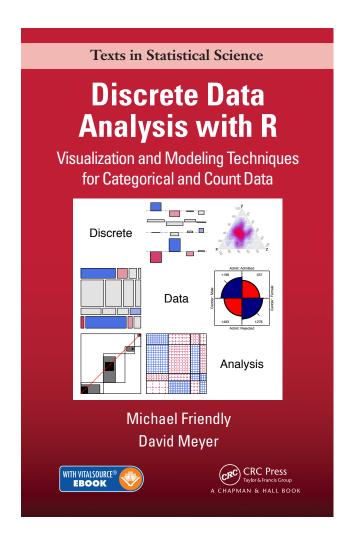
# Discrete Data Analysis with R: Solutions and Hints to Exercises

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## This document

This document is intended as an aid to instructors who wish to use *Discrete Data Analysis with R* in a course. It contains the text of the **Exercises** sections from all chapters, together with some solutions or hints for the various problems. Answers and commentary are indicated with the  $\bigstar$  symbol, and with text in this font.

All R code for the book, and other materials are available on the web site, http://ddar.datavis.ca.

# **Chapter 1** Introduction

★ These questions are all conceptual, or based on judgment. No individual solutions are provided. In general, students should come up with some interesting examples related to the questions and explain why they consider them to be good or bad graphic or tabular displays.

Some other sources that students might consult are:

- The Gallery of Data Visualization, http://datavis.ca/gallery/. A categorized collection of some of the best and worst of statistical graphics.
- Junk Charts, http://junkcharts.typepad.com/, a blog by Kaiser Fung. There is also a list of related blogs on graphics and data visualization at http://junkcharts.typepad.com/junk\_charts/other-graphics-blogs.html.
- Flowing Data, http://flowingdata.com/, by Nathan Yau. An eclectic collection of examples and blog posts encomposing a wide range from information visualization to statistical graphics.

Exercise 1.1 A web page, "The top ten worst graphs," http://www.biostat.wisc.edu/~kbroman/topten\_worstgraphs/ by Karl Broman lists his picks for the worst graphs (and a table) that have appeared in the statistical and scientific literature. Each entry links to graph(s) and a brief discussion of what is wrong and how it could be improved.

- (a) Examine a number of recent issues of a scientific or statistical journal in which you have some interest. Find one or more examples of a graph or table that is a particularly bad use of display material to summarize and communicate research findings. Write a few sentences indicating how or why the display fails and how it could be improved.
- (b) Do the same task for some popular magazine or newspaper that uses data displays to supplement the text for some story. Again, write a few sentences describing why the display is bad and how it could be improved.

**Exercise 1.2** As in the previous exercise, examine the literature in recent issues of some journal of interest to you. Find one or more examples of a graph or table that you feel does a *good* job of summarizing and communicating research findings.

- (a) Write a few sentences describing why you chose these displays.
- (b) Now take the role of a tough journal reviewer. Are there any features of the display that could be modified to make them more effective?

**Exercise 1.3** Infographics are another form of visual displays, quite different from the data graphics featured in this book, but often based on some data or analysis. Do a Google image search for the topic "Global warming" to see a rich collection.

- (a) Find and study one or two images that attempt some visual explanation of causes and/or effects of global warming. Describe the main message in a sentence or two.
- (b) What visual and graphic features are used in these to convey the message?

**Exercise 1.4** The Wikipedia web page en.wikipedia.org/wiki/Portal:Global\_warming gives a few data-based graphics on the topic of global warming. Read the text and study the graphs.

- (a) Write a short figure title for each that would announce the conclusion to be drawn in a presentation graphic.
- (b) Write a figure caption for each that would explain what is shown and the important graphical details for a reader to understand.

**Exercise 1.5** The R Graph Gallery, http://rgraphgallery.blogspot.com/, contains a large collection of examples of graphs in R, tagged by type or content, together with the R code to produce them. Explore this collection for the terms (a) association plot (b) bar chart (c) categorical data (d) fluctuation diagram (e) mosaic plot Find one or two you particularly like and write a few sentences saying why you do.

# **Chapter 2** Working with Categorical Data

**Exercise 2.1** The packages vcd (Meyer et al., 2015) and vcdExtra (Friendly, 2015) contain many data sets with some examples of analysis and graphical display. The goal of this exercise is to familiarize yourself with these resources.

You can get a brief summary of these using the function datasets () from vcdExtra. Use the following to get a list of these with some characteristics and titles.

```
> ds <- datasets(package = c("vcd", "vcdExtra"))
> str(ds, vec.len = 2)

'data.frame': 75 obs. of 5 variables:
    $ Package: chr "vcd" "vcd" ...
    $ Item : chr "Arthritis" "Baseball" ...
    $ class : chr "data.frame" "data.frame" ...
    $ dim : chr "84x5" "322x25" ...
    $ Title : chr "Arthritis Treatment Data" "Baseball Data" ...
```

- (a) How many data sets are there altogether? How many are there in each package?
  - ★ nrow() gives the number of rows in a data frame. table() for a single variable gives the frequencies for each level.

```
> ds <- datasets(package=c("vcd", "vcdExtra"))
> nrow(ds)
[1] 75
> table(ds$Package)
    vcd vcdExtra
    33    42
```

- (b) Make a tabular display of the frequencies by Package and class.
  - ★ Use table (), but now for Package and class.

- (c) Choose one or two data sets from this list, and examine their help files (e.g., help (Arthritis) or ?Arthritis). You can use, e.g., example (Arthritis) to run the R code for a given example.
  - ★ Run the following types of commands:

```
> ?Arthritis  # Help Files
> ?Baseball  # Help Files
> example (Arthritis)  # Example Syntax/Analysis
> example (Baseball)  # Example Syntax/Analysis
```

Exercise 2.2 For each of the following data sets in the vcdExtra package, identify which are response variable(s) and which are explanatory. For factor variables, which are unordered (nominal) and which should be treated as ordered? Write a sentence or two describing substantitive questions of interest for analysis of the data. (*Hint*: use data(foo, package="vcdExtra") to load, and str(foo), help(foo) to examine data set foo.)

- (a) Abortion opinion data: Abortion
  - ★ Support\_Abortion is the response, Sex and Status are binary, nominal explanatory variables. From help (Abortion), How does support for abortion depend on sex and status?

(b) Caesarian Births: Caesar

★ Infection is the response, Risk, Antibiotics and Planned are binary, nominal explanatory variables.

```
> data(Caesar, package="vcdExtra")
> str(Caesar)

table [1:3, 1:2, 1:2, 1:2] 0 1 17 0 1 1 11 17 30 4 ...
- attr(*, "dimnames")=List of 4
    ..$ Infection : chr [1:3] "Type 1" "Type 2" "None"
    ..$ Risk : chr [1:2] "Yes" "No"
    ..$ Antibiotics: chr [1:2] "Yes" "No"
    ..$ Planned : chr [1:2] "Yes" "No"
```

(c) Dayton Survey: DaytonSurvey

★ In DaytonSurvey, the variables cigarette, alcohol, and marijuana can all be treated as response variables. sex and race are potential explanatory variables. Potentially interesting questions are how each of the responses depend on sex and race, and how they vary jointly.

```
> data(DaytonSurvey, package="vcdExtra")
> str(DaytonSurvey)
```

(d) Minnesota High School Graduates: Hoyt

★ Status is the response, Rank, Occupation, and Sex are explanatory variables. Both Rank and Occupation are ordinal. How does Status vary with Rank, Occupation, and Sex?

```
> data(Hoyt, package="vcdExtra")
> str(Hoyt)
```

Exercise 2.3 The data set *UCBAdmissions* is a 3-way table of frequencies classified by Admit, Gender, and Dept.

(a) Find the total number of cases contained in this table.

★ For a table object, just use sum()

```
> data(UCBAdmissions)
> sum(UCBAdmissions)
[1] 4526
```

(b) For each department, find the total number of applicants.

★ Use margin.table(UCBAdmissions, 3) to find the marginal total for the third dimension (dept).

```
> margin.table(UCBAdmissions, 3)
Dept
A B C D E F
933 585 918 792 584 714
```

(c) For each department, find the overall proportion of applicants who were admitted.

 $\star$ 

```
> ucb.df <- as.data.frame(UCBAdmissions)
> abd <- xtabs(Freq ~ Dept + Admit, data=ucb.df)
> prop.table(abd, 1)

    Admit
Dept Admitted Rejected
    A 0.644159 0.355841
    B 0.632479 0.367521
    C 0.350763 0.649237
    D 0.339646 0.660354
    E 0.251712 0.748288
    F 0.064426 0.935574
```

(d) Construct a tabular display of department (rows) and gender (columns), showing the proportion of applicants in each cell who were admitted relative to the total applicants in that cell.

\*

Exercise 2.4 The data set DanishWelfare in vcd gives a 4-way,  $3 \times 4 \times 3 \times 5$  table as a data frame in frequency form, containing the variable Freq and four factors, Alcohol, Income, Status, and Urban. The variable Alcohol can be considered as the response variable, and the others as possible predictors.

- (a) Find the total number of cases represented in this table.
  - ★ This is a data set in the form of a frequency data.frame, so sum the Freq variable

```
> data("DanishWelfare", package="vcd")
> sum(DanishWelfare$Freq)
[1] 5144
```

- (b) In this form, the variables Alcohol and Income should arguably be considered *ordered* factors. Change them to make them ordered.
  - ★ Use ordered() or as.ordered() on the factor variable. str() will then show them as Ord.factor.

```
> levels(DanishWelfare$Alcohol)
[1] "<1" "1-2" ">2"

> DanishWelfare$Alcohol <- as.ordered(DanishWelfare$Alcohol)
> DanishWelfare$Income <- as.ordered(DanishWelfare$Income)
> str(DanishWelfare)

'data.frame': 180 obs. of 5 variables:
$ Freq : num 1 4 1 8 6 14 8 41 100 175 ...
$ Alcohol: Ord.factor w/ 3 levels "<1"<1-2"<">>2": 1 1 1 1 1 1 1 1 1 1 1 1 ...
$ Income : Ord.factor w/ 4 levels "0-50"<"50-100"<..: 1 1 1 1 1 1 1 1 1 1 ...
$ Status : Factor w/ 3 levels "Widow", "Married", ...: 1 1 1 1 2 2 2 2 2 2 ...
$ Urban : Factor w/ 5 levels "Copenhagen", "SubCopenhagen", ...: 1 2 3 4 5 1 2 3 4 5 ...</pre>
```

- (c) Convert this data frame to table form, DanishWelfare.tab, a 4-way array containing the frequencies with appropriate variable names and level names.
  - ★ Use xtabs() with Freq as the response.

```
> DanishWelfare.tab <-xtabs(Freq ~ ., data = DanishWelfare)
> str(DanishWelfare.tab)

xtabs [1:3, 1:4, 1:3, 1:5] 1 3 2 8 1 3 2 5 2 42 ...
- attr(*, "dimnames")=List of 4
    ..$ Alcohol: chr [1:3] "<1" "1-2" ">2"
    ..$ Income : chr [1:4] "0-50" "50-100" "100-150" ">150"
    ..$ Status : chr [1:3] "Widow" "Married" "Unmarried"
    ..$ Urban : chr [1:5] "Copenhagen" "SubCopenhagen" "LargeCity" "City" ...
- attr(*, "class")= chr [1:2] "xtabs" "table"
- attr(*, "call")= language xtabs(formula = Freq ~ ., data = DanishWelfare)
```

- (d) The variable Urban has 5 categories. Find the total frequencies in each of these. How would you collapse the table to have only two categories, City, Non-city?
  - ★ margin.table() handles the first part; collapse.table() is designed for the second part. It is arguable whether SubCopenhagen should be considered City or NonCity.

```
> margin.table(DanishWelfare.tab, 4)
Urban
   Copenhagen SubCopenhagen
                                  LargeCity
                                                                 Country
> DW2 <- vcdExtra::collapse.table(DanishWelfare.tab,
                                   Urban=c("City", "NonCity", "City", "City", "NonCity"))
> head(ftable(DW2))
                                   "Urban" "City" "NonCity"
 "Alcohol" "Income"
                      "Status"
 "<1"
           "0-50"
                      "Widow"
                                               1.0
                      "Married"
                                              155
                                                         183
                      "Unmarried"
                                               14
                                                          10
                      "Widow"
            "50-100"
                                               2.9
                      "Married"
                                              338
                      "Unmarried"
```

(e) Use structable() or ftable() to produce a pleasing flattened display of the frequencies in the 4-way table. Choose the variables used as row and column variables to make it easier to compare levels of Alcohol across the other factors.

\*

**Exercise 2.5** The data set *UKSoccer* in vcd gives the distributions of number of goals scored by the 20 teams in the 1995/96 season of the Premier League of the UK Football Association.

This two-way table classifies all  $20 \times 19 = 380$  games by the joint outcome (Home, Away), the number of goals scored by the Home and Away teams. The value 4 in this table actually represents 4 or more goals.

(a) Verify that the total number of games represented in this table is 380.

\*

```
> data("UKSoccer", package="vcd")
> sum(UKSoccer)
[1] 380
> margin.table(UKSoccer)
[1] 380
```

- (b) Find the marginal total of the number of goals scored by each of the home and away teams.
  - ★ Use margin.table() for each dimension:

```
> margin.table(UKSoccer, 1)
Home
    0     1     2     3     4
    76     142     90     45     27
> margin.table(UKSoccer, 2)
Away
    0     1     2     3     4
140     136     55     38     11
```

- (c) Express each of the marginal totals as proportions.
  - ★ Use prop.table() on the result of margin.table() for each dimension:

- (d) Comment on the distribution of the numbers of home-team and away-team goals. Is there any evidence that home teams score more goals on average?
  - ★ You could find the mean number of goals, weighted by their marginal frequencies. On average, home teams score about 0.4 more goals.

```
> weighted.mean(0:4, w=margin.table(UKSoccer,1))
[1] 1.4868
> weighted.mean(0:4, w=margin.table(UKSoccer,2))
[1] 1.0632
```

Graphically, you could also compare the marginal frequencies in a mosaic plot, or use agreementplot().

```
Goals
Team 0 1 2 3 4
home 76 142 90 45 27
away 140 136 55 38 11
> mosaic(margins, shade=TRUE)
```



Exercise 2.6 The one-way frequency table Saxony in VCd records the frequencies of families with 0, 1, 2, ... 12 male children, among 6115 families with 12 children. This data set is used extensively in Chapter 3.

```
> data("Saxony", package = "vcd")
> Saxony

nMales
0 1 2 3 4 5 6 7 8 9 10 11 12
3 24 104 286 670 1033 1343 1112 829 478 181 45 7
```

Another data set, <code>Geissler</code>, in the <code>vcdExtra</code> package, gives the complete tabulation of all combinations of <code>boys</code> and <code>girls</code> in families with a given total number of children (<code>size</code>). The task here is to create an equivalent table, <code>Saxony12</code> from the <code>Geissler</code> data.

```
> data("Geissler", package = "vcdExtra")
> str(Geissler)

'data.frame': 90 obs. of 4 variables:
$ boys : int 0 0 0 0 0 0 0 0 0 0 0 ...
$ girls: num 1 2 3 4 5 6 7 8 9 10 ...
$ size : num 1 2 3 4 5 6 7 8 9 10 ...
$ Freq : int 108719 42860 17395 7004 2839 1096 436 161 66 30 ...
```

(a) Use subset () to create a data frame, sax12 containing the Geissler observations in families with size==12.

★

```
> data("Saxony", package="vcd")
> data("Geissler", package="vcdExtra")
> sax12 <- subset(Geissler, size==12)</pre>
> sax12
     boys girls size Freq
12
         0
                  12
                         12
24
                            12
                                     24
                    11
                   10 12 104
9 12 286
8 12 670
45
                     8
54
           4
                            12 1033
```

```
69 6 6 12 1343

75 7 5 12 1112

80 8 4 12 829

84 9 3 12 478

87 10 2 12 181

89 11 1 12 45

90 12 0 12 7
```

(b) Select the columns for boys and Freq.

```
*
```

```
> sax12 <- subset(sax12, select=c("boys", "Freq"))
```

(c) Use xtabs () with a formula, Freq ~ boys, to create the one-way table.

```
*
```

```
> Saxony12<-xtabs(Freq~boys, data=sax12)
> Saxony12
boys
0  1  2  3  4  5  6  7  8  9  10  11  12
3  24  104  286  670  1033  1343  1112  829  478  181  45  7
```

(d) Do the same steps again to create a one-way table, Saxony11, containing similar frequencies for families of size==11.

```
*
```

```
> sax11 <- subset(Geissler, size==11, select = c("boys","Freq"))
> Saxony11 <- xtabs(Freq~boys, data=sax11)
> Saxony11
boys
0  1  2  3  4  5  6  7  8  9  10  11
8  72  275  837  1540 2161 2310 1801 1077  492  93  24
```

Exercise 2.7 \* Interactive coding of table factors: Some statistical and graphical methods for contingency tables are implemented only for two-way tables, but can be extended to 3+-way tables by recoding the factors to interactive combinations along the rows and/or columns, in a way similar to what ftable() and structable() do for printed displays.

