/>

52

# The Bayesian process



53

# Posterior = Prior × Likelihood

We have: Data, some model, some parameter(s) of interest,  $\theta$

Can calculate likelihood,  $p(Data|\theta)$

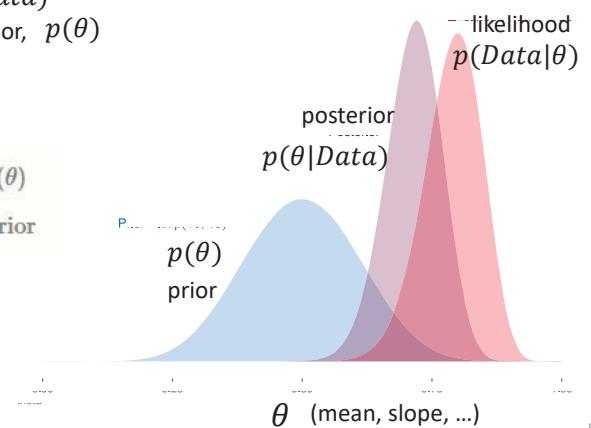
Want: posterior:  $p(\theta|Data)$

Previous research: some prior,  $p(\theta)$

Bayes theorem:

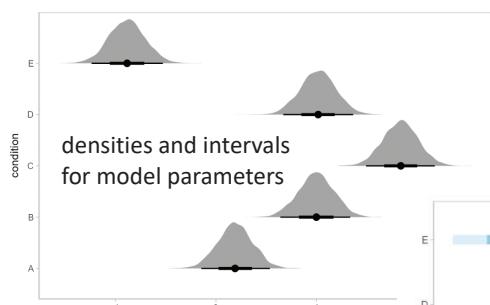
$$p(\theta|Data) \propto p(Data|\theta) \cdot p(\theta)$$

$$\text{posterior} \propto \text{likelihood} \cdot \text{prior}$$

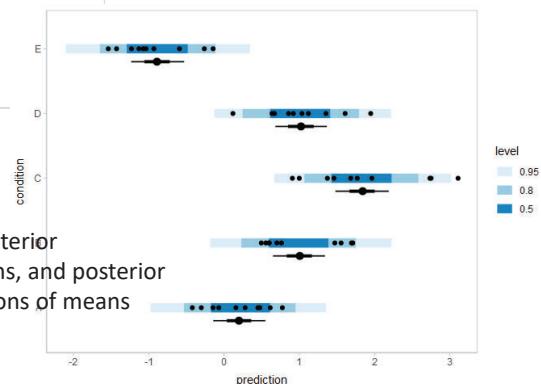


54

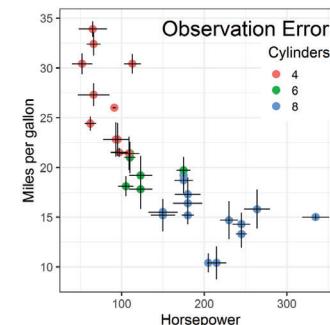
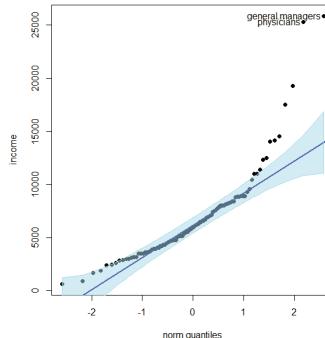
# tidybayes plots



Everything we can do for data with  
{ggdist} we can do for Bayesian  
models with {tidybayes}



