

# Assessing the Graphical Perception of Time and Speed on 2D+Time Trajectories

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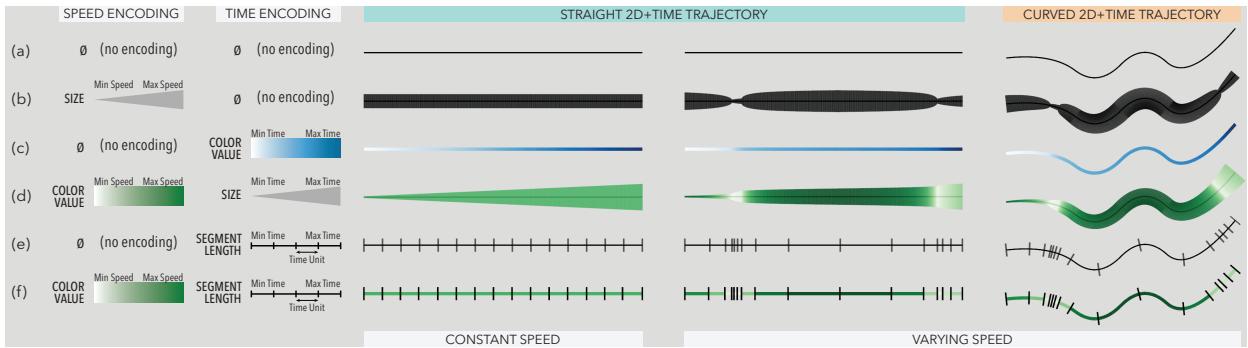


Fig. 1. Different encodings of time and speed for straight and curved 2D+time trajectories. Both constant speed and varying speed (two slow sections near the start and end, high speed in the middle) are shown. (a) Neither time nor speed are visually conveyed; (b) size (or stroke width) conveys speed; (c) color value conveys time elapsed; (d) color value conveys speed and size conveys time elapsed; (e) segment length (spacing between ticks) conveys time distribution, from which speed can be inferred (the closer two ticks, the slower); and (f) color value conveys speed on top of segment length. Results from studying nine visual encodings suggest that (e) and (f) are the best choices for conveying both time and speed and that (d) is the next best.

**Abstract**— We empirically evaluate the extent to which people perceive non-constant time and speed encoded on 2D paths. In our graphical perception study, we evaluate nine encodings from the literature for both straight and curved paths. Visualizing time and speed information is a challenge when the x and y axes already encode other data dimensions, for example when plotting a trip on a map. This is particularly true in disciplines such as time-geography and movement analytics that often require visualizing spatio-temporal trajectories. A common approach is to use 2D+time trajectories, which are 2D paths for which time is an additional dimension. However, there are currently no guidelines regarding how to represent time and speed on such paths. Our study results provide InfoVis designers with clear guidance regarding which encodings to use and which ones to avoid; in particular, we suggest using color value to encode speed and segment length to encode time whenever possible.

**Index Terms**—Trajectory visualization, visual encoding, movement data, graphical perception, quantitative evaluation.

## 1 INTRODUCTION

Temporal data is prevalent in many fields such as history, meteorology, finance, geography, industrial processes and social movements. The most common way of representing time is to assign it to one of the spatial axes using the positional variables  $x$  or  $y$  because position is the most powerful visual variable [55] – for example a line chart typically maps time to the  $x$ -axis. However, sometimes it is not possible to map time to position because positional variables are already encoding other dimensions of the data, such as geographical locations on a map or more abstract dimensions in a scatterplot.

In the simple example of travel, one could be interested in visualizing where one has travelled, where one is at a given point in time, and how time has passed while travelling. Typically, a 2D path drawn on a map would show the route taken, but might omit contextual details such as travel speed, allowable speed, or how long it takes to traverse a route.

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The literature is full of examples where encoding information about time and/or speed on these 2D paths, or *2D+time trajectories*, is important (see Figures 4, 6, and 7). Researchers and InfoVis designers have encoded speed and time along 2D paths using visual variables such as size (e.g., [1, 9, 10]), color brightness or value (e.g., [1, 9, 23, 32, 71]), and segment length (e.g., [10, 70]). Although InfoVis designers often have to decide how to visually encode time and speed information on 2D+time trajectories, so far no studies have been conducted to assess the relative interpretability of these visual encodings and no guidelines exist for helping designers in making such decisions.

To fill this gap, we studied the *graphical perception* [20] of nine time and speed encodings for 2D+time trajectories (Figure 1(b–f) shows five of these encodings). We selected these nine encodings based on our review of the visual variables that have been used in the literature to encode time and speed. 18 participants performed two tasks (perceiving speed and perceiving time), for two path shapes (straight and curved), and for all nine encodings, multiple times. For simple straight or curved paths (no complex shape, sharp angles or loops), and when it is important to estimate the speed and/or time value at a point on the path, our results in terms of accuracy and completion time indicate that:

1. The best choices are either to encode speed with brightness/color value and time with segment length, or to encode both time and speed using segment length only.
2. If using segment length is not desirable, the next best choice is to encode speed with color value and time with size.

2D+time trajectories range from very simple to very complex. Our results apply to the very simple cases, in situations where the shapes of the paths are either straight or smoothly curved. These results open the door to a wealth of opportunities for running studies towards developing a fuller understanding of how to best encode time and speed on 2D+time trajectories. In particular, now that we have identified encodings that should be avoided, future studies can build on this work and focus on encodings that perform well for new tasks and trajectory types.

## 2 BACKGROUND AND RELATED WORK

There exists a wide variety of techniques for representing time in visualization such as line graphs, small multiples, and animation (see [1, 6, 8] for comprehensive reviews). In this paper, we focus on a technique called time flattening. We first explain the space-time cube metaphor and time flattening. This provides a framework to describe 2D+time trajectories. Then, we survey the ways of encoding time and speed on 2D trajectories. Lastly, we discuss related graphical perception studies.

### 2.1 Space-Time Cube and Time Flattening

Spatio-temporal data in 2D consists of data points which have two spatial dimensions and a temporal dimension. When time cannot be mapped to either  $x$  or  $y$ , it can be mapped to the other spatial dimension,  $z$ . This creates a *space-time cube* [38], which refers to the treatment of time as a third dimension in addition to two spatial dimensions [54]. The space-time cube metaphor has often been taken literally to visualize 2D data in 3D, for example in geo-visualization (e.g., [34, 48, 50]) and for the visual analytics of movement data [4, 5, 7]. Bach et al. [8] clarify that “*a space-time cube does not need to involve spatial data.*” The space-time cube representation has in fact been used to visualize non-spatial, abstract data, such as in Configurable Spaces [44]. The 3D space-time cube, however, suffers from 3D visualizations problems, such as occlusion [75] and inconsistent perception across rotations [39].

Using the space-time cube conceptually instead of literally provides alternatives to 3D visualizations. Bach et al. [8] call *time-flattening* the technique that “aggregates a space-time volume into a plane orthogonal to the time axis” [8, page 9]. The result is similar to the one of long exposure photography, where several frames of an image changing over time in a time interval are collapsed into a single image.

Time flattening an object’s 2D position over time creates a 2D+time trajectory, such as a travel itinerary on a map. For example, Minard’s comparison of Hannibal’s and Napoleon’s campaigns [58] in Figure 2 shows the 2D position of armies over time. In this example, the thickness of the 2D path encodes the size of the army. Neither time nor speed are visually encoded. In a visualization context, spatial 2D+time trajectories where conveying temporal information is important are prominent in, e.g., movement analysis [7] and eye tracking data [14]. Time-flattening non-spatial data also creates 2D+time trajectories. For example, in Hans Rosling’s famous TED talk [62], the dots of scatter-plots change position according to time. Dots follow trajectories in two non-spatial dimensions, such as lifespan and income. Similarly, DimpVis [46] shows 2D+time trajectories of abstract data for navigating in time via the direct manipulation of graphical elements in visualizations. In this paper, we focus on time-flattened 2D+time trajectories.

### 2.2 Encoding Time and Speed on 2D+time Trajectories

When looking at 2D+time trajectories, both the absolute time, and the relative speed can be of interest. Here, we describe the visual variables that have been used to encode time and speed on 2D+time trajectories.