For the *UCBAdmissions* data, produce a two-way table object, UCB.tab2, that has the combinations of Admit and Gender as the rows, and Dept as its columns, to look like the result below:

```
Dept
Admit:Gender
                    Α
                        В
                             С
                                     Ε
                                         F
  Admitted:Female 89
                       17 202 131
                                    94
                                        24
  Admitted:Male
                  512 353 120 138
                                    53
                        8 391 244 299 317
  Rejected: Female 19
                  313 207 205 279 138 351
  Rejected: Male
```

(a) Try this the long way: convert UCBAdmissions to a data frame (as.data.frame()), manipulate the factors (e.g., interaction()), then convert back to a table (as.data.frame()).
★

(b) Try this the short way: both ftable() and structable() have as.matrix() methods that convert their result to a matrix.

## **Exercise 2.8** The data set VisualAcuity in vcd gives a $4 \times 4 \times 2$ table as a frequency data frame.

```
> data("VisualAcuity", package = "vcd")
> str(VisualAcuity)

'data.frame': 32 obs. of 4 variables:
$ Freq : num 1520 234 117 36 266 ...
$ right : Factor w/ 4 levels "1","2","3","4": 1 2 3 4 1 2 3 4 1 2 ...
$ left : Factor w/ 4 levels "1","2","3","4": 1 1 1 1 2 2 2 2 3 3 ...
$ gender: Factor w/ 2 levels "male","female": 2 2 2 2 2 2 2 2 2 ...
```

(a) From this, use xtabs () to create two  $4 \times 4$  frequency tables, one for each gender.

 $\star$ 

(b) Use structable() to create a nicely organized tabular display.

+

```
> structable(right ~ left + gender, data = va.tab)
                            3
           right 1 2
left gender
                             72
    male
                  821 116
                                   43
                 1520 234 117
112 494 151
     female
                                   36
2
     male
    female
                  266 1512
                            362
                  85 145 583
124 432 1772
    male
                                 179
    female
    male
                   35
                        27
                              87
                                  331
   female
                       78 205
               66
                                 492
```

(c) Use  $\mathtt{xtable}()$  to create a  $\mathtt{LMEX}$  or  $\mathtt{HTML}$  table.

```
> library(xtable)
> va.xtab <- xtable(va.tabm)
> print(va.xtab, type="html")
```

# **Chapter 3** Fitting and Graphing Discrete Distributions

**Exercise 3.1** The Arbuthnot data in HistData (Friendly, 2014a) (Example 3.1) also contains the variable Ratio, giving the ratio of male to female births.

(a) Make a plot of Ratio over Year, similar to Figure 3.1. What features stand out? Which plot do you prefer to display the tendency for more male births?

 $\star$ 



The plot is similar to Figure 3.1 in the text. If it is easier to think in terms of probability of a male birth, plotting that directly may be preferable.

(b) Plot the total number of christenings, Males + Females or Total (in 000s) over time. What unusual features do you see?

```
> # total number of Christenings
> with(Arbuthnot, {
    Total= Males + Females
+ plot(Year, Total, type='b', ylab="Total Christenings (Male + Female)")
+ Arb.smooth <- loess.smooth(Year, Total)
+ lines(Arb.smooth$x, Arb.smooth$y, col="blue", lwd=2)
+ })</pre>
```



There was a large decline in births between 1640–1660, corresponding to years of plague in England.

**Exercise 3.2** Use the graphical methods illustrated in Section 3.2 to plot a collection of geometric distributions for p = 0.2, 0.4, 0.6, 0.8, over a range of values of  $k = 0, 1, \dots 10$ .

(a) With xyplot(), try the different plot formats using points connected with lines, as in Figure 3.9, or using points and lines down to the origin, as in the panels of Figure 3.10.

```
> KL <- expand.grid(k = 0 : 10, p = c(0.2, 0.4, 0.6, 0.8))
> geom_df <- data.frame(KL, prob = dgeom(KL$k, KL$p))
> geom_df$p = factor(geom_df$p)
> library(lattice)
> mycol<-palette()[2:5]
> xyplot(prob ~ k | p , data = geom_df, type = c("b"),
+ pch = 16, lwd = 4, cex = 1.25,
+ xlab = list("Number of events (k)", cex = 1.25), layout = c(2,2),
+ ylab = list("Probability", cex = 1.25))
> xyplot(prob ~ k | p , data = geom_df, type = c("h", "p"),
+ pch = 16, lwd = 4, cex = 1.25,
+ xlab = list("Number of events (k)", cex = 1.25), layout = c(2,2),
+ ylab = list("Number of events (k)", cex = 1.25), layout = c(2,2),
+ ylab = list("Probability", cex = 1.25))
```



(b) Also with xyplot(), produce one version of a multi-line plot in a single panel that you think shows well how these distributions change with the probability p of success.

\*



(c) Do the same in a multi-panel version, conditional on p.

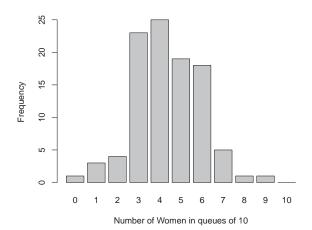
 $\star$ 



Exercise 3.3 Use the data set WomenOueue to:

(a) Produce plots analogous to those shown in Section 3.1 (some sort of bar graph of frequencies).

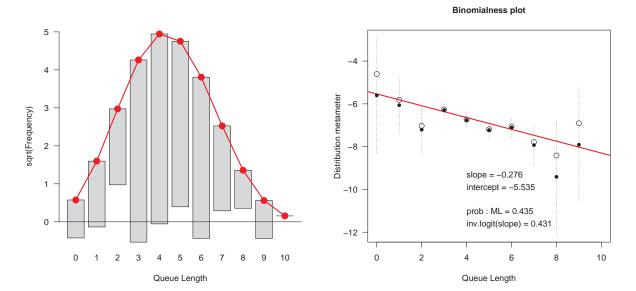
```
> data("WomenQueue", package = "vcd")
> barplot(WomenQueue, xlab="Number of Women in queues of 10", ylab= "Frequency")
```



(b) Check for goodness-of-fit to the binomial distribution using the goodfit () methods described in Section 3.3.2.  $\bigstar$  Note that with goodfit (), you should specify n=10 for the binomial distribution as the size parameter.

- (c) Make a reasonable plot showing departure from the binomial distribution.
  - ★ The simplest plot is the hanging rootogram. An alternative plot is a "binomialness" plot produced by distplot().

```
> plot(gf.women, xlab = "Queue Length")
> distplot(WomenQueue, type = "binomial", size=10, xlab = "Queue Length")
```



(d) Suggest some reasons why the number of women in queues of length 10 might depart from a binomial distribution, Bin(n = 10, p = 1/2).

\*

- Perhaps women (or men) are more prevalent in these queues, so  $p \neq 1/2$ .
- · People often join lines in groups, so the observations are unlikely to be independent.

**Exercise 3.4** Continue Example 3.13 on the distribution of male children in families in Saxony by fitting a binomial distribution,  $Bin(n = 12, p = \frac{1}{2})$ , specifying equal probability for boys and girls. [*Hint*: you need to specify both size and prob values for goodfit().]

(a) Carry out the GOF test for this fixed binomial distribution. What is the ratio of  $\chi^2/df$ ? What do you conclude?  $\bigstar$  Note that you need to specify both n and p as fixed parameters here.

The binomial model fits very badly.

(b) Test the additional lack of fit for the model  $Bin(n=12,p=\frac{1}{2})$  compared to the model  $Bin(n=12,p=\hat{p})$  where  $\hat{p}$  is estimated from the data.

```
\star
```

This fits much better, but still not a good fit.

(c) Use the plot.gootfit() method to visualize these two models.



```
> plot(Saxony_gf, main = "Fit for p=0.5", xlab = "Number of Male Children (Out of 12)")
> plot(Saxony_gf2, main = "Fit for p=phat", xlab = "Number of Male Children (Out of 12)")
```

Fit for p=0.5

Fit for p=phat





**Exercise 3.5** For the Federalist data, the examples in Section 3.3.1 and Section 3.3.2 showed the negative binomial to provide an acceptable fit. Compare this with the simpler special case of geometric distribution, corresponding to n = 1.

(a) Use goodfit () to fit the geometric distribution. [Hint: use type="nbinomial", but specify size=1 as a parameter.]

 $\star$ 

```
> fdfit1 <- goodfit(Federalist, type = "binomial", par = list(size=6))</pre>
> fdfit1
Observed and fitted values for binomial distribution
with parameters estimated by `ML'
 count observed
                   fitted pearson residual
    0 156 1.3072e+02
                            2.21074
-3.39860
             63 9.6362e+01
            29 2.9597e+01
                                   -0.10972
                                  1.43139
5.31624
             8 4.8483e+00
            4 4.4673e-01
1 2.1954e-02
                                    6.60094
             1 4.4953e-04
                                  47.14399
> fdfit2 <- goodfit(Federalist, type = "nbinomial", par = list(size=1))</pre>
Observed and fitted values for nbinomial distribution
with parameters estimated by `ML with size fixed'
                   fitted pearson residual
 count observed
                            -0.172219
           156 158.16590
             63 62.68326
                                  0.040006
             29 24.84221
                                  0.834194
             8 9.84530
4 3.90182
                                 -0.588102
                                 0.049702
     4
                  1.54635
                                 -0.439353
                             -0.015044
                  0.61284
```

(b) Compare the negative binomial and the geometric models statistically, by a likelihood-ratio test of the difference between these two models.

 $\star$ 

(c) Compare the negative binomial and the geometric models visually by hanging rootograms or other methods.

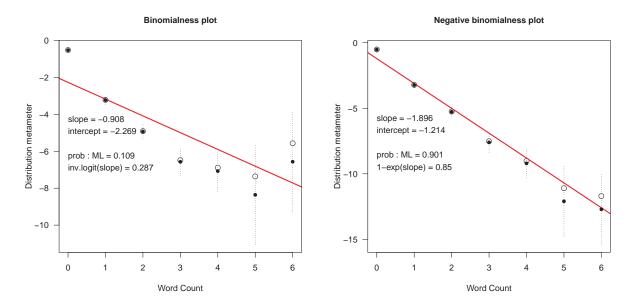
★ Hanging rootograms:

```
> plot(fdfit1)
> plot(fdfit2)
```



#### Distribution-ness plots:

```
> distplot(Federalist, type = "binomial", size=6, xlab = "Word Count")
> distplot(Federalist, type = "nbinomial", size=6, xlab = "Word Count")
```



**Exercise 3.6** Mosteller and Wallace (1963, Table 2.4) give the frequencies,  $n_k$ , of counts  $k = 0, 1, \ldots$  of other selected marker words in 247 blocks of text known to have been written by Alexander Hamilton. The data below show the occurrences of the word *upon*, that Hamilton used much more than did James Madison.

```
> count <- 0 : 5
> Freq <- c(129, 83, 20, 9, 5, 1)
```

- (a) Read these data into R and construct a one-way table of frequencies of counts or a matrix or data frame with frequencies in the first column and the corresponding counts in the second column, suitable for use with
  - \*\pmodfit() requires its first argument to be either a one-way table (from xtabs()), or a data.frame with frequencies in the *first* column and the corresponding counts in the second column. Both of the following forms will work.

(b) Fit and plot the Poisson model for these frequencies.

```
*
```

```
> (up0 <- goodfit(Upon, type="poisson"))</pre>
Observed and fitted values for poisson distribution
with parameters estimated by `ML'
             rved fitted pearson residual 129 121.61816 0.66937
                               0.66937
-0.34115
              83 86.16671
               20
                   30.52465
                                      -1.90494
                   7.20892
                                      0.66708
                    1.27688
                                        3.29481
                    0.18094
                                       1.75800
> summary(up0)
 Goodness-of-fit test for poisson distribution
X^2 df P(> X^2)
Likelihood Ratio 13.139 4 0.010617
> plot (up0)
```



(c) Fit and plot the negative binomial model for these frequencies.

```
*
```

```
83
                 73.89421
                                   1.059285
             20
                 28.41547
                                  -1.578705
                  9.25319
                                  -0.083233
                  2.74068
                                   1.364738
                  0.76332
                                   -0.036432
> summary (up1)
Goodness-of-fit test for nbinomial distribution
                    X^2 df P(> X^2)
Likelihood Ratio 6.0306 3 0.11013
> plot (up1)
```



#### (d) What do you conclude?

★ The negative binomial model fits better than the Poisson.

**Exercise 3.7** The data frame *Geissler* in the vcdExtra package contains the complete data from Geissler's (1889) tabulation of family sex composition in Saxony. The table below gives the number of boys in families of size 11.

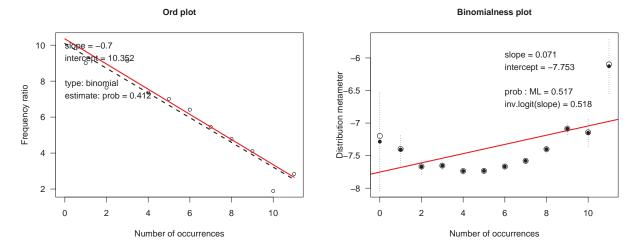
boys	0	1	2	3	4	5	6	7	8	9	10	11
Freq	8	72	275	837	1,540	2,161	2,310	1,801	1,077	492	93	24

- (a) Read these data into R.
  - ★ See Exercise 2.6, which calculates sax11 in the form of a data frame.
- (b) Following Example 3.13, use goodfit () to fit the binomial model and plot the results. Is there an indication that the binomial does not fit these data?
   ★ The binomial distribution fits badly, where the extremes are under-fitted, and the middle values are over-fitted.

```
> sax11.tab <- xtabs(Freq ~ boys, data=sax11)
> goodfit(sax11.tab, type="binomial", par=list(size=11))
Observed and fitted values for binomial distribution
with parameters estimated by `ML'
 count observed
                       fitted pearson residual
                       3.5616
                8
                                           2.3518
                    41.9479 224.5724
                                            4.6400
              275
                                           3.3650
             837 721.3629
1540 1544.7559
                                           4.3055
                                           -0.1210
             2161 2315.6023
2310 2479.3627
                                           -3.2128
                                           -3.4013
             1801 1896.2173
                                           -2.1866
      8
              1077 1015.1593
                                            1.9409
              492
                    362.3173
                                            6.8130
     10
               93
                      77.5881
                                            1.7497
                       7.5523
                                            5.9850
> summary(goodfit(sax11.tab, type="binomial", par=list(size=11)))
```

- (c) Diagnose the form of the distribution using the methods described in Section 3.4.
  - ★ The Ord plot indicates that the closest distribution according to its heuristics is the binomial; the binomialness distribution plot, however, shows this is not an acceptable model, as was also seen in the text for families of size 12.

```
> Ord_plot(sax11.tab)
> distplot(sax11.tab, type="binomial", size=11)
```



(d) Try fitting the negative binomial distribution, and use distplot () to diagnose whether the negative binomial is a reasonable fit. **TODO**: Something wrong here.

```
> goodfit(sax11.tab, type="nbinomial", par=list(size=11))
Observed and fitted values for nbinomial distribution with parameters estimated by `ML with size fixed' \,
 count observed
                    fitted pearson residual
                8
                   109.12
                                       -9.6801
               72
                    409.11
                                      -16.6667
              275 836.63
                                      -19.4171
              837 1235.67
                                      -11.3414
                                        1.7171
             1540 1474.07
             2161 1507.26
                                       16.8389
             2310 1369.95
                                       25.3981
             1801 1133.97
                                       19.8082
             1077
                   869.62
                                        7.0323
                   625.73
              492
                                       -5.3462
                   426.55
               93
                                      -16.1500
                                      -25.3999
```

**Exercise 3.8** The data frame *Bundesliga* gives a similar data set to that for UK soccer scores (*UKSoccer*) examined in Example 3.9, but over a wide range of years. The following lines calculate a two-way table, BL1995, of home-team and away-team goals for the 306 games in the year 1995.

```
4 3 5 3 0 0 0 0
5 4 1 0 1 0 0 0
```

- (a) As in , find the one-way distributions of HomeGoals, AwayGoals, and TotalGoals = HomeGoals +
  - ★ There are several ways to do this, but as illustrated in the text for Example 3.9, create the one-way variables in a data.frame, and then use xtabs () to get their marginal distributions.

```
> BL.df <- as.data.frame(BL1995, stringsASFactors=FALSE)
> BL.df <- as.data.rrame(BL1995, strings)
BL.df <- within(BL.df, {
+ HomeGoals <- as.numeric(HomeGoals)
+ AwayGoals <- as.numeric(AwayGoals)
+ TotalGoals <- HomeGoals + AwayGoals
      # marginal distributions
> (BL.home <- xtabs(Freq ~ HomeGoals, data=BL.df))
HomeGoals

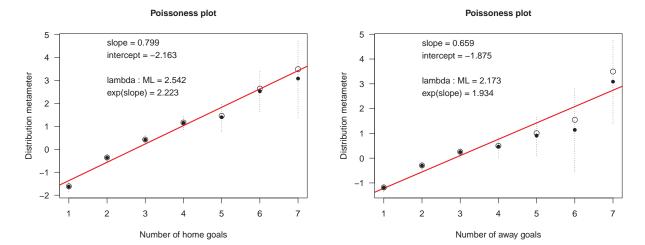
1 2 3 4 5 6 7
61 107 78 41 11 6 2
> (BL.away <- xtabs(Freq ~ AwayGoals, data=BL.df))</pre>
       2 3 4
 1 2 3 4 5 6 7
94 114 66 21 7 2 2
> (BL.total <- xtabs(Freq ~ TotalGoals, data=BL.df))
TotalGoals
2 3 4 5 6 7 8 9 10 11 12 13 14 26 35 98 62 39 29 10 4 2 1 0 0 0
```

- (b) Use goodfit () to fit and plot the Poisson distribution to each of these. Does the Poisson seem to provide a reasonable fit?
  ★ The Poisson distribution has a bad fit for all of these.

```
> summary(goodfit(BL.home))
Goodness-of-fit test for poisson distribution
                   X^2 df P(> X^2)
Likelihood Ratio 70.722 5 7.2516e-14
> summary(goodfit(BL.away))
Goodness-of-fit test for poisson distribution
                   X^2 df P(> X^2)
Likelihood Ratio 97.973 5 1.4131e-19
> summary(goodfit(BL.total))
Goodness-of-fit test for poisson distribution
                   X^2 df P(> X^2)
Likelihood Ratio 72.558 8 1.5185e-12
```

- (c) Use distplot () to assess fit of the Poisson distribution.
  - ★ The distribution plots for home goals looks better than that for away goals, which shows a systematic departure from the red line.

```
> distplot(BL.home, xlab="Number of home goals")
> distplot(BL.away, xlab="Number of away goals")
```



- (d) What circumstances of scoring goals in soccer might cause these distributions to deviate from Poisson distributions?
  - ★ The Poisson distribution relies on (a) independent events with (b) constant probabilities. The probability of scoring a goal is almost certainly not constant over all pairs of teams.

