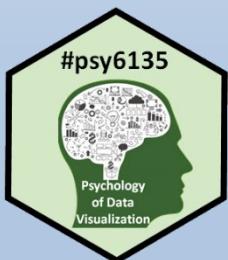


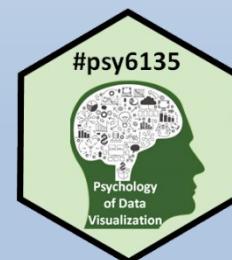
The Language of Graphs: from Bertin to GoG to ggplot2



Michael Friendly

Psych 6135

<https://friendly.github.io/6135/>



Orienting questions

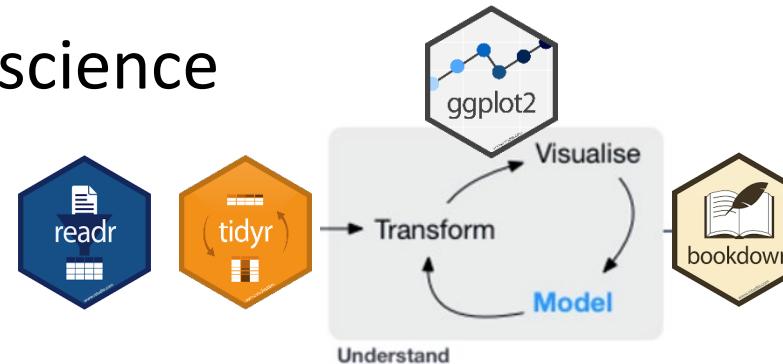
- How did we get from early ideas of graph types (line, bar, pie charts, scatterplots, ...) to expressing those in modern software?



- What new thinking was required?
- How to formalize different kinds of graphs and their attributes?
- How to make the language of a graph express what we want to see?
- How to do that most simply, elegantly, or generalizable?

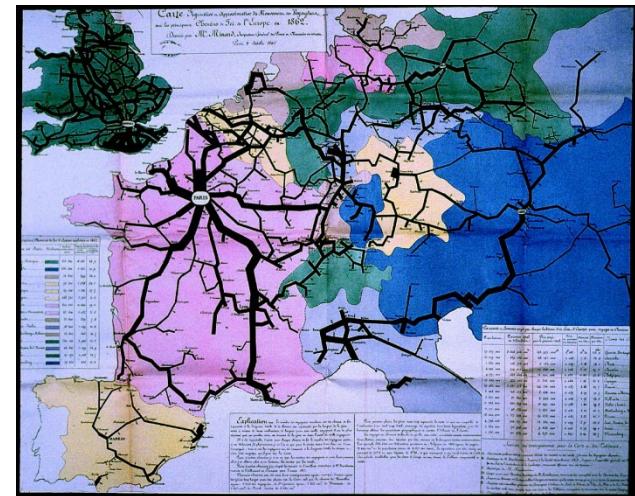
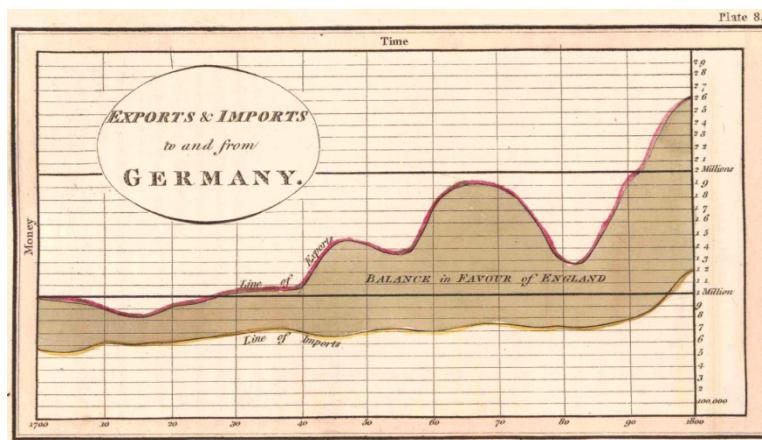
Topics

- Idea: Graphs as visual language
 - Early attempts at standardization of graphs
- Jacques Bertin: *Semiology of Graphics*
 - Mapping of visual properties to data relations
- Graphics programming languages:
 - Goal: power & elegance
- Lee Wilkinson: *Grammar of Graphics*
- Hadley Wickham: ggplot2
- Graphs in data science



Metaphor: Graphs as visual language

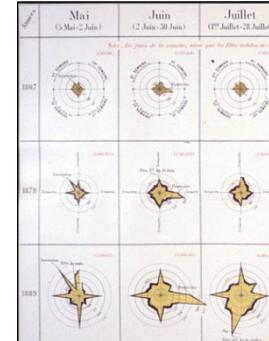
- Playfair, Guerry, Minard and others described their fundamental insight that **graphical displays** convey quantitative data more directly than **numbers**.
 - Playfair (1802)
 - *“Regarding numbers and proportions, the best way to catch the imagination is to speak to the eyes”*
 - Minard (1861)
 - *“The aim of my carte figurative is ... to convey promptly to the eye the relation not given quickly by numbers requiring mental calculation.”*



Metaphor: Graphs as visual language



- Émile Cheysson (1890) took this further:
 - “When a law is contained in figures, it is buried like metal in an ore; it is necessary to extract it. This is the work of graphical representation.
 - It points out the coincidences, the relationships between phenomena, their anomalies, and we have seen what a powerful means of control it puts in the hands of the statistician to verify new data, discover and correct errors with which they have been stained.”



Willard C. Brinton: An ode to graphs

MAGIC IN GRAPHS

THERE is a magic in graphs. The profile of a curve reveals in a flash a whole situation —the life history of an epidemic, a panic, or an era of prosperity. The curve informs the mind, awakens the imagination, convinces.

Graphs carry the message home. A universal language, graphs convey information directly to the mind. Without complexity there is imaged to the eye a magnitude to be remembered. Words have wings, but graphs interpret. Graphs are pure quantity, stripped of verbal sham, reduced to dimension, vivid, unescapable.

Graphs are all inclusive. No fact is too slight or too great to plot to a scale suited to the eye. Graphs may record the path of an ion or the orbit of the sun, the rise of a civilization, or the acceleration of a bullet, the climate of a century or the varying pressure of a heart beat, the growth of a business, or the nerve reactions of a child.

The graphic art depicts magnitudes to the eye. It does more. It compels the seeing of relations. We may portray by simple graphic methods whole masses of intricate routine, the organization of an enterprise, or the plan of a campaign. Graphs serve as storm signals for the manager, statesman, engineer; as potent narratives for the actuary, statistician, naturalist; and as forceful engines of research for science, technology and industry. They display results. They disclose new facts and laws. They reveal discoveries as the bud unfolds the flower.

W. C. Brinton,
*Graphic
Presentation,*
1939

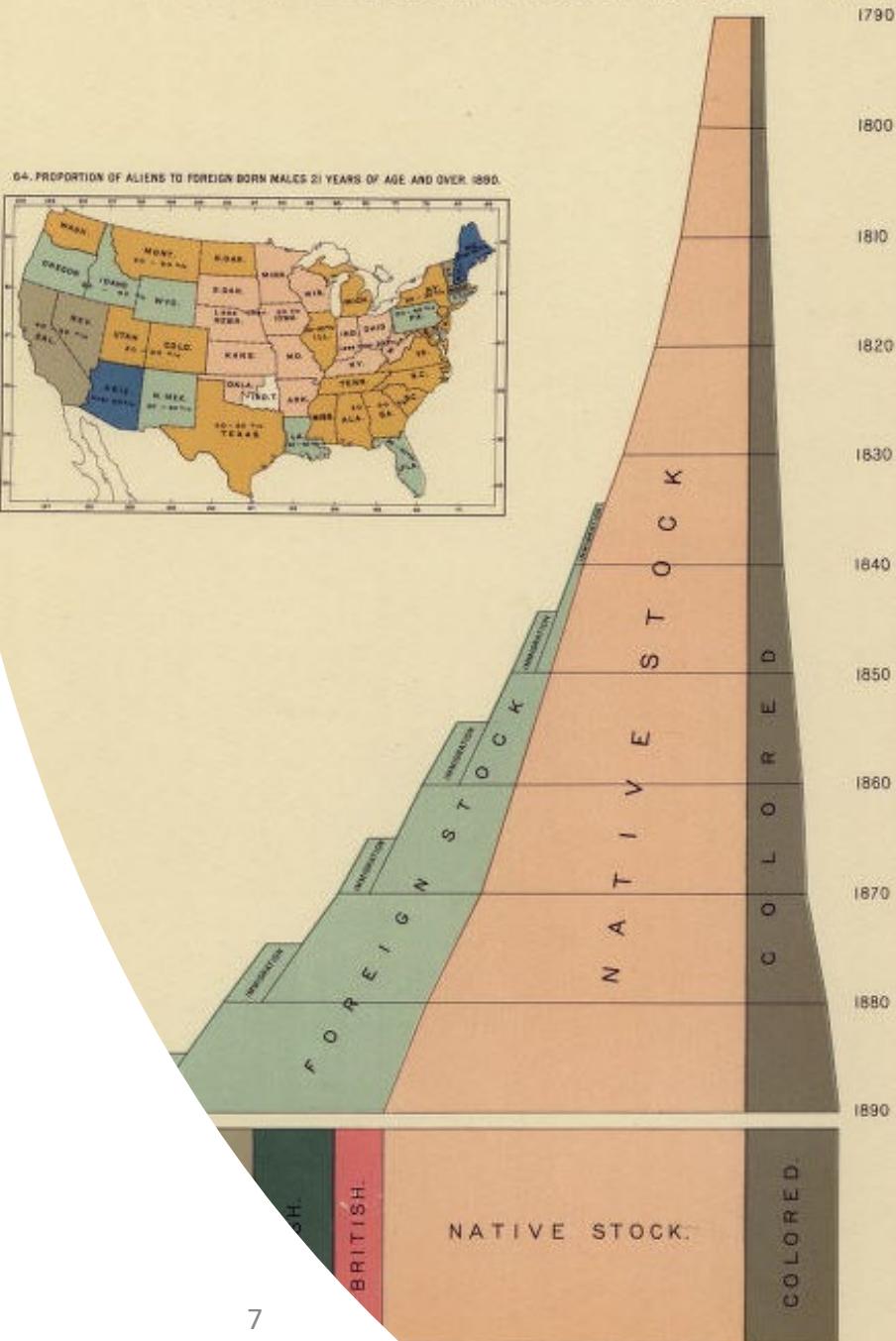


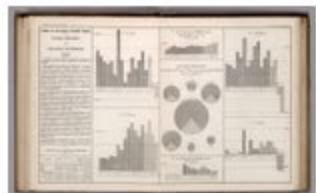
Context: Statistical albums, 1870-1910

From ~1870–1910, statistical albums of official statistics on topics of population, trade, moral & political issues became widespread throughout Europe and the U.S.

- France: *Album de Statistique Graphique*: 1879-1899 (trade, commerce & other topics)
- USA: Census atlases: 1870/80/90--
- Switzerland: *Atlas graphique de la Suisse*: 1897, 1914

64. PROPORTION OF ALIENS TO FOREIGN BORN MALES 21 YEARS OF AGE AND OVER, 1890.

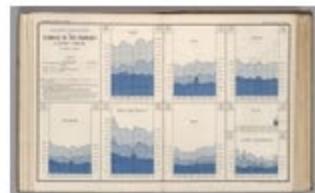




Ministere des T...
Statistical Dia...
1882
National Atlas
Related (32)



Ministere des T...
Statistical Dia...
1882
National Atlas
Related (32)



Ministere des T...
Statistical Dia...
1883
National Atlas
Related (34)



Ministere des T...
Statistical Dia...
1883
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Ministere des T...
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Ministere des T...
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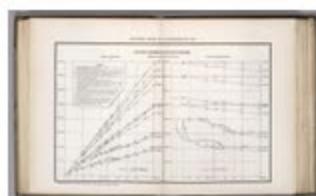
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Statistical Dia...
1885
National Atlas
Related (31)



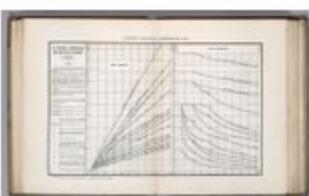
Ministere des T...
Statistical Dia...
1884
National Atlas
Related (31)



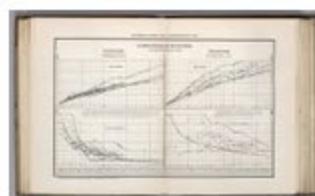
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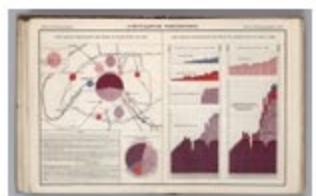
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Statistical Dia...
1884
National Atlas
Related (31)



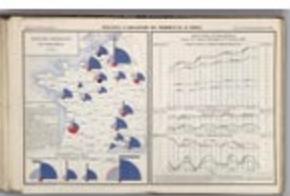
Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



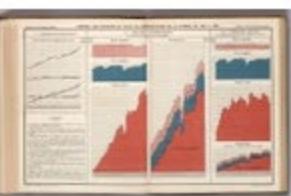
Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)



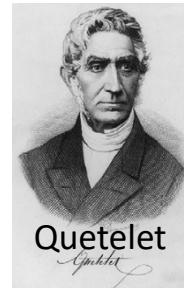
Ministere des T...
Statistical Dia...
1886
National Atlas
Related (2)

Need for standardization

- Beautiful graphics: Yes, but all separate designs
 - Can anything be compared across countries?
- Émile Cheysson (1878)
 - *"The time will come when Science has to lay down general principles and decide on well-defined standards. We can no longer tolerate this sort of anarchy"*
- International statistical meetings (ISI)
 - 1852 (Brussels), 1857 (Vienna), 1869 (The Hague), 1872 (St. Petersburg), 1876 (Budapest) ...
 - Participants: Quetelet, Cheysson, Levasseur (France), Ernest Engel, Gustav von Mayr, Hans Schwabe (Germany), Francis Walker (U.S.), ...



Cheysson



Quetelet
Gedächtnis



Levasseur



von Mayr



Walker

No consensus, but the germ of an idea

- ISI St. Petersburg (1872) resolutions:
 - *"The Congress accepts that it is not worth going into details about the choice of methods or facts for graphical representation".*
 - *"no strict rule can be imposed on authors, because the only real problem is that of applying the graphical method to data that is comparable".*
- Standardize the data before the graphs!
- Most of the debate had to do with thematic maps
 - number of class intervals for a quantitative variable
 - number and variety of shading colors
- Yet, the idea of a **visual language** had been accepted, along with the need for some **theory of graphs**



Fast forward

Bertin: Modern theory of data graphics

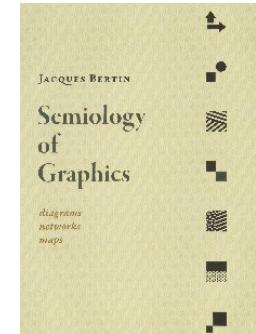
A Semiology of graphics:

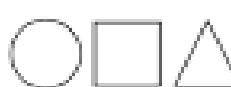
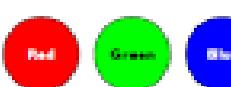
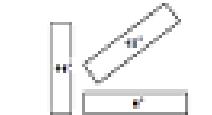
- Visual variables
 - Decoding: Reading levels of a graph
 - Reorderable matrix

	Selective	Associative	
Visual Variables	Position	• .	•• .•
	Size	• ●	•• ●●
	Shape		
	Value	○●○○○○	○●○●●○○●
	Color	• .	●○●●○●●●●
	Orientation	\\ /	
	Texture		

Bertin: *Semiology of graphics* (1967)

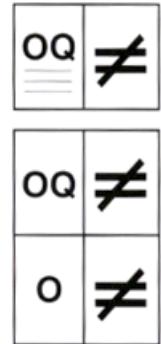
- Defines a system of “grammatical elements” of graphs and relations among visual attributes that give **meaning** (semantics) from perceptual features
 - **Planar** variables: (x,y) coordinates
 - **Retinal** variables: shape, size, color, ...