55



- QQplots
- Model fit plots

## Uncertainty in fits & curves

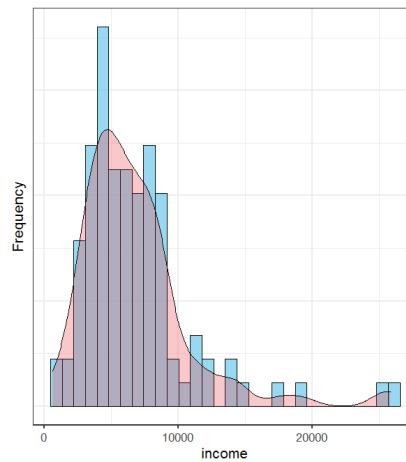
56

## QQ plots

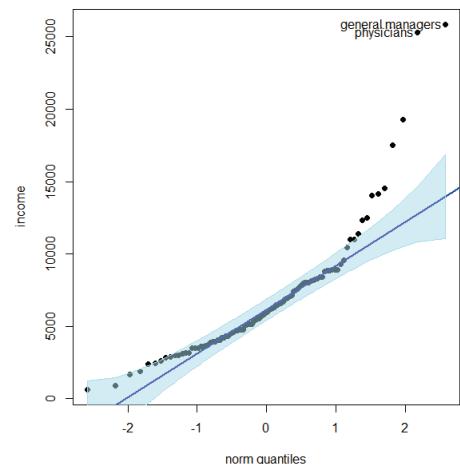
- How close is my data to a {Normal | exponential |  $\chi^2$ } distribution?
- There are lots of statistical tests, but these don't tell **why** or **where** a distribution is rejected.
- These tests are also overly sensitive to small departures
- Plot observed Quantiles vs. theoretical Quantiles
  - If observed  $\sim$  theoretical with **slope = 1**, OK
  - **Confidence bands** help to identify deviation from model & outliers
- Use cases:
  - Is a single variable reasonably normally distributed?
  - Are the residuals from my linear model Normal?
  - Outliers in multivariate data?  $D^2 \sim \chi^2$   $\rightarrow$  chisq QQ plot

## Prestige data: income

Income is clearly **positively** skewed.  
(But normality is not required for predictors.)



This shows up as a **U-shaped** pattern  
The 95 % confidence band shows  
greatest departure in the upper tail

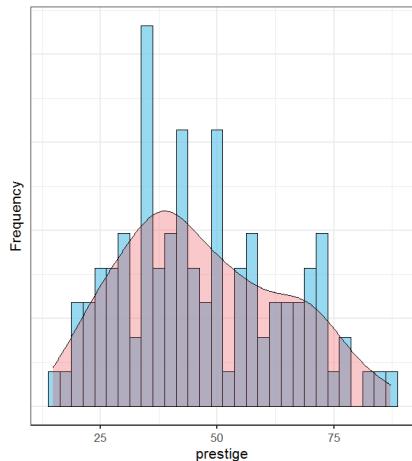


57

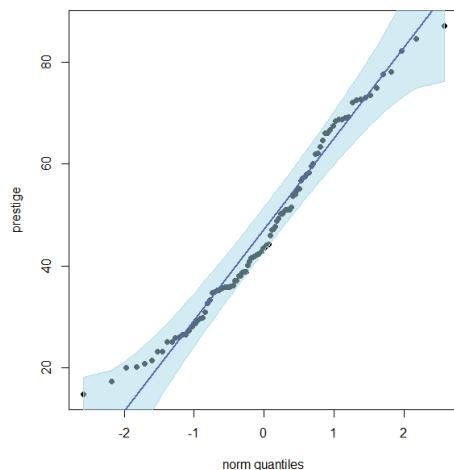
58

## Prestige data: prestige

Occupational prestige doesn't look precisely normal, but not that bad.



The 95% confidence band includes all the observations



59

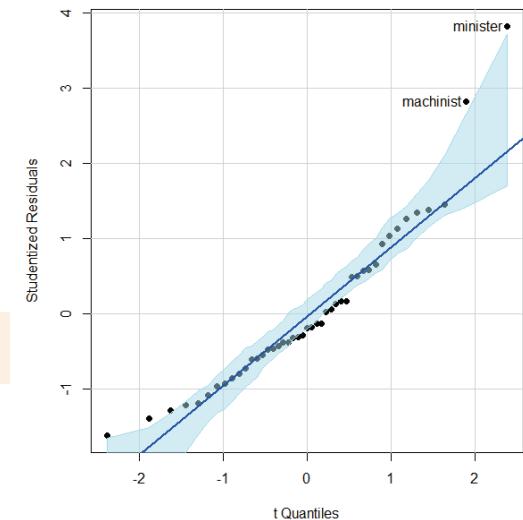
## Prestige data: residuals

Normality of **residuals** is more important for linear models

Some small evidence of + skew

Confidence bands help to identify potential outliers – badly fitted pts

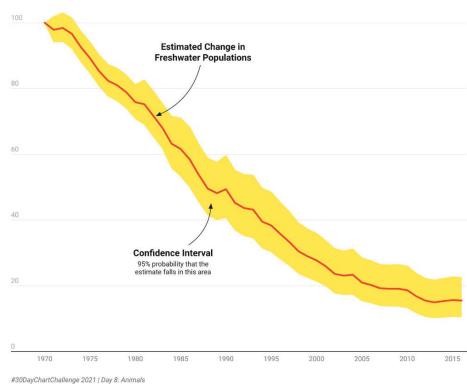
```
qqPlot(lm(prestige ~ income + education + type, data=Duncan))
```



60

## Curves + Uncertainty

Humanity has wiped out 60% of animal populations since 1970 – and freshwater habitats are the worst hit with populations having collapsed by more than 80%  
The Living Planet Index, produced for WWF by the Zoological Society of London, uses data on 16,704 populations of mammals, birds, fish, reptiles and amphibians to track the decline of wildlife. It underlines how the vast and growing consumption of food and resources by the global population is destroying the web of life upon which human society ultimately depends on.



Cederic Scherer used this graphic to argue about the decline of animal & freshwater populations.