**Size** is sometimes used to encode time on 2D+time trajectories [1]. Figure 3 (left) shows time being mapped to the size (or stroke-width) of the path. Bertin provides examples of using size for encoding *movement* data [10] (see Figure 5(b)). Bach et al. [9] discussed encoding the duration between two consecutive events on a trajectory by varying the curve’s thickness. In that case, a thicker curve represents a long time interval between two events, which corresponds to a slow speed.

**Color value/brightness** (we refer to *value* in this paper) frequently encodes time on 2D+time trajectories [1]. Figure 3 (right) illustrates mapping time to brightness. Figure 4 shows *colored time flattening* [8], where each data item on a *Time Curve* [9] has been assigned a color

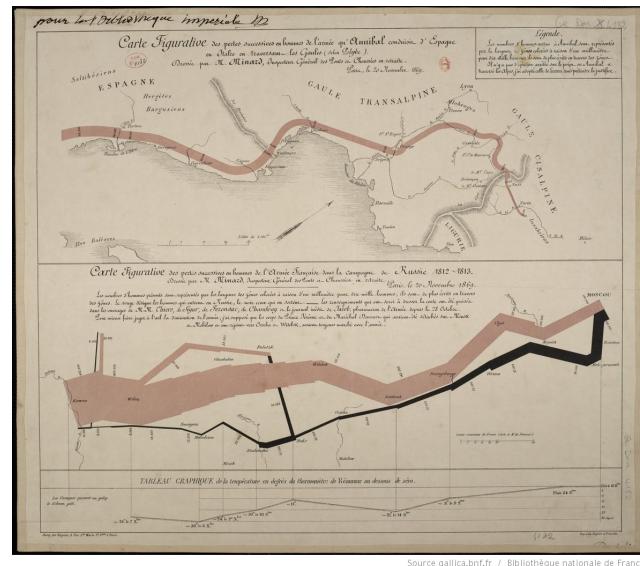


Fig. 2. Minard’s comparison of Hannibal’s second Punic war campaign (top) and Napoleon’s Russian campaign (bottom) [58].

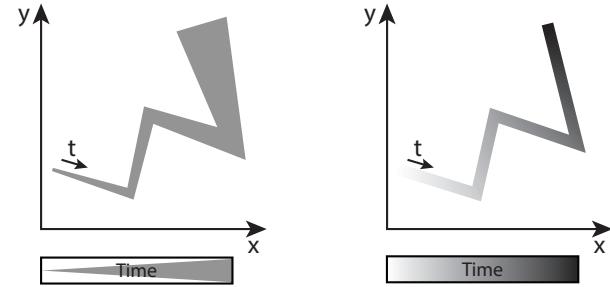


Fig. 3. Encoding time on a 2D trajectory using line-width (left) and brightness (right). Reproduction from [1] with added Legends.

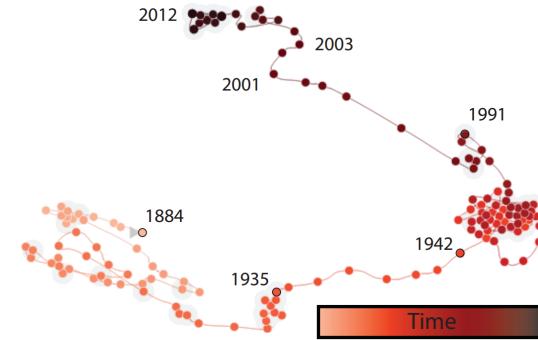


Fig. 4. Evolution of temperature over time using time curves [9]. The color of time points encodes their timestamp. Legend was added.

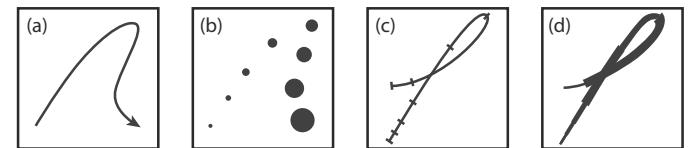


Fig. 5. Examples reproduced from Bertin [10]: (a) neither time nor speed are encoded; (b) size encodes time; (c) time units with tick marks convey time and speed (the longer a segment, the faster), and (d) ticks are combined with size (the thicker a segment, the faster).

according to its timestamp. Time has also been color-coded on strokes to show the stroke order when writing Chinese characters [77], and in abstract graphs [23]. Value has also been used to encode speed, either in grayscale [32] or on a color ramp [18, 71] (see Figure 6 for an example created using Tableau). It is also common to map velocity in a vector field to a color ramp in flow visualization (e.g., [12, 73]).

**Segment length** has been used in *time geography* to create Linear Cartograms. Linear Cartograms move spatial points on maps such that travel time between two points is encoded by the length of the link between these two points [13, 43, 64]. As a result, Linear Cartograms do not remain faithful to the spatial position – or 2D attributes – of data points. To solve this issue, Linear Cartograms with Fixed Vertex Positions [16] maintain the position of points, and instead, create sinusoidal links whose length represents time between points. Visits [70] uses a similar approach: position along the horizontal axis represents time, while sizes of the circles encode duration. Bertin [10] proposed marking time units using ticks to convey both time and speed without distorting the 2D path: time ticks convey time elapsed, and spaces between ticks (i.e., segment lengths) convey speed (see Figure 5(c)). Bertin [10] used time ticks to convey speeds of ships on a map (see Figure 7). He also proposed using segment length and size simultaneously to double encode speed, as shown in Figure 5(d). Arrows, which are often used to encode velocity in vector fields in flow visualization [51], could encode time/speed along a trajectory. Although we did not find an example of such, arrows would be similar to time ticks (segment length), except they would also provide direction along the trajectory.

**Summary.** Several visual variables (size, color value, and length) have been used for encoding either the time or speed of 2D+time trajectories in time flattened visualizations. However, to our knowledge, no study has assessed the graphical perception of these visual encodings.

### 2.3 Studies on Graphical Perception

Graphical perception is “the visual decoding of the quantitative and qualitative information encoded on graphs” [19], or more generally, the ability to understand the visual encoding of information [53].

Graphical perception studies for statistical data graphics have a long history with studies of factors such as types of representations and shapes dating back to 1926 [26, 27, 33, 61]. Researchers in cartography have ranked the effectiveness of visual variables (e.g., [54, 65]). Similarly, in information visualization during the 1980s and early 1990s, Cleveland and McGill [19–22] and Spence [67] conducted graphical perception studies to experimentally rank visual variables. These studies confirmed Bertin’s [10] rankings of visual variables according to their effectiveness for encoding nominal, ordinal, and quantitative data. This method has been used to study how people use bar charts, pie charts, scatterplots, and tables [52, 66, 68, 72] among others. It has been used to study changes of variable rankings when looking at large displays [11, 76], the perception of uncertainty in visualizations [15, 37, 63], and the perception of mean and error representations [24].

Guidelines that are derived from these studies provide guidelines about which encodings to use, thus inform to the grammar of visualization [78] and are used in automatic presentation software [55, 56] such as Tableau. Our new graphical perception study adds guidance on how to best encode time and speed on 2D paths.

## 3 STUDY RATIONALE

Many factors could play a role in the graphical perception of time and speed on 2D+time trajectories, including:

- The choice of tasks participants are asked to perform.
- The 2D path, including its curvature, direction, length, range of angles (abrupt changes of direction), and crossings.
- The time function (ranges of speeds and time distributions).
- The background, with its color and texture (e.g., a map).

Our goal was to establish which visual encodings to use to encode time and speed on 2D+time trajectories. Therefore, we prioritized the number of encodings to study at the cost of constraining other factors in order to i) limit the influence of confounding factors; ii) ensure consistent difficulty between datasets and tasks; and iii) limit the duration of the experiment.



Fig. 6. A dual color ramp encodes speed along the path of a bike ride.

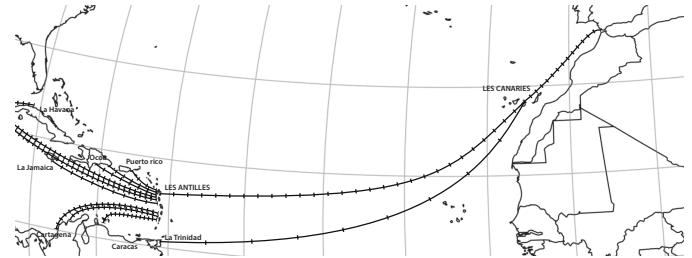


Fig. 7. Reproduction of Bertin’s [10] map where time ticks convey both the total travel time and the speed of a ship.