Exercise 3.9 \* Repeat the exercise above, this time using the data for all years in which there was the standard number (306) of games, that is for Year>1965, tabulated as shown below.

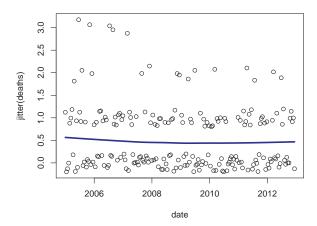
```
> BL <- xtabs(~ HomeGoals + AwayGoals, data = Bundesliga,
                subset = (Year > 1965))
> BL
          AwayGoals
HomeGoals
                                                                   9
                                                              8
                  590
             868
                        458
                              206
                                     88
                                           22
                                                        2
                                                                   0
                                                 12
            1049 1550
                        589
                              360
                                    121
                                           34
                                                  8
                                                        6
                                                                   1
            1039
                        810
                                     95
                                           26
                                                                   0
             712
                  793
                        392
                              187
                                     43
                                            8
                                                                   0
                   388
                        245
                                     26
                                                                   0
             128
                  164
                        106
                               34
                                                              0
                                                                   0
                   63
                          38
                               10
                                                                   0
        8
                                                                   0
                                                              0
                                                                   0
                     0
        10
        11
                     2
                           0
                                0
                                      0
                                                                   0
```

★ The steps are similar to those in the previous problem, but now there are more years, and the range of both home goals and away goals is larger.

Exercise 3.10 Using the data CyclingDeaths introduced in Example 3.6 and the one-way frequency table CyclingDeaths.tab = table (CyclingDeaths\$ deaths),

- (a) Make a sensible plot of the number of deaths over time. For extra credit, add a smoothed curve (e.g., using lines (lowess(...))).
  - ★ The number of deaths in a given fortnight are discrete, taking values 0:3. A barplot, using plot (deaths ~ date, type="h", data=CyclingDeaths) is one option. Perhaps slightly better is to jitter the number of deaths.

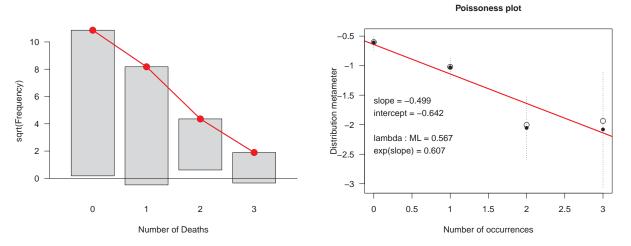
```
> data("CyclingDeaths", package="vcdExtra")
> CyclingDeaths.tab <- table(CyclingDeaths$deaths)
> plot(jitter(deaths) ~ date, data=CyclingDeaths)
> with(CyclingDeaths, {lines(lowess(date, deaths), lwd=3, col="blue")})
```



 $(b) \ \ Test \ the \ goodness \ of \ fit \ of \ the \ table \ \texttt{CyclingDeaths.tab} \ to \ a \ Poisson \ distribution \ statistically \ using \ \texttt{goodfit} \ () \ .$ 

(c) Continue this analysis using a rootogram() and distplot().

```
> plot(gf, xlab="Number of Deaths")
> distplot(CyclingDeaths.tab)
```



(d) Write a one-paragraph summary of the results of these analyses and your conclusions.

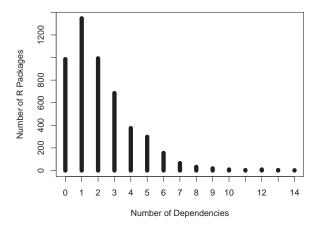
Exercise 3.11 \* The one-way table, *Depends*, in vcdExtra and shown below gives the frequency distribution of the number of dependencies declared in 4,983 R packages maintained on the CRAN distribution network on January 17, 2014. That is, there were 986 packages that had no dependencies, 1,347 packages that depended on one other package, . . . up to 2 packages that depended on 14 other packages.

Depends	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
# Pkgs	986	1,347	993	685	375	298	155	65	32	19	9	4	9	4	2

(a) Make a bar plot of this distribution.

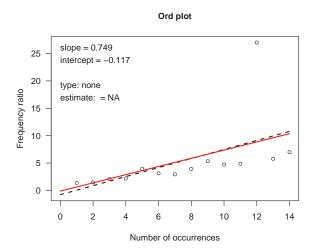
\*

```
> data("Depends", package="vcdExtra")
> plot(Depends, xlab="Number of Dependencies", ylab="Number of R Packages", lwd=8)
```

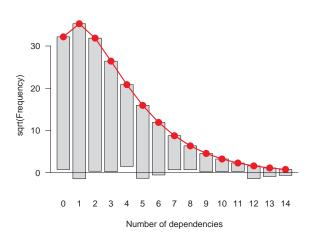


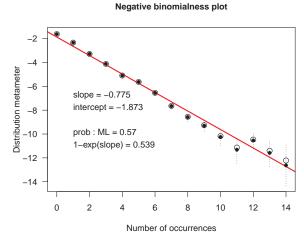
- (b) Use Ord\_plot () to see if this method can diagnose the form of the distribution.
  - $\bigstar$  This turns out to be a case where the Ord plot method, as implemented in Ord\_plot () does not determine the form of the distribution. According to Table 3.11, the log series distribution is the only one with positive slope b and negative intercept a, but this requires a=-b.

> Ord\_plot (Depends)



- (c) Try to fit a reasonable distribution to describe dependencies among R packages.
  - \* Among the distributions described in Chapter 3 and implemented in the vcd package the negative binomial may be the best to try.





So, this does not fit well, particularly for the packages with many dependencies, but it is not altogether terrible. The remaining differences come from further heterogeneity for which we haven't got any covariates in the data set. For example, the age of the package would seem like a natural candidate: older packages probably have fewer dependencies.

Exercise 3.12 \* How many years does it take to get into the baseball Hall of Fame? The Lahman (Friendly, 2014b) package provides a complete record of historical baseball statistics from 1871 to the present. One table, HallOfFame, records the history of players nominated to the Baseball Hall of Fame, and those eventually inducted. The table below, calculated in help (HallOfFame, package="Lahman"), records the distribution of the number of years taken (from first nomination) for the 109 players in the Hall of Fame to be inducted (1936–present). Note that years==0 does not, and cannot, occur in this table, so the distribution is restricted to positive counts. Such distributions are called *zero-truncated distributions*. Such distributions are like the ordinary ones, but with the probability of zero being zero. Thus the other probabilities are scaled up (i.e., divided by 1 - Pr(Y = 0)) so they sum to 1.

years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
inducted	46	10	8	7	8	4	2	4	6	3	3	1	4	1	2

(a) For the Poisson distribution, show that the zero-truncated probability function can be expressed in the form

$$\Pr\{X = k \mid k > 0\} = \frac{1}{1 - e^{-\lambda}} \times \frac{e^{-\lambda} \lambda^k}{k!} \qquad k = 1, 2, \dots$$

★ The standard Poisson distribution has the probability mass function

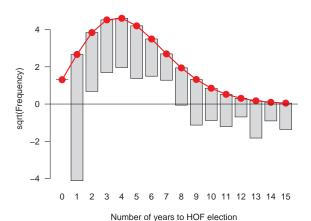
$$\Pr\{X = k\} = \frac{e^{-\lambda} \lambda^k}{k!} \quad k = 1, 2, \dots.$$

For this,  $\Pr\{X=0\}=e^{-\lambda}$ . The formula given in the problem scales the standard probability to account for zero-truncation, i.e., by the factor  $\frac{1}{1-\Pr\{X=0\}}$ .

(b) Show that the mean is  $\lambda/(1 - \exp(-\lambda))$ .

- (c) Enter these data into R as a one-way table, and use goodfit () to fit the standard Poisson distribution, as if you hadn't encountered the problem of zero truncation.
  - ★ The rootogram below shows why zero truncation needs to be taken into account.

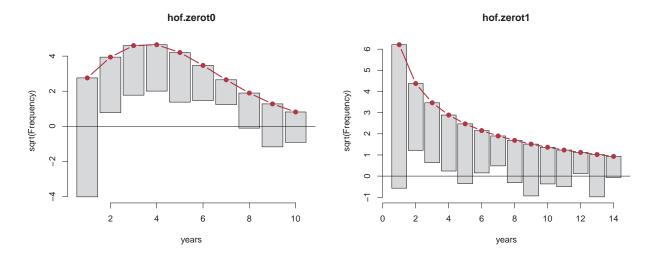
```
> years <- 1:15
> inducted <- c(46, 10, 8, 7, 8, 4, 2, 4, 6, 3, 3, 1, 4, 1, 2)
> HOF.df <- data.frame(years, inducted)
> HOF.tab <- xtabs(inducted ~ years, data=HOF.df)
> goodfit(HOF.tab)
Observed and fitted values for poisson distribution with parameters estimated by `ML' \,
                           fitted pearson residual
                      1.7081050
                  0
                                                -1.30694
                  46 7.0988218
                                               14.60056
                  10 14.7512214
                                                -1.23706
                  8 20.4351783
                                               -2.75082
                   7 21.2319627
                                                -3.08866
                  8 17.6478516
                                                -2.29660
                   4 12.2239706
                                               -2.35221
                  2 7.2574819
4 3.7702285
6 1.7409924
                                               -1.95157
       8
                                                0.11833
                                                3.22782
       9
                   3 0.7235500
     1.0
                                                2.67623
                   3 0.2733679
     11
                                                5.21498
                  1 0.0946756
4 0.0302668
                                                 2.94229
                                                22.81803
      14
                   1 0.0089848
                                                10.45503
     15
                   2 0.0024894
                                                34.54549
> summary(HOF.tab)
Number of cases in table: 109
Number of factors: 1
> plot(goodfit(HOF.tab), xlab='Number of years to HOF election')
```



Though not asked in the problem, the zero-truncated Poisson distribution can be fit using vglm() in the VGAM (Yee, 2015) package.

Models for count data, taking account of excess zeros or zero truncation are examined in more detail in Chapter 11. There, we use the countreg (Zeileis and Kleiber, 2014) package, that provides a zerotrunc() function for these problems. The zero truncated negative binomial provides a better fit than the zero truncated Poisson, but that is not great either.

```
> library(countreg)
> hof.zerot0 <- zerotrunc(years ~ 1, weights = inducted, data=HOF.df)
> hof.zerot1 <- zerotrunc(years ~ 1, weights = inducted, dist="negbin")
> countreg::rootogram(hof.zerot0)
> countreg::rootogram(hof.zerot1)
```



## **Chapter 4** Two-Way Contingency Tables

**Exercise 4.1** The data set fat, created below, gives a  $2 \times 2$  table recording the level of cholesterol in diet and the presence of symptoms of heart disease for a sample of 23 people.

(a) Use chisq.test (fat) to test for association between diet and disease. Is there any indication that this test may not be appropriate here?



(b) Use a fourfold display to test this association visually. Experiment with the different options for standardizing the margins, using the margin argument to fourfold(). What evidence is shown in different displays regarding whether the odds ratio differs significantly from 1?



(c) oddsratio(fat, log = FALSE) will give you a numerical answer. How does this compare to your visual impression from fourfold displays?



(d) With such a small sample, Fisher's exact test may be more reliable for statistical inference. Use fisher.test(fat), and compare these results to what you have observed before.



(e) Write a one-paragraph summary of your findings and conclusions for this data set.



Exercise 4.2 The data set Abortion in vcdExtra gives a  $2 \times 2 \times 2$  table of opinions regarding abortion in relation to sex and status of the respondent. This table has the following structure:

```
> data("Abortion", package = "vcdExtra")
> str(Abortion)

table [1:2, 1:2, 1:2] 171 152 138 167 79 148 112 133
- attr(*, "dimnames")=List of 3
..$ Sex : chr [1:2] "Female" "Male"
..$ Status : chr [1:2] "Lo" "Hi"
..$ Support_Abortion: chr [1:2] "Yes" "No"
```

(a) Taking support for abortion as the outcome variable, produce fourfold displays showing the association with sex, stratified by status.

```
\star
```



(b) Do the same for the association of support for abortion with status, stratified by sex.

```
*
```

```
> Abortion3<-aperm(Abortion, c(2,3,1))
> fourfold(Abortion3)
```



(c) For each of the problems above, use oddsratio() to calculate the numerical values of the odds ratio, as stratified in the question.

```
\star
```

```
# Sex by support for abortion, stratified by status
> summary(oddsratio(Abortion2))
z test of coefficients:
4.18 2.9e-05
                                                 0.91
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
     # Status by support for abortion, stratified by sex
> summary(oddsratio(Abortion3))
z test of coefficients:
                  Estimate Std. Error z value Pr(>|z|) 0.563 0.186 3.03 0.0025
                  0.563
Lo:Hi/Yes:No|Female
Lo:Hi/Yes:No|Male
                    -0.201
                               0.164
                                       -1.23
                                              0.2199
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- (d) Write a brief summary of how support for abortion depends on sex and status.
  - ★ Regardless of status, more women say no to abortion and more men say yes. Regardless of gender, more low status say no and more high status say yes.

**Exercise 4.3** The *JobSat* table on income and job satisfaction created in Example 2.5 is contained in the vcdExtra package.

(a) Carry out a standard  $\chi^2$  test for association between income and job satisfaction. Is there any indication that this test might not be appropriate? Repeat this test using simulate.p.value = TRUE to obtain a Monte Carlo test that does not depend on large sample size. Does this change your conclusion?



(b) Both variables are ordinal, so CMH tests may be more powerful here. Carry out that analysis. What do you conclude?



**Exercise 4.4** The Hospital data in vcd gives a  $3 \times 3$  table relating the length of stay (in years) of 132 long-term schizophrenic patients in two London mental hospitals with the frequency of visits by family and friends.

(a) Carry out a  $\chi^2$  test for association between the two variables.

 $\star$ 

```
> data("Hospital", package="vcd")
> chisq.test(Hospital)

Pearson's Chi-squared test

data: Hospital
X-squared = 35.2, df = 4, p-value = 4.3e-07
```

(b) Use association () to compute association statistics. How would you describe the strength of association here?

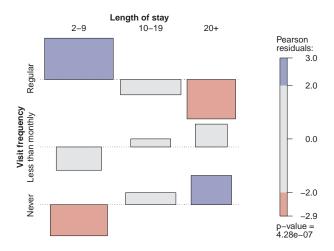
 $\star$ 

By the contingency coefficient, there is moderately strong association between the length of stay long-term schizophrenic patients and the frequency of visits by family and friends.

(c) Produce an association plot for these data, with visit frequency as the vertical variable. Describe the pattern of the relation you see here.

 $\star$ 

> assoc(Hospital, shade=TRUE)



(d) Both variables can be considered ordinal, so CMHtest () may be useful here. Carry out that analysis. Do any of the tests lead to different conclusions?