PLANAR VARIABLES	RETINAL VARIABLES		
Horizontal Position 	Shape 	Size 	Colour 
Vertical Position 	Value 	Orientation 	Texture 

Bertin: *Semiology of graphics*

- Defines a system of mapping of retinal variables to properties of data variables for perception of **relations**
 - Association (\equiv) – marks are perceived as **similar**
 - Selection (\neq) – marks are perceived as **forming classes**
 - Order (O) – marks are perceived as **showing order**
 - Quantity (Q) – marks are perceived as **proportional**
- This is the first theory of graphs relating visual attributes (encoding) to perceptual characteristics (decoding).
- It comprises nearly all known graph and thematic map types in a **general system**



The retinal variables and relationship types can be “implanted” in various symbol types in the plane (X,Y)

—VARIABLES OF THE IMAGE		POINT	LINE	AREA (ZONE)
XY	2 DIMENSIONS OF THE PLANE			
Z	SIZE			
	VALUE			
—DIFFERENTIAL VARIABLES		POINT	LINE	AREA (ZONE)
	TEXTURE			
	COLOR			
	ORIENTATION			
	SHAPE			

Visual variables & data characteristics

Visual variables differ in the kinds of information they can convey

Visual Variables	Characteristics				
	Selective	Associative	Quantitative	Order	Length
<i>Position</i>	• .	••• ••	↑	↑	Theoretically Infinite
<i>Size</i>	• ●	••●●		●>●>●>●	Selection: ~5 Distinction: ~20
<i>Shape</i>					Theoretically Infinite
<i>Value</i>	○●○○○○	○●○●○○●		○<○<○<○<○<●	Selection: <7 Distinction: ~10
<i>Color</i>	● ○	●○●○●○●			Selection: <7 Distinction: ~10
<i>Orientation</i>	\\ /				Theoretically Infinite
<i>Texture</i>	○○	○○○○			Theoretically Infinite

(≠) (≡) (Q) (O)

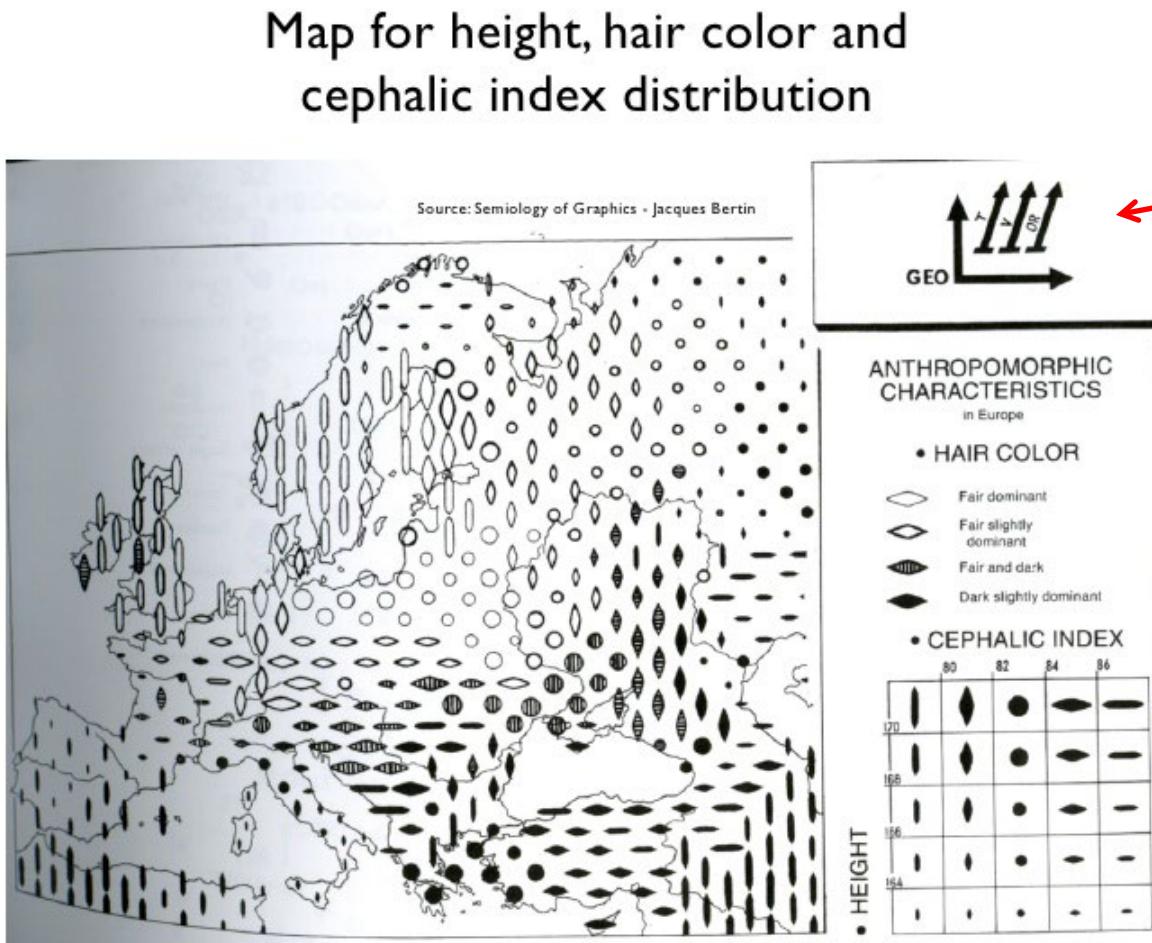
Some recommendations

Various authors have used Bertin's system to make recommendations for the best attributes to use with different symbol types

	Points	Lines	Areas	Best to show
Shape		possible, but too weird to show	cartogram	qualitative differences
Size			cartogram	quantitative differences
Color Hue				qualitative differences
Color Value				quantitative differences
Color Intensity				qualitative differences
Texture				qualitative & quantitative differences

Visual Variables

Retinal variables allow **several** variables to be encoded.
Bertin's system provides a general framework for thematic mapping, allowing multiple variables to be shown simultaneously in a single map.

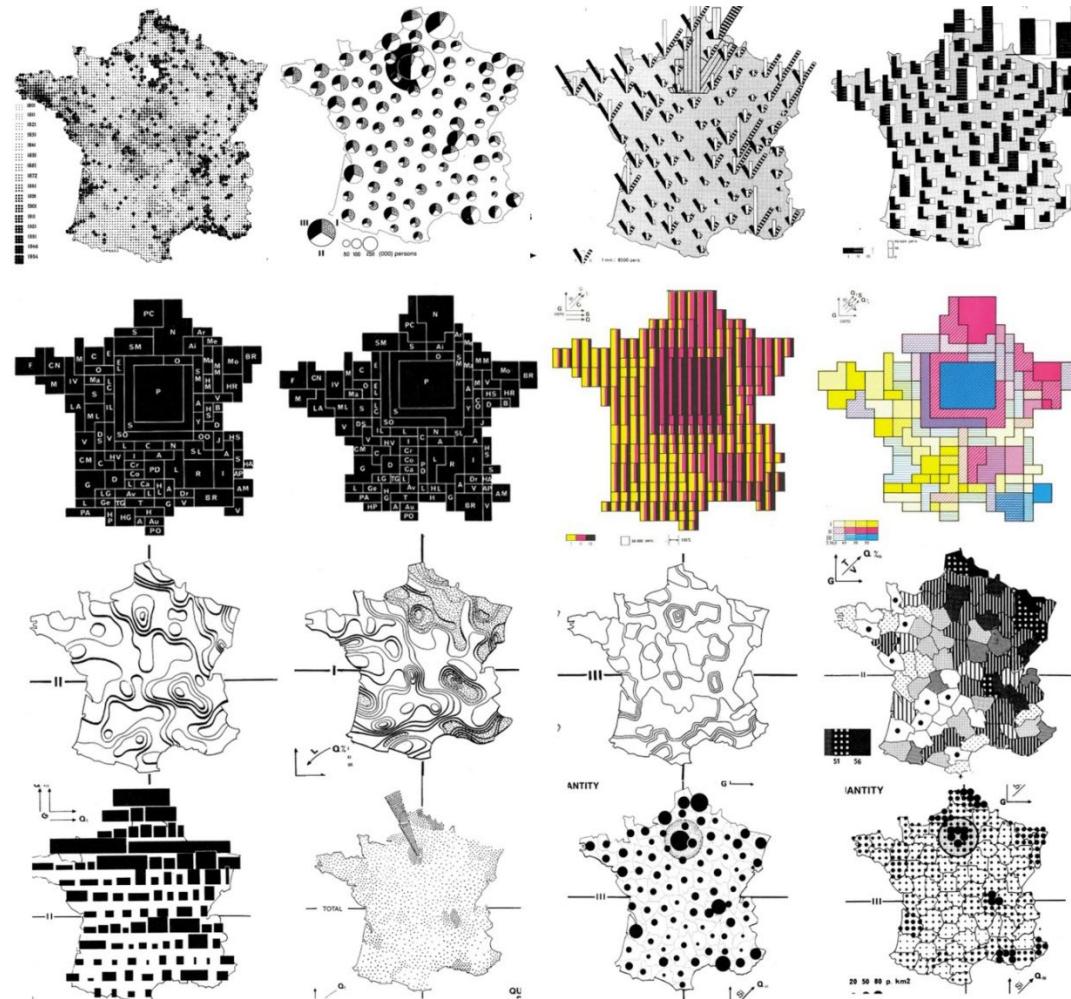


For Bertin, the legend is a symbolic description of the coordinate system and the variables displayed.

Various maps of France, encoding quantitative and categorical variables in a wide number of different ways.

This semiology is **productive**, as is the semiology of language.

Allows one to **imagine** new graphic encodings.



Decoding: Reading a graphic



How successful is a graph for transmitting information?

Bertin defines three **stages** for reading a graphic:

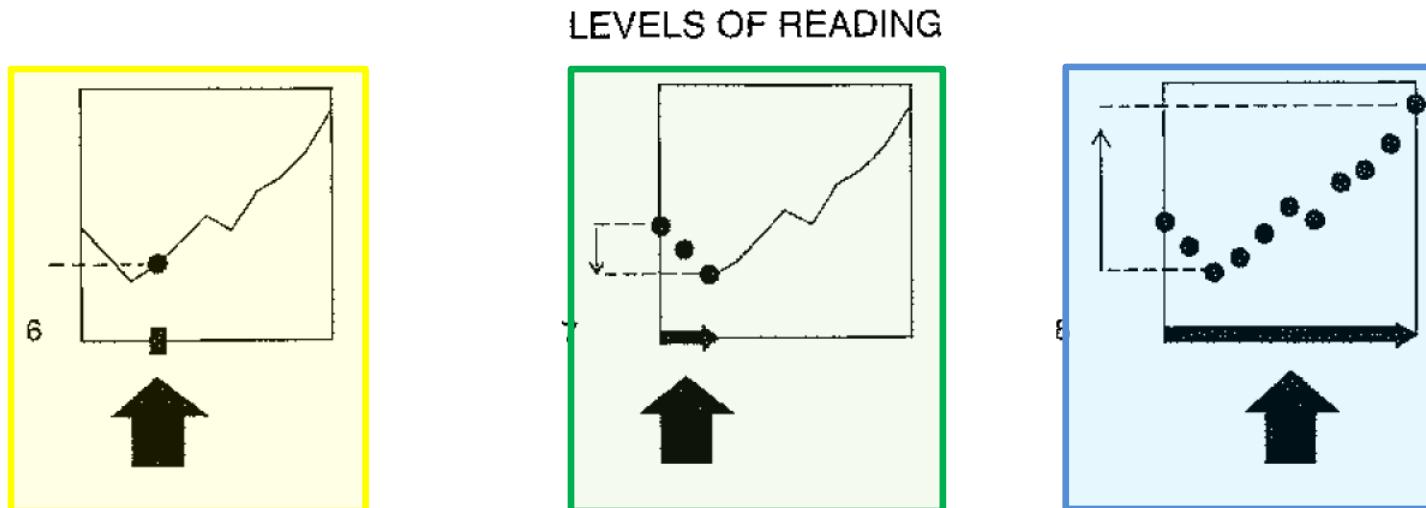
- **External:** What is the overall context?
 - Graph title, axis labels
- **Internal:** What visual variables are used to represent the components in the graphic?
 - points, lines, ...
 - size, shape, color:hue, color:intensity, texture, ...
- **Relationships:**
 - How are these components related?
 - What questions can I ask of this graphic?
 - What can I learn?

Research topic: Have there been any studies of this ordering in graph perception?

Reading levels

Questions a graph should answer:

- **Elementary**: find some specific value
- **Intermediate**: make comparisons, see a trend
- **Overall**: what is the general message or overall trend?

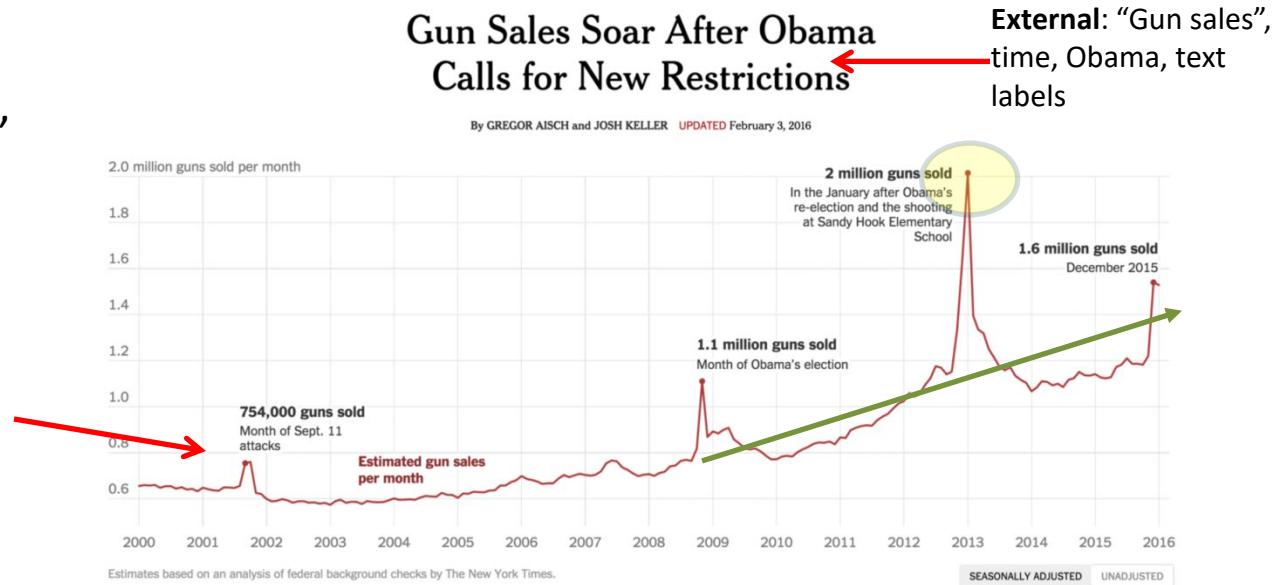


These ideas provided the beginnings of a theory of graphs related to graph perception.

Reading levels: Example

Graph from the NY Times,
Feb. 3, 2016

Internal: lines, points for
labeled events
Relationships: what is the
message?



Reading tasks:

- **Elementary:** “How many guns were sold in January of 2013?”
- **Intermediate:** “What’s the trend in gun sales since President Obama was elected?”
- **Overall:** “What’s the overall trend in gun sales in America since the year 2000?”

From: <https://medium.com/@karlsluis/before-tufte-there-was-bertin-63af71ceaa62>

Bertin: The reorderable matrix

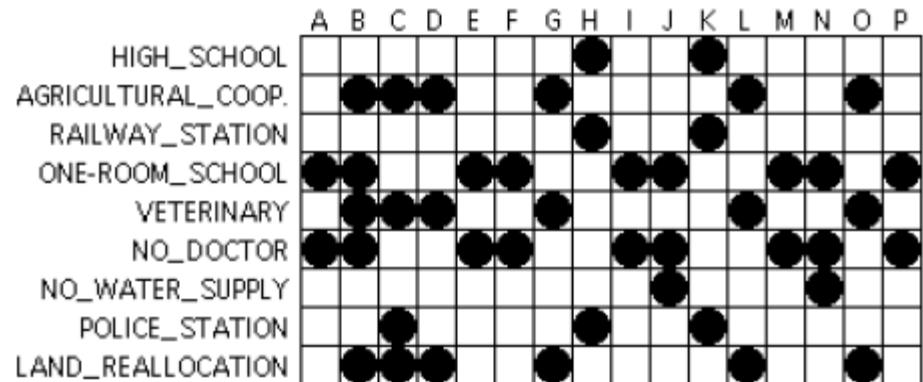
A data table: objects by characteristics

n		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	High School	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
2	Agricultural Cooperative	0	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0
3	Railway Station	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
4	One Room School	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	1
5	Veterinary	0	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0
6	No Doctor	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	1
7	No Water Supply	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
8	Police Station	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0
9	Land Reallocation	0	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0

Both rows and columns are
reorderable (\neq, \neq)

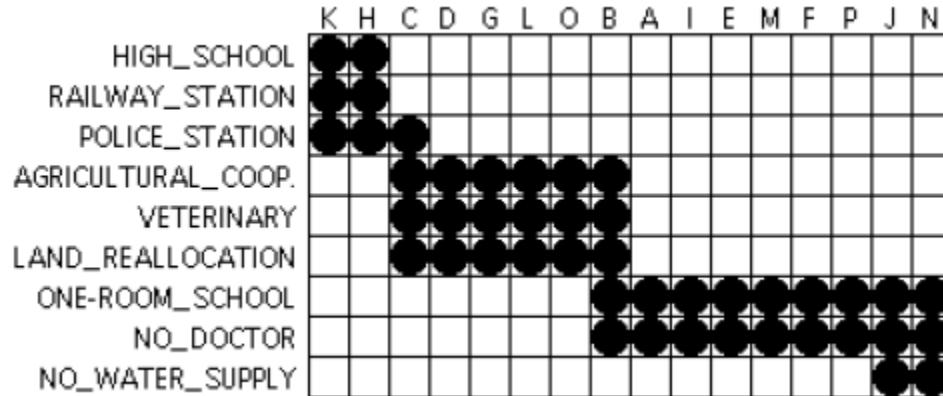
Encode each value by visual attributes

Overall relation can be
discovered by permuting rows,
cols



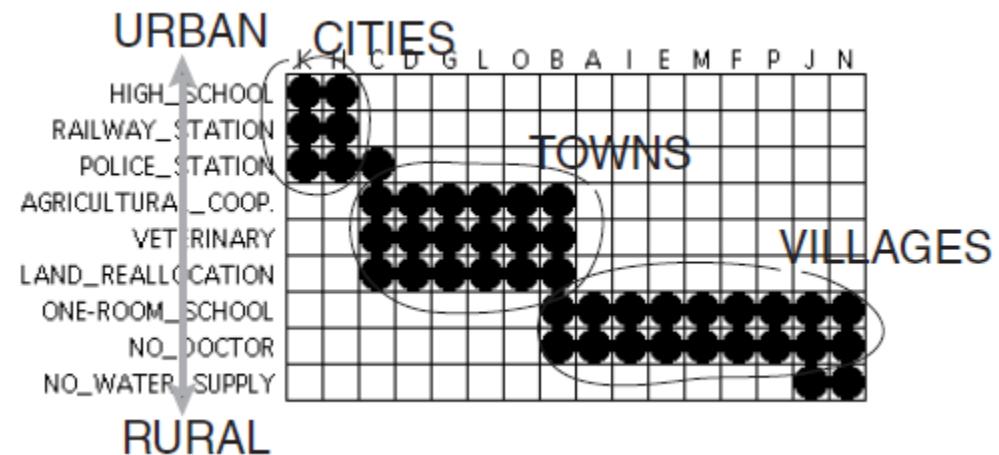
The reorderable matrix

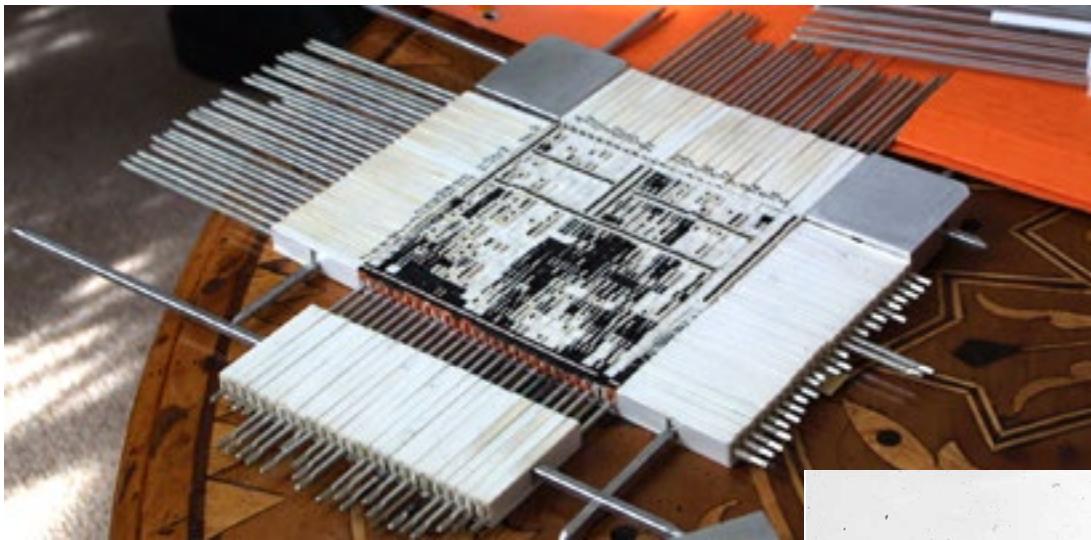
Permute rows and columns to put like with like