### 3.1 Choice of Tasks

Graphical perception tasks are often either value comparison tasks (e.g., [20, 41, 67]) or value estimation tasks (e.g., [61, 66]). We decided for the latter, specifically inverse lookup elementary tasks in Andrienko and Andrienko’s [6] task taxonomy for time-varying data.

In the **TIME** task, participants had to find the point on the trajectory that represents a certain amount of time elapsed (e.g., 50% of the total time). In the **SPEED**, task, they had to find the point where the speed is maximal or minimal. These two low-level tasks are often involved in compound higher-level real world tasks. For example, the **SPEED** task could be used to determine where a cyclist was struggling up a hill or comparing where two race cars reach their maximum speed, since this involves finding points on a trajectory where speed is at its lowest or highest. To illustrate an example where the **TIME** task is useful, when conducting eye tracking experiments, it is helpful to know at which point in time a participant was looking at a particular feature in a visualization, or how long a participant took to find an object.

These standard tasks were well suited to our study for three reasons. First, the purpose of this study was to assess how people can quickly read time and speed (bottom-up process), as opposed to more complex tasks with higher cognitive load (such as comparing the speed of two segments). For this reason, we selected the simplest tasks that require reading information about time and speed, which lay the ground for studying more complex tasks [2]. Second, the same input method can be used to perform both time and speed related tasks (clicking a point on the path), limiting participants’ overhead of learning different input methods. Finally, time is monotonically increasing along the 2D path. This ensures that there is a unique correct answer for each task. Because speed is not monotonous, we used the task where participants have to find extrema as it ensures that there is a unique answer to each task.

### 3.2 Paths and Time Functions Generation

As explained in the study rationale, many factors could be studied. As we chose to study many different encodings, we could not also study many trajectory variations due to concerns about experiment duration and experimental power. Although some 2D+time trajectories can sometimes be complex (e.g., the ones in TimeCurves [9] and DimpVis [46]), they can also often be very simple. An itinerary on a map is very unlikely to contain loops (see Figures 2, 6, and 7), and spatial trajectories of large vessels like a plane or the ship in Figure 7 are also likely to be smooth and not contain sharp angles due to physical limitations. We kept the trajectories relatively simple, restricting possible 2D

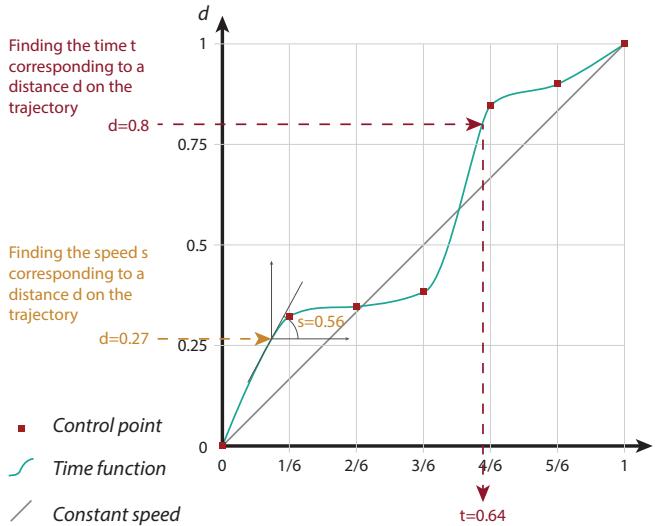


Fig. 8. A sample time function. Control points are evenly distributed along the time axis  $t$ , but are assigned random, increasing heights along the distance axis  $d$ . Shown are approximations for finding time  $t$  given distance along the path  $d$ , and the speed (i.e., tangent).

path shapes to only straight or smoothly curved. We included this factor because changes in curvature are common in 2D+time trajectories and may change the perception of visual encodings. We tested both straight paths and curved paths with a fixed direction from left to right, no sharp angles and no crossings (see Figure 1).

We generated curved paths using B-Splines with six control points to produce smooth curves with simple variations. We evenly spaced control points by 140px horizontally and assigned each a random  $y$  value between -200 and 200px. We used a fixed increment for the  $x$  value of control points for two reasons: it ensures the path always travels from left to right, so the *direction* of the path is clear, and it ensures the path does not have loops or self-intersections. We fixed the  $y$  coordinate of control points to be in a 400px range so that the maximum bounding box of the paths was  $700 \times 400$ px.

The constraints for curved paths informed the constraints for the straight ones, specifically to limit the confounding factor of path *length*. We determined the range of lengths for straight paths by generating one million curved paths. The distribution of their lengths was pseudo-normal and 86% of the lengths were in the range 750–1000px. Thus, we constrained the straight paths to have a random length in the range 750–1000px so that straight and curved paths had comparable lengths.

We generated time functions  $T : t \mapsto d$ , with  $t \in [0, 1]$  the time value, and  $d \in [0, 1]$  the distance along the path for  $t$ . We used B-Splines with seven control points to generate time functions (see Figure 8). To ensure  $T$  are monotone increasing functions (the distance along the curve increases with time), we generated control points as follows: 1) create a set  $D$  of values  $d_i \in D$ , with  $|D| = 7$ ,  $d_0 = 0$ ,  $d_6 = 1$ , and  $d_{1\dots 5}$  is a random value in  $[0, 1]$ ; 2) sort the values in  $D$  in ascending order; 3) create seven control points,  $C$ , with each  $C_i = (i/6, d_i)$ . This results in monotone increasing functions whose control points are evenly distributed along the  $t$  axis, with  $T(0) = 0$  and  $T(1) = 1$ .  $T$  are smooth functions with no sharp corners, but they can vary wildly with moments of extreme speed or slowness, depending on the values in  $D$ . To retrieve  $t$  given  $d$ , we simply apply  $T^{-1}$ , the inverse of  $T$ . To retrieve the speed  $s(t)$  at  $(t, d)$ , we find the slope of the tangent of the time function at  $(t, d)$ . Because  $T$  is monotonically increasing,  $s(t) \in [0, \pi/2]$ .

Initially, approximately 40% of the generated time functions had minimum or maximum speed at  $t = 0$  or  $t = 1$ , which would bias the results for speed questions. To remove this bias, we constrained the time functions for speed-related tasks to have minimum and maximum speeds within  $t \in [0.03, 0.97]$ .

### 3.3 Choice of Encodings

While many encodings can be used to encode time and/or speed on 2D+time trajectories, we chose to study encodings that have been used in the literature to encode real-world data. As a result, we study the encodings that academics and practitioners in the field have considered appropriate to encode time and/or speed, which we presented in Section 2: *size*, color *value*, and segment *length*. To refer to an encoding, our notation uses the letter S or T to indicate speed or time, followed by a icon showing its specific visual variable. For example, encoding speed using color value is referred to as **SVALUE**.

We also studied both single encodings (either time or speed is encoded using one visual variable) and double encodings (both time and speed are encoded using two visual variables). This decision was driven by two reasons. First, segment length conveys both time and speed (Figure 5). This made it a requirement to compare this encoding to encodings that also convey both time and speed, i.e., double encodings. Second, time and speed are dependant variables. Studying double encodings made it possible to explore the effects of encoding one variable on top of the other and how these two visual variables may interact with each other. We call encodings that can show two data dimensions *double encodings*. This contrasts with *redundant encodings* [47] that show the same data dimension. For example, the double encoding **SVALUE TLENGTH** encodes speed using color value, and time using segment length. If speed or time is not encoded, we use **SØ** or **TØ**.

Figure 9 shows the nine studied encodings. They are combinations of encoding speed and/or time, using value, size, and length.

- Speed-only encodings: color value **SVALUE**, and size **S<sub>SIZE</sub>**.
- Time-only encodings: color value **TVALUE**, size **T<sub>SIZE</sub>** and segment length **T<sub>LENGTH</sub>**.
- Speed and time double encodings:
  - Speed as color with time as size **SVALUE T<sub>SIZE</sub>**,
  - Speed as color with time as segment length **SVALUE T<sub>LENGTH</sub>**,
  - Speed as size with time as color **S<sub>SIZE</sub> TVALUE**,
  - Speed as size with time as segment length **S<sub>SIZE</sub> T<sub>LENGTH</sub>**.