Details aside, the confidence band gives visual evidence that the decline is systematic.

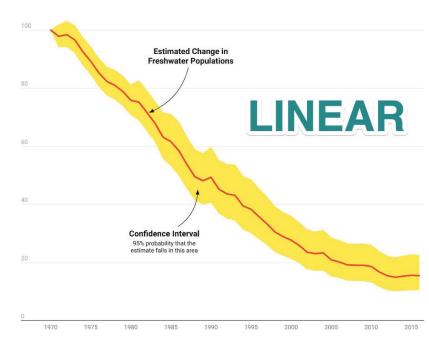
From: <https://twitter.com/CedScherer/status/1380211291466399744>

61

## Curves + Uncertainty: Scale

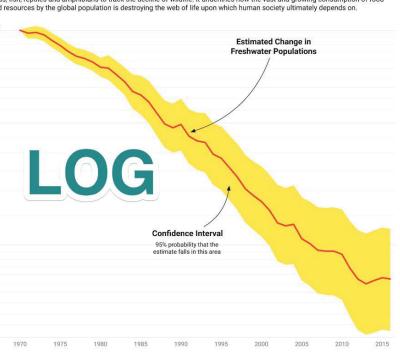
Arguably, **percent** reduction in animal population should be viewed on a **log scale**. Transformed uncertainty intervals are here the logs of the Upper/Lower levels

Humanity has wiped out 60% of animal populations since 1970 – and freshwater habitats are the worst hit with populations having collapsed by more than 80%  
The Living Planet Index, produced for WWF by the Zoological Society of London, uses data on 16,704 populations of mammals, birds, fish, reptiles and amphibians to track the decline of wildlife. It underlines how the vast and growing consumption of food and resources by the global population is destroying the web of life upon which human society ultimately depends on.



#30dayChartChallenge 2021 Day 8: Animals  
Cedric Scherer - Source: World Wildlife Fund (WWF) and Zoological Society of London - Created with Datawrapper

Humanity has wiped out 60% of animal populations since 1970 – and freshwater habitats are the worst hit with populations having collapsed by more than 80%  
The Living Planet Index, produced for WWF by the Zoological Society of London, uses data on 16,704 populations of mammals, birds, fish, reptiles and amphibians to track the decline of wildlife. It underlines how the vast and growing consumption of food and resources by the global population is destroying the web of life upon which human society ultimately depends on.

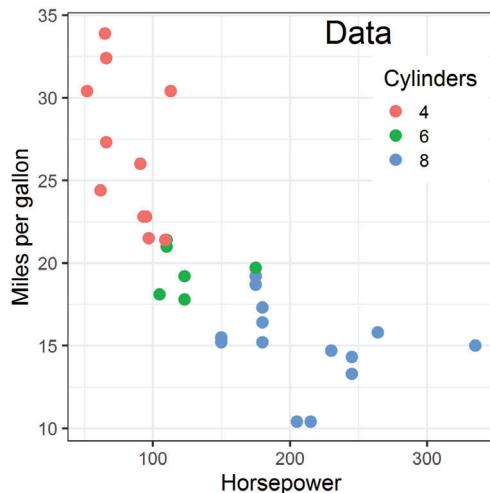


#30dayChartChallenge 2021 Day 8: Animals  
Cedric Scherer - Source: World Wildlife Fund (WWF) and Zoological Society of London - Created with Datawrapper

62

## Fitted curves

Data on gas mileage of *Motor Trend* 1974 cars



### Sources of uncertainty:

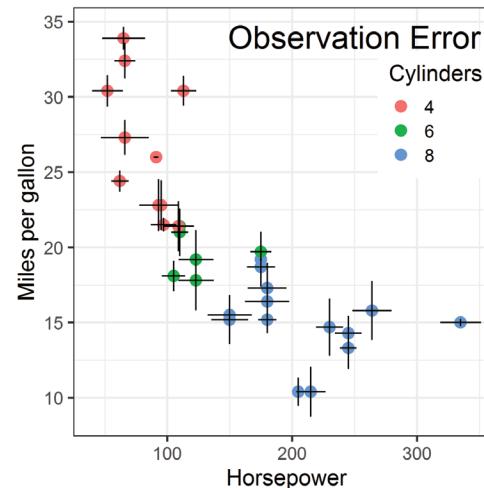
- Observations: **measurement error** in MPG and/or HP?
- Model form**: Linear? Quadratic? Interaction with cylinders
- Model fit uncertainty**: normal theory CIs? Bootstrap? Bayesian?

63

## Measurement uncertainty

Sometimes, we can quantify the uncertainty ("error") in values of  $x$  and or  $y$ .

e.g., each point is the average of  $n > 1$  cars.



Fitted models allow for errors in  $y$ :  
 $y = f(x) + \text{error}$   
and find estimates to minimize error

Most fitted models assume  $x$  is measured w/o error.