This is an early example of what I called “effect ordering” for data display

Interpret row/col order & clusters





A physical device
implementing matrix
reordering

This was used by Bertin and others in a large number of applied projects

Bertin was to visual data analysis in France what Tukey was to EDA in N. America



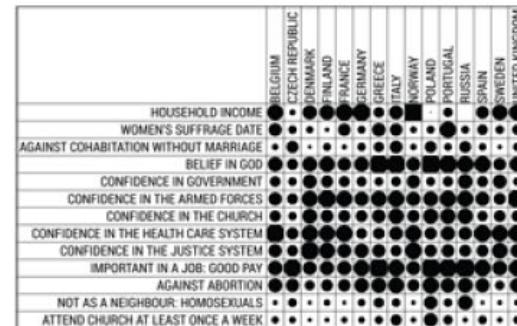
Bertifier

Bertifier: A web app implementing Bertin's idea of the reorderable matrix
 See: <http://www.aviz.fr/bertifier>

a

	Belg	Czech	Dene	Finla	Fran	Gem	Grec	Itali	Non	Polai	Port	Russ	Sd	Swee	United
Household income	2687	16957	2468	2571	2831	2879	204	243145	1537	1936	1529	22	2524	26904	
Women's suffrage date	1948	1920	1915	1906	1944	1918	1952	19	1913	1918	1976	1916	16	1921	1928
Against cohabitation	12	42	4	18	8	20	30	46	12	39	17	39	16	6	19
Belief in God	61	36	63	69	52	63	93	91	56	96	86	77	76	46	65
Confidence in Government	32	21	55	42	34	29	22	28	51	23	30	60	36	54	19
Confidence in the arms	50	34	72	83	73	58	70	75	57	63	75	73	57	41	89
Confidence in the church	36	20	63	47	41	40	52	67	44	65	67	67	31	39	36
Confidence in the health	91	42	75	73	78	34	39	54	74	44	58	51	79	75	80
Confidence in the justice	50	38	67	73	56	58	50	36	78	44	48	41	42	69	51
Important in a job: good pay	60	85	54	58	58	73	94	76	56	93	86	93	77	62	75
Against abortion	56	51	28	40	44	60	65	72	42	75	61	63	57	25	57
Not as a neighbour: hc	7	22	5	12	5	16	30	21	6	52	21	61	5	7	10
Attend church at least	15	13	5	7	11	12	19	35	9	54	25	8	21	9	17

b



c

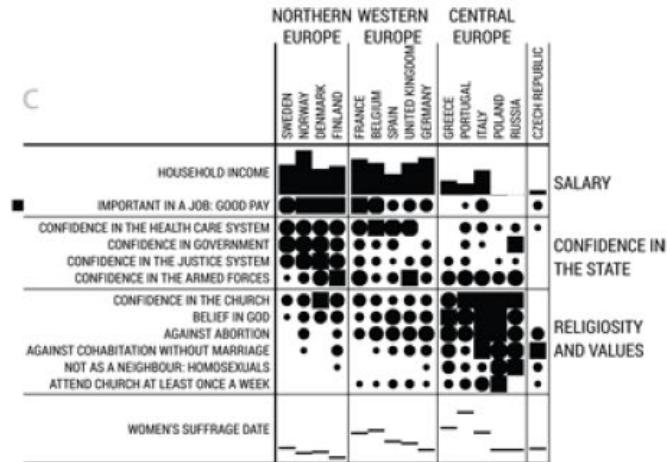


table: Attitudes and attributes by country

Values encoded by size and shape

Sorted and grouped by themes and country regions

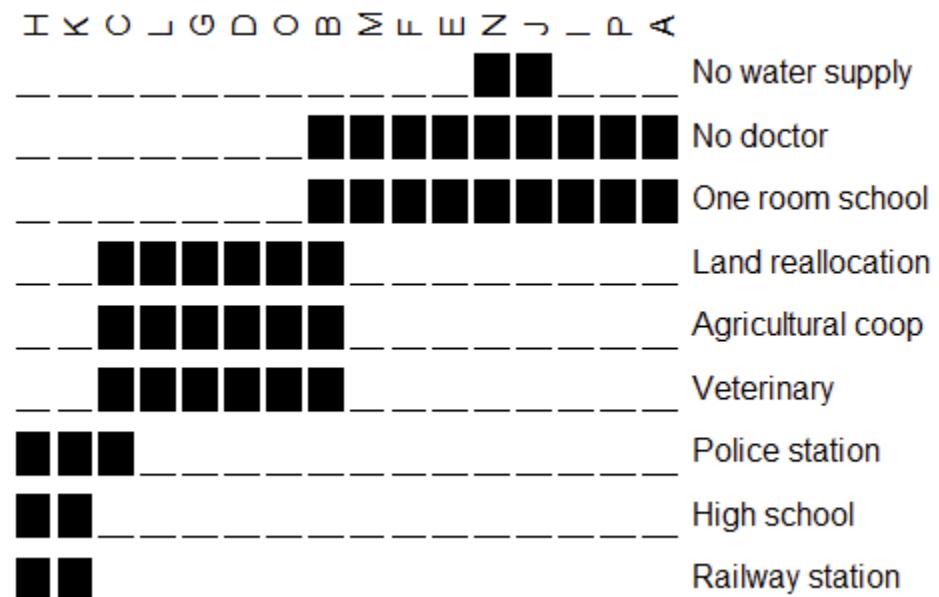
Watch: Youtube video of Bertifier, http://youtu.be/tJxAF_a_yBQ

seriate package

Matrix reordering is now recognized as a general problem, with criteria for many different goals, implemented in the `seriate` package

```
> list_seriation_methods(kind="matrix")
[1] "AOE"      "BEA"      "BEA_TSP"    "BK_unconstrained"   "CA"        "Heatmap"
[7] "Identity"  "LLE"      "Mean"      "PCA"                  "PCA_angle"  "Random"    "Reverse"
```

```
library(seriation)
data("Townships")
order <- seriate(Townships,
  method = "BEA_TSP")
bertinplot(Townships, order)
```



Heatmaps

Heatmaps are a re-invention of Bertin's ideas:

- Cluster analysis to reorder rows/cols
- Shading cells to show some variable

This example shows a microarray analysis of 128 leukemia patients using 12625 genes.

- The goal is to distinguish two types of leukemia
- The shading variable is a z-score for how well a given gene distinguishes the two types.
- Several clusters of high association are discovered!

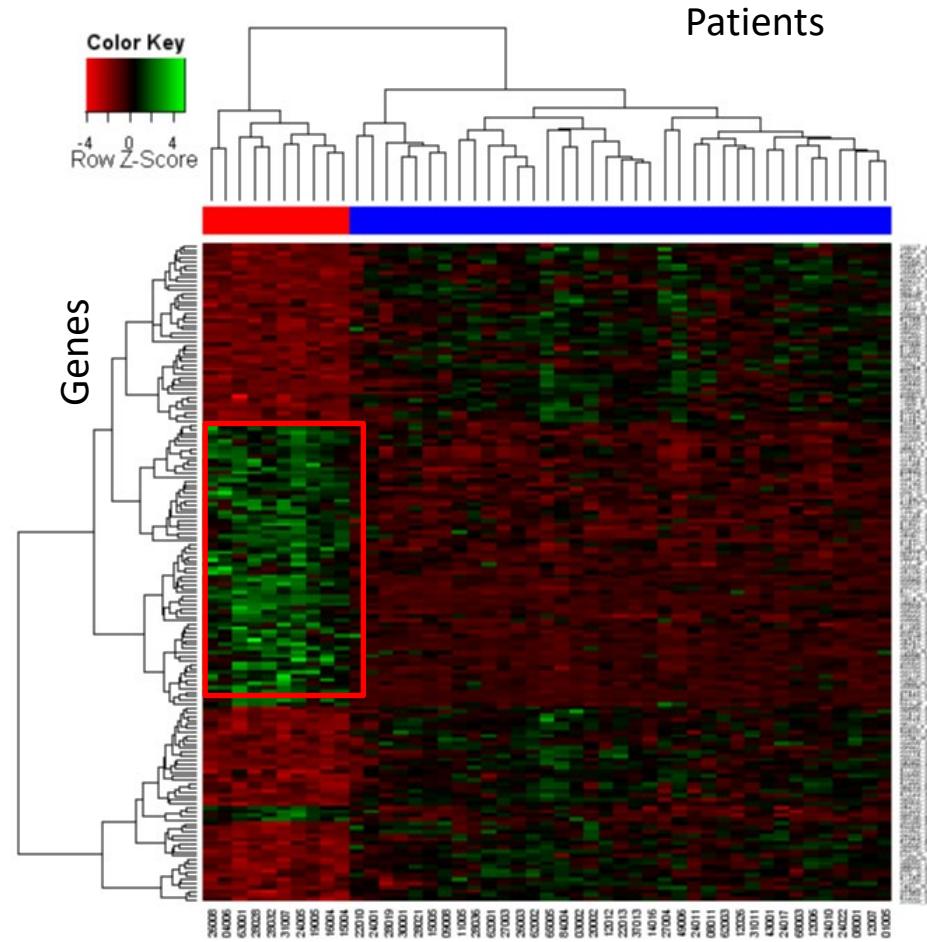


Image source: https://warwick.ac.uk/fac/sci/moac/people/students/peter_cock/r/heatmap/

See also: Wilkinson & Friendly, *The History of the Cluster Heat Map*, *The American Statistician*, 2009, 63, 179-184

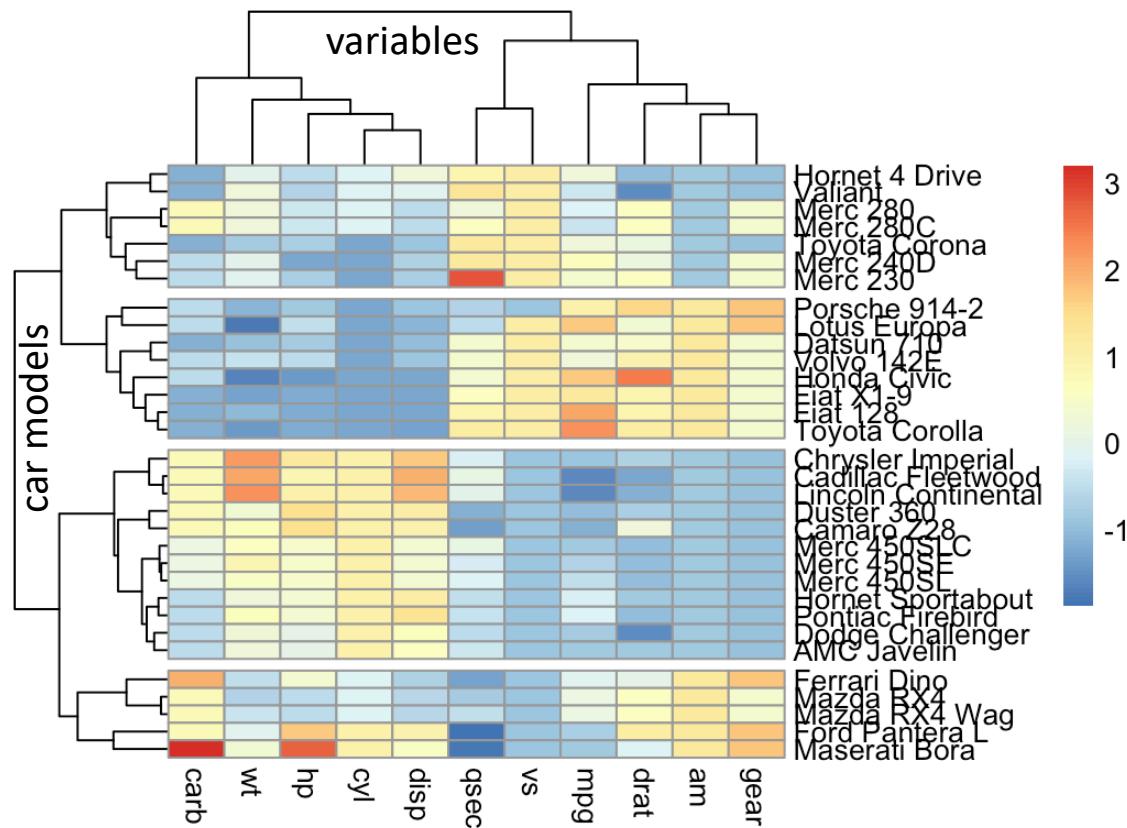
Heatmaps: the devil is in the details

There are many implementations of “heatmaps”

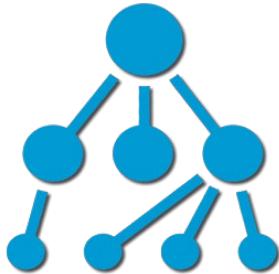
They differ importantly in the details of: clustering, shading scheme

This example shows a data set of 11 measures on 32 cars from the 1974 Motor Trends magazine

- Each variable was converted to z-scores
- The value was shaded using a bipolar color scheme
- Clusters of cars are slightly separated
- The very high and low values stand out



Software for computer graphics



data



How to ask a computer to
draw a graph?

```
ELEMENT: point(position(x*y),  
COORD: rect(dim(1,2))  
SCALE: linear(dim(1))  
SCALE: linear(dim(2))  
GUIDE: axis(dim(1), label("Sepa  
GUIDE: axis(dim(2), label("Sepa
```

code



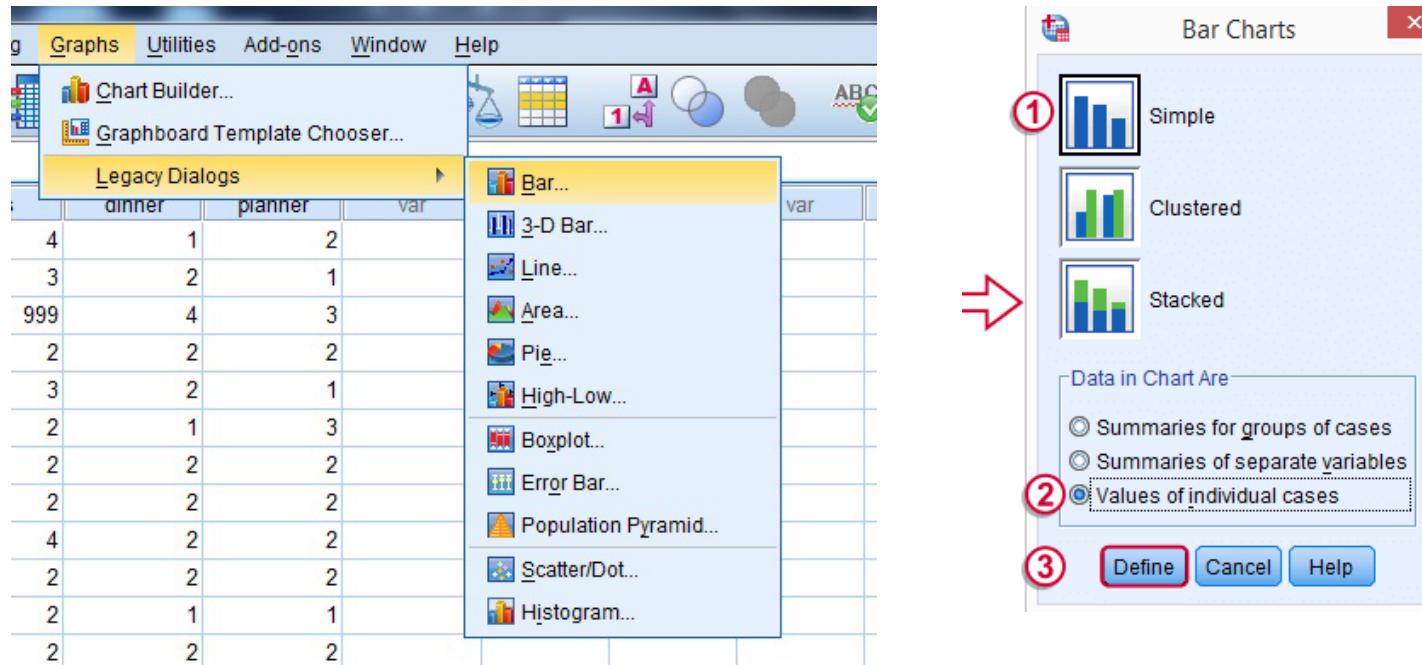
BEG: “Pretty please,
Mr. Computer, draw
me a graph”



graphical output

Making graphs: menus vs. syntax

Menu-driven graphics provide a wide range of graph types, with options
What's wrong with that?



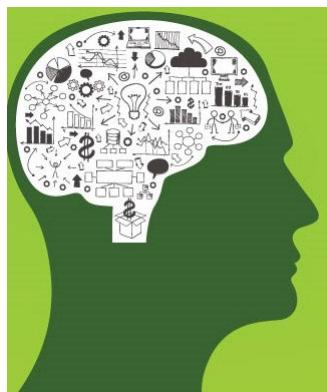
WYSIAYG: What you see is **all** you get. No way to do something different

Not reproducible: Change the data → Re-do manually from scratch

Often designed by programmers with little sense of data vis

Programming languages: Power & elegance

- **CS view:** All programming languages can be proved to be equivalent (to a Turing machine)
- **Cognitive view:** Languages differ in:
 - **expressive power:** ease of translating what you want to do into the results you want
 - **elegance:** how well does the code provide a human-readable description of what is done?
 - **extensibility:** ease of generalizing a method to wider scope
 - **learn-ability:** your learning curve (rate, asymptote)



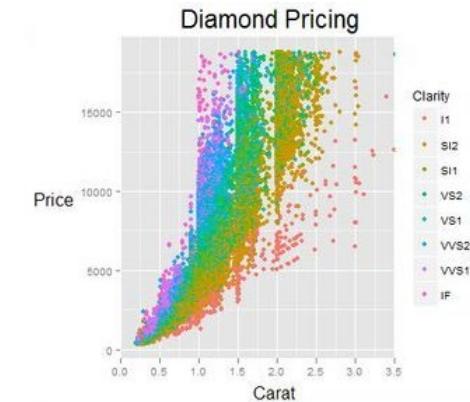
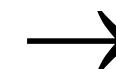
+

```
library(ggplot2)
source("plots/formatPlot.R")

View(diamonds)
summary(diamonds)

summary(diamonds$price)
avesize <- round(mean(diamonds$carat), 4)
clarity <- levels(diamonds$clarity)

p <- qplot(carat, price,
           data=diamonds, color=clarity,
           xlab="Carat", ylab="Price",
           main="Diamond Pricing")
```



Programming languages: Power & elegance

My journey

What did I learn
along the way?