```
> CMHtest (Hospital)
Cochran-Mantel-Haenszel Statistics for Visit frequency by Length of stay
                 AltHypothesis Chisq Df
           Nonzero correlation
                                29.1
                                         6.74e-08
cor
rmeans Row mean scores differ
                                34.4
                                        3.40e-08
                                29.6
                                      2 3.72e-07
        Col mean scores differ
cmeans
                                34.9
                                      4 4.86e-07
           General association
general
```

The results of all four tests lead to the same conclusion that there is a significant association between the visit frequency and the length of stay. The test for non-zero correlation, trteating both variables as ordinal, has the largest ratio of  $\chi^2/df$ .

## Exercise 4.5 Continuing with the Hospital data:

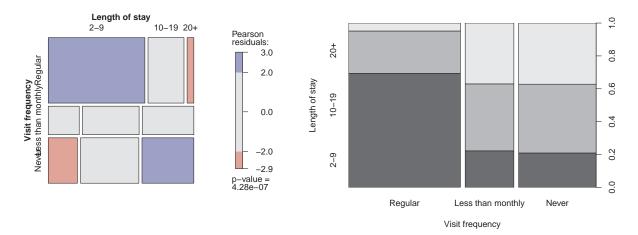
Visit frequency

ģ

(a) Try one or more of the following other functions for visualizing two-way contingency tables with this data: plot(), tile(), mosaic(), and spineplot(). [For all except spineplot(), it is useful to include the argument shade=TRUE].  $\star$ 

```
> plot(Hospital, shade=TRUE)
  > tile (Hospital, shade=TRUE)
                                                                                                                     Length of stay
                                                                                                       2-9
                                                                                                                                                20+
                                     Hospital
                                                                                                                          10-19
                                                                                                                                                                      Pearson
                                                                                                                                                                      residuals:
                                                                                         Visit frequency
Less than monthly Regular
                  Regular
                                 Less than monthly
                                                     Never
                                                                                                                                                                               3.0
                                                                                                                                                                               2.0
                                                                           2:4
                                                                           0:5
     2-9
                                                                      4-2 -2:0
Length of stay
                                                                                                                                                                               0.0
                                                                                                                                                                              -2.0
                                                                                                                                                                              -2.9
                                                                                                                                                                      p-value = 4.28e-07
```

```
> mosaic(Hospital, shade=TRUE)
> spineplot(Hospital)
```



(b) Comment on the differences among these displays for understanding the relation between visits and length of stay.

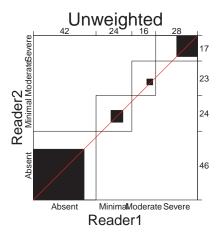


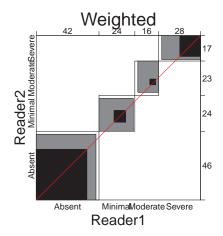
**Exercise 4.6** The two-way table *Mammograms* in vcdExtra gives ratings on the severity of diagnosis of 110 mammograms by two raters.

- (a) Assess the strength of agreement between the raters using Cohen's  $\kappa$ , both unweighted and weighted.
  - $\star$  Both unweighted and weighted  $\kappa$  indicate substantial agreement. Fleiss-Cohen weights give greater weight to the "near-misses," so gives a larger value.

- (b) Use agreementplot() for a graphical display of agreement here.
  - $\bigstar$  The default for agreementplot () shows the weighted display, so you can use weights=1 for the unweighted version.

```
> agreementplot (Mammograms, main="Unweighted", weights=1)
> agreementplot (Mammograms, main="Weighted")
```





- (c) Compare the Kappa measures with the results from assocstats(). What is a reasonable interpretation of each of these measures?

  The contingency coefficient and Cramer's V assess only association, so these could be large when
  - ★ The contingency coefficient and Cramer's V assess only association, so these could be large when there is little agreement. The values of these statistics are not directly comparable.

**Exercise 4.7** Agresti and Winner (1997) gave the data in Table 4.1 on the ratings of 160 movies by the reviewers Gene Siskel and Roger Ebert for the period from April 1995 through September 1996. The rating categories were Con ("thumbs down"), Mixed, and Pro ("thumbs up").

Table 4.1: Movie ratings by Siskel & Ebert, April 1995–September 1996. Source: Agresti and Winner (1997)

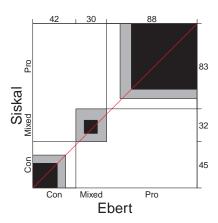
			Ebert		
		Con	Mixed	Pro	Total
	Con	24	8	13	45
Siskel	Mixed	8	13	11	32
	Pro	10	9	64	83
	Total	42	30	88	160

(a) Assess the strength of agreement between the raters using Cohen's  $\kappa$ , both unweighted and weighted.

(b) Use agreementplot() for a graphical display of agreement here.

★ The agreement plot shows that both Siskal and Ebert most commonly gave "thumbs up" ratings where they largely agreed. Their ratings differed most when one of them gave a Mixed rating.

```
> agreementplot(ratings)
```



(c) Assess the hypothesis that the ratings are *symmetric* around the main diagonal, using an appropriate  $\chi^2$  test. *Hint*: Symmetry for a square table T means that  $t_{ij} = t_{ji}$  for  $i \neq j$ . The expected frequencies under the hypothesis of symmetry are the average of the off-diagonal cells,  $E = (T + T^{\mathsf{T}})/2$ .

\*

```
> T <- (ratings + t(ratings))/2
> (Chisq <- sum((ratings - T)^2 / T))
[1] 0.5913
> df <- nrow(T) * (nrow(T)-1) /2
> pchisq(Chisq,df, lower.tail = FALSE)
[1] 0.89842
```

(d) Compare the results with the output of mcnemar.test().

 $\star$ 

```
> mcnemar.test(ratings)
McNemar's Chi-squared test
data: ratings
McNemar's chi-squared = 0.591, df = 3, p-value = 0.9
```

### **Exercise 4.8** For the *VisualAcuity* data set:

(a) Use the code shown in the text to create the table form, VA.tab.

\*

```
> data("VisualAcuity", package = "vcd")
> VA <- xtabs(Freq ~ right + left + gender, data = VisualAcuity)
> dimnames(VA)[1:2] <- list(c("high", 2, 3, "low"))
> names(dimnames(VA))[1:2] <- paste(c("Right", "Left"), "eye grade")</pre>
```

(b) Perform the CMH tests for this table.

```
*
```

```
> CMHtest (VA)

$`gender:male`
Cochran-Mantel-Haenszel Statistics for Right eye grade by Left eye grade
in stratum gender:male

AltHypothesis Chisq Df Prob
cor Nonzero correlation 1555 1 0
rmeans Row mean scores differ 1556 3 0
```

```
cmeans Col mean scores differ 1557 3 0
general General association 3303 9 0

$`gender:female`
Cochran-Mantel-Haenszel Statistics for Right eye grade by Left eye grade
in stratum gender:female

AltHypothesis Chisq Df Prob
cor Nonzero correlation 3691 1 0
rmeans Row mean scores differ 3709 3 0
cmeans Col mean scores differ 3724 3 0
general General association 8096 9 0
```

- (c) Use the woolf\_test() described in Section 4.3.2 to test whether the association between left and right eye acuity can be considered the same for men and women.
  - ★ The Woolf test gives no evidence that the association differs for men and women.

```
> woolf_test(VA)
Woolf-test on Homogeneity of Odds Ratios (no 3-Way assoc.)
data: VA
X-squared = 0.0892, df = 1, p-value = 0.77
```

**Exercise 4.9** The graph in Figure 4.23 may be misleading, in that it doesn't take into account of the differing capacities of the 18 life boats on the *Titanic*, given in the variable cap in the *Lifeboats* data.

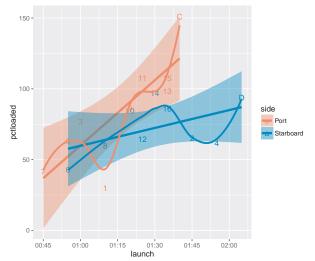
(a) Calculate a new variable, pctloaded, as the percentage loaded relative to the boat capacity.

 $\star$ 

```
> Lifeboats$pctloaded <- with(Lifeboats, 100*total/cap)</pre>
```

(b) Produce a plot similar to Figure 4.23, showing the changes over time in this measure.

★ Boats on the port side began loading earlier, but were initally less than half full. Boats launched from the starbord side were more consistent over time. With percent loaded (0–100) as the response, a linear model is only a rough approximation.



Test a model allowing different slopes and intercepts for port and starboard sides:

```
> summary(lm(pctloaded ~ side * as.numeric(launch), data=Lifeboats))
```

#### **Chapter 5** Mosaic Displays for n-Way Tables

Exercise 5.1 The data set criminal in the package logmult (Bouchet-Valat, 2015) gives the  $4 \times 5$  table below of the number of men aged 15–19 charged with a criminal case for whom charges were dropped in Denmark from 1955-1958.

```
> data("criminal", package = "logmult")
> criminal
Year
  1955 141 285 320 441 427
  1956 144 292 342 441 396
  1957 196 380 424 462 427
 1958 212 424 399 442 430
```

- (a) Use loglm() to test whether there is an association between Year and Age. Is there evidence that dropping of charges in relation to age changed over the years recorded here?
  - $\bigstar$  There is a significant association between Year and Age, so the row profiles of proportions differ over year.

```
> loglm(~Year + Age, data=criminal)
Call:
loglm(formula = ~Year + Age, data = criminal)
Statistics:
Likelihood Ratio 38.245 12 0.00014004
                 38.410 12 0.00013155
```

- (b) Use mosaic() with the option shade=TRUE to display the pattern of signs and magnitudes of the residuals. Compare this with the result of mosaic () using "Friendly shading," from the option gp=shading\_Friendly. Describe verbally what you see in each regarding the pattern of association in this table.
  - ★ It is helpful here to display all the residual contributions to association in the mosaic display using labeling=labeling\_residuals.

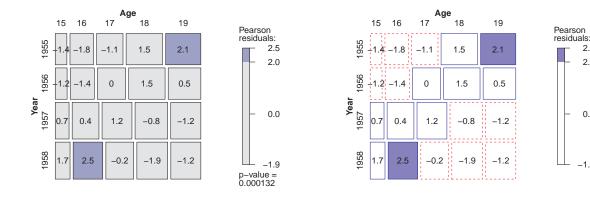
```
> mosaic(criminal, shade=TRUE,
         labeling=labeling_residuals, suppress=0)
 mosaic (criminal, gp=shading_Friendly,
         labeling=labeling_residuals, suppress=0)
```

2.5

20

0.0

-1.9



Although only two residuals exceed the default  $|r_{ij}| > 2$  threshold for shading, there is clearly a systematic association between year and age shown by the signs of the residuals.

The Friendly shading option here gives a better picture of the pattern of associations, showing positive and negative residuals in the diagonally opposite corners of the plot. See Exercise 6.2 for further analysis of this data.

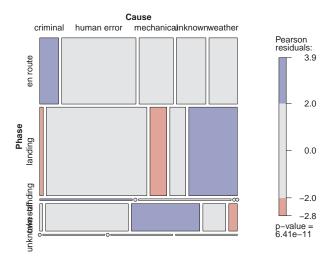
Exercise 5.2 The data set AirCrash in vcdExtra gives a database of all crashes of commercial airplanes between 1993-2015, classified by Phase of the flight and Cause of the crash. How can you best show is the nature of the association between these variables in a mosaic plot? Start by making a frequency table, aircrash.tab:

```
> data("AirCrash", package = "vcdExtra")
> aircrash.tab <- xtabs(~ Phase + Cause, data = AirCrash)</pre>
```

(a) Make a default mosaic display of the data with shade=TRUE and interpret the pattern of the high-frequency cells.



mosaic(aircrash.tab, shade=TRUE)



Four cells stand out as having greater than expected frequencies, if Phase and Cause were independent. Both take-off and en-route are positively associated with criminal activities. Crashes in landing are more associated with weather. It is difficult to interpret the unknown cells.

- (b) The default plot has overlapping labels due to the uneven marginal frequencies relative to the lengths of the category labels. Experiment with some of the labeling\_args options (abbreviate, rot\_labels, etc.) to see if you can make the plot more readable. Hint: a variety of these are illustrated in Section 4.1 of vignette("strucplot")
  - ★ Here are two alternatives that reduce the overplotting of labels:

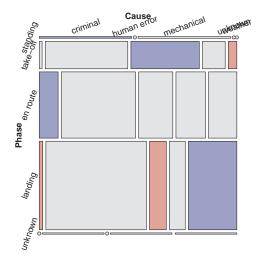
```
> mosaic(aircrash.tab, shade=TRUE, rot_labels=c(20,90,0,70), legend=FALSE)
> mosaic(aircrash.tab, shade=TRUE, alternate_labels=TRUE, legend=FALSE)
```





- (c) The levels of Phase and Cause are ordered alphabetically (because they are factors). Experiment with other orderings of the rows/columns to make interpretation clearer, e.g., ordering Phase temporally or ordering both factors by their marginal frequency.
  - ★ Ordering by Phase is slightly easier to interpret. Ordering both variables by marginal frequencies is also slightly better than the default, except that it leads to more overplotting of the labels.

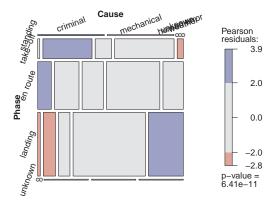
```
> # reorder Phase temporarally
> roworder <- c(3, 4, 1, 2, 5)
> mosaic(aircrash.tab[roworder,], shade=TRUE, rot_labels=c(20,90,0,70), legend=FALSE)
> # marginal frequencies
> roworder <- order(rowSums(aircrash.tab))
> colorder <- order(colSums(aircrash.tab))
> mosaic(aircrash.tab[roworder, colorder], shade=TRUE, rot_labels=c(20,90,0,70), legend=FALSE)
```





The best general approach, as was illustrated in Figure 1.10, uses *effect ordering* to order the factors according to their associations. One easy method for this (Friendly and Kwan, 2003) is to order the factor levels according to their scores on the first dimension of a correspondence analysis solution as illustrated below. This maximizes an opposite corner pattern of the residuals.

```
0.051548 27.5 93.1 *******
0.012340 6.6 99.7 **
0.000562 0.3 100.0
 Total: 0.187452 100.0
> # reorder by CA coordinates on Dim 1
> revorder <- order(aircrash.ca$rowcoord[,"Dim1"])
> colorder <- order(aircrash.ca$colcoord[,"Dim1"])</pre>
> aircrash.tab[roworder, colorder]
Phase
            criminal mechanical unknown human error weather
                        2 0
24 8
  standing
  take-off
                                                          29
  en route
                                  29
                                                          63
                                            18
                                                         114
  landing
                                  0
                                                           1
                                            1
  unknown
> mosaic(aircrash.tab[roworder, colorder], shade=TRUE, rot_labels=c(20,90,0,70))
```



**Exercise 5.3** The Lahman package contains comprehensive data on baseball statistics for Major League Baseball from 1871 through 2012. For all players, the *Master* table records the handedness of players, in terms of throwing (L, R) and batting (B, L, R), where B indicates "both." The table below was generated using the following code:

```
> library(Lahman)
> data("Master", package = "Lahman")
> basehands <- with(Master, table(throws, bats))</pre>
```

	Bats		
Throws	В	L	R
L	177	2640	527
R	924	1962	10442

- Use the code above, or else enter these data into a frequency table in R.
  - ★ These notes use a later version of the Lahman package (v. 4.0-1) with the code above, so the numbers used in the plots don't correspond to those in the table. The current version of the table is shown below.

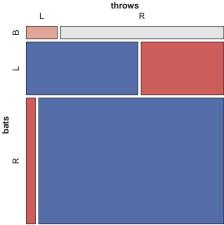
```
> bats
throws B L R
L 182 2742 550
R 956 2031 10814
```

• Construct mosaic displays showing the relation of batting and throwing handedness, split first by batting and then by throwing.

 $\bigstar$  By default, a two-way contingency table is split first by the row variable, then by the column variable. So, to split the other way, you can use t () on the table argument.

```
> mosaic(basehands, shade=TRUE, labeling=labeling_residuals(), legend=FALSE)
> mosaic(t(basehands), direction=c("h", "v"), shade=TRUE, legend=FALSE)
```





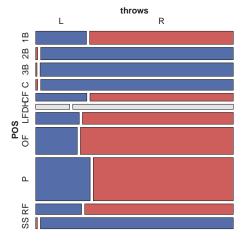
- From these displays, what can be said about players who throw with their left or right hands in terms of their batting handedness?
  - ★ Players who throw with their left or right hands are most likely to bat in the same way. From the values of the cell residuals, left handers are more likely to be uni-handers than righties.

Exercise 5.4 \* A related analysis concerns differences in throwing handedness among baseball players according to the fielding position they play. The following code calculates such a frequency table.