#### 3.3.1 Color Value

We picked two color ramps from <http://colorbrewer2.org/>, using green for speed (**SVALUE**) and blue for time (**TVALUE**), in order to avoid confusion when changing from a time-color encoding to a speed-color encoding.

**Speed (SVALUE):** The color value at any point on the path conveys the speed at this point. A paler green means slower speed; a darker green means faster speed, no variations in color means constant speed.

The **SVALUE** mapping function required particular attention. We generated 10 million time functions like Figure 8, and found that there was a pseudo-normal distribution of speeds (angles). Due to the difficulty in perceiving slight changes in color, we used *histogram equalization*, a technique which is recommended for lookup tasks [1] such as our SPEED task. Histogram equalization consists of subdividing the value range into  $n$  uniform bins and counting the number of data values in each bin. The color scale is sampled according to the cumulative frequencies of the bins, i.e., a bin containing many data values is attributed a bigger slice of the color ramp. Using histogram equalization ensured that the scale of colors reflected the data's value distribution, and also improved discriminability of values, especially in high density regions.

**Time (TVALUE):** The color value at any point conveys the time at this point. The palest blue means 0% of time elapsed; the darkest blue means 100% of time elapsed; a linear gradient change means linear time. In contrast to **SVALUE**, **TVALUE** did not need histogram equalization as all values of time are shown equally.

#### 3.3.2 Size

The visual variable size changes the thickness of the path.

**Speed (S<sub>SIZE</sub>):** The path thickness at any point conveys the speed at this point. A thinner path means slower speed; a thicker path means faster speed; no variation in thickness means constant speed.

**Time (T<sub>SIZE</sub>):** The thickness at any point conveys the time at this point. The thinnest point means 0% of time elapsed; the thickest point means 100% of time elapsed; a linear thickness change means linear time.

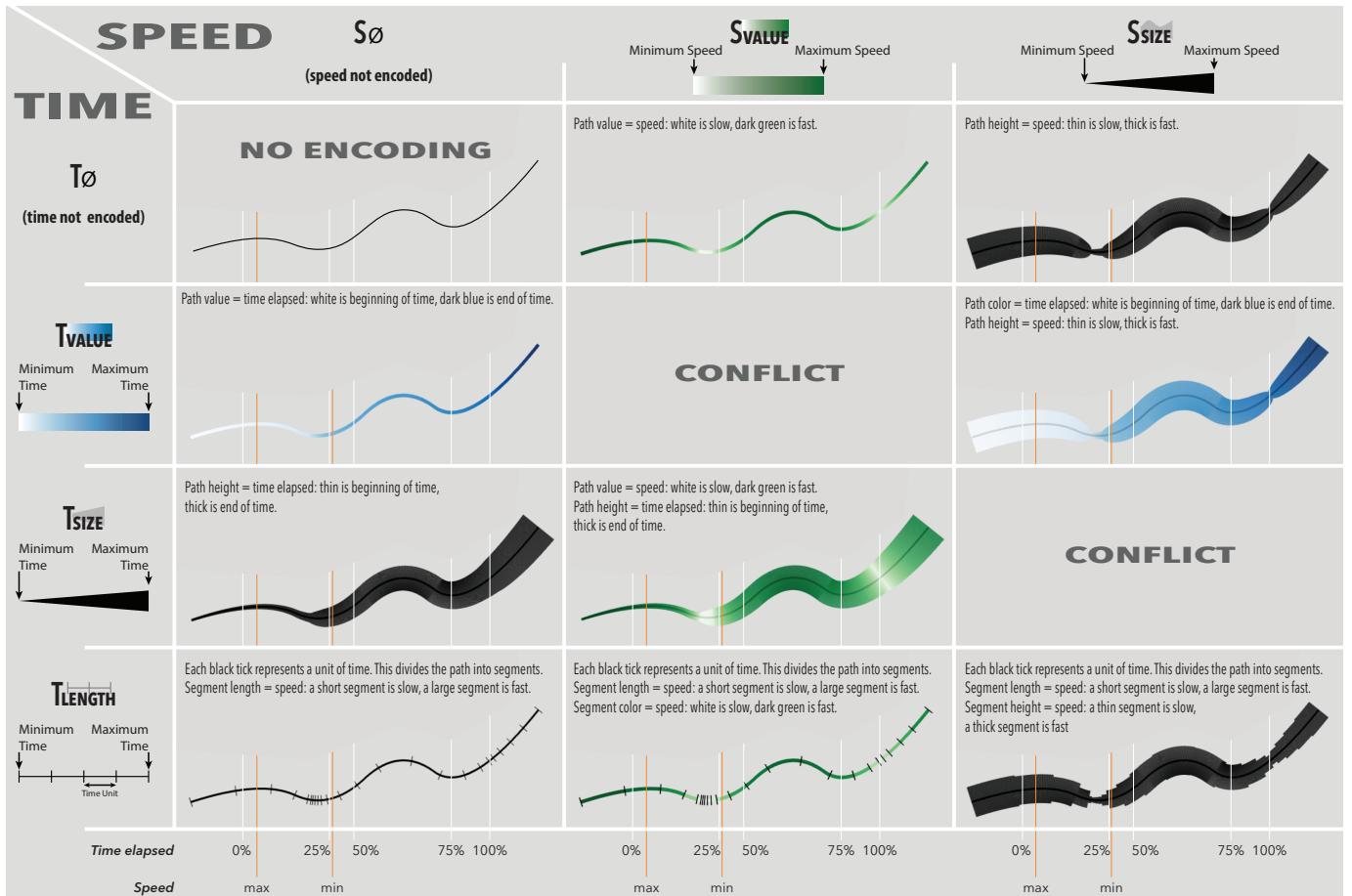


Fig. 9. Time and speed encodings on 2D paths. Speed can be: not encoded ( $S\emptyset$ ) or encoded using value ( $S_{VALUE}$ ) or size ( $S_{SIZE}$ ). Time can be: not encoded ( $T\emptyset$ ) or encoded using value ( $T_{VALUE}$ ), size ( $T_{SIZE}$ ), or length ( $T_{LENGTH}$ ). We tested all nine encodings that convey at least one of speed and time.

### 3.3.3 Segment Length

We used the visual variable length by creating time ticks orthogonal to the path (similar to Figure 7). These ticks resemble Linear Cartograms with Fixed Vertex Positions [16], but they do not distort the path.  $T_{LENGTH}$  is a particular encoding in that it conveys both time and speed.

*Speed* can be inferred by assessing a segment's length. The space between two ticks represents the distance travelled in each of these time "units". A larger gap between two ticks means that more distance is covered within one time unit, implying faster speed. A smaller gap implies slower speed.

*Time* can be inferred by assessing the distribution and frequency of ticks. Evenly distributed ticks means linear time. We chose to sample time at 23 evenly spaced intervals and to draw ticks at each resulting position on the path. We chose 23 ticks for two reasons: i) it is a sufficient number of ticks to indicate small and large trends in the data; and ii) it is prime, ensuring that no ticks will fall exactly on common milestones such as one-third or one-half of the time elapsed.

### 3.3.4 Null Encodings

Mapping speed or time to the null encoding means that this information is not explicitly encoded. Five of the nine encodings do not explicitly encode either speed ( $S\emptyset$ ) or time ( $T\emptyset$ ).

### 3.3.5 Double Encodings

Among the encodings shown in Figure 9, four double encode speed and time. These four encodings are  $S_{VALUE} T_{SIZE}$ ,  $S_{VALUE} T_{LENGTH}$ ,  $S_{SIZE} T_{VALUE}$ , and  $S_{SIZE} T_{LENGTH}$ . We did not study the  $S\emptyset T\emptyset$  encoding, which encodes neither speed nor time. Although such a condition can sometimes be used as a baseline, in our study, asking participants to assess time and speed without any indication would have resulted in a random baseline not suited for comparison.

## 4 STUDY OF ENCODINGS

The purpose of the study was to determine the differences between the nine visual encodings we presented, in performing time- and speed-related tasks on 2D+time trajectories.

### 4.1 Experimental Design

The three factors were ENCODING (the nine visual encodings), TASK (TIME and SPEED), and SHAPE of the path (STRAIGHT and CURVED). We used a within-participant design, where all participants perform the exact same trials. We counterbalanced ENCODING and TASK in order to mitigate learning effects.