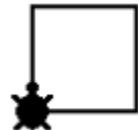
Language	Features: Tools for thinking?
FORTRAN	Subroutines – reusable code Subroutine libraries (e.g., BLAS)
<i>APL</i> , <i>APL2STAT</i>	N-way arrays, nested arrays Generalized reduction, outer product Function operators
Logo	Turtle graphics Recursion, list processing
Lisp, LispStat, <i>ViSta</i>	Object-oriented computing Functional programming
Perl	Regular expressions Search, match, transform, apply
SAS	Data steps, PROC steps, BY processing SAS macros, Output Delivery system
R	Object-oriented methods, tidyverse: dplyr, ggplot2, ...



Programming languages: Elegance - Logo

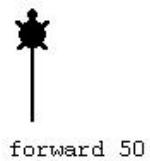
Features:

- Based on Lisp, but tuned to young minds
 - Papert: *Mindstorms: Children, Computers, and Powerful Ideas* (1980)
- Turtle graphics: draw by directing a turtle, not by (x,y) coordinates
 - Analytic geometry rests on a coordinate system.
 - Turtle geometry is "body syntonic": Tell turtle what to do.
- Data types:
 - words, lists, arrays, property lists
- Lists & list processing: inherited from Lisp, but with gentler syntax.
 - Lists are infinitely expandable & nestable.
- Recursion rather than iteration is the natural method to process lists
- Extensions:
 - multiple, animated turtles (sprites);
 - object-oriented programming (message passing) -> SmallTalk



Logo : Turtle graphics

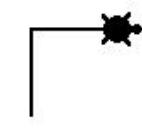
Turtle primitives: forward, back, left, right, penup, pendown, ...



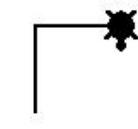
forward 50



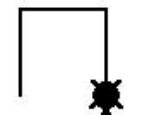
right 90



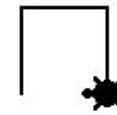
forward 50



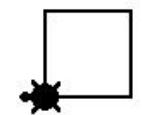
right 90



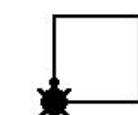
forward 50



right 90



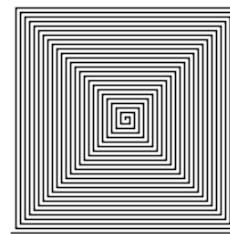
forward 50



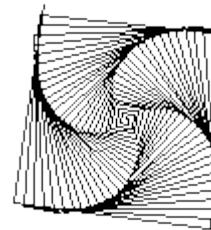
right 90

Recursive procedures:

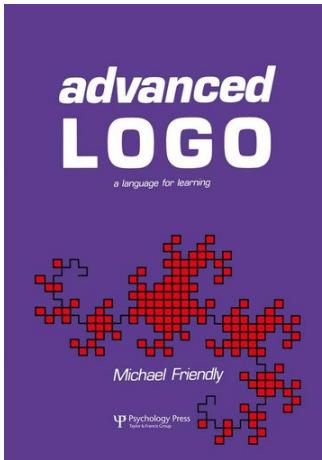
```
> to spiral :size :angle  
if :size > 100 [stop]  
forward :size  
right :angle  
spiral (:size + 2) :angle  
end
```



> spiral 0 90



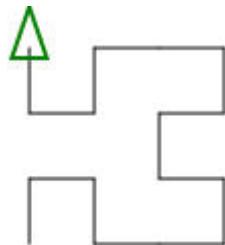
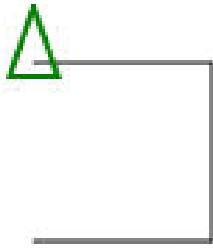
> spiral 0 91



Michael Friendly

UP
University of Toronto Press

Logo : Hilbert curves



Logo was more than just pretty pictures
It was graphics & mathematics for young
minds: **A language for learning**

```
to Hilbert0 :turn :size
right :turn
forward :size
left :turn
forward :size
left :turn
forward :size
right :turn
end
```

Start with some basic shape

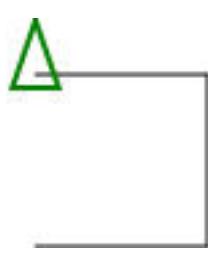
What happens if you replace each **line** with a smaller copy
of the basic shape?

What happens if you continue this process?

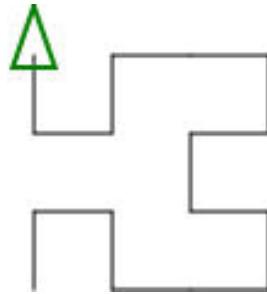
What happens if you choose a different basic shape?

Logo : Hilbert curves

depth: 1



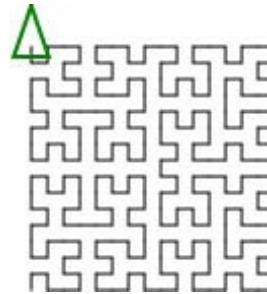
depth: 2



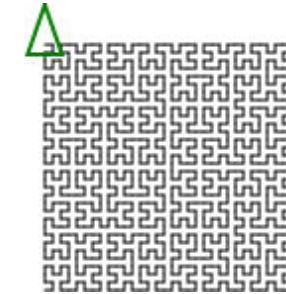
depth: 3



depth: 4



depth: 5



```
to Hilbert :depth :turn :size
if :depth = 0 [stop]
right :turn
Hilbert (:depth-1) -:turn :size
forward :size
left :turn
Hilbert (:depth-1) :turn :size
forward :size
Hilbert (:depth-1) :turn :size
left :turn
forward :size
Hilbert (:depth-1) -:turn :size
right :turn
end
```

Hilbert curve: A continuous, space-filling fractal, of Hausdorff dimension 2

Theorem (Hilbert, 1891): The euclidean length of the n-th depth Hilbert curve, H_n is $2^n - \frac{1}{2^n}$

Proof (by enumeration): Redefine forward to calculate total turtle path length

```
to forward.length :size
make "total.length :total.length + :size
forward :size
end
```

Logo: Tower of Hanoi

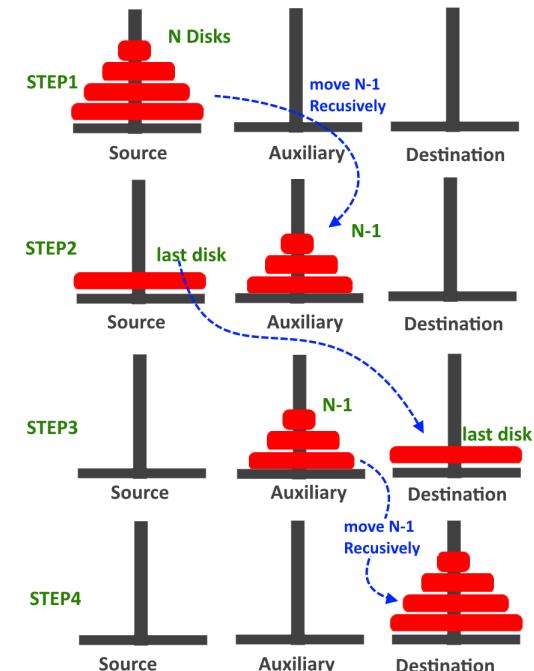
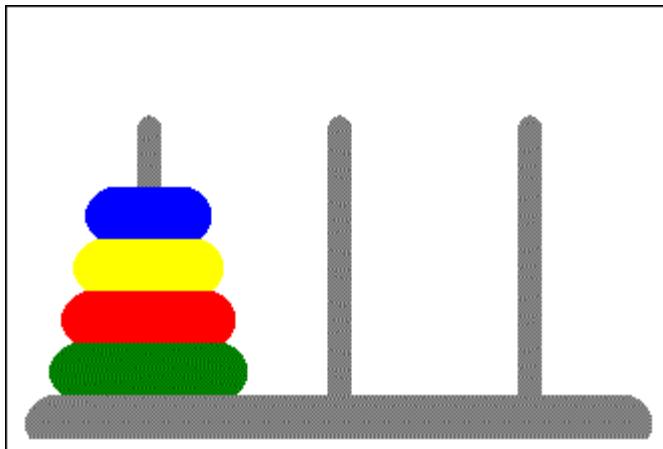
Move N disks from one pole to another, with no disk ever resting on a disk smaller than itself.

```
to Hanoi :n :start :goal :spare  
if :n=0 [stop]  
Hanoi :n-1 :start :spare :goal  
move :n :start :goal  
Hanoi :n-1 :spare :goal :start  
end
```

```
# move disks 1:n from START to GOAL  
# are we done?  
# move disks 1:n-1 from START to SPARE  
# move disk n from START to GOAL  
# move disks 1:n-1 from SPARE to GOAL
```

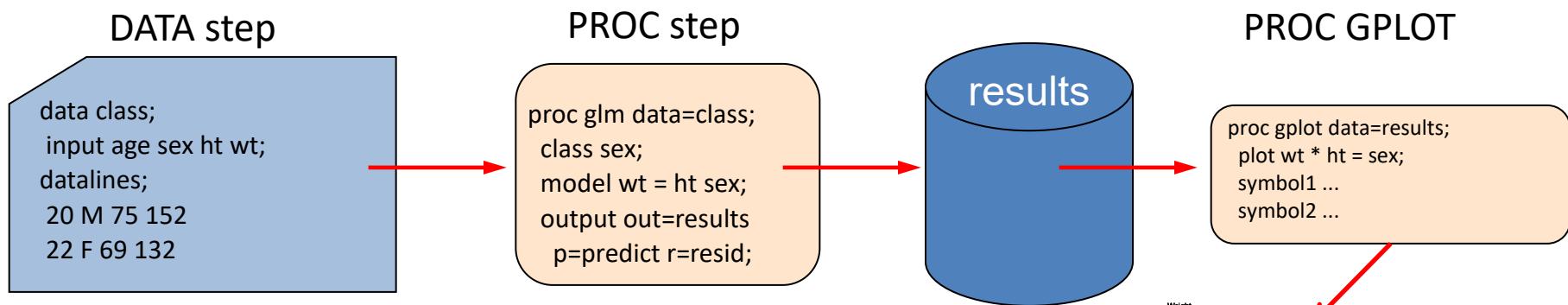
A direct translation
of an algorithm into
code

The Tower of Hanoi problem has an elegant solution in Logo
Change the 'move' instruction to render on screen or by a
robot!



Graphics programming languages: SAS

- SAS: procedures + annotate facility + macros
 - PROC GPLOT (x,y plots), PROC GCHART, PROC GMAP, ...
 - Annotate: data set with instructions (move, draw, text, fonts, colors)
 - Macros: Create a new, generic plot type, combining PROC steps and DATA steps.



Why I gave up SAS: This works well, and is very powerful, but lacks elegance

SAS thinking : many languages



ODS graphics

- template language

Output delivery system (ODS)

%macro language

proc iml

- matrix language, graphics

• procs, Annotate language

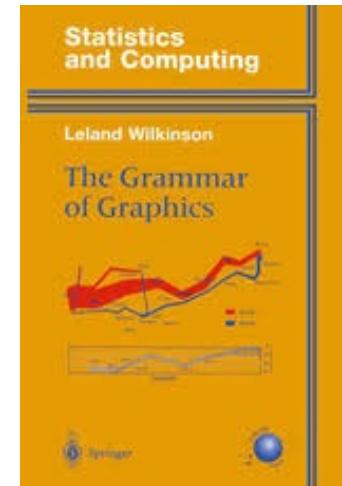
SAS/Graph:

Base SAS, SAS/STAT

- data step, proc steps

Wilkinson: Grammar of Graphics

- Natural language:
 - Grammar/syntax: What are the **minimal, complete** set of rules to describe **all** well-formed sentences?
 - John ate the big red apple ✓
 - John big apple red apple ate the ✗
 - Semantics: How to distinguish meaning, nonsense, poetry in well-formed sentences?
 - Large green trucks carry garbage ✓
 - Colorless green ideas sleep furiously ??
- How to apply these ideas to graphics?
 - Grammar: Algebra, scales, statistics, geometry, ...
 - Semantics: Space, time, uncertainty, ...
 - Needed: a complete **formal theory** of graphs & **computational graphics language**



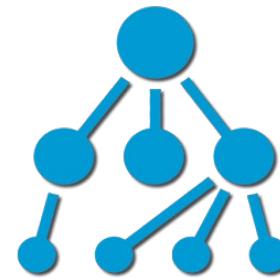
Wilkinson: Grammar of Graphics

- A complete system, describing the components of graphs and how they combine to produce a finished graphic
 - *“The grammar of graphics takes us beyond a limited set of charts (words) to an almost unlimited world of graphical forms (statements)”* (Wilkinson, 2005, p. 1).
 - *“... describes the **meaning** of what we do when we construct statistical graphics ... more than a taxonomy”*
 - *“This system is capable of producing some **hideous** graphics ... This system cannot produce a **meaningless** graphic, however.”*
- This is a general theory for **producing** graphs.
 - the foundation of most modern software systems;
 - not connected with a theory for **reading** graphs à la Bertin.

Wilkinson: Grammar of Graphics

- Components:
 - **specification**: a formal language for composing graphs
 - **assembly**: coordination of attributes
 - internal: a data structure for a graphical “object”
 - **rendering**: producing a graphic on a display system
 - low level: device drivers for screen, PDF, PNG, SVG, ...

```
ELEMENT: point(position(x*y),  
COORD: rect(dim(1,2))  
SCALE: linear(dim(1))  
SCALE: linear(dim(2))  
GUIDE: axis(dim(1), label("Sepa  
GUIDE: axis(dim(2), label("Sepa
```



code

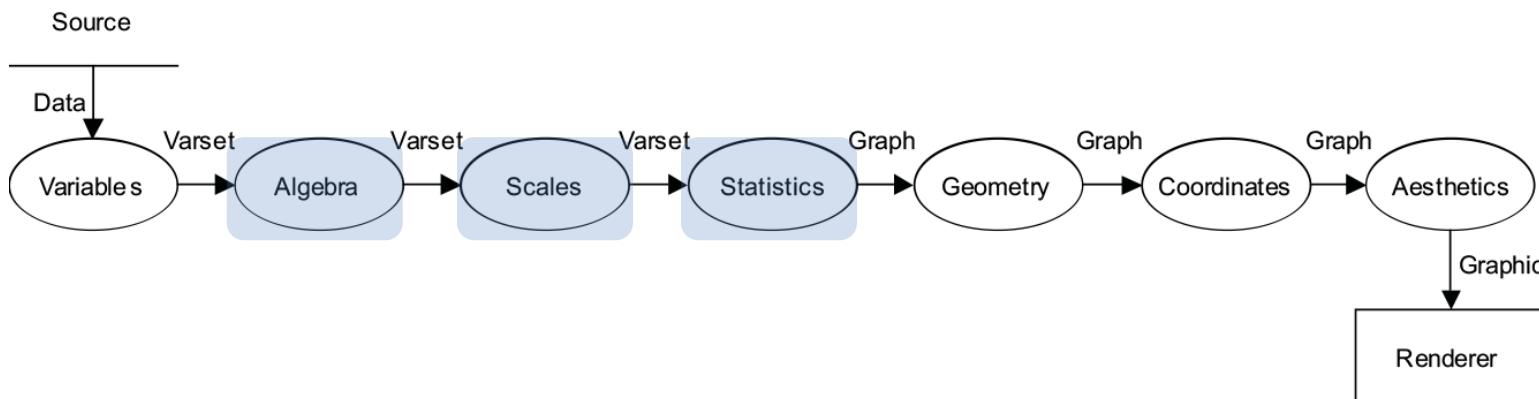


data structure

graphical output

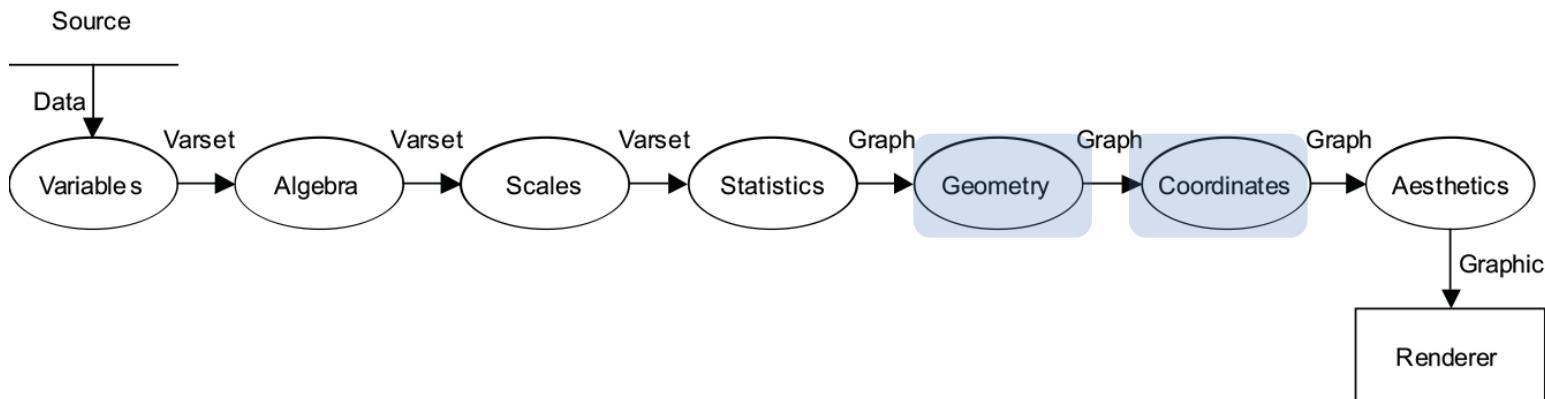
Grammar of Graphics: Specification

- **Algebra:** combine variables into a data set to be plotted
 - cross (A^*B), nest (A/B), blend ($A+B$), filter, subset, ...
think: dplyr
- **Scales:** how variables are represented
 - categorical, linear, log, power, logit, ...
SCALE: `linear(dim(1))`
- **Statistics:** computations on the data
 - binning, summary (mean, median, sd), region (CI), smoothing



Grammar of Graphics: Specification

- **Geometry:** Creation of geometric objects from variables
 - Functions: point, line, area, interval, path, ...
 - Partitions: polygon, contour,
 - Networks: edge
 - Collision modifiers: stack, dodge, jitter
- **Coordinates:** Coordinate system for plotting
 - transformations: translation, rotation, dilation, shear, projection
 - mappings: Cartesian, polar, map projections, warping, Barycentric
 - 3D+: spherical, cylindrical, dimension reduction (MDS, SVD, PCA)



Grammar of Graphics: Specification

- **Aesthetics:** mapping of qualitative and quantitative scales to sensory attributes (extends Bertin)
 - **Form:** position, size, shape (polygon, glyph, image), rotation, ...
 - **Surface:** color (hue, saturation, brightness), texture (pattern, orientation), blur, transparency
 - **Motion:** direction, speed, acceleration
 - **Sound:** tone, volume, rhythm, voice, ...
 - **Text:** label, **font**, **size**, ...
- **Facets:** Construct multiplots (“small multiples”) by partitioning, blending or nesting
- **Guides:** Allow for reading the encodings of variables mapped to aesthetics
 - **scales:** axes, legend (labels: size, shape, color, ...)
 - **annotations:** (title, footnote, line, arrow, ellipse, text, ...)