```
> library(Lahman)
> MasterFielding <- data.frame(merge(Master, Fielding, by = "playerID"))
> throwPOS <- with(MasterFielding, table(POS, throws))</pre>
```

- (a) Make a mosaic display of throwing hand vs. fielding position.
  - ★ There is clearly a very strong association between throwing hand and fielding position. A peculiarity of the data is that designated hitters (DH) do not play a fielding position, but instead fill in for the pitcher in the batting order, so throwing hand is not really relevant here. This position might arguably be deleted from this analysis.

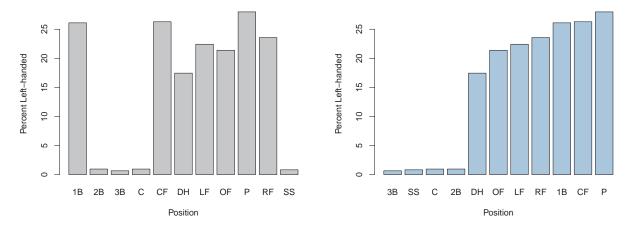
> mosaic(throwPOS, shade=TRUE, legend=FALSE)



- (b) Calculate the percentage of players throwing left-handed by position. Make a sensible graph of this data.
  - ★ A barplot is simple and reasonable here. However, the levels of fielding position are ordered alphabetically, which makes interpretation harder. Sorting by pctLeft is better.

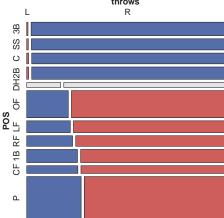
```
> pctLeft <- 100 * throwPOS[,1] / rowSums(throwPOS)
> pctLeft

   1B   2B   3B   C   CF   DH   LF   OF   P   RF   SS
26.100   0.955   0.650   0.952  26.284  17.449  22.419  21.385  27.984  23.571   0.818
> ord <- order(pctLeft)
> barplot(pctLeft, xlab="Position", ylab="Percent Left-handed")
> barplot(pctLeft[ord], xlab="Position", ylab="Percent Left-handed", col="lightblue")
```



- (c) Re-do the mosaic display with the positions sorted by percentage of left-handers.
  - > mosaic(throwPOS[ord,], shade=TRUE, legend=FALSE)

    throws
    L R



- (d) Is there anything you can say about positions that have very few left-handed players?
  - $\bigstar$  All infield positions except for 1<sup>st</sup> base have a very small percentage of players who throw left-handed. Given the marginal distributions of handeness and position, outfielders, pitchers and 1<sup>st</sup> basemen are more likely to throw left-handed than if these variables were independent.

**Exercise 5.5** For the Bartlett data described in Example 5.12, fit the model of no three-way association,  $H_4$  in Table 5.2.

(a) Summarize the goodness of fit for this model, and compare to simpler models that omit one or more of the two-way terms.



(b) Use a mosaic-like display to show the lack of fit for this model.

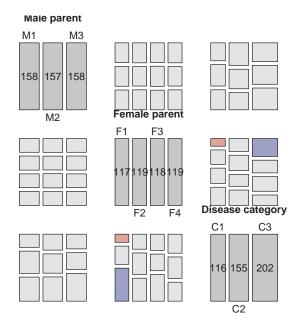


**Exercise 5.6** Red core disease, caused by a fungus, is not something you want if you are a strawberry. The data set jansen.strawberry from the agridat (Wright, 2015) package gives a frequency data frame of counts of damage from this fungus from a field experiment reported by Jansen (1990). See the help file for details. The following lines create a  $3 \times 4 \times 3$  table of crossings of 3 male parents with 4 (different) female parents, recording the number of plants in four blocks of 9 or 10 plants each showing red core disease in three ordered categories, C1, C2, or C3.

```
> data("jansen.strawberry", package = "agridat")
> levels(dat$male) <- paste0("M", 1:3)
> levels(dat$female) <- paste0("F", 1:4)</pre>
  jansen.tab <- xtabs(count ~ male + female + category, data = dat)</pre>
> names (dimnames (jansen.tab)) <- c("Male parent", "Female parent",
                                   "Disease category")
> ftable(jansen.tab)
                          Disease category C1 C2 C3
Male parent Female parent
                                            6 13 20
M1
            F1
                                           8 15 17
13 10 16
            F4
                                            8 21 11
M2
            F1
                                            5 13 21
                                           9 16 14
                                           16 9 15
                                           12 13 14
            F4
МЗ
                                            5 10 24
            F2
                                           13 12 15
                                            3 14 22
```

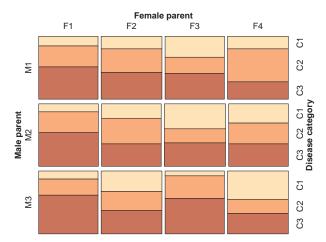
- (a) Use pairs (jansen.tab, shade=TRUE) to display the pairwise associations among the three variables. Describe how disease category appears to vary with male and female parent. Why is there no apparent association between male and female parent?
  - ★ This was a designed experiment, with male and female parents completely crossed to create 12 populations. Disease categories seem to be associated with female parents, with more serious disease (C3) more prevalent in parent F4.

```
> library(vcd)
> pairs(jansen.tab, shade=TRUE)
```

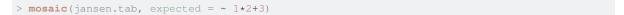


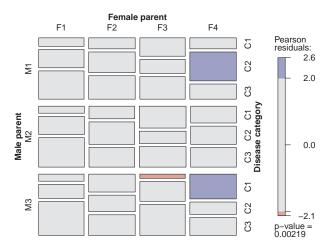
(b) As illustrated in Figure 5.6, use mosaic() to prepare a 3-way mosaic plot with the tiles colored in increasing shades of some color according to disease category. Describe the pattern of category C3 in relation to male and female parent. (Hint: the highlighting arguments are useful here.)

```
> cols <- c("moccasin", "lightsalmon1", "indianred")
> mosaic(jansen.tab, highlighting=3, highlighting_fill=cols)
```



(c) With category as the response variable, the minimal model for association is [MF][C], or  $\sim 1 \pm 2 + 3$ . Fit this model using loglm() and display the residuals from this model with mosaic(). Describe the pattern of lack of fit of this model.





**Exercise 5.7** The data set caith in MASS (Ripley, 2015) gives another classic  $4 \times 5$  table tabulating hair color and eye color, this for people in Caithness, Scotland, originally from Fisher (1940). The data is stored as a data frame of cell frequencies, whose rows are eye colors and whose columns are hair colors.

```
> data("caith", package = "MASS")
> caith
        fair red medium dark black
blue
         326 38
688 116
                     241 110
584 188
                           188
light
                                    4
medium
         343 84
                      909
                           412
                                   26
dark
          98
              48
                      403
                           681
                                   85
```

(a) The loglm() and mosaic() functions don't understand data in this format, so use Caith <- as.matrix(caith) to convert to array form. Examine the result, and use names(dimnames(Caith)) <-c() to assign appropriate names to the row and column dimensions.

```
> Caith <- as.matrix(caith)
> dimnames(Caith)

[[1]]
[1] "blue" "light" "medium" "dark"

[[2]]
[1] "fair" "red" "medium" "dark" "black"
> names(dimnames(Caith)) <- c("Eye", "Hair")</pre>
```

(b) Fit the model of independence to the resulting matrix using loglm().

(c) Calculate and display the residuals for this model.

```
> residuals(caith.mod)
```

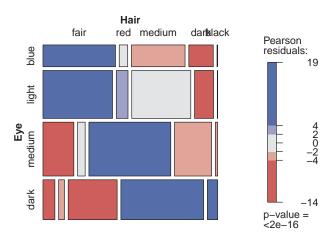
```
Hair

Eye fair red medium dark black
blue 8.63 -0.0193 -2.67 -6.00 -3.94
light 11.60 3.3125 -1.73 -12.19 -6.63
medium -6.56 -1.0691 7.40 -2.19 -2.20
dark -16.19 -2.7683 -5.41 16.28 8.46
```

(d) Create a mosaic display for this data.

\*

```
> mosaic(Caith, shade=TRUE)
```



Exercise 5.8 The HairEyePlace data in VcdExtra gives similar data on hair color and eye color, for both Caithness and Aberdeen as a  $4 \times 5 \times 2$  table.

(a) Prepare separate mosaic displays, one for each of Caithness and Aberdeen. Comment on any difference in the pattern of residuals.

 $\star$ 

(b) Construct conditional mosaic plots, using the formula ~ Hair + Eye | Place and both mosaic() and cotabplot(). It is probably more useful here to suppress the legend in these plots. Comment on the difference in what is shown in the two displays.



Exercise 5.9 Bertin (1983, pp. 30–31) used a 4-way table of frequencies of traffic accident victims in France in 1958 to illustrate his scheme for classifying data sets by numerous variables, each of which could have various types and could be assigned to various visual attributes. His data are contained in Accident in vcdExtra, a frequency data frame representing his  $5 \times 2 \times 4 \times 2$  table of the variables age, result (died or injured), mode of transportation, and gender.

```
> data("Accident", package = "vcdExtra")
> str(Accident, vec.len=2)

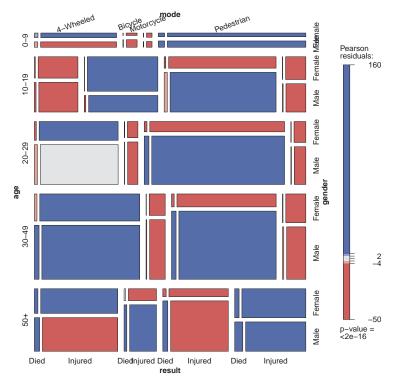
'data.frame': 80 obs. of 5 variables:
$ age : Ord.factor w/ 5 levels "0-9"<"10-19"<...: 5 5 5 5 5 ...
$ result: Factor w/ 2 levels "Died", "Injured": 1 1 1 1 1 ...
$ mode : Factor w/ 4 levels "4-Wheeled", "Bicycle",...: 4 4 2 2 3 ...
$ gender: Factor w/ 2 levels "Female", "Male": 2 1 2 1 2 ...
$ Freq : int 704 378 396 56 742 ...</pre>
```

(a) Use loglm() to fit the model of mutual independence, Freq ~ age+mode+gender+result to this data set.
 ★ You can use loglm() directly on the frequency data frame, with Freq as the response:

Or, convert to an array first with xtabs ()

(b) Use mosaic() to produce an interpretable mosaic plot of the associations among all variables under the model of mutual independence. Try different orders of the variables in the mosaic. (*Hint*: the abbreviate component of the labeling\_args argument to mosaic() will be useful to avoid some overlap of the category labels.) ★ In this data set, mode is arguably an ordered factor, and better results will come from reordering its levels, from Pedestrian to 4-Wheeled vehicle. The order of variables given in xtabs() gives a reasonable result. The label overlap can be avoided by rotating the labels for mode.

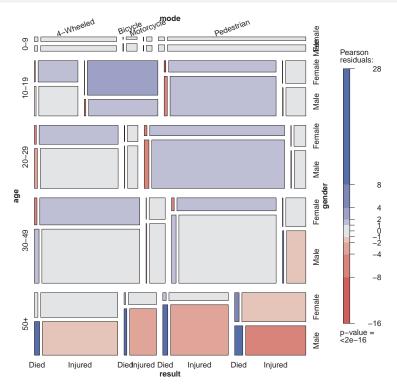
```
> Accident$mode <- ordered(Accident$mode,
+ levels=levels(Accident$mode)[c(4,2,3,1)])
> mosaic(accident.tab, shade=TRUE, rot_labels = c(20, 90, 00, 90))
```



- (c) Treat result ("Died" vs. "Injured") as the response variable, and fit the model
   Freq ~ age\*mode\*gender + result that asserts independence of result from all others jointly.
   ★ This fits much better than the mutual independence model, but still has a terrible fit. There still remain important associations between result and the other variables
  - > loglm (Freq ~ age \* mode \* gender + result, data = Accident)

- (d) Construct a mosaic display for the residual associations in this model. Which combinations of the predictor factors are more likely to result in death?
  - ★ The largest postive residuals appear in the 50+ age group, where males are more likely to have died, regardless of mode. It can also be seen that in the 30-49 age group, more males die in bicycle and motorcycle accidents. Other permutations of the table variables or other displays like doubledecker plots can highlight other features.

```
> mosaic(accident.tab, expected = ~age * mode * gender + result,
+ shade=TRUE, rot_labels = c(20, 90, 00, 90),
+ gp_args=list(interpolate=c(1,2,4,8)))
```



Exercise 5.10 The data set Vietnam in vcdExtra gives a  $2 \times 5 \times 4$  contingency table in frequency form reflecting a survey of student opinion on the Vietnam War at the University of North Carolina in May 1967. The table variables are sex, year in school, and response, which has categories: (A) Defeat North Vietnam by widespread bombing and land invasion; (B) Maintain the present policy; (C) De-escalate military activity, stop bombing and begin negotiations; (D) Withdraw military forces immediately. How does the chosen response vary with sex and year?

```
> data("Vietnam", package = "vcdExtra")
> str(Vietnam)

'data.frame': 40 obs. of 4 variables:
$ sex : Factor w/ 2 levels "Female", "Male": 1 1 1 1 1 1 1 1 1 1 1 1 1 ...
$ year : int 1 1 1 1 2 2 2 2 3 3 ...
$ response: Factor w/ 4 levels "A", "B", "C", "D": 1 2 3 4 1 2 3 4 1 2 ...
$ Freq : int 13 19 40 5 5 9 33 3 22 29 ...
```

(a) With response (R) as the outcome variable and year (Y) and sex (S) as predictors, the minimal baseline loglinear model is the model of joint independence, [R][YS]. Fit this model, and display it in a mosaic plot.

\*

(b) Construct conditional mosaic plots of the response versus year separately for males and females. Describe the associations seen here.

(c) Follow the methods shown in Example 5.10 to fit separate models of independence for the levels of sex, and the model of conditional independence,  $R \perp Y \mid S$ . Verify that the decomposition of  $G^2$  in Eqn. (5.6) holds for these models.

(d) Construct a useful 3-way mosaic plot of the data for the model of conditional independence.

 $\star$ 

## **Exercise 5.11** Consider the models for 4-way tables shown in Table 5.3.

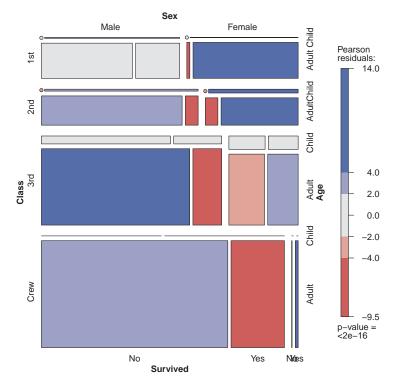
- (a) For each model, give an independence interpretation. For example, the model of mutual independence corresponds to  $A \perp B \perp C \perp D$ .
  - ★ The basic idea of the notation is that terms in separate []s are said to be independent under a given model. Variables within a [] term are allowed to be associated
    - mutual: [A] [B] [C] [D]  $\leftrightarrow A \perp B \perp C \perp D$
    - joint: [ABC] [D]  $\leftrightarrow$   $(ABC) \perp D$
    - conditional: [AD] [BD] [CD]  $\leftrightarrow$   $(AD) \perp (BD) \perp (CD)$
    - markov (order 1): [AB] [BC] [CD]  $\leftrightarrow$   $(AB) \perp (BC) \perp (CD)$
    - markov (order 2): [ABC] [BCD]  $\leftrightarrow$  (ABC)  $\perp$  (BCD)
    - · saturated: no independence relationship
- (b) Use the functions shown in the table together with loglin2formula() to print the corresponding model formulas for each.
  ★ The model generating functions, mutual(), joint(), etc. provide a simple way to specify loglin
  - ear models for loglm() and mosaic().

```
> loglin2formula(mutual(4, factors=LETTERS[1:4]))
\sim A + B + C + D
> loglin2formula(joint(4, factors=LETTERS[1:4]))
~A:B:C + D
> loglin2formula(joint(4, factors=LETTERS[1:4], with=1))
~B:C:D + A
> loglin2formula(conditional(4, factors=LETTERS[1:4]))
~A:D + B:D + C:D
> loglin2formula(conditional(4, factors=LETTERS[1:4], with=1))
~B:A + C:A + D:A
> loglin2formula(markov(4, factors=LETTERS[1:4]))
~A:B + B:C + C:D
> loglin2formula(markov(4, factors=LETTERS[1:4], order=2))
~A:B:C + B:C:D
> loglin2formula(saturated(4, factors=LETTERS[1:4]))
~A:B:C:D
```

Exercise 5.12 The dataset Titanic classifies the 2,201 pasengers and crew of the Titanic by Class (1st, 2nd, 3rd, Crew), Sex, Age, and Survived. Treating Survived as the response variable,

(a) Fit and display a mosaic plot for the baseline model of joint independence, [CGA][S]. Describe the remaining pattern of associations.