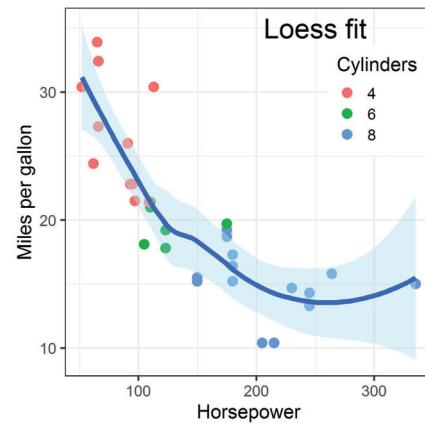
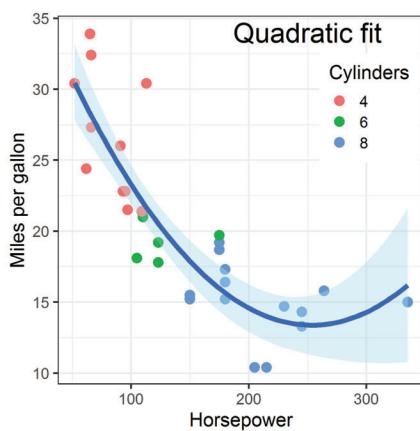
Big problem if  $\text{error} \sim f(x, \text{other } xs)$

64

## Model forms: nonlinear fits

When a relation is clearly non-linear, we can fit alternative models.

The CI bands tell us where the data is too thin to rely on the predicted value.



`p1 + geom_smooth(method = lm, formula = y~poly(x,2), ...)`

`p1 + geom_smooth(method = loess, formula = y~x, ...)`

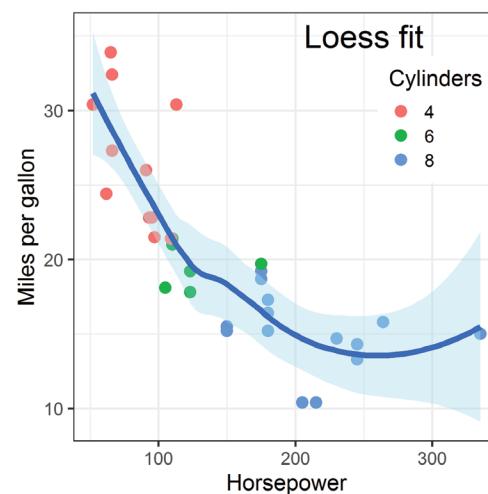
65

## Fitted curves: smoothers

In each case, the confidence band gives visual evidence for uncertainty of the predicted values.

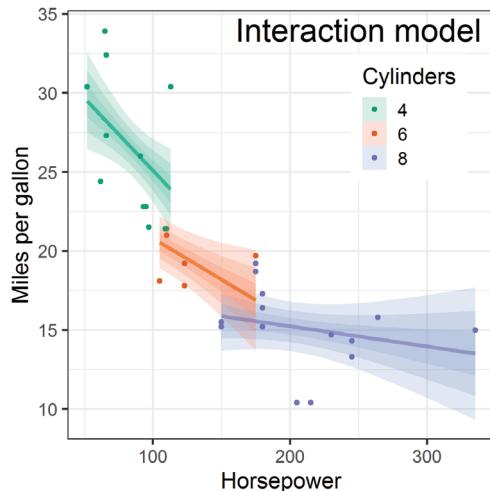
But, uncertainty may be expressed differently.

- a formula for std. error based on normal/large sample theory
- envelope of (normal) simulations
- Bayesian predictive distribution



66

## Interaction models



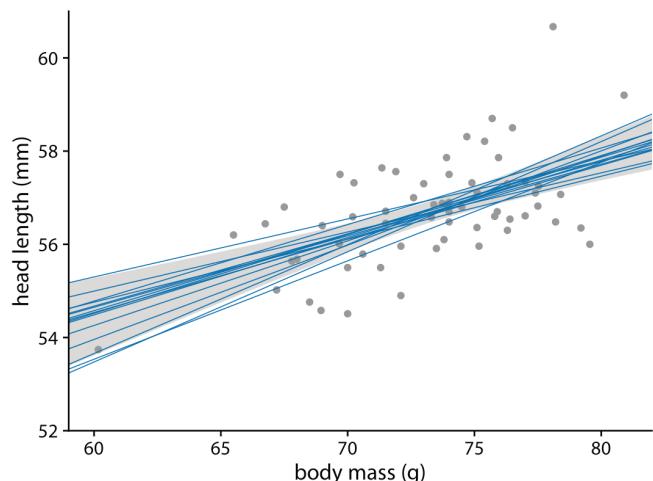
The non-linear relation between hp & mpg can (arguably) be better explained by a model that allows different slopes for 4, 6, 8 cylinders.