Grammar of Graphics: Implementation

- Wilkinson illustrates the GoG with a programming language (GPL: the *Graphics Production Language*)
- GPL statements
 - **DATA**: expressions that create variables to display from data sets
 - **TRANS**: variable transformations prior to plotting (e.g., ranking the data points)
 - **ELEMENT**: define graphical elements (e.g., points, lines, ...) and their aesthetic attributes (e.g., shape, color, ...) to use in the display
 - **SCALE**: apply scale transformations to the plot (e.g., square root or log)
 - **COORD**: select the coordinate system for use in the graphic (e.g., Cartesian, polar)
 - **GUIDE**: guides to aid interpretation (axes, legends)

GPL example: scatterplot

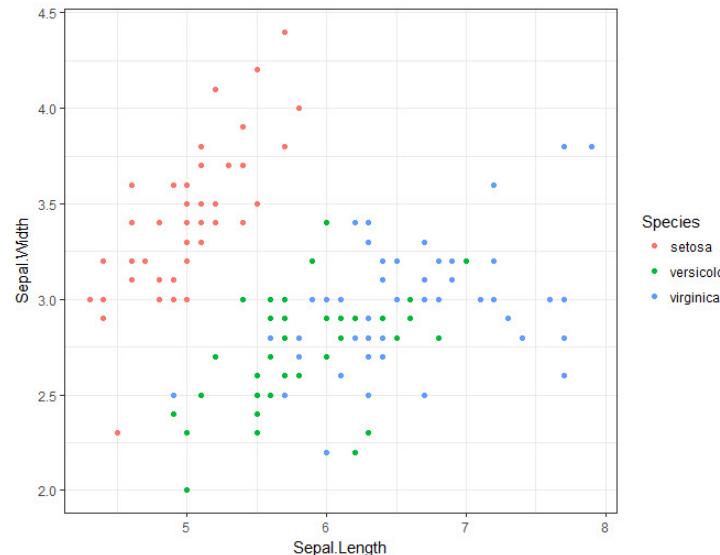
A simple scatterplot of the Iris data, points colored by species

```
DATA: x = "SepalLength"  
DATA: y = "SepalWidth"  
DATA: z = "Species"  
TRANS: x = x  
TRANS: y = y  
ELEMENT: point(position(x*y), color(z)) ←  
COORD: rect(dim(1,2))  
SCALE: linear(dim(1))  
SCALE: linear(dim(2))  
GUIDE: axis(dim(1), label("Sepal Length"))  
GUIDE: axis(dim(2), label("Sepal Width"))
```

TRANS, SCALE, COORD and GUIDE all show the defaults & aren't necessary here.

The key one is ELEMENT, specifying points, positioned by (x*y) and colored by z

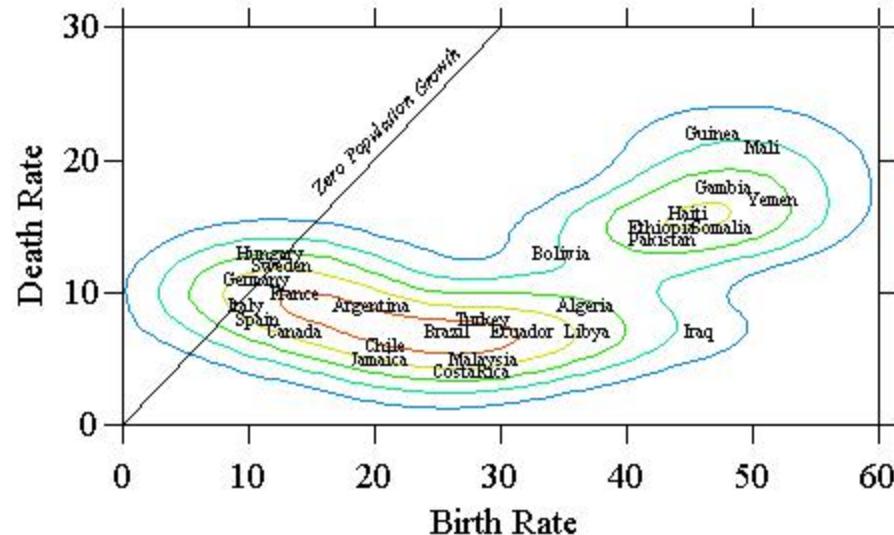
SPSS graphics now use GPL as the backend (syntax) for their graphics engine



GPL example: contour plot

A smoothed contour plot of birth rate vs. death rate for selected countries

```
ELEMENT: point(position(birth*death), label(country))
ELEMENT: contour(position(smooth.kernel.density(birth*death)), color.hue())
GUIDE: form.line(position((0,0), (30,30)), label("Zero population growth"))
GUIDE: axis(dim(1), label("Birth rate"))
GUIDE: axis(dim(2), label("Death rate"))
```



Wilkinson, *Grammar of Graphics*, Fig 1.1

GPL syntax

The essential features of a graph are described by **ELEMENT**

- The geometrical objects (point, line, interval, ...) are specified within this
- Their visual properties (position, color) and statistical summaries are given as well

Some typical graph types:

<i>Graph</i>	<i>Syntax</i>
<i>scatterplot</i>	ELEMENT: <i>point</i> (<i>position</i> (<i>d*r</i>))
<i>line chart</i>	ELEMENT: <i>line</i> (<i>position</i> (<i>d*r</i>))
<i>bar chart</i>	ELEMENT: <i>interval</i> (<i>position</i> (<i>d*r</i>))
<i>hor. bar chart</i>	COORD: <i>rotate</i> (270) ELEMENT: <i>point</i> (<i>position</i> (<i>d*r</i>))
<i>clustered bar chart</i>	ELEMENT: <i>interval.dodge</i> (<i>position</i> (<i>d*r</i>) , <i>color</i> (<i>c</i>))
<i>stacked bar chart</i>	ELEMENT: <i>interval.stack</i> (<i>position</i> (<i>summary.proportion</i> (<i>r</i>) , <i>color</i> (<i>c</i>)))
<i>histogram</i>	ELEMENT: <i>interval</i> (<i>position</i> (<i>summary.count</i> (<i>bin.rect</i> (<i>y</i>))))

GPL in SPSS syntax

GGRAPH

```
/GRAPHDATASET NAME="graphdataset" VARIABLES=read write  
/GRAPHSPEC SOURCE=INLINE.
```

BEGIN GPL

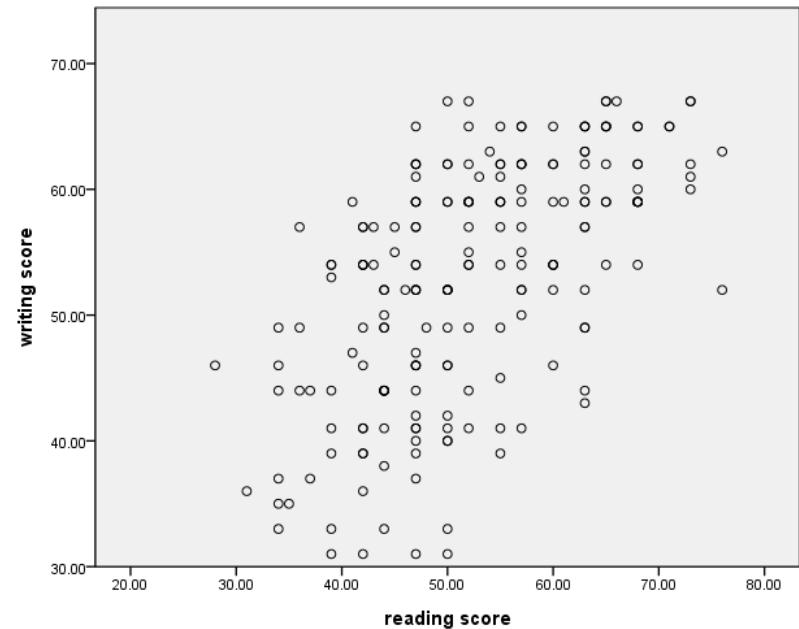
```
SOURCE: s=userSource(id("graphdataset"))  
DATA: read=col(source(s), name("read"))  
DATA: write=col(source(s), name("write"))  
GUIDE: axis(dim(1), label("reading score"))  
GUIDE: axis(dim(2), label("writing score"))  
ELEMENT: point(position(read*write))
```

END GPL.

SPSS menu choices → GPL code

You can:

- Extract the code
- Tweak it
- Save to make it reproducible



Facets & frames

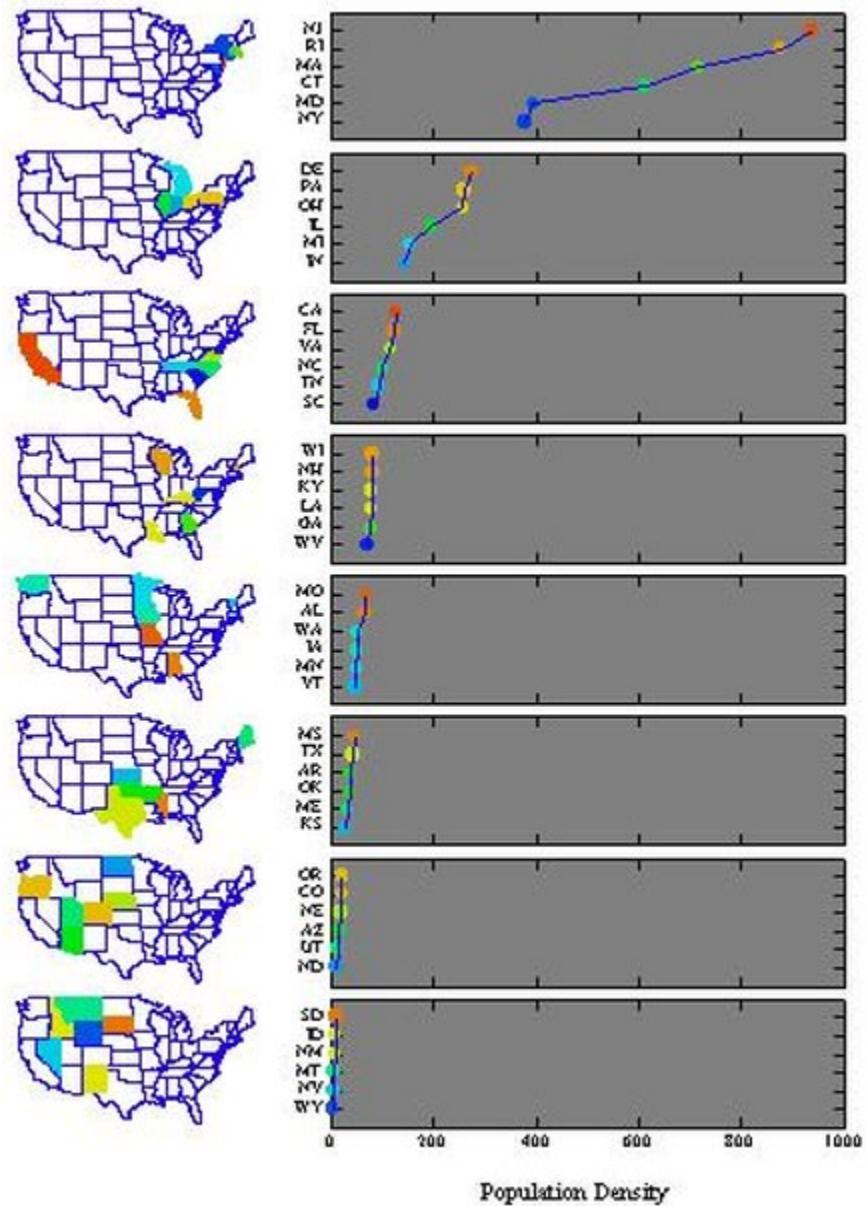
Tables of graphs:

- Facets: → graphs of subset
- Frames: → separate graphs

Linked micromap:

- Population density of US, divided in octiles
- States in each octile shown separately

GoG was a coherent language for specifying and producing nearly all known graphic forms.



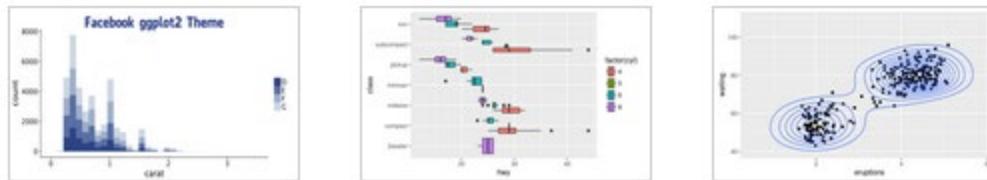
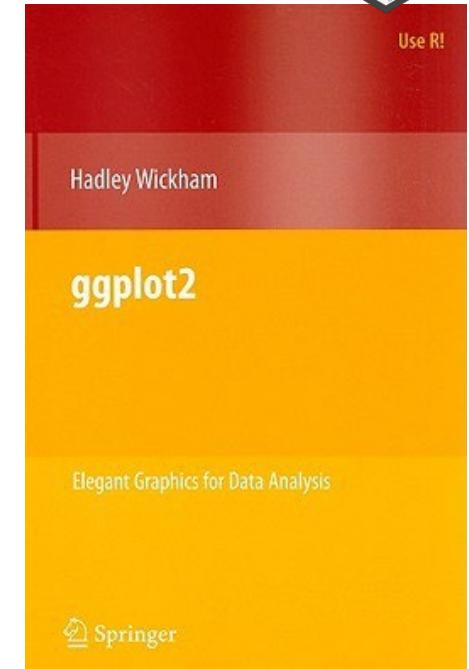
Colorless green graphs sleep furiously

- JSM 2017: Dinner with Lee Wilkinson, Howard Wainer, Paul Vellman, & others
- The great debate:
 - LW: The GoG is a **complete** theory, a formal mathematical model comprehending **all** graphs.
"Beauty is truth, truth beauty,"--that is all Ye know on earth, and all ye need to know.
 - MF: There is more--
 - **Implementation matters:** translating a graphic idea into a finished graph should be facilitated by the **language** of graphic code.
 - A productive language for graphs should encompass the steps of **data analysis**
 - Pere Milán: A truly expressive graphic language should recommend the right graphic(s) to “get the message home”



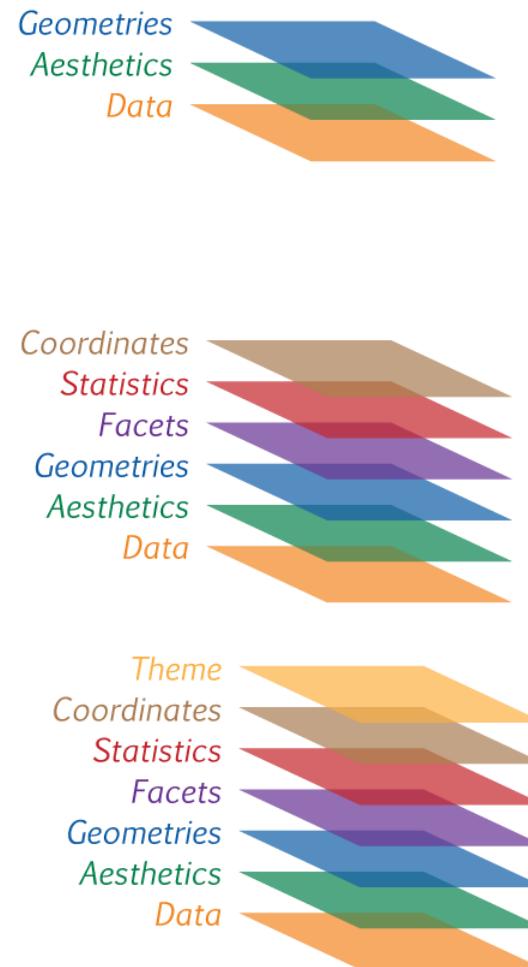
Wickham: ggplot2

- *ggplot2: Elegant graphics for data analysis*
 - a computational language for thinking about & constructing graphs
 - sensible, aesthetically pleasing defaults
 - + themes: default, bw, journal, tufte, ...
 - infinitely extendable
 - ggplot extensions:
<https://exts.ggplot2.tidyverse.org/>



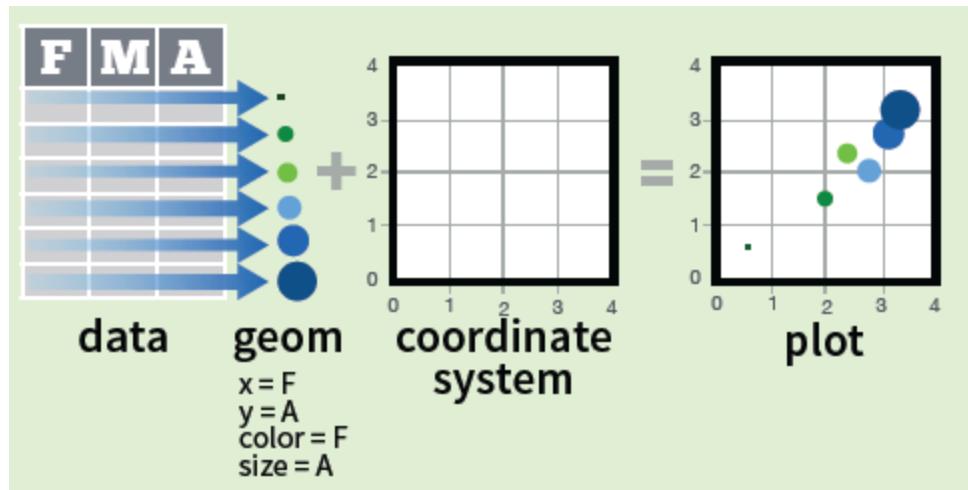
Wickham: ggplot2

- Implementation of GoG in R as **layers of a graphic**
 - Basic layers:
 - Data,
 - Aesthetics (data → plot mapping)
 - Geoms (points, lines, bars, ...),
 - Statistics: summaries & models
 - Coordinates: plotting space
 - Facets: partition into sub-plots
 - Themes: define the general features of **all** graphical elements



ggplot2: data + geom = graph

- Every graph can be described as a combination of independent building blocks, connected by “+” (read: “and”)
 - **data**: a data frame: quantitative, categorical; local or data base query
 - **aesthetic mapping** of variables into visual properties: size, color, x, y
 - **geometric objects (“geom”)**: points, lines, areas, arrows, ...
 - **coordinate system (“coord”)**: Cartesian, log, polar, map,