The experiment consisted of 9 encoding blocks. Each encoding block was split into two task blocks. Each task block consisted of training trials followed by 12 recorded repetitions (6 STRAIGHT, then 6 CURVED). A participant always performed the task blocks in the same order for all encoding blocks. To summarize, the experiment consisted of  $18 \text{ participants} \times 9 \text{ ENCODING} \times 2 \text{ TASK (TIME, SPEED)} \times 2 \text{ SHAPE (STRAIGHT, CURVED)} \times 6 \text{ repetitions} = 3888 \text{ trials}$ .

### 4.2 Dataset and Tasks

We generated a dataset for measured trials. For all 36 (ENCODING  $\times$  TASK  $\times$  SHAPE) combinations, we created six repetitions with a 2D path, a time function, and a value the participant had to locate on the path, each randomly generated. This resulted in 216 tuples {ENCODING, TASK, SHAPE, 2D path, time function, value}, that were used by all participants. For TIME, the values to locate were 25%, 50%, and 75% of the total elapsed time, with each value appearing twice. For SPEED, the values to locate were minimum and maximum, with each value appearing three times. Within each TASK  $\times$  SHAPE block, the six repetitions appeared in random order.

### 4.3 Dependent Measures: Time and Accuracy

For each trial, we measured the time participants took to complete the task and the error of their answers.

The error for each trial is the absolute value of the difference between a participant's answer and the correct answer. Because all possible values of time  $t \in [0, 1]$  are visible and encoded for TIME, this measure of error is consistent across TIME trials. In contrast, only a subset of speed values  $s \in [0, \pi/2]$  are encoded for each SPEED trial, according to the time function  $T$ . To ensure consistency of measure across SPEED trials, we normalized the error measurement for each trial according to the minimum and maximum speed for the current trial:

$$\text{error} = \left| \frac{p\text{Answer} - c\text{Answer}}{\max - \min} \right|, \quad \text{with}$$

$p\text{Answer}$  the participant answer  
 $c\text{Answer}$  the correct answer  
 $\min$  the trial minimum time or speed value  
 $\max$  the trial maximum time or speed value

Because for TIME,  $\min = 0$  and  $\max = 1$  for all trials,  $\text{error} = |p\text{Answer} - c\text{Answer}|$  for TIME trials.

### 4.4 Hypotheses

Our hypotheses for this experiment were:

**Hnull speed worse** For SPEED, we expect encodings that do not encode speed ( $S_0 T_{\text{VALUE}}$  and  $S_0 T_{\text{SIZE}}$ ) to result in larger errors than  $S_0 T_{\text{LENGTH}}$  (from which speed can be deduced), which in turn should result in larger errors than encodings that encode speed.

**Hnull time worse** For TIME, we expect encodings that do not encode time ( $S_{\text{VALUE}} T_0$  and  $S_{\text{SIZE}} T_0$ ) to result in larger errors than those that encode time.

**Hvalue time bad** For TIME, we expect encodings mapping time to  $T_{\text{VALUE}}$  to result in larger errors than those mapping time to either  $T_{\text{SIZE}}$  or  $T_{\text{LENGTH}}$ , as perceiving small color variations is delicate.

### 4.5 Apparatus and Participants

The setup consisted of a desktop computer equipped with a mouse, a keyboard, and a 24" LCD display with a resolution of 1920x1080 pixels. Trajectories were shown in a  $700 \times 400$  pixel area. To make sure the color ramps were equally visible at each end of the scale, we used a light gray background. Participants sat at a distance of approximately 65 cm from the display.

We recruited 18, non-color blind, participants (12 females, 5 males, 1 chose not to say) aged 18–45 (mean 25.4), via posters displayed in the university. There were 16 students, 3 of which were studying Computer Science. 5 participants had prior knowledge of information visualization (see Figure 11).

### 4.6 Procedure

**1. Introduction.** Participants filled out a demographic questionnaire. They were then asked to follow the instructions from the study software on the screen. The software consisted of an introduction followed by perception tasks for the 9 ENCODING blocks. Participants progressed through the software by pressing the “N” key in instruction screens, and pressing the “space” key in trial screens. They were instructed to answer as accurately as possible, and to make their best guess whenever they did not know the answer.

The first introduction screens consisted of a series of images showing a person driving home from work along a route displayed on a map. The text explained how stopping at traffic lights and speeding along a highway impacted the time and speed along the journey. The goal of the explanation was to ensure that participants would correctly interpret the paths as 2D+time trajectories, not as time axes.

**2. Perception tasks.** At the beginning of each of the 18 ENCODING  $\times$  TASK blocks, an instruction screen explained how to read the encoding, with three example images illustrating either TIME or SPEED on i) a STRAIGHT path with a linear time function; ii) a STRAIGHT path with a non-linear time function; and iii) a CURVED path with a non-linear

time function. Each instruction screen was followed by a minimum of two training trials (one STRAIGHT, one CURVED), with randomly generated paths and time functions. Thus participants were asked to practice for every task for every encoding.

During the training trials, an encoded path was shown alongside a statement explaining the task. For TIME, the statement was “Click the point on the path where the time that has elapsed is: X”, with X being one of 25%, 50%, and 75%. For SPEED, it was “Click the point on the path where the speed is: X”, with X being either fastest or slowest.

Each trial displayed the encoding for 10 seconds, with a timer bar shown on screen. When 10 seconds had elapsed, the path remained on the screen but the encoding disappeared and the participant answered with their best estimation. We set this 10 second limit for two reasons. First, it ensured that the study would be completed within a reasonable amount of time to prevent participant fatigue. Second, we fixed this limit because the goal of this study was to assess the *immediate* graphical perception of visual encodings. Our pilot studies revealed that completion times longer than 10 seconds occurred only when participants were attempting to measure the size of the path or to count the number of ticks very carefully, and perception, not careful reading, was the focus of this study. To answer, the participant moved the mouse along the path and clicked when they were satisfied. An orange circle tracked the mouse position along the path to make it explicitly clear which point on the path the participant is about to select.

To ensure that each participant understood the encoding while training, the software showed the correct answer's location on the path after they gave an answer. For SPEED, the software also showed their accuracy (i.e., 1 - error), since answers that are very close to correct may not be physically near the location of the true maximum or minimum speed. Participants were encouraged to generate as many new training trials as they liked by pressing the “R” key.

Once participants completed the training for any TASK  $\times$  ENCODING, the software warned them that measured trials would start. Participant then performed the six STRAIGHT trials, followed by the six CURVED trials. Measured trials were identical to training trials, except participants were not allowed to retry and the software did not provide any information about the correct answer or the answer's accuracy. We recorded the participant's answer and the time spent for each trial.

**3. Concluding the study.** After completing all trials, participants indicated how effective each encoding was for performing each task on a 1–5 Likert scale (1: very bad, 3: neutral, 5: very good). We reminded them of the encodings using images. Participants scored each technique twice, for TIME and SPEED. The whole experiment took approximately one hour, and participants received \$20 remuneration.

### 4.7 Results

To report the results of our study, we follow the recommendation from APA [3] and base our analyses on *estimation* using bootstrapped [45] confidence intervals [28] instead of p-values. A 95% confidence interval contains the true mean 95% of the time and conveys effect sizes [28], making it possible to *estimate* differences between encodings. This approach has been recommended for reporting statistical results in HCI over the traditional null hypothesis significance testing (with p-values only), which leads to dichotomous thinking [31]. It has seen increased use recently in HCI and visualization (e.g., see [17, 30, 42, 69, 74, 79]).

We prespecified analyses before conducting the experiment and tested on pilot data. For 3% of the trials (118/3888), participants reached the 10 seconds timeout. We discarded from the analysis the 138 SPEED trials and the 19 TIME trials with  $\text{error} > .5$ . The rationale for discarding these trials is that such large errors indicate that participants performed the inverse task of what was expected, e.g., they identified the point at which the speed was minimum while they were asked to identify the point at which it was maximum.