The graph shows normal theory CIs at 95%, 90%, and 80% for each cylinder level

67

## Simulations to convey uncertainty

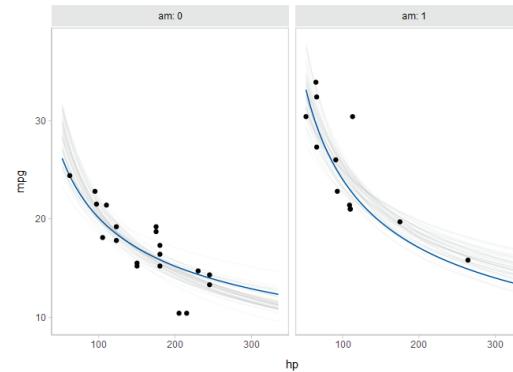
Simulating fits from the data (e.g., bootstrap, Bayesian estimation) shows the variability. Doesn't rely on classical, normal theory.



69

## Hypothetical Outcome Plots

- Rather than showing a complete distribution or point estimate and error bars, Hypothetical Outcome Plots (HOPs) visualize a set of **draws** from a distribution
  - each draw is shown as a new plot in either a small multiples or animated form.



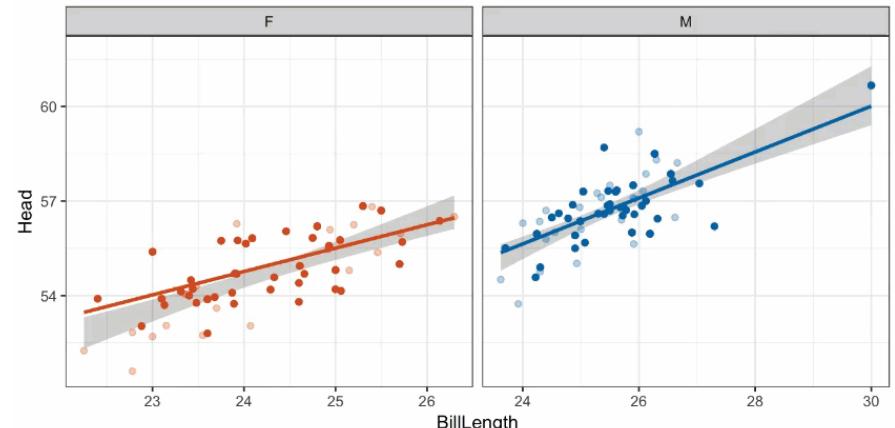
68

## Animation to understand uncertainty

All assessments of uncertainty rely on a comparison: data vs. what could have been

- Sampling distributions, simulations, Bayesian posterior distributions, ...

Sometimes useful to appreciate the variability with animated graphics



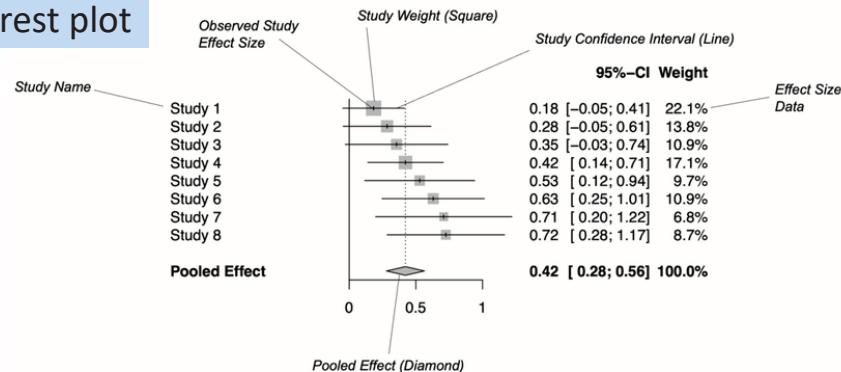
70

# Replication uncertainty: Meta analysis

In meta analysis, we have multiple studies reporting the same effect.

- How to visualize/compare effect sizes?
- How to calculate a **pooled**, overall effect?
- How to assess **heterogeneity** of effects?