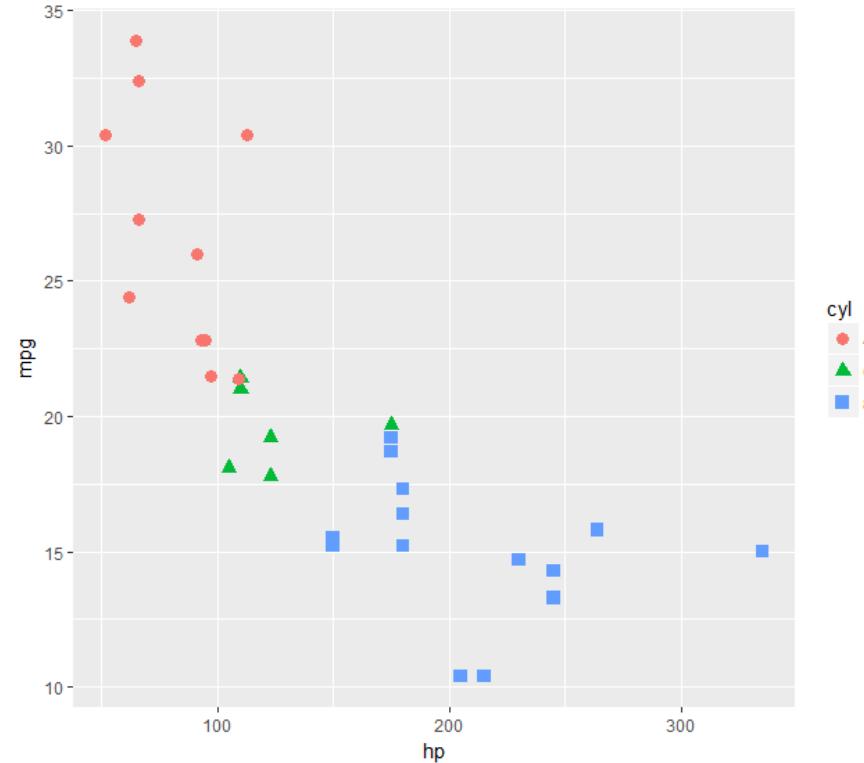
```
ggplot(FMA,  
       aes(x=F, y=A, color=F, size=A) +  
       geom_point())
```

ggplot2: data + geom = graph

```
ggplot(data=mtcars,  
       aes(x=hp, y=mpg,  
           color=cyl, shape=cyl)) +  
  geom_point(size=3)
```

In this call:

- **data=mtcars**: data frame ①
- **aes(x=, y=)**: plot X,Y variables ②
- **aes(color=, shape=)**: attributes ③
- **+ geom_point()**: what to plot ④
- the coordinate system is taken to be the standard Cartesian (x,y)
- a corresponding legend is automatically generated

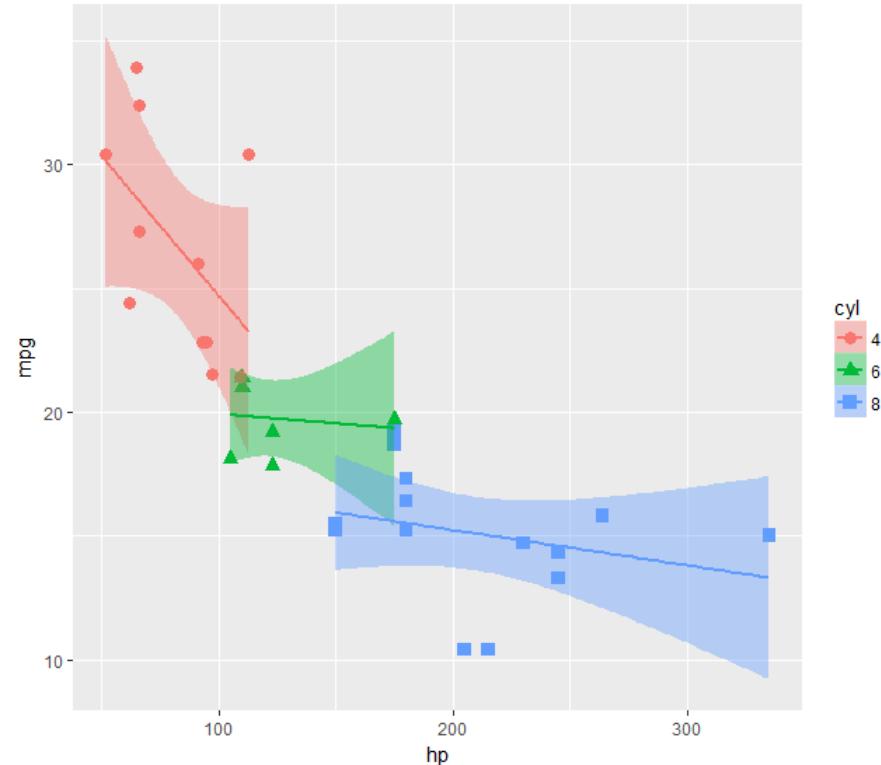


ggplot2: geoms

Wow! I can really see something there.

How can I enhance this visualization?

Easy: add a `geom_smooth()` to fit linear regressions for each level of cyl



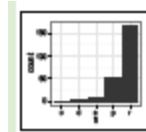
```
ggplot(mtcars, aes(x=hp, y=mpg, color=cyl, shape=cyl)) +  
  geom_point(size=3) +  
  geom_smooth(method="lm", aes(fill=cyl))
```

ggplot2: GoG -> graphic language

- The implementation of GoG ideas in ggplot2 for R created a more expressive language for data graphs
 - **layers**: graph elements combined with “+” (read: “and”)

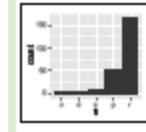
```
ggplot(mtcars, aes(x=hp, y=mpg)) +  
  geom_point(aes(color = cyl)) +  
  geom_smooth(method ="lm") +
```

- **themes**: change graphic elements consistently



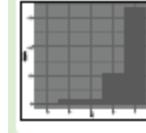
r + theme_bw()

White background
with grid lines



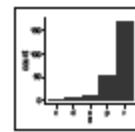
r + theme_gray()

Grey background
(default theme)



r + theme_dark()

dark for contrast



r + theme_classic()

r + theme_light()

r + theme_linedraw()

r + theme_minimal()

Minimal themes

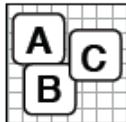
r + theme_void()

Empty theme

ggplot2: more geoms

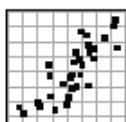
Continuous X, Continuous Y

```
e <- ggplot(mpg, aes(cty, hwy))
```



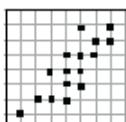
e + geom_label(aes(label = cty), nudge_x = 1, nudge_y = 1, check_overlap = TRUE)

x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust



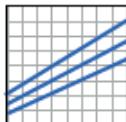
e + geom_jitter(height = 2, width = 2)

x, y, alpha, color, fill, shape, size



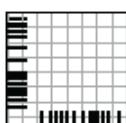
e + geom_point()

x, y, alpha, color, fill, shape, size, stroke



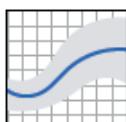
e + geom_quantile()

x, y, alpha, color, group, linetype, size, weight



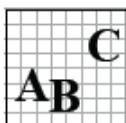
e + geom_rug(sides = "bl")

x, y, alpha, color, linetype, size



e + geom_smooth(method = lm)

x, y, alpha, color, fill, group, linetype, size, weight



e + geom_text(aes(label = cty), nudge_x = 1, nudge_y = 1, check_overlap = TRUE)

x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

ggplot2 facilitates graphical thinking by making a clear separation among:

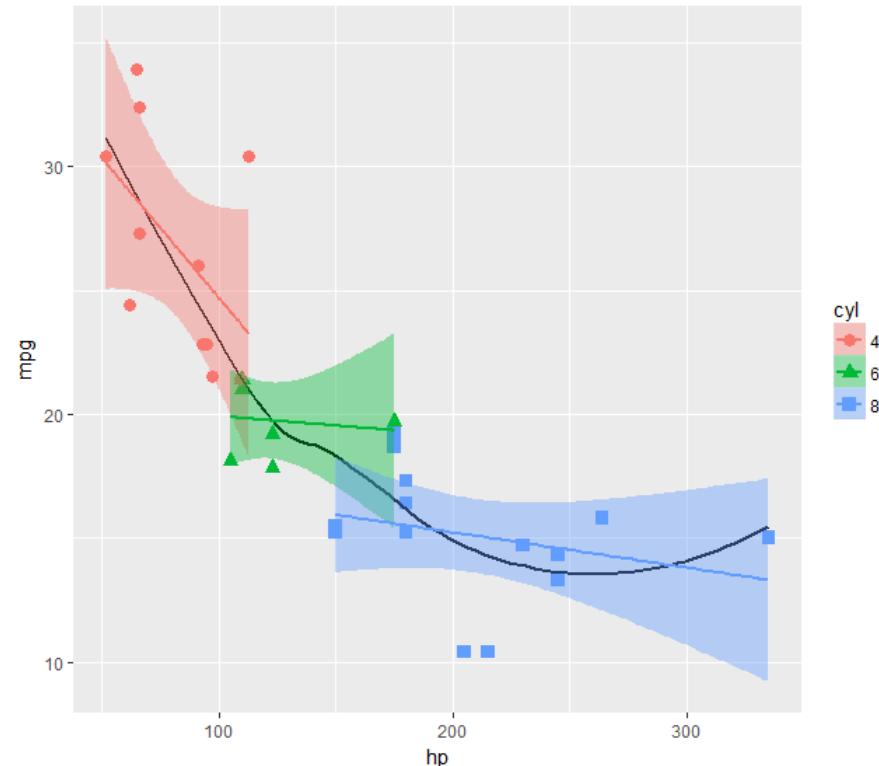
- mapping data variables to plot features (**aes()**);
- geometric objects (**geom_()**)
- statistical summaries (**stat_()**)

ggplot2: layers & aes()

Aesthetic attributes in the `ggplot()` call are inherited by `geom_()` layers

Other attributes can be passed as
constants (`size=3, color="black"`) or
with `aes(color=, ...)` in different layers

This plot adds an overall loess smooth to
the previous plot



```
ggplot(mtcars, aes(x=hp, y=mpg)) +  
  geom_point(size=3, aes(color=cyl, shape=cyl)) +  
  geom_smooth(method="lm", aes(color=cyl, fill=cyl)) +  
  geom_smooth(method="loess", color="black", se=FALSE)
```

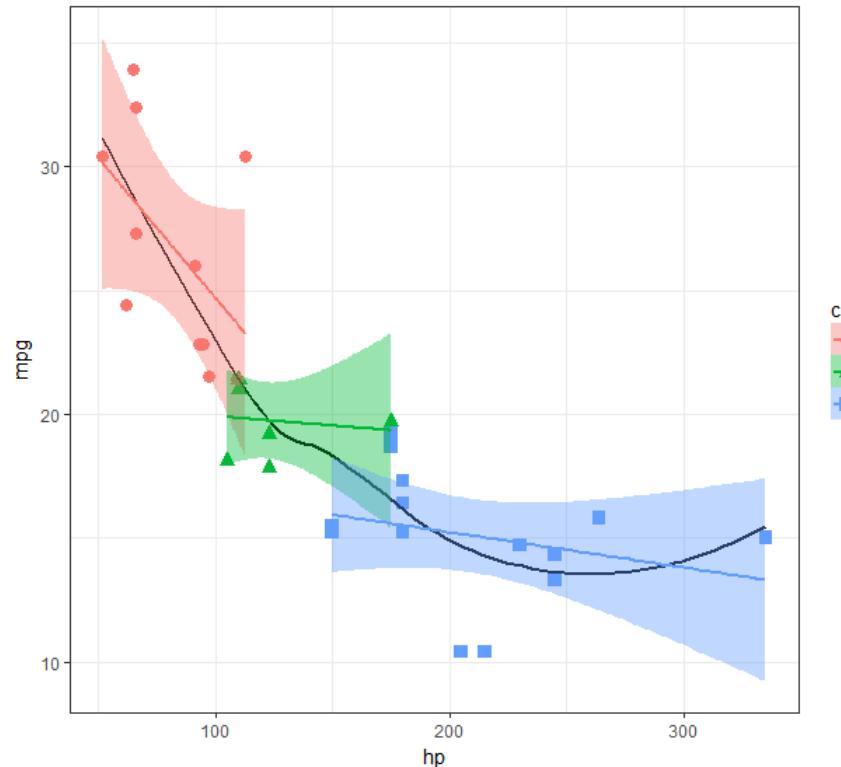
ggplot2: themes

All the graphical attributes of ggplot2 are governed by themes – settings for all aspects of a plot

A given plot can be rendered quite differently just by changing the theme

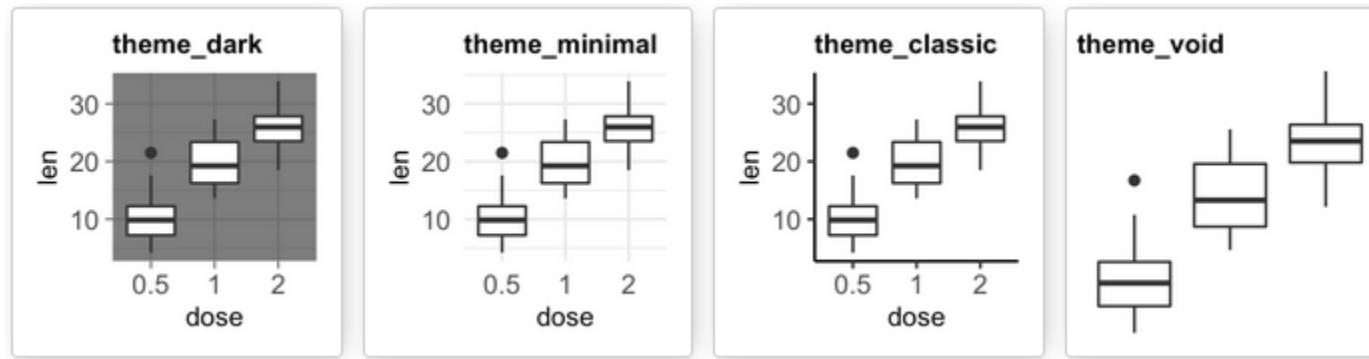
If you haven't saved the ggplot object, `last_plot()` gives you something to work with further

`last_plot() + theme_bw()`

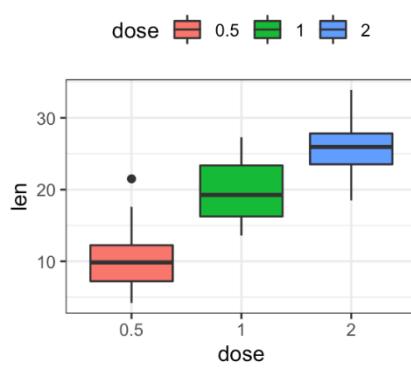


ggplot2: themes

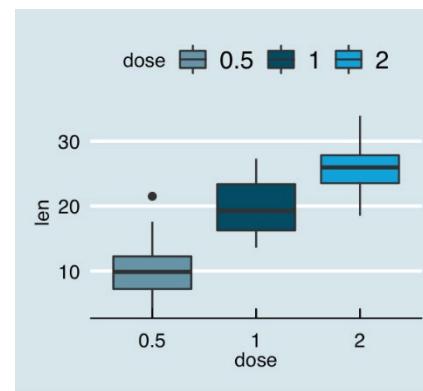
Built-in ggplot themes provide a wide variety of basic graph styles



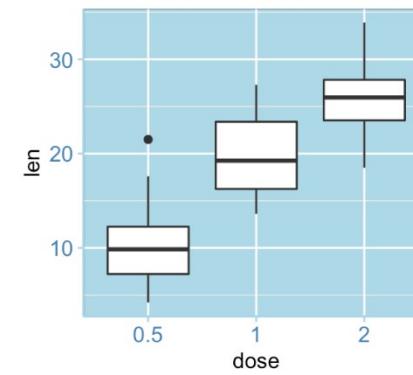
Other packages provide custom themes, or you can easily define your own



`theme_hc()`



`theme_economist()`



`theme_bluewhite()`

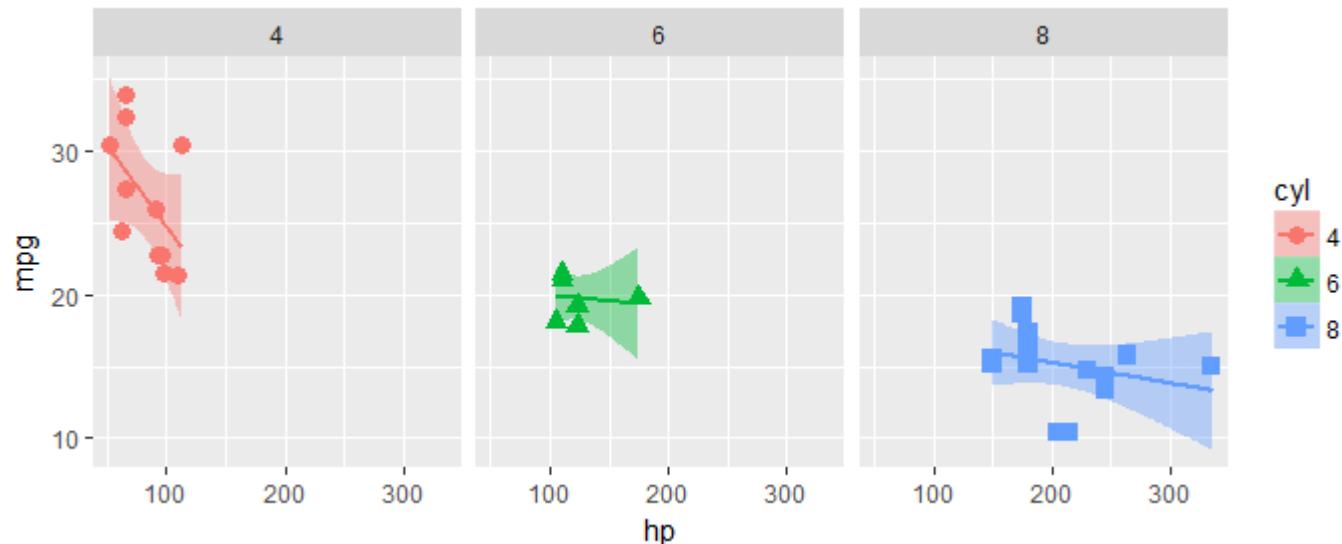
ggplot2: facets

Facets divide a plot into separate subplots based on one or more discrete variables

```
plt <-  
  ggplot(mtcars, aes(x=hp, y=mpg, color=cyl, shape=cyl)) +  
    geom_point(size=3) +  
    geom_smooth(method="lm", aes(fill=cyl))  
  
plt + facet_wrap(~gear)
```