```
> # what is the formula for the joint independence here?
> form1 <- loglin2formula(joint(4, factors=names(dimnames(Titanic))))</pre>
```



- (b) Do the same for a "main effects" model that allows two-way associations between each of C, G, and A with S.
  - $\star$
- (c) What three-way association term should be added to this model to allow for greater survival among women and children? Does this give an acceptable fit?
  - $\star$
- (d) Test and display models that allow additional three-way associations until you obtain a reasonable fit.
  - $\star$

# **Chapter 6** Correspondence Analysis

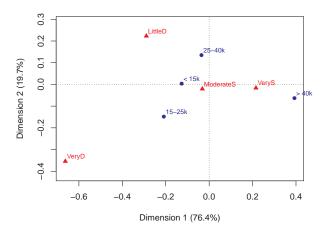
These solutions use an updated version of the ca package, v. 0.64 or greater. In particular, MCA plots are now simpler, using a new mcaplot() function, and lines can be added to MCA plots using multilines().

Exercise 6.1 The JobSat data in vcdExtra gives a 4 × 4 table recording job satisfaction in relation to income.

- (a) Carry out a simple correspondence analysis on this table. How much of the inertia is accounted for by a one-dimensional solution? How much by a two-dimensional solution?
  - ★ The 1D solution accounts for 76.4

(b) Plot the 2D CA solution. To what extent can you consider the association between job satisfaction and income "explained" by the ordinal nature of these variables?

```
> plot(jobsat.ca)
```



Job satisfaction is ordered as expected by its ordinal levels along Dimension 1. The levels of income in this plot do not appear to be ordered according to the quantitative levels they represent.

**Exercise 6.2** Refer to Exercise 5.1 in Chapter 5. Carry out a simple correspondence analysis on the  $4 \times 5$  table *criminal* from the logmult package.

(a) What percentages of the Pearson  $\chi^2$  for association are explained by the various dimensions?

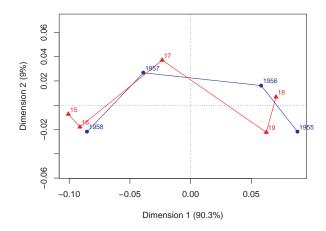
```
> data("criminal", package = "logmult")
> criminal.ca <- ca(criminal)
>  # just show the scree plot
> summary(criminal.ca, rows=FALSE, columns=FALSE)
Principal inertias (eigenvalues):
```

```
dim value % cum% scree plot
1 0.004939 90.3 90.3 **************
2 0.000491 9.0 99.3 **
3 3.8e-050 0.7 100.0
------
Total: 0.005468 100.0
```

(b) Plot the 2D correspondence analysis solution. Describe the pattern of association between year and age.

```
*
```

```
> plot(criminal.ca, lines=TRUE)
```



The category points for both year and age vary systematically over Dimension 1. There were more younger men in later years, and more older in earlier years.

**Exercise 6.3** Refer to Exercise 5.2 for a description of the AirCrash data from the vcdExtra package. Carry out a simple correspondence analysis on the  $5 \times 5$  table of Phase of the flight and Cause of the crash.

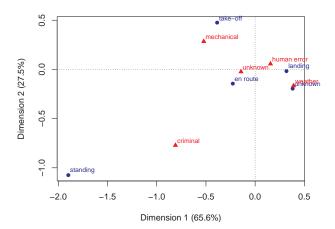
(a) What percentages of the Pearson  $\chi^2$  for association are explained by the various dimensions?  $\bigstar$  aircrash.tab was calculated in Exercise 5.2.

```
> aircrash.tab
Phase
           criminal human error mechanical unknown weather
  en route
                              63
                                          29
                                                  25
                 16
  landing
                  4
                             114
                                          19
                                                   18
                                                           55
  standing
                              2.9
                                          2.4
                                                    8
  take-off
                   0
  unknown
> aircrash.ca <- ca(aircrash.tab)</pre>
       # just show the scree
> summary(aircrash.ca, rows=FALSE, columns=FALSE)
Principal inertias (eigenvalues):
                        C11m%
 dim
        value
                               scree plot
                  65.6 65.6
        0.123002
                               *******
                  27.5 93.1
6.6 99.7
        0.051548
        0.012340
                               * *
                    0.3 100.0
        0.000562
Total: 0.187452 100.0
```

(b) Plot the 2D correspondence analysis solution. Describe the pattern of association between phase and cause. How would you interpret the dimensions?

```
\star
```

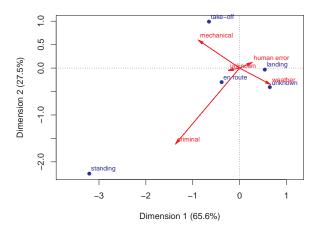
```
> plot (aircrash.ca)
```



Dimension 1 largely distinguishes standing from landing, where the main causes are human error and weather problems. Dimension 2 largely distinguishes standing from take-off, where mechanical problems are more prevalent.

(c) The default plot method uses map="symmetric" with points for both rows and columns. Try using map="symbiplot" with vectors (arrows=) for either rows or columns. (Read help(plot.ca) for a description of these options.)

```
★
> plot(aircrash.ca, map="symbiplot", arrows=c(FALSE, TRUE))
```



**Exercise 6.4** The data set *caith* in MASS gives a classic table tabulating hair color and eye color of people in Caithness, Scotland, originally from Fisher (1940).

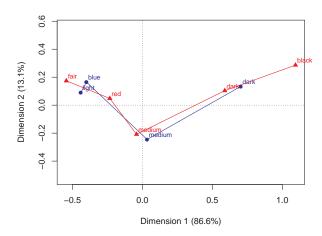
(a) Carry out a simple correspondence analysis on this table. How many dimensions seem necessary to account for most of the association in the table?

One could argue that a 1D solution is adequate here. The 2D solution is essentially complete.

(b) Plot the 2D solution. The interpretation of the first dimension should be obvious; is there any interpretation for the second dimension?

```
\star
```

```
> plot(caith.ca, lines=TRUE)
```



Dimension 1 is the obvious light–dark dimension for hair color and eye color (but not that blue and light eyes have quite similar positions). Dimension 2 could be called "extremes vs. middle", but in CA results with largely 1D association, this effect is common, often called the "horseshoe" or "arch" effect.

**Exercise 6.5** The same data, plus a similar table for Aberdeen, are given as a three-way table as *HairEyePlace* in vcdExtra.

(a) Carry out a similar correspondence analysis to the last exercise for the data from Aberdeen. Comment on any differences in the placement of the category points.



(b) Analyze the three-way table, stacked to code hair color and place interactively, i.e., for the loglinear model [Hair Place][ Eye]. What does this show?

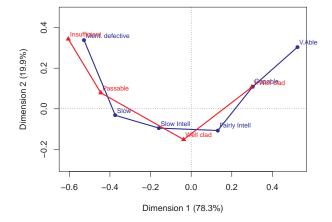


**Exercise 6.6** The data set Gilby in vcdExtra gives a classic (but now politically incorrect)  $6 \times 4$  table of English schoolboys classified according to their clothing and their teacher's rating of "dullness" (lack of intelligence).

(a) Compute and plot a correspondence analysis for this data. Write a brief description and interpretation of these results.

```
*
```

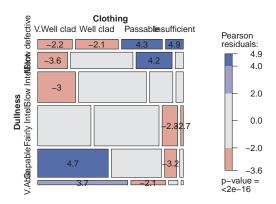
```
name
             mass
                   qlt
                          inr
                                      cor ctr
    Mntd
                         292
                                      707
                                           264
                                                   337
                                                       289 426
                   943
                         187
                                      936
                                          223
                                                   -32
2 3 4
    Slow
                                            75
                                                   -95
    SlwI
              236
                   938
                          85
                                 -159
                                      691
                                                       247
                                                            106
    FrlI
              310
                          88
                                 131
                                      601
                                            67
                                                  -107
                                                       398
                                                            176
5
                                                   109
                                                       115
             217
                  1000
                         223
                                 304 885 253
                                                            129
    Cpbl
                         125
                                          118
               34
                   990
                                 524
                                      740
                                                   304
                                                       250
                                                            157
Columns:
                                                   k=2 cor ctr
107 113 208
            mass
369
                                  k=1
    name
                   qlt
                          inr
                                      cor ctr
                                 299
1 2
    VWll
                         366
                                      886 414
                         106
                                                  -151 934 496
             435
                   989
                                       56
                                             8
    Wllc
3
                                 -445 946 383
    Pssb
             154
                   976
                         317
                                                    79
                                                             47
                                                   344 233 248
    Insf
               42
                   955
                         212
                                -605 722 195
```



The association between clothing and dullness is largely one dimensional, but there is a moderately large horseshoe effect. On Dimension 1, the categories of both variables are approximately equally spaced.

(b) Make an analogous mosaic plot of this table. Interpret this in relation to the correspondence analysis plot.

> mosaic(Gilby, shade=TRUE, labeling=labeling\_residuals)



The mosaic nearly shows the opposite corner pattern associated with a unidimensional association of two ordered variables, but the largest residuals are not systematically confined to the diagonally opposite cells. The mosaic shows the marginal frequencies of dullness and the cell frequencies by the

area of the tiles, while this information is not available in the CA plot.

**Exercise 6.7** For the mental health data analyzed in Example 6.2, construct a shaded sieve diagram and mosaic plot. Compare these with the correspondence analysis plot shown in Figure 6.2. What features of the data and the association between SES and mental health status are shown in each?

### \*

Exercise 6.8 Simulated data are often useful to help understand the connections between data, analysis methods, and associated graphic displays. Section 6.3.1 illustrated interactive coding in R, using a simulated 4-way table of counts of pets, classified by age, color, and sex, but with no associations because the counts had a constant Poisson mean,  $\lambda = 15$ .

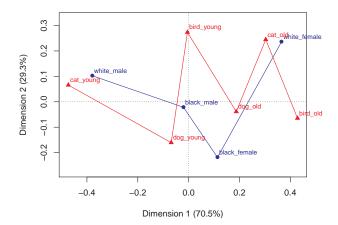
- (a) Re-do this example, but in the call to rpois(), specify a non-negative vector of Poisson means to create some associations among the table factors.
  - ★ First, create a data.frame of the factor levels. Then you can use these variables to create the Poisson means in a way that varies across some of the category combinations to create associations. Here we try to creates associations between Pet and Color (more black dogs than other combinations) and between Age and Sex (more young male pets). These cell means are then used in the call to rpois().

```
> dim <- c(3, 2, 2, 2)
/ dlm <- c(3, 2, 2, 2)
/ factors <- expand.grid(Pet=c("dog", "cat", "bird"),
/ Age=c("young", "old"),
/ Color=c("black", "white"),
/ Sex=c("male", "female"))</pre>
> means <-
+ with (factors, {
             ifelse(Pet=="dog", 10, 0) +
ifelse((Pet=="dog" & Color=="black"), 10, -10) +
             ifelse((Age=="young" & Sex=="male"), 5, -5)
> set.seed(1234)
> tab <- array(rpois(prod(dim), means), dim=dim)
> dimnames(tab) <- list(Pet=c("dog", "cat", "bird"),</pre>
                                   Age=c("young", "old"),
Color=c("black", "white"),
Sex=c("male", "female"))
> # stack
> as.matrix(ftable(Pet + Age ~ Color + Sex, tab))
                    Pet Age
Color_Sex
                      dog_young dog_old cat_young cat_old bird_young bird_old
  black_male
                            36
                                            30
                                                         16
                                                                      6
                                                                                        16
                                 30
                                                                         4
                                                                                                       5
   black_female
                                             25
   white_male
                                 22
                                                            18
   white_female
```

(b) Use CA methods to determine if and how the structure you created in the data appears in the results.

\*

```
> library(ca)
> pets.ca <- ca(as.matrix(ftable(Pet + Age ~ Color + Sex, tab)))
> plot(pets.ca, lines=TRUE)
```



**Exercise 6.9** The TV data was analyzed using CA in Example 6.4, ignoring the variable Time. Carry out analyses of the 3-way table, reducing the number of levels of Time to three hourly intervals as shown below.

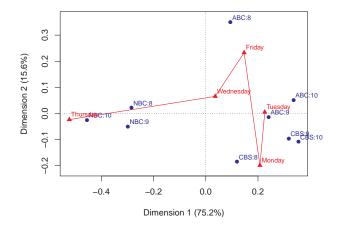
```
> data("TV", package="vcdExtra")
> # reduce number of levels of Time
> TV.df <- as.data.frame.table(TV)
> levels (TV.df$Time) <- rep(c("8", "9", "10"), c(4, 4, 3))
> TV3 <- xtabs (Freq ~ Day + Time + Network, TV.df)
> structable (Day ~ Network + Time, TV3)
                 Day Monday Tuesday Wednesday Thursday Friday
Network Time
                          536
ABC
                         1401
                                    1205
                                                              682
                                                                       907
                                   1044
                                                                       711
           10
                          910
                                                  668
                                                              349
CBS
          8
                         1167
                                                  550
                                                              680
                                                                       509
                                     646
           9
                          967
                                     959
                                                  409
                                                              385
                                                                       544
                                                                       426
           10
                          789
                                     798
                                                              270
                                                  324
NBC
                                    1090
                                                  512
                                                             1927
                          858
                                                                       823
           8
                                                                       590
                          946
                                     890
                                                  831
                                                             1858
          10
                          825
                                                  869
                                                             2101
```

(a) Use the stacking approach (Section 6.3) to perform a CA of the table with Network and Time coded interactively. You can create this using the as.matrix() method for a "structable" object.

```
> TV3S <- as.matrix(structable(Day ~ Network + Time, TV3), sep=":")
*
> TV3S.ca <-ca(TV3S)
> summary(TV3S.ca, rows=FALSE, columns=FALSE)
Principal inertias (eigenvalues):
 dim
        value
                        cum%
                               scree plot
                  75.2
        0.089629
                        75.2
 1
                               *******
                  15.6 90.8
7.5 98.3
1.7 100.0
        0.018576
                        90.8
                               ****
        0.008992
                               * *
        0.002001
 Total: 0.119198 100.0
```

- (b) What loglinear model is analyzed by this approach?
  - ★ The model is the joint independence model, [Day][Network Time], asserting that the frequencies of watching the combinations of networks in different time slots do not vary with day of the week.
- (c) Plot the 2D solution. Compare this to the CA plot of the two-way table in Figure 6.4.

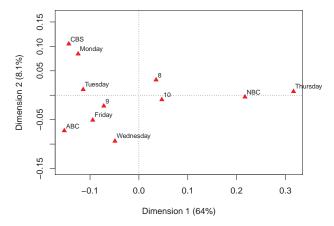
```
> plot(TV3S.ca, lines=c(FALSE, TRUE))
```



Overall, the plot is somewhat similar to that of Figure 6.4 except for the (arbitrary) reversal of Dimension 1. The category points for NBC cluster near Thursday, while those for ABC and CBS are at the other end of that dimension and Dimension 2 is related to the difference in viewership to those channels.

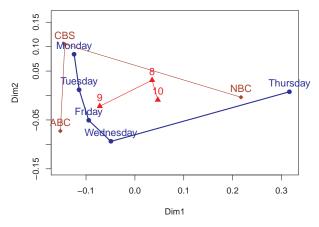
- (d) Carry out an MCA analysis using mjca() of the three-way table TV3. Plot the 2D solution, and compare this with both the CA plot and the solution for the stacked three-way table.
  - ★ The MCA analysis differs in that it includes all pairwise associations of Day, Time and Network rather than just the joint independence model analyzed by the stacking approach.

```
> TV3.mca <- mjca(TV3)
> plot(TV3.mca, collabels="level")
```



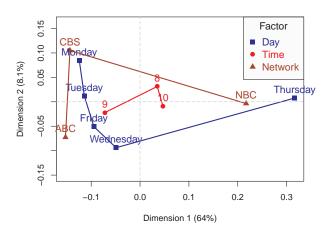
The default plot is somewhat difficult to read because the factor points s aren't distinguished by color or shape. A customized plot can be constructed as follows (as illustrated in the text, e.g., for Figure 6.10).

```
> res <- plot(TV3.mca, labels=0, pch='.', cex.lab=1.2)
> coords <- data.frame(res$cols, TV3.mca$factors)
> nlev <- rle(as.character(coords$factor))$lengths
> fact <- unique(as.character(coords$factor))
>
> cols <- c("blue", "red", "brown")
> lwd <- 2
> plot(Dim2 ~ Dim1, type='n', data=coords, asp=1)
> points(coords[,1:2], pch=rep(16:18, nlev), col=rep(cols, nlev), cex=1.2)
> text(coords[,1:2], labels=coords$level, col=rep(cols, nlev), pos=3, cex=1.2, xpd=TRUE)
> multilines(coords[, c("Dim1", "Dim2")], group=coords$factor, col=cols, lwd=lwd)
```



A similar plot can now be produced more simply using:

```
> mcaplot(TV3.mca, legend=TRUE)
```



**Exercise 6.10** Refer to the MCA analysis of the *PreSex* data in Example 6.8. Use the stacking approach to analyze the stacked table with the combinations of premarital and extramarital sex in the rows and the combinations of gender and marital status in the columns. As suggested in the exercise above, you can use as .matrix (structable ()) to create the stacked table.