## Forrest plot



From: Harrer, [Doing Meta Analysis in R](#)

71

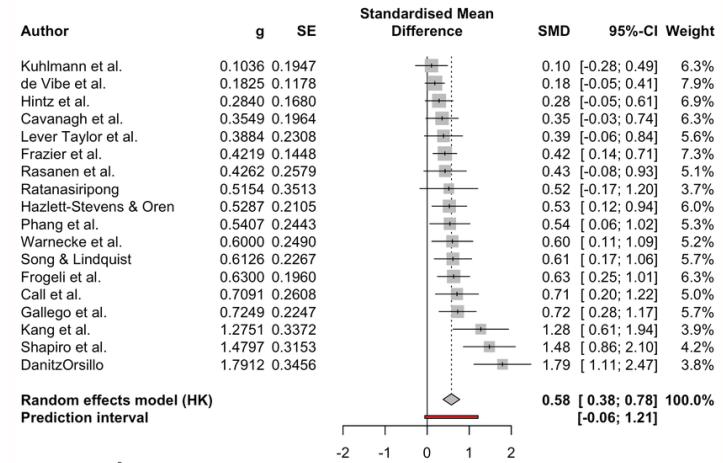
# Forrest plots

Effect size: std. measure of size of effect (g)

Study weight:  $\sim 1 / \text{SE}^2$

Pooled effect: weighted average of effect sizes

`meta::forrest()`



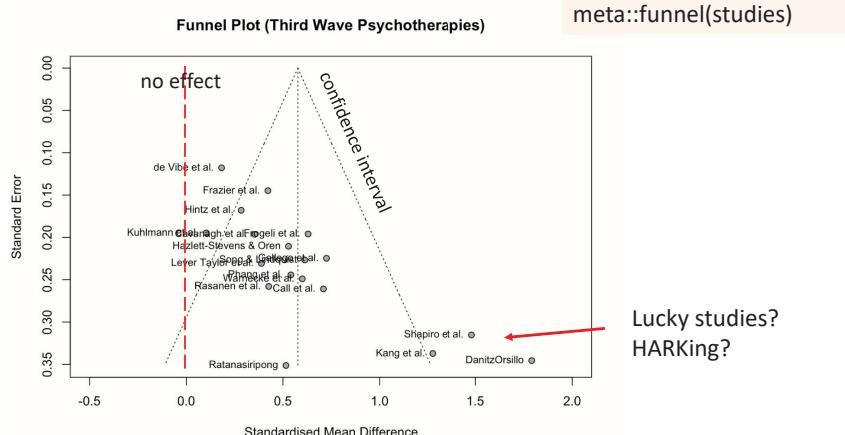
72

# Publication bias: Funnel plots

Publication bias: NS studies less likely to be published → effect overestimated

Funnel plot: Plot std. error vs. observed effect size

Should be **symmetric** when no publication bias

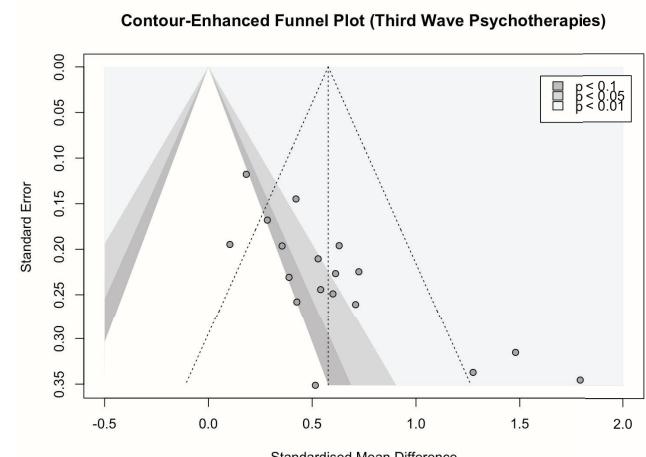


73

# Contour-enhanced funnel plots

Funnel plots can be enhanced by showing contours of p-values

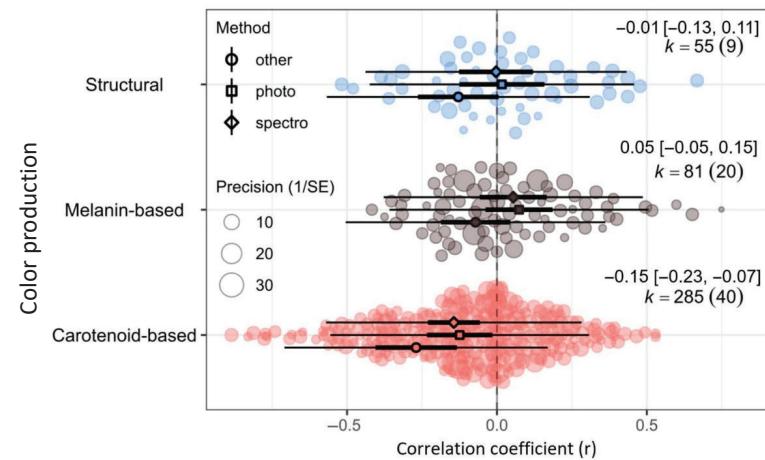
`meta::funnel(studies, contour = c(0.9, 0.95, 0.99))`



From: Harrer, [Doing Meta Analysis in R](#)

# Orchard plots

Plot for interaction (moderator) effects between mechanism of colour production and pollutant type



Janas et al (2024) Avian colouration in a polluted world: a meta-analysis/ <https://doi.org/10.1111/brv.13067>