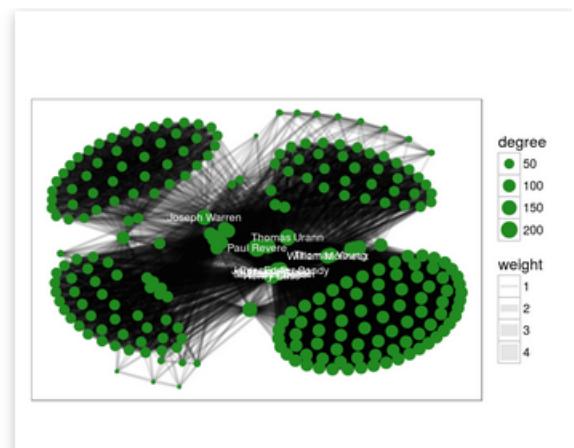
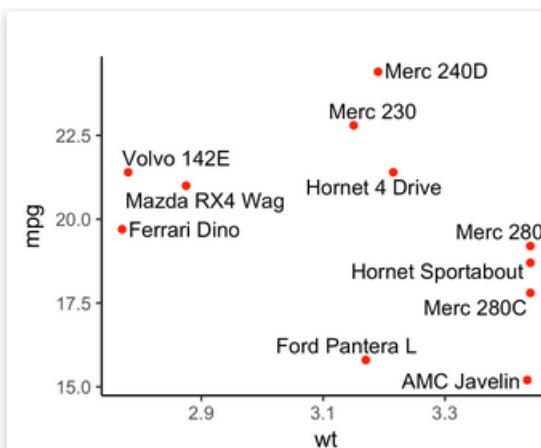
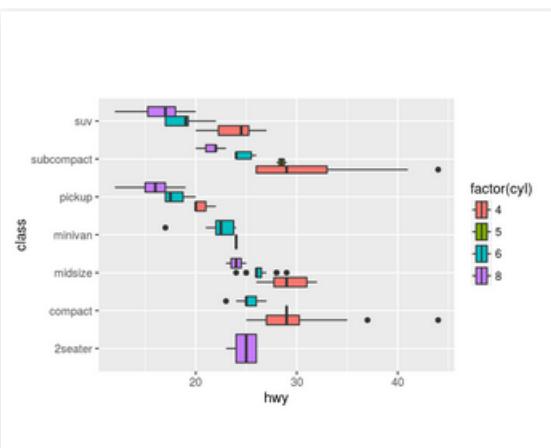
Syntax:

facet_wrap(rowvar ~ colvar)

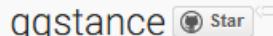


ggplot2: extensions

ggplot2 provides a **prototype** system for implementing new geoms, stats, themes, ...
Many of these are listed at <https://exts.ggplot2.tidyverse.org/>



ggstance



ggstance implements horizontal versions of common ggplot2 geoms.

▪ **author:** [lionel-](#)

▪ **tags:** [visualization](#), [general](#)

▪ **js libraries:**

ggrepel



Repel overlapping text labels away from each other.

▪ **author:** [slowkow](#)

▪ **tags:** [visualization](#), [general](#)

▪ **js libraries:**

ggraph



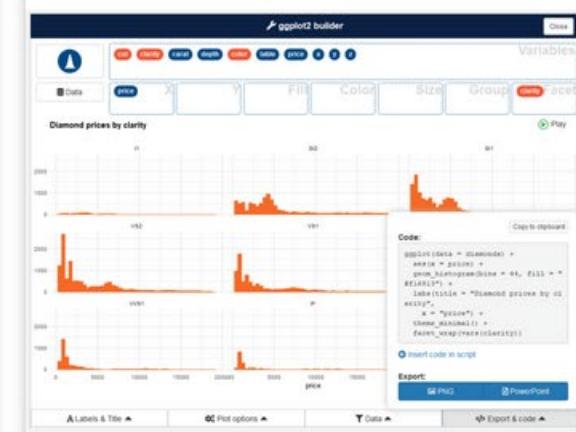
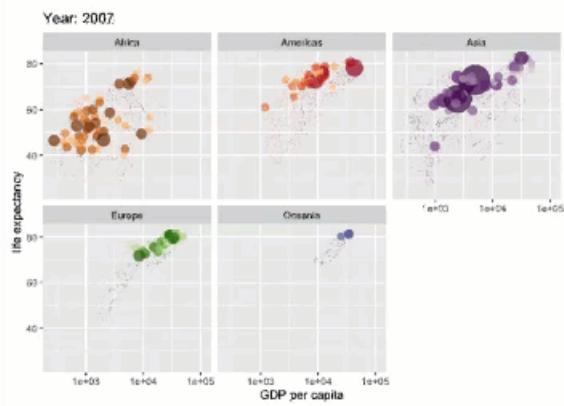
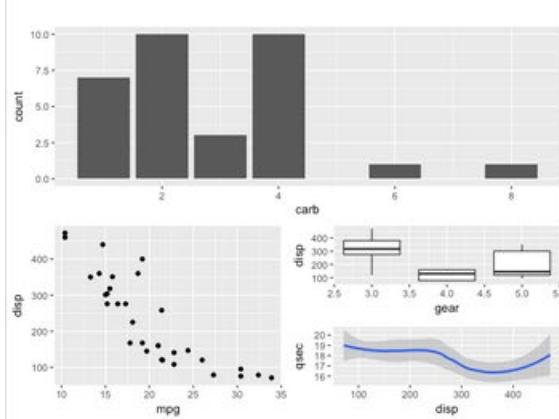
ggraph is tailored at plotting graph-like data structures (graphs, networks, trees, hierarchies...).

▪ **author:** [thomasp85](#)

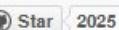
▪ **tags:** [visualization](#), [general](#)

ggplot2: extensions

ggplot2 provides a **prototype** system for implementing new geoms, stats, themes, ...
Many of these are listed at <https://exts.ggplot2.tidyverse.org/>



patchwork



2025

Easy composition of ggplot plots using arithmetic operators

■ author: thomasasp85

■ tags: visualization, composition

gganimate



1765

A Grammar of Animated Graphics.

■ author: thomasasp85

■ tags: visualization, general

■ js libraries:

esquisse



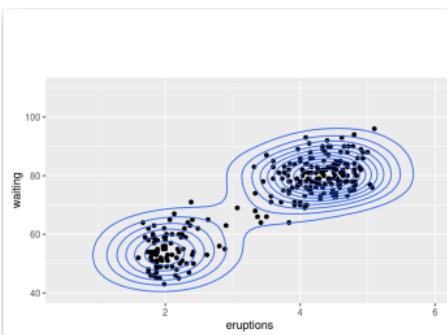
1433

Explore and Visualize Your Data Interactively with ggplot2

■ author: dreamrs

■ tags: visualization, interface

ggplot2: extensions



ggalt

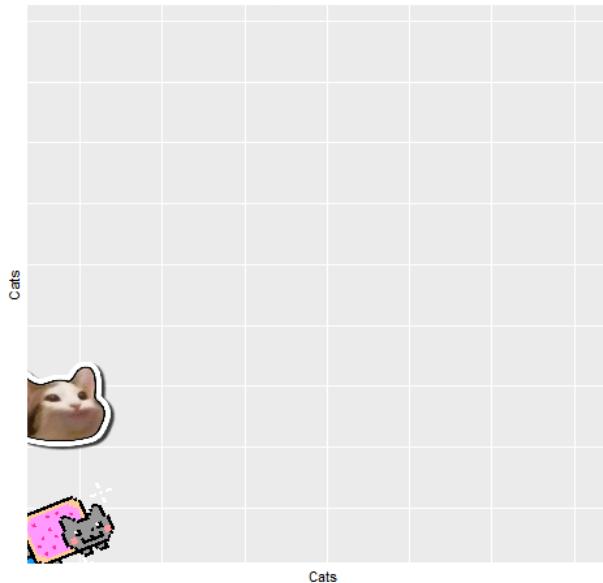
A compendium of 'geoms', 'coords' and 'stats' for 'ggplot2'.

■ author: hrbrmstr

■ tags: visualization, general

■ js libraries:

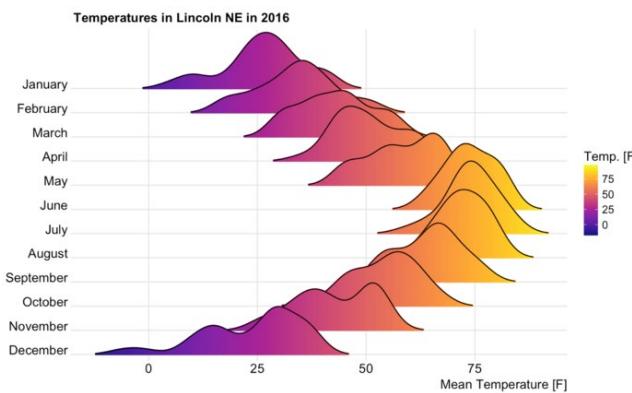
ggcats, a core package of the memeverse



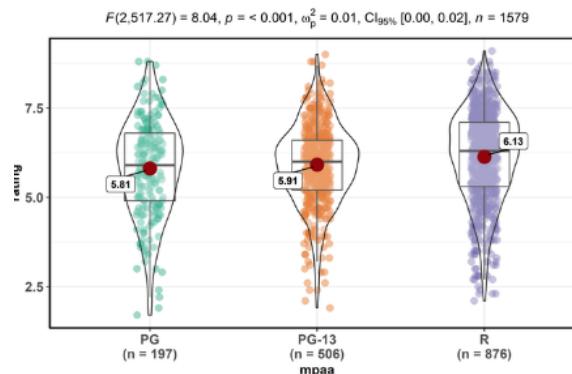
ggwordcloud



ggridges



ggstatsplot



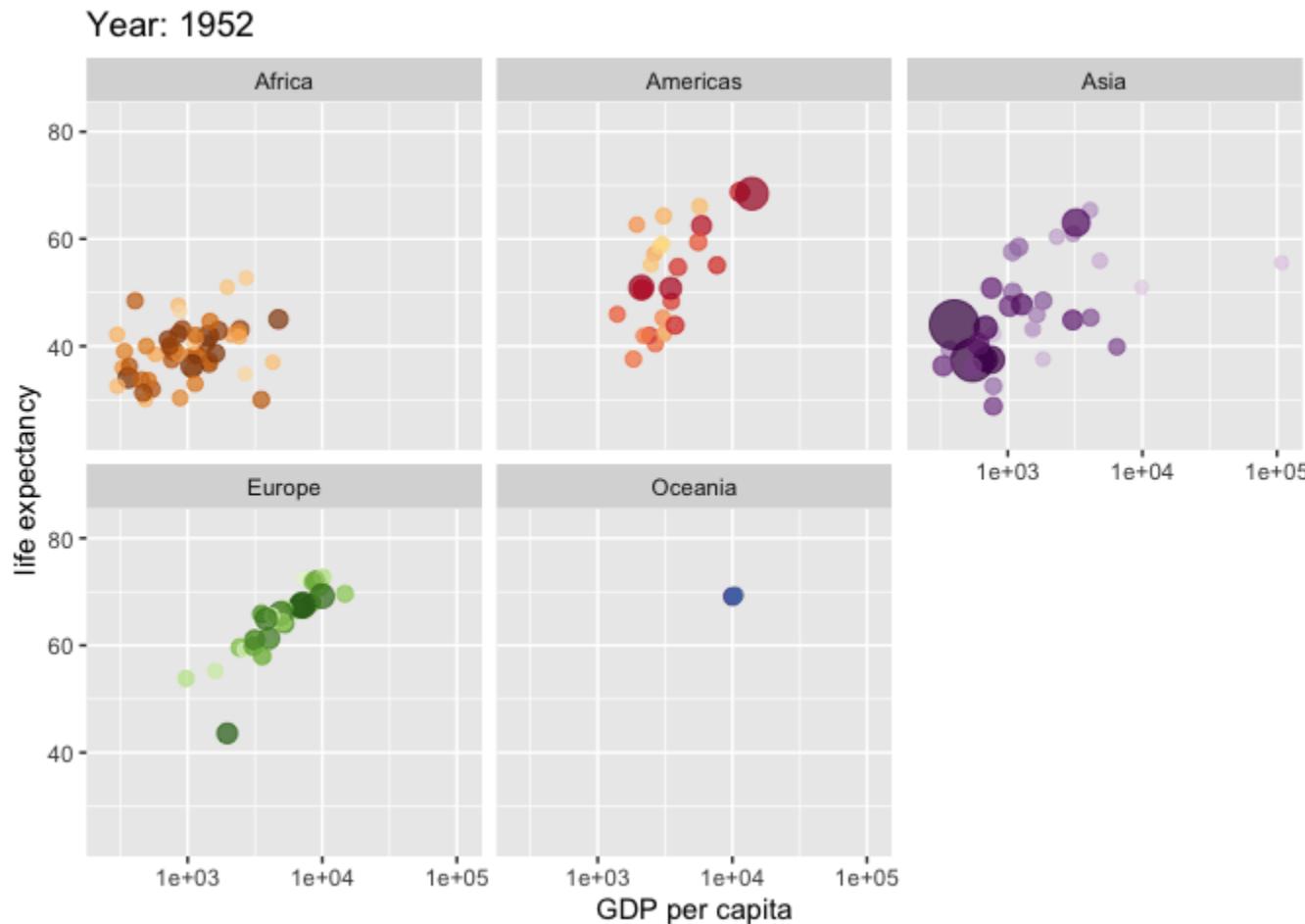
The wide range of extensions indicates the power of ggplot2 as a general framework for data graphics

ganimate: A grammar of animation

- **ganimate** extends `ggplot2` grammar to include a structured description of animation.
- → New grammar classes added to a plot object specify how it should change with time.
 - `transition_*`() how data should change and how it relates to itself across time.
 - `view_*`() how positional scales should change along the animation.
 - `enter_*/exit_*`() how new data appear, and old data disappear over the animation.
 - `ease_aes()` defines how different aesthetics should change over transitions

Goal: Produce an animation of Rosling's gapminder data, showing how life expectancy varies with GDP per capita.

- Stratify by continent: **facet_wrap(~continent)**
- Animate this by Year: **transition_time(year)**



Basic bubble plot by continent: `lifeExp ~ gdp`;

- `size` ~ population;
- `facet` ~ continent

```
library(gapminder)
ggplot(gapminder, aes(gdpPercap, lifeExp, size = pop, colour = country)) +
  geom_point(alpha = 0.7, show.legend = FALSE) +
  scale_colour_manual(values = country_colors) +
  scale_size(range = c(2, 12)) +
  scale_x_log10() +
  facet_wrap(~continent) +
```

Animate this:

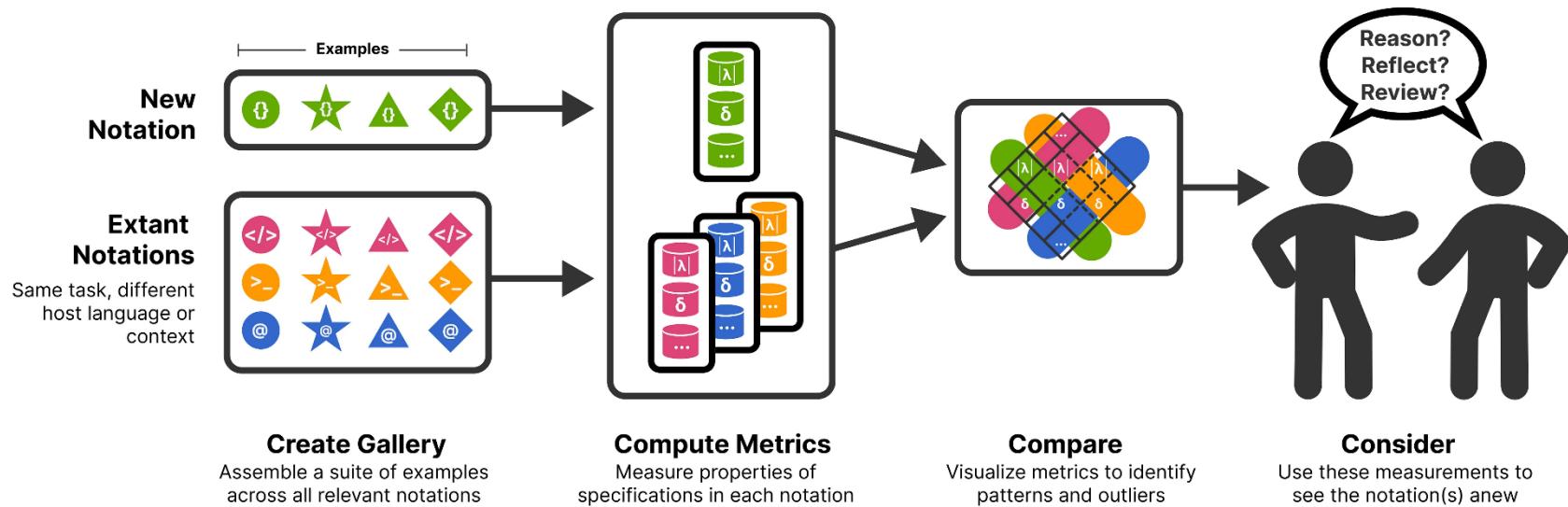
- change frame title;
- transition over year;
- interpolate linearly

```
labs(title = 'Year: {frame_time}',
      x = 'GDP per capita', y = 'Life expectancy') +
  transition_time(year) +
  ease_aes('linear')
```

interpolate linearly

Going Meta: Graphic notation

How do different software graphic languages make it easier or harder to produce the graph I want?

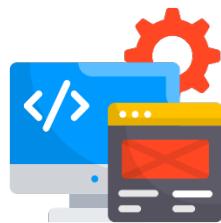


From: Nicolas Kruchten, [Usability of Visualization Notations](#)

Meta: Comparing graphic notation

- Graphs can be produced in a variety of software languages:
 - D3, ggplot2, Vega-Lite, matplotlib, Seaborn, Plotly, ...
- How do they differ in ease of use, efficiency of expression?
- **Cognitive dimensions** of notations?
 - **viscosity** (how easy to make changes to specifications),
 - **abstraction** (how easy to extend the notation),
 - **closeness of mapping** (how similar notation to target domain),
 - **progressive evaluation** (how easy to check work done to date),
 - **hard mental operations** (how demanding notation is of working memory).