```
as.matrix(structable(PremaritalSex + ExtramaritalSex ~ Gender + Marital, PreSex), sep=":")
> presexS<-
> presexS
                    PremaritalSex:ExtramaritalSex
Gender: Marital Status Yes: Yes: No No: Yes No: No
      Women:Divorced
                                  54
                                         36
                                              214
      Women:Married
                                          4
                                               322
      Men:Divorced
                           28
                                  60
                                         17
                                                68
      Men:Married
```

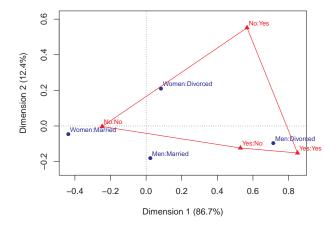
- (a) What loglinear model is analyzed by this approach? Which associations are included and which are excluded in this analysis?
  - ★ The model is that of independence between the combinations of the row variables and the column variables, i.e., [Pre Extra][Gender Marital]. The associations that remain indicate how the combinations of pre- and extra-marital sex are related to the the combinations of gender and marital status. The

- association between pre-marital sex and extra-marital sex is excluded, as is the association between gender and marital status.
- (b) Plot the 2D CA solution for this analysis. You might want to draw lines connecting some of the row points or column points to aid in interpretation.
  - ★ The 2D solution accounts for 99.1% of association between these sets of variables.

```
> presexS.ca <- ca(presexS)</pre>
> summary (presexS.ca)
Principal inertias (eigenvalues):
 dim
         value
                      오
                           cum%
                                   scree plot
                    86.7 86.7
         0.153126
         0.021929
                     12.4
                            99.1
                                    * * *
                     0.9 100.0
         0.001552
 Total: 0.176606 100.0
Rows:
    name
             mass
                    qlt
995
                          inr
                                  k=1 cor ctr
                                                    k=2 cor ctr
1
              310
                                   84
                                       138
                                             14
                                                    210 857 622
    WmnD
2
              343
                    995
                          376
                                  -437 984 427
                                                              34
    WmnM
                                                    -46 11
                    996
                                                    -97
    MnDv
              167
                          494
                                  715
                                       978
                                           558
4
                    878
                           39
    MnMr
              181
Columns:
                                                    k=2 cor ctr
-152 30 61
-124 51 122
552 486 817
             mass
58
                    qlt
979
                         inr
250
                                        cor ctr
948 274
    name
                                  850
    YsYs
              175
                                        937 320
2
                    989
                          296
                                  530
    YesN
3
                                  567
               59 1000
                          209
                                        514 124
    NoYs
              708 1000
                                 -247 1000 283
    NoNo
                          245
```

Here is one version of a plot, drawing lines connecting the pairs of Gender and Marital status.

```
> res <- plot(presexS.ca)
> # join pairs of column points
> lines(res$cols[1:2,], col="red")
> lines(res$cols[3:4,], col="red")
> lines(res$cols[c(1,3),], col="red")
> lines(res$cols[c(2,4),], col="red")
```



- (c) How does this analysis differ from the MCA analysis shown in Figure 6.10?
  - ★ The MCA analysis treats all four factors individually, analyzing all bivariate associations. The stacked approach here treats them in two sets, analyzing only the associations *between* sets.

Exercise 6.11 Refer to Exercise 5.10 for a description of the Vietnam data set in vcdExtra.

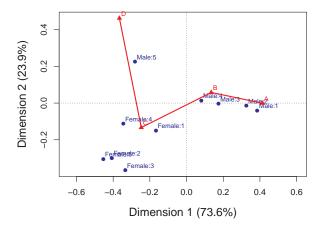
(a) Using the stacking approach, carry out a correspondence analysis corresponding to the loglinear model [R][YS], which asserts that the response is independent of the combinations of year and sex.

 $\bigstar$  Two dimensions account for 97.5% of the association between response and the combinations of year and sex.

```
> data(Vietnam, package="vcdExtra")
> vietnam.tab <- xtabs (Freq ~ sex + year + response, data=Vietnam)
> vietnam.stacked <- as.matrix(structable(response ~ sex + year, vietnam.tab), sep=":")
> vietnam.ca <- ca(vietnam.stacked)</pre>
> summary(vietnam.ca, rows=FALSE, columns=FALSE)
Principal inertias (eigenvalues):
                   응
 dim
        value
                        cum%
                               scree plot
        0.085680 73.6 73.6
0.027881 23.9 97.5
 1
                               **********
                   2.5 100.0
        0.002854
Total: 0.116415 100.0
```

(b) Construct an informative 2D plot of the solution, and interpret in terms of how the response varies with year for males and females.

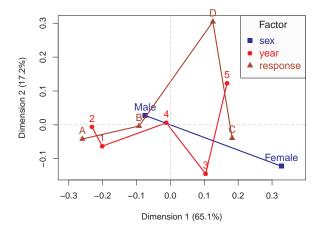
```
> plot(vietnam.ca, cex.lab=1.3, lines=c(FALSE, TRUE), lwd=2)
```



Dimension 1 corresponds to the ordering of the response categories, from the "dove" response D ("Withdraw military forces immediately") to the "hawk" response A ("Defeat North Vietnam by widespread bombing ..."). Males are ordered progressively from hawk to dove by their years in schools, with a big gap between year 4 and 5 (graduate students). Females cluster around response C ("De-escalate military activity ...") with no obvious trend over year in school.

- (c) Use mjca() to carry out an MCA on the three-way table. Make a useful plot of the solution and interpret in terms of the relationship of the response to year and sex.
  - ★ The MCA solution only accounts for 82.3% of the bivariate associations in 2D, and not much better in 3D.

```
> vietnam.mca <- mjca(vietnam.tab)</pre>
> summary (vietnam.mca, rows=FALSE, columns=FALSE)
Principal inertias (eigenvalues):
                   용
 dim
        value
                       cum%
                              scree plot
        0.028219 65.1
 1
                       65.1
                              *******
        0.007445
                  17.2
                       82.3
                              ****
        0.000380
                   0.9
                        83.2
 4
        1e-06000
                   0.0
                        83.2
 Total: 0.043317
> mcaplot(vietnam.mca, legend=TRUE)
```



The plot of the MCA solution is somewhat difficult to interpret, because neither the response categories nor year are ordered as one would expect. This turns out to be an example where MCA is not that useful, because the association of response and year is different for males and females— i.e., there is a three-way association in this table. MCA, however, only accounts for pairwise associations among the table variables.

**Exercise 6.12** Refer to Exercise 5.9 for a description of the *Accident* data set in vcdExtra. The data set is in the form of a frequency data frame, so first convert to table form.

```
> accident.tab <- xtabs(Freq ~ age + result + mode + gender, data=Accident)
```

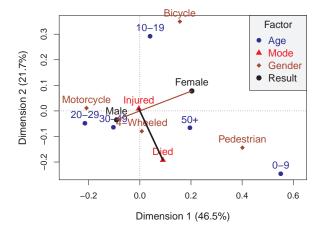
(a) Use mjca() to carry out an MCA on the four-way table accident.tab.

```
> accident.mca <- mjca(accident.tab)</pre>
> summary(accident.mca)
Principal inertias (eigenvalues):
        value
                                scree plot
        0.025429
                   46.5
                        46.5
        0.011848
                   21.7
                         68.1
        0.001889
                         71.6
                    0.9
        0.000491
                         72.5
 Total: 0.054700
Columns:
                                            k=1 cor ctr
                                                            k=2 cor ctr
                               qlt
672
                                    inr
                        mass
                name
             age:0-9
                                     107
                                            551 561 152
                                                           -246 111
                                                                     65
                         13
2
           age:10-19
                          49
                               678
                                     91
                                                            292 665 354
                                             40
                                                 1.3
                                           -215 747 102
           age:20-29
                               784
                                     8.5
                          56
                                                            -48
                                                                      11
4
           age:30-49
                          76
                               546
                                      75
                                           -103396
                                                      32
                                                            -63 149
                                                                      26
5
             age:50+
                                                            -67
                          56
                               687
                                     8.5
                                           196 616
                                                      84
                                                                 72.
                               515
515
                                                           -192 422
6
         result:Died
                          1.1
                                             90 92
                                                                      34
                                                 92
      result:Injured
                                             -4
                                                                422
                                     73
                                                            -80 228
8
      mode: 4-Wheeled |
                          81
                               230
                                              8
                                                      0
                                                                      43
                                            156 127
        mode:Bicycle
                          31
                               762
                                     98
                                                            349 635
10
     mode: Motorcycle
                          99
                               686
                                     70
                                           -209 684 170
                                                             11
11
     mode:Pedestrian
                           38
                               677
                                     100
                                            401 600
                                                    241
                                                           -144
12
       gender:Female
                               788
                                     77
                                            203 686 126
                                                             78 101
                               788
                                            -91 686
                                                            -35 101
```

The adjusted inertias indicate that a 2D solution accounts for only 68.1% of the pairwise associations. The qualities (qlt) of the factor levels indicate that the categories are only moderately well-represented in a 2D plot.

(b) Construct an informative 2D plot of the solution, and interpret in terms of how the variable result varies in relation to the other factors.

```
*
```



In the figure above, one interpretation of the dimensions is in terms of the age categories: Dimension 1 for young adults vs. old and young, Dimension 2 for teenage vs. the rest. In these terms, Dimension 1 shows associations among males, aged 20–49, riding a motorcycle or 4-wheeled vehicle and more likely to be injured, vs. females, either old or very young, as pedestrians and more likely to have died. Dimension 2 contrasts bicycle accidents involving youth aged 10–19 who are more likely to be just injured against the other categories.

**Exercise 6.13** The *UCBAdmissions* data was featured in numerous examples in Chapter 4 (e.g., Example 4.11, Example 4.15) and Chapter 5 (e.g., Example 5.14, Example 5.18).

(a) Use mjca() to carry out an MCA on the three-way table UCBAdmissions.

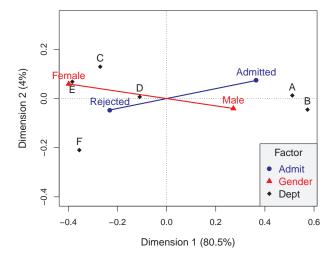


```
> ucb.mca <- mjca (UCBAdmissions)
> summary(ucb.mca)
Principal inertias (eigenvalues):
        value
                                scree plot
        0.114945
                   80.5
                        80.5
        0.005694
                   4.0
                         84.5
                   0.0
                         84.5
                         84.5
                    0.0
                         84.5
 Total: 0.142840
Columns:
                             qlt
911
                                          k=1 cor ctr
                                                           k=2 cor ctr
                       mass
                                   inr
                                                               36 123 |
36 78 |
                                          365 875 150
                                                           74
     Admit:Admitted
                        129
                                    93 |
                             911
                                    59
                        204
                                         -231 875
                                                   95
                                                           -47
     Admit:Rejected
                                    95 i
                        135
                             863
                                         -399 845 187
                                                           59
      Gender:Female |
                                                               19
                                                                   8.4
                        198
                                    6.5
                                          272 845 127
        Gender:Male
                             863
```

```
5
6
7
                                 838
                                       117
                                               512 837 156
               Dept:A
                                 829
                                       124
                                               573
                                                                 -45
                                                                           15
                            43
                                                    824 123
               Dept:B
                                                                 130 137
                                                                          199
               Dept:C
                                 731
                                       108
                                              -270
                                                    594
                                                         43
                            68
8 9
                                 832
                                       106
117
                                             -110 828
-384 787
                                                         6
55
               Dept:D
                            58
                                                                        3
                                                                            0
                                                                  69
                                                                      25
                                                                           35
                            43
               Dept:E
                                 812
                                                               -210 190 406
10
               Dept:F
                                 737
                                       116 |
                                             -355 547
```

(b) Plot the 2D MCA solution in a style similar to that shown in Figure 6.10 and Figure 6.11.

> **par** (op)



- (c) Interpret the plot. Is there some interpretation for the first dimension? What does the plot show about the relation of admission to the other factors?
  - ★ The first dimension largely corresponds to Admission, showing the overall association of Males more likely to be admitted, Females more likely to be rejected. Note that the departments, labeled A–F, were actually ordered by overall rate of admission, but this ordering does not appear along Dimension 1 in the plot.

# **Chapter 7** Logistic Regression Models

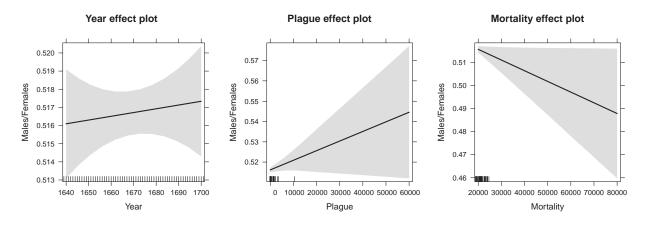
**Exercise 7.1** Arbuthnot's data on the sex ratio of births in London was examined in Example 3.1. Use a binomial logistic regression model to assess whether the proportion of male births varied with the variables Year, Plague, and Mortality in the Arbuthnot data set. Produce effect plots for the terms in this model. What do you conclude?

★ For the binomial logistic model, use cbind (Males, Females) for the response variable in the model.

```
> data(Arbuthnot, package="HistData")
> arbuth.mod <- glm(cbind(Males, Females) ~ Year + Plague + Mortality,
                    data=Arbuthnot, family=binomial)
> summary (arbuth.mod)
Call:
glm(formula = cbind(Males, Females) ~ Year + Plague + Mortality,
    family = binomial, data = Arbuthnot)
Deviance Residuals:
  Min
            1Q Median
                                   Max
       -0.996
                -0.005
                         0.850
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)
           -4.07e-02
                        3.09e-01
                                    -0.13
             8.28e-05
                        1.93e-04
                                    0.43
                                             0.668
Year
Plague
             1.91e-06
                        1.13e-06
                                    1.68
                                             0.093
                        9.26e-07
                                             0.045 *
Mortality
            -1.86e-06
                                    -2.01
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 169.74 on 81 degrees of freedom
Residual deviance: 156.31 on 78 degrees of freedom
AIC: 963.8
Number of Fisher Scoring iterations: 3
> LRstats (arbuth.mod)
Likelihood summary table:
           AIC BIC LR Chisq Df Pr(>Chisq)
arbuth.mod 964 973
                        156 78
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The effects of Year and Plague are small and non-significant. The Male/Female proportion appears to decrease with increasing Mortality.

```
> library(effects)
> arbuth.eff <- allEffects(arbuth.mod)
> plot(arbuth.eff, ylab="Males/Females", rows=1, cols=3)
```



In the plots for Plague and Mortality, it is apparent that both are extremely skewed. One alternative is to represent these as log(Plague+1) and log(Mortality) in the model. Overall, these effects are quite small, but the main effects model arbuth.mod is better than the null model.

**Exercise 7.2** For the Donner Party data in *Donner*, examine Grayson's 1990 claim that survival in the Donner Party was also mediated by the size of the family unit. This takes some care, because the family variable in the *Donner* data is a simplified grouping based on the person's name and known alliances among families from the historical record. Use the following code to compute a family.size variable from each individual's last name:

```
> data("Donner", package="vcdExtra")
> Donner$survived <-factor(Donner$survived, labels=c("no", "yes"))
> # use last name for family
> lname <-strsplit(rownames(Donner), ",")
> lname <-sapply(lname, function(x) x[[1]])
> Donner$family.size <-as.vector(table(lname)[lname])</pre>
```

(a) Choose one of the models (donner.mod4, donner.mod6) from Example 7.9 that include the interaction of age and sex and nonlinear terms in age. Fit a new model that adds a main effect of family.size. What do you conclude about Grayson's claim?

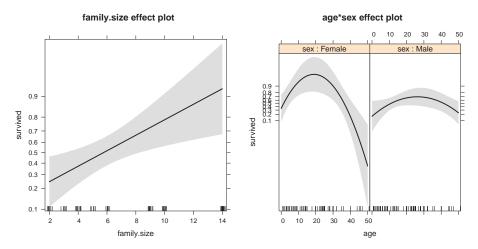
\*

Family size seems to have an effect on survival.

(b) Produce an effect plot for this model.