75

# Geographic Uncertainty

Historical maps  
Hurricane maps  
Coding maps for uncertainty  
Empirical studies



76

# Uncertainty in historical maps

Abraham Ortelius, *Theatrum Orbis Terrarum*, 1570

Monsters and other imagined creatures used to mark or mask unexplored areas

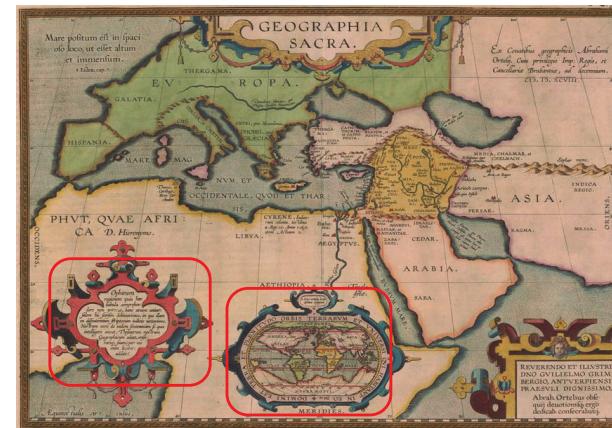


Here Be  
Dragons!

77

# Uncertainty in historical maps

Abraham Ortelius, *Geographia Sacra*, 1603



Source of the Nile?  
• Don't know!  
• Hide in a cartouche

Location of Ophir  
(mentioned in Bible)  
• text describes 4 possible locations

Van Duzer, Some Methods for Indicating Cartographic Uncertainty, Fifteenth through Eighteenth Centuries,  
<https://dx.doi.org/10.55283/jhk.13795>

78

# Uncertainty in historical maps

John Smith's map, Virginia, 1612



Uses ● markers to distinguish places he has visited, vs. those he just heard about

Van Duzer, Some Methods for Indicating Cartographic Uncertainty, Fifteenth through Eighteenth Centuries,  
<https://dx.doi.org/10.5283/1hk.13795>

79

## What is the Cone of Uncertainty?

As seen on TV:

- The center is meant to track the average prediction, either over models or history
- The cone size generally represents some “2/3 confidence interval”
- Does this mean I am safe if I lived in Tallahassee FL ⭐ in 2005? 2020?



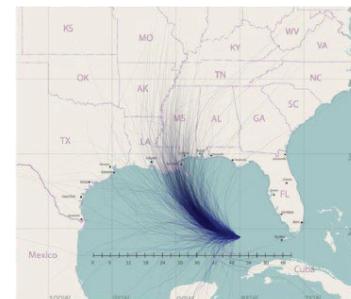
From: <https://www.youtube.com/watch?v=nE6XaHtpm04>

81

# Geographic uncertainty

Predicting the path of hurricanes:

- Given what we can measure today (location, wind speed, direction, ...) where is **this** hurricane likely to be in 1 day, 3 days, 5 days?
- Most forecasts are based on an **ensemble of predictions**, representing the uncertainty in initial conditions, model physics, ...
- Often this is represented as a “cone of uncertainty”



(a) Storm path ensemble



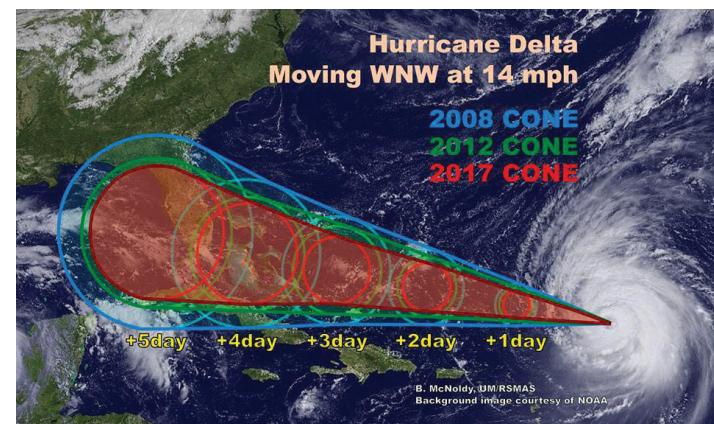
(b) Uncertainty cone.

80

## The Incredible Shrinking Cone

Changes in presumed accuracy are often shown as below

- The cone represents the probable track of the center of a tropical cyclone, formed by enclosing the area swept out by a set of circles along the forecast track (at 12, 24, 36 hours, etc).
- The size of each circle is set so that two-thirds of historical official forecast errors over a 5-year sample fall within the circle.