Meta: Comparing graphic notation



- Graphs can be produced in a variety of software languages:
 - R, ggplot2, D3, Vega-Lite, matplotlib, Seaborn, Plotly, ...
- How do they differ in ease of use, efficiency of expression?
- Cognitive dimensions of notations?
 - **viscosity** (how easy to make changes to specifications),
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 - **progressive evaluation** (how easy to check work done to date),
 - **hard mental operations** (how demanding notation is of working memory).



https://en.wikipedia.org/wiki/Cognitive_dimensions_of_notations

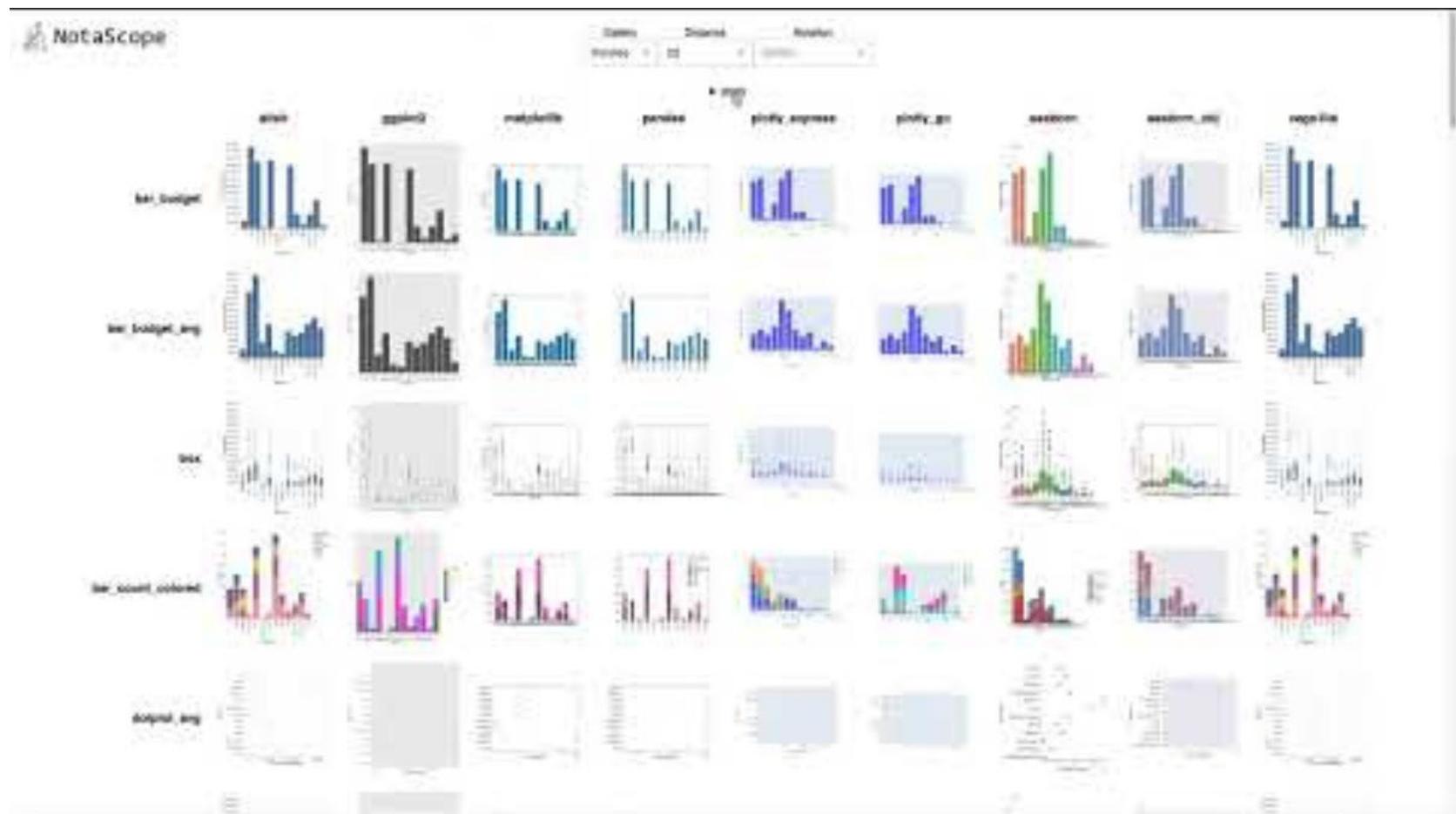
Software metrics

- Generate a collection of graph types
- Code each in a variety of specification languages & implementations
- Calculate metrics for each:
 - **Terseness**: # characters in code for given graph
 - **Economy**: Size of vocabulary (operators, functions, ...) to combine/add new stuff
 - **Viscosity**: How hard to change one notation to another?



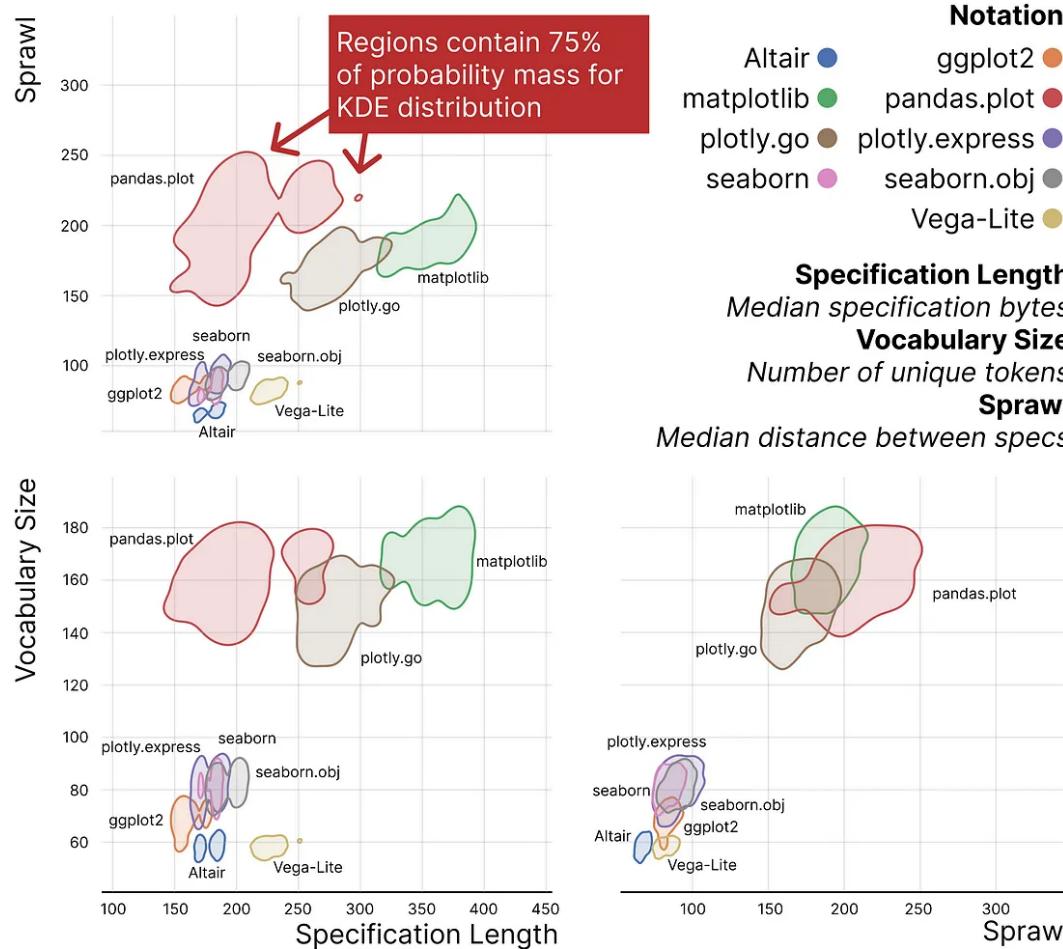
Notascope

<https://app.notascope.io/> - Online tool to demonstrate the metric-driven approach to graphic software evaluation



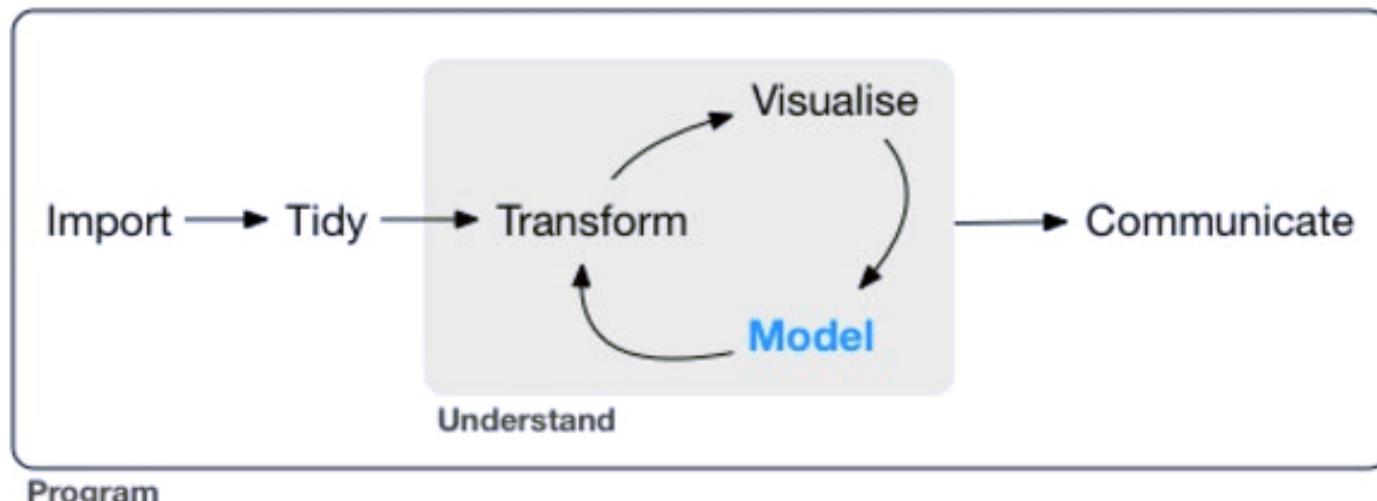
Evaluate, Analyze

Given a collection of graphs, implementations and metrics, we can better understand the how software languages differ in translation from IDEA → CODE → GRAPH



A larger view: Data science

- Data science treats statistics & data visualization as parts of a larger process
 - Data import: text files, data bases, web scraping, ...
 - Data cleaning → “tidy data”
 - Model building & visualization
 - Reproducible report writing

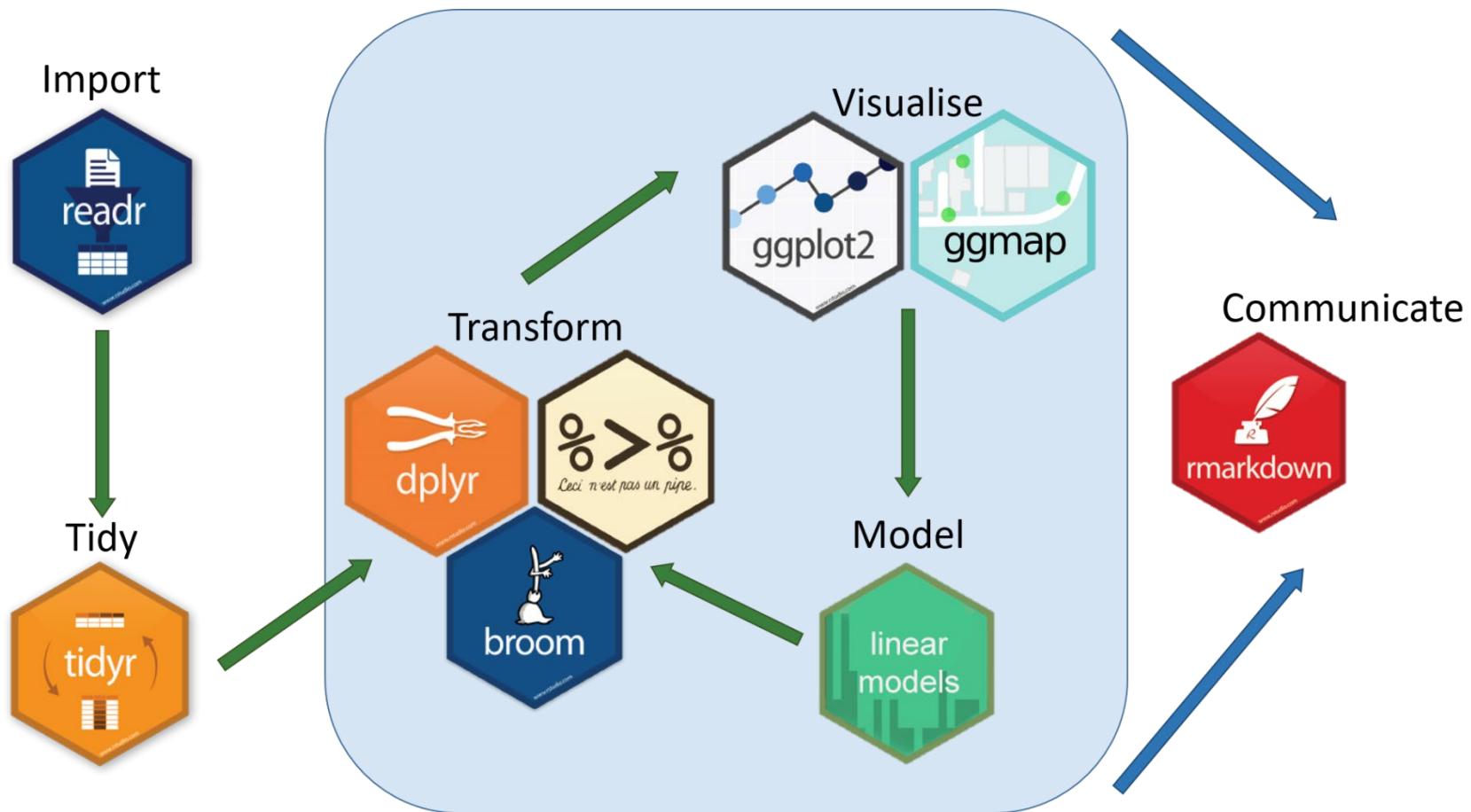




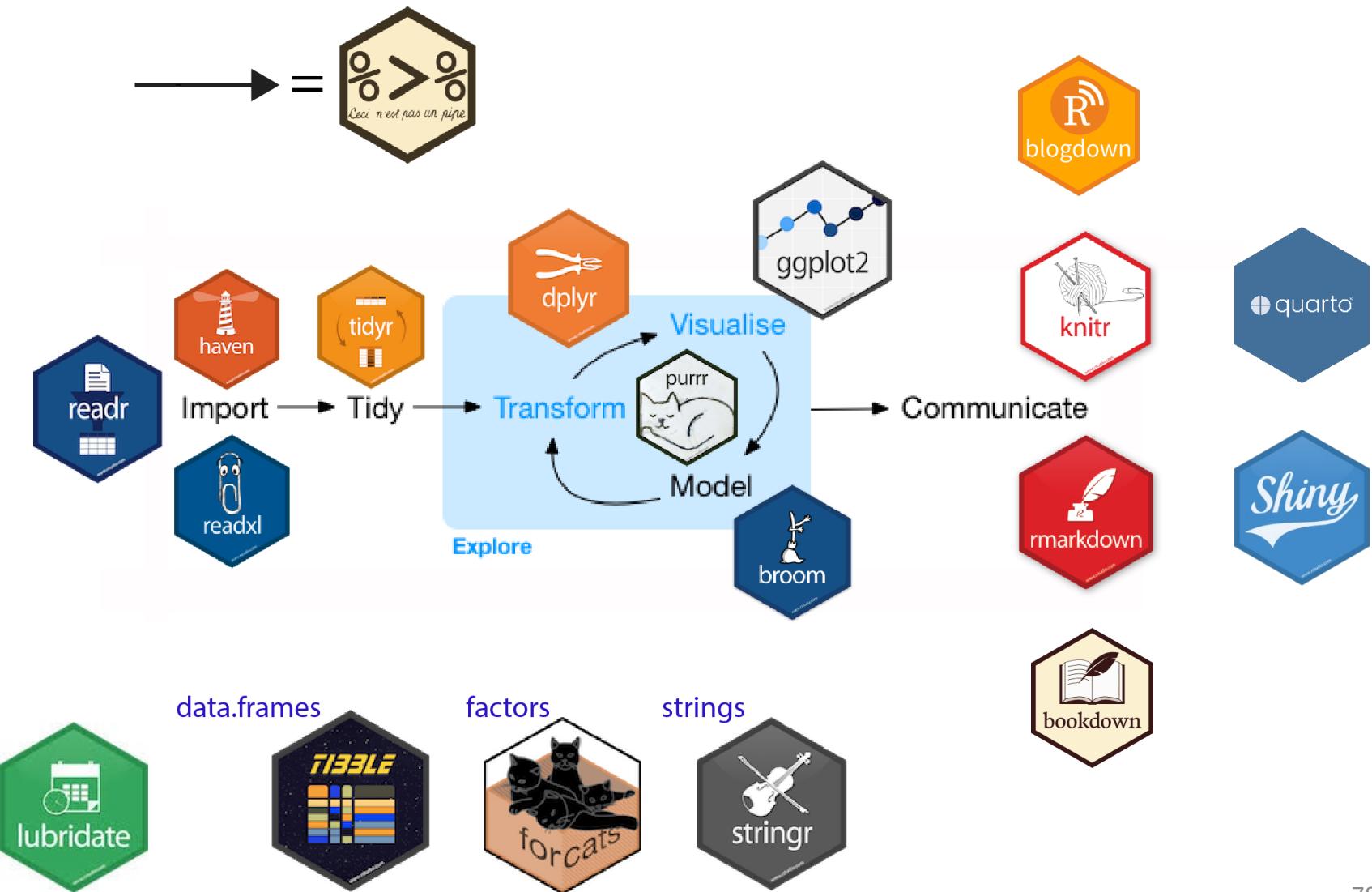
The tidyverse of R packages



These ideas inspire a larger view of data analysis and graphics based on tidy principles.



The tidyverse expands



Summary

- Graphical developers in the Golden Age recognized the idea of “graphic language,” but could not define it.
- Bertin first formalized the relations between graphical features (“retinal variables”), data attributes (O , Q , \neq , \equiv), and “reading levels”
- Wilkinson, in GoG, created a comprehensive syntax and algebra to define any **syntactically correct** graph
- Wickham, in ggplot2, created an **expressive** language to ease the translation of graphic ideas into plots.
- More general views can evaluate **usability** of graphic notations
- Tidyverse ideas place data analysis & graphics within a communication-oriented, reproducible research framework.