As our experiment consists of many conditions, we used Cousineau's [25] approach for reporting confidence intervals in within-participant designs. This approach removes individual differences (variance) between participants when testing multiple conditions. We explain the procedure for reporting errors (it is similar for reporting completion times):

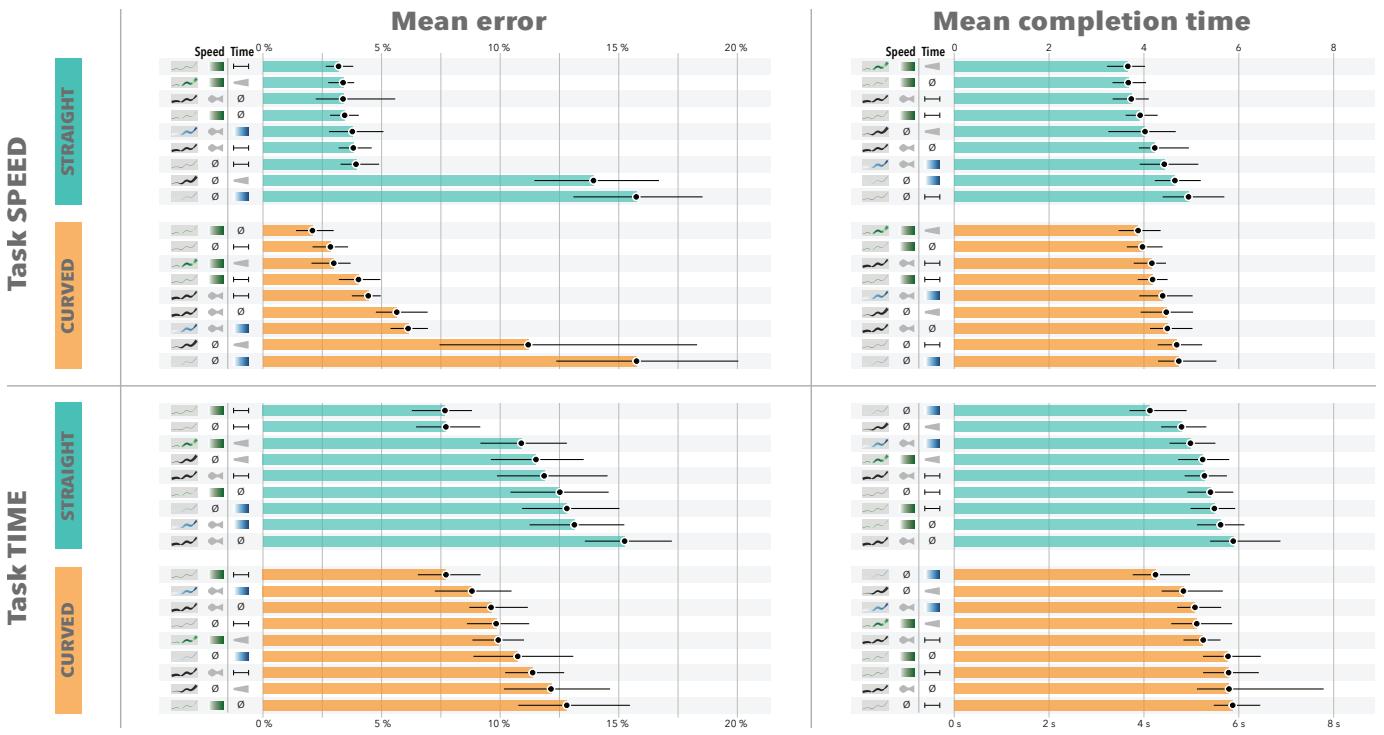


Fig. 10. Error and completion time mean 95% confidence intervals for each visual encoding by TASK and SHAPE, sorted according to mean estimate. Confidence intervals were computed according to Equation  $E_1$  to remove individual differences [25].

### 1. Computing participant estimates for each condition.

Let  $e_{p,\{\text{ENCODING},\text{TASK},\text{SHAPE}\}}$  be the error estimate for participant  $p$  for each  $\text{ENCODING} \times \text{TASK} \times \text{SHAPE}$  condition. The error estimate is the mean error of the six trials for this condition.

### 2. Computing participant mean across conditions.

Let  $E_{p,\{\text{TASK},\text{SHAPE}\}}$  be the mean error for participant  $p$  for each  $\text{TASK} \times \text{SHAPE}$  condition, i.e., across all  $\text{ENCODING}$ .

### 3. Computing overall mean across conditions.

Let  $E_{\{\text{TASK},\text{SHAPE}\}}$  be the mean error for all participants for each  $\text{TASK} \times \text{SHAPE}$  condition, i.e., across all  $\text{ENCODING}$ .

### 4. Removing individual differences.

We compute the adjusted error for each participant and each condition using Equation  $E_1$ .

$$\begin{aligned} e_{p,\{\text{ENCODING},\text{TASK},\text{SHAPE}\}} &= \\ e_{p,\{\text{ENCODING},\text{TASK},\text{SHAPE}\}} - E_{p,\{\text{TASK},\text{SHAPE}\}} + E_{\{\text{TASK},\text{SHAPE}\}} &= \end{aligned} \quad (E_1)$$

Figure 10 shows error and completion times adjusted using Equation  $E_1$ , by  $\text{ENCODING}$ ,  $\text{TASK}$ , and  $\text{SHAPE}$  using 95% bootstrapped confidence intervals. Black dots are mean point estimate, i.e., the best guess, and the black lines represent confidence intervals, whose length conveys effect sizes. Figure 11 shows participants' demographics and Likert Scale answers for each encoding. Note that the encoding ratings for P1 and P2 are missing due to a technical error in the data collection of the post-questionnaire.

## 5 DISCUSSION

Completion times show that participants were much faster than 10 seconds to complete both tasks, regardless of the encodings. Completion times are also less discriminating than errors (almost all confidence intervals overlap in Figure 10 – Mean completion time). Therefore, we focus on the error measure to analyze the results and mention completion time wherever there is a notable result. We use the notation  $A > B$  to express that participants made smaller errors with encoding  $A$  than with encoding  $B$ , and  $A \geq B$  to express that there are indications that there may be a small difference between encoding  $A$  and encoding  $B$ . We first discuss the perception of speed (SPEED task) then the perception of time (TIME task). We provide overall recommendations, discuss the limitations and indicate possible future work.

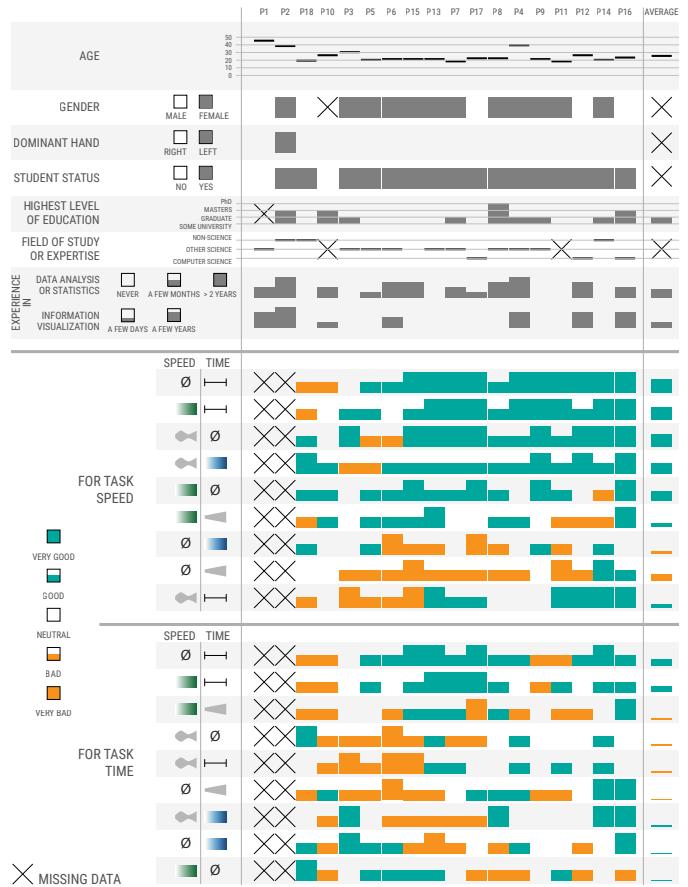


Fig. 11. Participant demographics and 1–5 Likert scale answers for each encoding and task. Participants were reordered using Bertifier [59] according to the similarity of the scores they gave to each encoding. Encodings were vertically reordered independently for SPEED and for TIME according to the similarity of the scores they received.