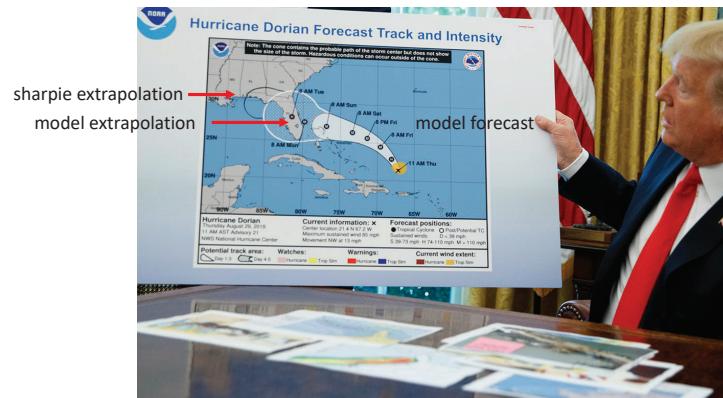
B. McNoldy, UM/RSMAS  
Background image courtesy of NOAA

82

# Sharpiegate

In Sept. 2019, Donald Trump went live with “extrapolated” predictions of the path of Hurricane Dorian.

- He had earlier predicted it would hit Alabama & Georgia.
- Let it be said, let it be written (with a sharpie)



From: <https://www.theguardian.com/us-news/2019/sep/05/trump-hurricane-dorian-alabama-map-sharpiegate>

83

## Studies

Cheong et al. (2016)

- Participants presented with a house location (“X”)
- Asked if they would stay or leave based on one of the wildfire hazard communication techniques shown here

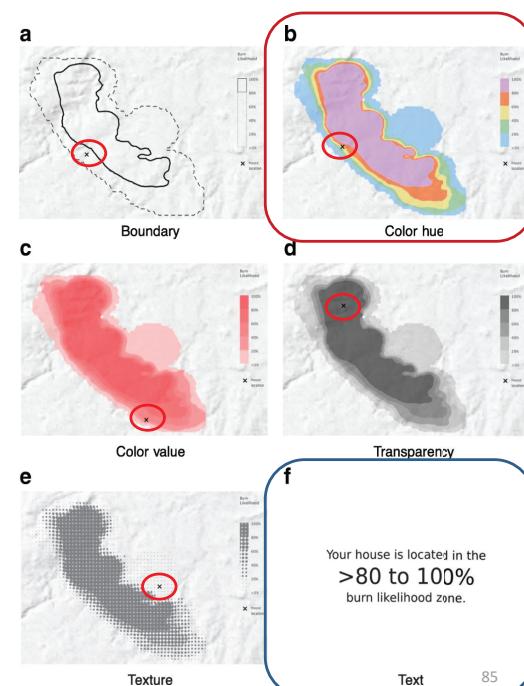
Results:

Display type mattered little when given 30 sec. to decide

With time pressure (5 sec.):

- color hue was best
- Text & simple boundary worst

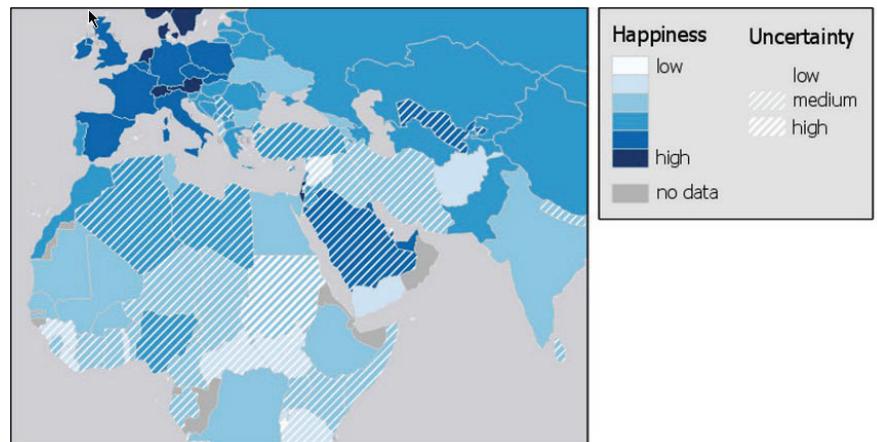
Cheong, et al. (2016). Evaluating the impact of visualization of wildfire hazard upon decision making under uncertainty. *International Journal of Geographical Information Science*, 30(7), 1377–1404.



85

# Coding maps for uncertainty

In choropleth maps we can show uncertainty with another visual attribute



From: <https://www.e-education.psu.edu/geog486/node/693>

84

## Summary

- Uncertainty is fundamental to data analysis & models
- Showing variation in distributions a basic problem
  - histograms, density plots, boxplots
  - Better: violin, raincloud, ...
  - {ggdist} offers many alternatives
- Error bars: many flavors; can show multiple intervals
- Bayesian methods, bootstrap, simulation
  - Different methods, but similar ways to show uncertainty
- Geographic data: need to be careful about interpretations

86