## 5.1 Perceiving Speed on 2D+Time Trajectories

For perceiving speed on 2D+time trajectories,  $S_{VALUE} >= S_{SIZE} > S_\emptyset$ . This confirms **Hnull speed worse**. Encodings that do not encode speed directly ( $S_\emptyset T_{VALUE}$  and  $S_\emptyset T_{SIZE}$ ) resulted in much larger errors than those that do. This also means that inferring speed from an encoding that maps time to either  $T_{SIZE}$  or  $T_{VALUE}$  is difficult. We also found that encodings with  $S_{VALUE}$  tend to result in smaller errors (as well as faster completion times) than those with  $S_{SIZE}$ . While the differences are not definitive and small for STRAIGHT, they are pronounced for CURVED.

However, participants were able to reasonably deduce speed from  $S_\emptyset T_{LENGTH}$  for STRAIGHT, and they made only small errors with this encoding with CURVED paths (similar errors to encodings with  $S_{VALUE}$ ). This seems to contradict our findings that  $S_\emptyset$  is the worst for perceiving speed, but  $T_{LENGTH}$  is a particular encoding in that it also shows *linearly discretized* speeds, which overcomes the lack of a direct speed encoding. Participants also performed better with  $S_\emptyset T_{LENGTH}$  than with encodings using  $S_{SIZE}$  for CURVED. This indicates that  $T_{LENGTH}$  is more robust to the shape of the trajectory than  $S_{SIZE}$ . This is not surprising as the size of the trajectory is greatly distorted on curved paths.

For STRAIGHT paths,  $S_\emptyset T_{LENGTH}$  also resulted in longer completion times than the three encodings with  $S_{VALUE}$  and the  $S_{SIZE} T_{LENGTH}$  encoding. This result is likely due to  $T_{LENGTH}$  not being a direct mapping of speed. Results for CURVED paths are less definite but there may be a similar, weaker effect. One intriguing result is that for CURVED,  $S_\emptyset T_{LENGTH} >= S_{VALUE} T_{LENGTH}$ . The difference is both small and weak, but might indicate that while both  $S_{VALUE}$  and  $T_{LENGTH}$  accurately convey speed, combining both may result in a less accurate encoding. It is difficult to speculate on the reasons for this possible effect without conducting a new study, as there is no obvious explanation.

Interestingly, adding  $T_{LENGTH}$  on top of  $S_{SIZE}$  did not improve the reading of speed ( $S_{SIZE} T_{LENGTH} \approx S_{SIZE} T_\emptyset$ ). This may be because consecutive segments blend together when they have similar speeds.

Participants gave the best ratings for performing SPEED (see Figure 11) for the two encodings that also resulted in the smallest errors:  $S_{VALUE} T_{LENGTH}$  and  $S_\emptyset T_{LENGTH}$ . Also, while participants made small errors with  $S_{VALUE} T_{SIZE}$ , they found this encoding to be bad for speed-related tasks. Unsurprisingly, participants gave the lowest scores to encodings that do not show speed. They also gave low scores to  $S_{SIZE} T_{LENGTH}$ , possibly due to how this encoding may mask the time divisions when consecutive segments have similar speed.

## 5.2 Perceiving Time on 2D+Time Trajectories

For reading time on 2D+time trajectories,  $T_{LENGTH} > T_{SIZE} >= T_{VALUE} > T_\emptyset$ . This agrees with **Hnull time worse** and **Hvalue time bad**. Participants made larger errors with encodings that do not explicitly encode time than with those that do (**Hnull time worse**). Among the encoding that explicitly encode time, they made larger errors with  $T_{VALUE}$ , although not confidently.  $T_{LENGTH}$  clearly led to the lowest errors for STRAIGHT, and led to lower-or-similar errors than other encodings for CURVED. We found two exceptions to this high-level result. These involve the  $S_{SIZE} T_\emptyset$  and  $S_{SIZE} T_{LENGTH}$  encodings.

First, **Hnull time worse** is confirmed except for  $S_{SIZE} T_\emptyset$ . As we expected, participants made large errors with this encoding which does not encode time. However, for the CURVED paths, they made relatively low errors. Because this result was unexpected, we examined the six tuples that corresponded to this ENCODING  $\times$  TASK  $\times$  SHAPE block. While each tuple had a non-trivial time function that appeared to add complexity to the task, the correct answer for five of six trials occurred very close to the correct answer if it were a linear time function. The remaining trial had an average accuracy of 79%, with only one participant having less than 10% error. While we do not discard these results, this suggests that the encoding did not play a role in aiding the perceptibility of time, as participants would make errors lower than expected simply by answering as if the encoding was not there.

Second,  $T_{LENGTH}$  did not always result in low errors for TIME. While participants made small errors with  $S_{VALUE} T_{LENGTH}$  and  $S_\emptyset T_{LENGTH}$ ,  $S_{SIZE} T_{LENGTH}$  resulted in large errors for both STRAIGHT and CURVED. The interaction between  $S_{SIZE}$  and  $T_{LENGTH}$  that we found for SPEED also occurs for TIME:  $S_{SIZE} T_{LENGTH}$  is worse than  $S_\emptyset T_{LENGTH}$ .

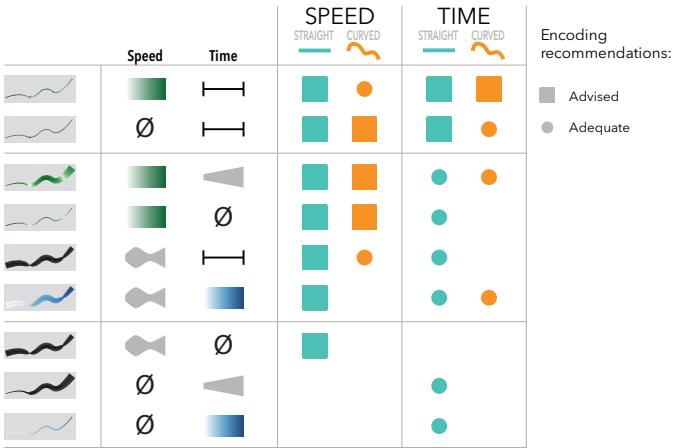


Fig. 12. Summary of our recommendations for encoding time and speed on 2D+time trajectories, according to the shape of the path and the information to convey.

One noteworthy result is that  $T_{VALUE}$  is not good at conveying time. This confirms our hypothesis that discriminating small variations in color value is difficult. It is worth noting that the TIME task, where this result occurs, asks for participants to find specific non-extremum values along an increasing gradient. In that sense, it is different from the SPEED task which asks only for a maximum or minimum value on a path. The nature of the tasks may explain why the value encoding is much worse for TIME than SPEED, but the properties of the data (time monotonically increases while speed can go up and down) encourage different tasks. Despite our findings, the literature is full of examples where time is mapped to value (e.g., [9, 23, 77]). In contrast,  $T_{SIZE}$ , which leads to more accurate perception of time than  $T_{VALUE}$ , is almost never used for encoding time on 2D paths. Interestingly, participants were the fastest with  $S_\emptyset T_{VALUE}$ , while making large errors. One explanation is that participants gave up and rapidly gave a best guess once they had realized that reading time is difficult with this encoding.

It is also important to note that while participants made small errors with  $S_{VALUE} T_{LENGTH}$  and  $S_\emptyset T_{LENGTH}$ , their completion times with these encodings were slightly higher than with some other encodings. While the result are not definitive and the differences are small, it raises questions about how humans decode  $T_{LENGTH}$ . We can safely assume that comparing multiple segment sizes requires more effort than e.g. finding the darkest color value with  $S_{VALUE}$ . This is particularly true for CURVED, where segments get distorted and become harder to compare as they can have varying curvatures.

Participants (see Figure 11) scored high two encodings with  $T_{LENGTH}$ :  $S_{VALUE} T_{LENGTH}$  and  $S_\emptyset T_{LENGTH}$ . As for SPEED,  $S_{SIZE} T_{LENGTH}$  got low scores. Interestingly, they slightly preferred  $S_\emptyset T_{LENGTH}$  over  $S_{VALUE} T_{LENGTH}$ , while it resulted in slightly larger errors for CURVED; this may be because adding  $S_{VALUE}$  on top of  $T_{LENGTH}$  adds visual clutter.

## 5.3 Recommendations

Most of the differences between encodings are small. However, they are consistent across TASK and SHAPE. Participants' subjective judgements broadly align with quantitative findings. They found encodings with  $T_{LENGTH}$  to be very good at conveying both time and speed, with the exception of  $S_{SIZE} T_{LENGTH}$ . They also found  $T_{VALUE}$  to be bad at conveying time. While the subjective preferences are consistent across participants for SPEED, they are less clear for TIME, as no encoding unanimously received positive judgement despite some clear differences in terms of error measure. This suggests that people may be less conscious of the efficacy of encodings when estimating time than when estimating speed. This may also be due to the fact that for the SPEED task, participants were always looking for an extremum value, while for the TIME task they were also looking for intermediate values. As a result, clear recommendations for encoding time and speed on 2D+time trajectories emerge. Figure 12 presents a summary of these recommendations.

To best encode speed only:

- RS<sub>1</sub>** We advise encoding speed with **value S<sub>VALUE</sub>**. If value cannot be used (e.g., color is already used extensively), we advise conveying speed by mapping time to **segment length T<sub>LENGTH</sub>**.
- RS<sub>2</sub>** If a visualization contains **straight** paths only and the important information to convey is **speed**, then any speed encoding (and **T<sub>LENGTH</sub>**) can be used. If it contains **curved** paths, we discourage encoding speed with **size S<sub>SIZE</sub>**.

To best encode time only:

- RT<sub>1</sub>** We advise encoding time with **segment length T<sub>LENGTH</sub>**, in the form of time ticks.
- RT<sub>2</sub>** We discourage encoding time with **value T<sub>VALUE</sub>**.

To best encode both time and speed simultaneously:

- RTS<sub>1</sub>** We advise using **segment length T<sub>LENGTH</sub>** whenever possible to convey both time and speed. Encoding speed with **value S<sub>VALUE</sub>** on top of **T<sub>LENGTH</sub>** can improve perceiving time, but may slightly interfere with perceiving speed.
- RTS<sub>2</sub>** If the number of available variables is limited, we advise using **segment length** alone **S<sub>0</sub> T<sub>LENGTH</sub>** as this encoding conveys both time and speed.
- RTS<sub>3</sub>** If using segment length **T<sub>LENGTH</sub>** is not possible or not desirable within the context of a visualization, we advise encoding speed with **value** and time with **size S<sub>VALUE</sub> T<sub>SIZE</sub>**.

Our results are consistent with geography guidelines [49] and urban planning and traffic maps [36]. They provide quantifiable evidence that confirms existing empirical knowledge, such as that size should not be used to encode time. Also, although we do not explicitly rank the visual variables, our results are consistent with Mackinlay's ranking of visual variables [55]. One difference is that because our results are not the same for straight and curved paths, we expect this ranking to change according to both the shape of the 2D path and the time distribution of more complex 2D+time trajectories.

## 5.4 Limitations and Future Work

This first empirical evaluation of visual encodings of time and speed on 2D+time trajectories allowed us to draw recommendations regarding which encoding to use for conveying time and speed independently or in combination. More than providing Infovis designers with a set of clear rules, these results open the door to a wealth of opportunities for running new studies and developing a fuller understanding of the challenges of encoding temporal data on 2D paths.

As for any perceptual study, our results and recommendations are valid within the scope of the study. In particular, we favored the diversity of encodings at the expense of constraining other factors. Our findings apply to simple shapes and value estimation tasks. However, like other focused controlled studies, our results provide precision rather than generalizability [57] and may not generalize to more complex tasks and path shapes. Perhaps the most generalizable results are those where encodings poorly conveyed either time or speed; if these encodings fail for simple tasks and paths, they should also fail for more complex tasks and path shapes. Specifically, building on our results, future studies can discard **T<sub>VALUE</sub>** and **T<sub>SIZE</sub>** for encoding time and avoid **S<sub>SIZE</sub>** for encoding speed. We think the following factors are worth studying in the future.

**The path** of a 2D trajectory can be complex in many ways. We tested path curvature, finding different results for straight and curved paths. Other path characteristics that could affect the accuracy of encodings include path direction, length, angles, and crossings:

**Path Direction.** While the paths in our study all started from the left and ended on the right, this is unlikely to be the case in a real-world context. Representing the direction of a 2D+time trajectory is often important (e.g., in connected scatterplots [40] and for eye-tracking scanpaths [35]), and related work indicates that humans have a leftward bias of attention [29]. This makes direction worth studying in the context of 2D+time trajectories. One could also study how time encodings provide direction on complicated paths. For example, **T<sub>SIZE</sub>** and **T<sub>VALUE</sub>** provide direction, while **T<sub>LENGTH</sub>** does not. To convey direction when mapping time to **T<sub>LENGTH</sub>**, one could also map time to **T<sub>VALUE</sub>**, although

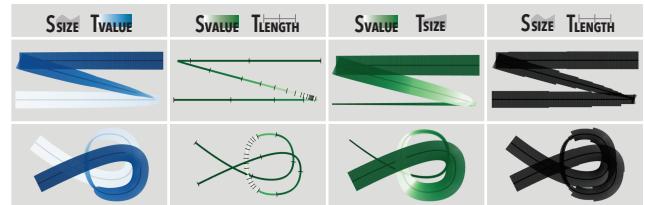


Fig. 13. Some encodings for paths with sharp corners and loops.

this would prevent using **S<sub>VALUE</sub>** to encode speed. An alternative is to mark time ticks with arrowheads instead of perpendicular lines.

**Path Length.** We tested paths with similar lengths. Further studies could assess the effect of path length on the perception of encodings. We envision that encodings which show the absolute value of time at each point along the path in a continuous manner (**T<sub>SIZE</sub>** and **T<sub>VALUE</sub>**) will lead to larger errors as the length of the path increases, as this puts further stress on our ability to perceive small variations. One could increase the maximum size of the path with **T<sub>SIZE</sub>**, but this would clutter the visualization. In contrast, we expect **T<sub>LENGTH</sub>** – which scales well since it shows relative time – to be robust to longer paths.

**Path Angles.** Our paths had smooth angles. While this can be the case (e.g., Figures 2 and 7), paths often have sharp corners, e.g., road trips can feature 90 degree turns. Sharp corners encoded with **S<sub>SIZE</sub>** or **T<sub>SIZE</sub>** may be less useful due to visual artifacts and overlaps (Figure 13).

**Path crossings.** We tested individual paths that do not cross. The perception of encodings will be affected if one or more paths cross (see Figure 13). For example, **S<sub>VALUE</sub>** and **T<sub>VALUE</sub>** will be difficult to read when they overlap. **S<sub>SIZE</sub>** and **T<sub>SIZE</sub>** suffer from the same problem if the encoding is fully opaque, but transparency may improve the result. We expect **T<sub>LENGTH</sub>** to be more robust for paths that cross.

**The context** surrounding a 2D+time trajectory plays an important role in the perception of graphical encodings. We used a light gray background so that color ramps can contain white. In many cases, mapping time or speed to color will not be an option if surrounding elements already make use of a variety of colors. To a lesser extent, the context can affect **T<sub>LENGTH</sub>**. For example, latitude lines on a map (see Figure 7) or contour lines on a contour map would conflict with time ticks and make it more difficult to decode time and speed. Also, **S<sub>SIZE</sub>** and **T<sub>SIZE</sub>** are likely to impact the surroundings because they take much screen real estate. Finally, when objects are represented on 2D+time trajectories, such as players on a soccer field [60], fixation times on eye-tracking scanpaths [35], or the data points in Figure 4, the manner of their representation may constrain which encoding can be used.

**The time function** of a 2D+time trajectory can affect the perception of encodings. In this study, we used a variety of time functions. Future work could study the differences between various time distributions and ranges of speeds. Some encodings may be better for small and smooth variations, some for large and abrupt variations, and others may be more robust to the whole spectrum of possible distributions.

## 6 CONCLUSIONS

Results of our graphical perception study of nine visual encodings provide advice for encoding time and speed on simple 2D+time trajectories. However, it is important to remember that 2D+time trajectories are more general than spatio-temporal trajectories. Our findings apply not only to spatial data, but also to abstract representations that need to convey speed and/or time but already make use of the two dimensions of the plane. This is the case for example with connected scatterplots [40], eye-tracking data [14] and other time-evolving abstract data graphics [46]. We hope that these initial results will help researchers design new studies to develop a fuller understanding of how to best visually encode 2D+time trajectories.

## 7 ACKNOWLEDGMENTS

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