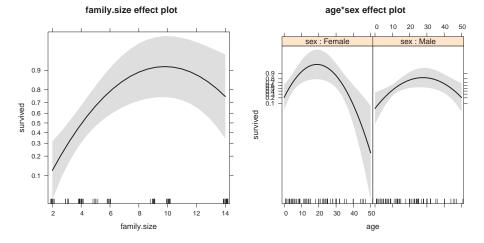
 $\star$ 

```
> library(effects)
> donner.eff4a <- allEffects(donner.mod4a, xlevels=list(age=seq(0,50,5)))
> plot(donner.eff4a, ticks=list(n=8))
```



(c) Continue, by examining whether the effect of family size can be taken as linear, or whether a nonlinear term should be added.

```
> Anova (donner.mod4b)
Analysis of Deviance Table (Type II tests)
Response: survived
                  LR Chisq Df Pr(>Chisq)
                    17.64
                               0.00015
poly(age, 2)
                     2.33
                              0.12688
poly(family.size, 2)
                    24.81
                               4.1e-06 ***
poly(age, 2):sex
                    12.25
                               0.00219 **
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> donner.eff4b <- allEffects(donner.mod4b, xlevels=list(age=seq(0,50,5)))
> plot(donner.eff4b, ticks=list(n=8))
```



Exercise 7.3 Use component+residual plots (Section 7.5.3) to examine the additive model for the ICU data given by

(a) What do you conclude about the linearity of the (partial) relationship between age and death in this model?

 $\star$ 

(b) An alternative strategy is to allow some nonlinear relation for age in the model using a quadratic (or cubic) term like poly(age, 2) (or poly(age, 3)) in the model formula. Do these models provide evidence for a nonlinear effect of age on death in the ICU?

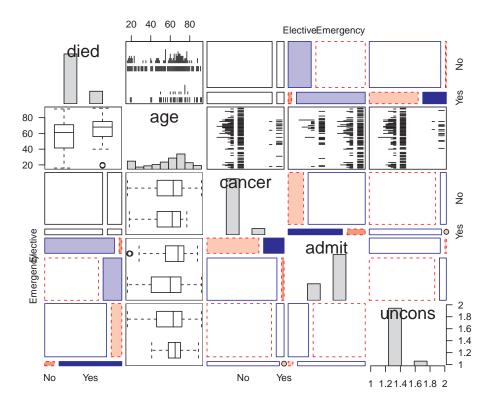


**Exercise 7.4** Explore the use of other marginal and conditional plots to display the relationships among the variables predicting death in the ICU in the model icu.glm2. For example, you might begin with a marginal gpairs () plot showing all bivariate marginal relations, something like this:

```
> library(gpairs)
> gpairs(ICU[,c("died", "age", "cancer", "admit", "uncons")],
+ diag.pars=list(fontsize=16, hist.color="lightgray"),
+ mosaic.pars=list(gp=shading_Friendly,
+ gp_args=list(interpolate=1:4)))
```

## ★ First, begin with the gpairs() plot:

```
> library(gpairs)
> gpairs(ICU[,c("died", "age", "cancer", "admit", "uncons")],
+ diag.pars=list(fontsize=16, hist.color="lightgray"),
+ mosaic.pars=list(gp=shading_Friendly,
+ gp_args=list(interpolate=1:4)))
```



Exercise 7.5 The data set Caesar in VcdExtra gives a  $3 \times 2^3$  frequency table classifying 251 women who gave birth by Caesarian section by Infection (three levels: none, Type 1, Type2) and Risk, whether Antibiotics were used, and whether the Caesarian section was Planned or not. Infection is a natural response variable. In this exercise, consider only the binary outcome of infection vs. no infection.

```
> data("Caesar", package="vcdExtra")
> Caesar.df <- as.data.frame(Caesar)
> Caesar.df$Infect <- as.numeric(Caesar.df$Infection %in%
+ c("Type 1", "Type 2"))</pre>
```

(a) Fit the main-effects logit model for the binary response Infect. Note that with the data in the form of a frequency data frame you will need to use weights=Freq in the call to glm(). (It might also be convenient to reorder the levels of the factors so that "No" is the baseline level for each.)

\*

- (b) Use summary () or car (Fox and Weisberg, 2015)::Anova () to test the terms in this model.
  - ★ By both the Wald tests from summary() and the Type II LR tests from car::Anova all three factors have significant effects on the probability of infection.

```
> library(car)
> summary(caesar.glm)
Deviance Residuals:
          1Q Median
                            3Q
  Min
-6.747 -0.44\overline{3} 0.000 3.23\overline{4} 5.420
Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.793 0.478 -1.66 0.097.
RiskYes 1.827 0.436 4.19 2.8e-05 ***
AntibioticsYes -3.001 0.459 -6.53 6.4e-11 ***
PlannedYes -0.906 0.408 -2.22 0.026 *
RiskYes
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
Null deviance: 300.85 on 16 degrees of freedom Residual deviance: 236.36 on 13 degrees of freedom
AIC: 244.4
Number of Fisher Scoring iterations: 6
> Anova (caesar.glm)
Analysis of Deviance Table (Type II tests)
Response: Infect
     LR Chisq Df Pr(>Chisq)
             20.6 1 5.8e-06 ***
56.5 1 5.7e-14 ***
5.2 1 0.022 *
Antibiotics
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- (c) Interpret the coefficients in the fitted model in terms of their effect on the odds of infection.
  - ★ From the coefficients in the model given above, Risk factors increase the log odds of infection by 1.83; treatment with Antibiotics decreases the log odds by 3.0; a planned C-section descreases the log odds of infection by 0.91.

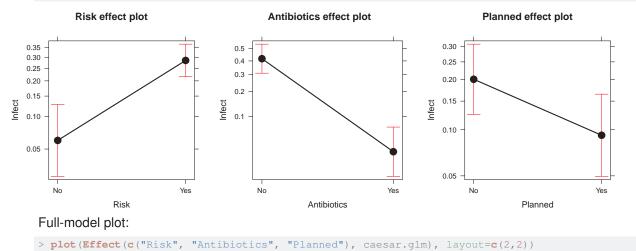
These effects can perhaps be more easily interpreted in terms of the odds ratios calculated below, that give the multiple of the odds for the Yes group compared to the No group. For example, Risk can be said to multiply the odds by 6.22; Antibiotics multiplies the odds by 0.05, or a decrease of 95%; a planned C-section multiplies the odds of infection by 0.40, a decrease of 60%.

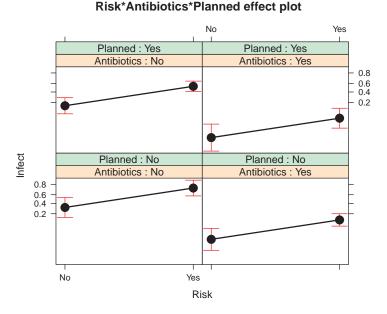
```
> exp(cbind(OddsRatio=coef(caesar.glm),
+ confint(caesar.glm)))

OddsRatio 2.5 % 97.5 %
(Intercept) 0.452263 0.170362 1.13401
RiskYes 6.215158 2.736586 15.35990
AntibioticsYes 0.049734 0.019257 0.11742
PlannedYes 0.403978 0.176257 0.88092
```

- (d) Make one or more effects plots for this model, showing separate terms, or their combinations.
  - ★ Two kinds of effect plots are useful here: allEffects(), that gives plots for each of the (main effect) terms in the model, and a full-model plot, showing predicted log odds of infection for all combinations of Risk, Antibiotics and Planned.

```
> library(effects)
> plot(allEffects(caesar.glm), rows=1, cols=3)
```

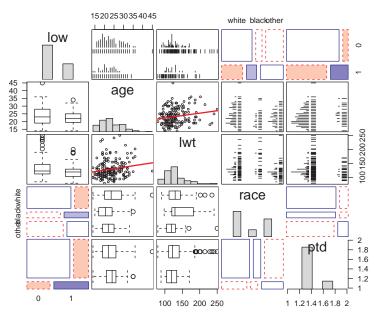




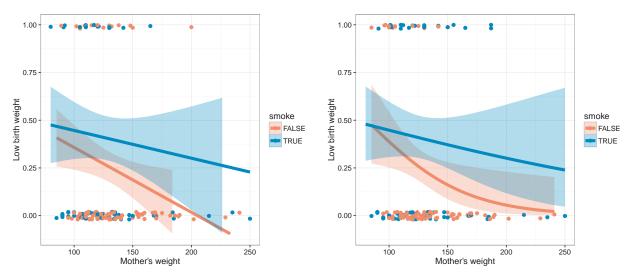
**Exercise 7.6** The data set birthwt in the MASS package gives data on 189 babies born at Baystate Medical Center, Springfield, MA during 1986. The quantitative response is bwt (birth weight in grams), and this is also recorded as low, a binary variable corresponding to bwt < 2500 (2.5 Kg). The goal is to study how this varies with the available predictor variables. The variables are all recorded as numeric, so in R it may be helpful to convert some of these into factors and possibly collapse some low frequency categories. The code below is just an example of how you might do this for some variables.

```
> data("birthwt", package="MASS")
> birthwt <- within(birthwt, {
    low <- factor(low)
+    race <- factor(race, labels = c("white", "black", "other"))
+    ptd <- factor(ptl > 0)  # premature labors
+    ftv <- factor(ftv)  # physician visits
+    levels(ftv)[-(1:2)] <- "2+"
+    smoke <- factor(smoke>0)
+    ht <- factor(ht>0)
+    ui <- factor(ui>0)
+    })
```

- (a) Make some exploratory plots showing how low birth weight varies with each of the available predictors. In some cases, it will probably be helpful to add some sort of smoothed summary curves or lines.
  - ★ There are a wide variety of plots one could make for this data set. It is not unreasonable to start with a <code>gpairs()</code> plot for an overview. The first row and column shows the relations of low birth weight to the predictors. From this we can see that low birth weight (<code>low==1</code>) is associated with lower age, lower mother's weight (<code>lwt</code>), <code>race=="black"</code>, and previous premature labors (<code>ptd</code>).



Other plots can explore particular relationships with low birth weight more thoroughly. Here we just show some example plots using ggplot2 (Wickham and Chang, 2015) for the relationship between low and mother's weight (lwt) conditioned by smoking status.



- (b) Fit several logistic regression models predicting low birth weight from these predictors, with the goal of explaining this phenomenon adequately, yet simply.
  - ★ Here, we just start with the main effects model, then eliminate non-significant terms. A more general analysis could use MASS::stepAIC(), test for non-linear relations, and the presence of interaction effects.

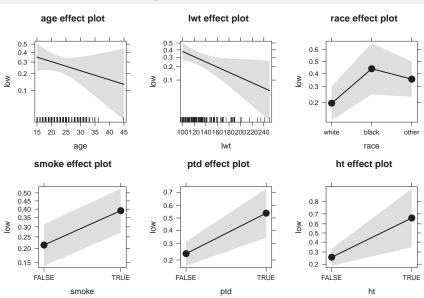
```
> # quick check on important effects
> bwt.mod0 <- glm(low ~ age + lwt + race + smoke + ptd + ht + ui + ftv,</pre>
                     data=birthwt, family = binomial)
> summary (bwt.mod0)
glm(formula = low ~ age + lwt + race + smoke + ptd + ht + ui +
ftv, family = binomial, data = birthwt)
Deviance Residuals:
         1Q Median
-0.807 -0.501
                                 3Q
   Min
                                         Max
                             0.884
-1.704
                                       2.215
Coefficients:
              Estimate Std. Error z value Pr(>|z|)
                            1.24471
0.03870
0.00708
(Intercept) 0.82302
                                                0.5085
                                         0.66
              -0.03723
                                         -0.96
age
                                                  0.3360
              -0.01565
                                        -2.21
7 w/t
                                                  0.0271
               1.19241
0.74068
                            0.53596
                                         2.22
raceblack
                                                  0.0261 *
                            0.46174
raceother
                                          1.60
                                                  0.1087
               0.75553
smokeTRUE
                             0.42502
                                          1.78
                                                  0.0755
ptdTRUE
               1.34376
                             0.48062
                                          2.80
                                                  0.0052 **
htTRUE
               1.91317
                             0.72074
                                          2.65
                                                  0.0079 **
uiTRUE
               0.68020
                             0.46434
                                          1.46
                                                  0.1430
ftv1
              -0.43638
                             0.47939
                                         -0.91
                                                  0.3627
               0.17901
                             0.45638
                                          0.39
                                                  0.6949
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
Null deviance: 234.67 on 188 degrees of freedom Residual deviance: 195.48 on 178 degrees of freedom
AIC: 217.5
Number of Fisher Scoring iterations: 4
> car::Anova(bwt.mod0)
Analysis of Deviance Table (Type II tests)
Response: low
       LR Chisq Df Pr(>Chisq)
                          0.3318
age
            0.94
            5.47
5.75
                          0.0193 *
1 wt.
                          0.0564 .
race
                          0.0737 .
smoke
            3.20
            8.11
                          0.0044 **
ptd
ht
            7.46
                          0.0063 **
```

```
2.11 1
ui
                    0.1463
          1.36
                2
                       0.5071
ftv
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> # remove NS terms
> bwt.mod1 <- update(bwt.mod0, . ~ . - ui - ftv)</pre>
> Anova (bwt.mod1)
Analysis of Deviance Table (Type II tests)
Response: low
      LR Chisq Df Pr(>Chisq)
          1.33
                       0.2487
age
lwt
          5.40
                       0.0201
          6.24
                       0.0441 *
race
smoke
          4.62
                       0.0316
          8.73
                       0.0031 **
ptd
          6.46
                       0.0111 *
ht
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
> anova(bwt.mod1, bwt.mod0, test="Chisq")
Analysis of Deviance Table
Model 1: low ~ age + lwt + race + smoke + ptd + ht
Model 2: low ~ age + lwt + race + smoke + ptd + ht + ui + ftv
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)
        181
                    199
                    196
```

(c) Use some graphical displays to convey your findings.

\*

> plot(allEffects(bwt.mod1), ci.style="bands")



Exercise 7.7 Refer to Exercise 5.9 for a description of the Accident data. The interest here is to model the probability that an accident resulted in death rather than injury from the predictors age, mode, and gender. With glm(), and the data in the form of a frequency table, you can use the argument weight=Freq to take cell frequency into account.

- (a) Fit the main effects model, result == "Died" ~ age + mode + gender. Use car::Anova() to assess the model terms.
  - \* For analysis using glm() it is useful to consider the ordering of factors for the interpretation of coefficients, contrasts and plots. In the Accident data, mode is an unordered factor, but it is probably more useful to reorder the levels to make Pedestrian the reference level.

- (b) Fit the model that allows all two-way interactions. Use anova() to test whether this model is significantly better than the main effects model.
   ★ In the all two-way model, all terms are significant and the addition of these terms is a significant
  - ★ In the all two-way model, all terms are significant and the addition of these terms is a significant improvement over the main effects model.

```
> acc.mod2 <- update(acc.mod1, . ~ .^2)</pre>
> Anova (acc.mod2)
Analysis of Deviance Table (Type II tests)
Response: result == "Died"
             LR Chisq Df Pr(>Chisq)
                 1101 4
136 3
419 1
                             < 2e-16 ***
age
                              < 2e-16 ***
mode
gender
                             < 2e-16 ***
age:mode
                  122 12
                             < 2e-16 ***
age:gender
                             1.6e-09 ***
                   47
                       4
mode:gender
                   22 3
                             6.7e-05 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> anova(acc.mod1, acc.mod2, test="Chisq")
Analysis of Deviance Table
Model 1: result == "Died" ~ age + mode + gender
Model 2: result == "Died" ~ age + mode + gender + age:mode + age:gender +
   mode:gender
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
          71
                   64599
2
          52
                   64384 19
                                  214 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- (c) Fit the model that also allows the three-way interaction of all factors. Does this offer any improvement over the two-way model?
  - ★ In the three-way model, the term age:mode:gender is not significant, and anova() shows that it does not improve the two-way model. Using LRstats(), the two-way model has the best AIC and BIC.

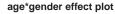
```
> acc.mod3 <- update(acc.mod1, . ~ .^3)</pre>
> Anova (acc.mod3)
Analysis of Deviance Table (Type II tests)
Response: result == "Died"
                LR Chisq Df Pr(>Chisq)
                    1101 4
                             < 2e-16 ***
mode
                     136
                                < 2e-16 ***
                     419 1
gender
                                < 2e-16 ***
age:mode
                     122 12
                                < 2e-16 ***
age:gender
                      47 4
22 3
                               1.6e-09 ***
                      22
mode:gender
                                6.7e-05 ***
age:mode:gender
                      13 12
                                  0.37
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> anova(acc.mod1, acc.mod2, acc.mod3, test="Chisq")
Analysis of Deviance Table
Model 1: result == "Died" ~ age + mode + gender
Model 2: result == "Died" ~ age + mode + gender + age:mode + age:gender +
```

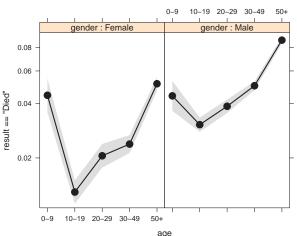
```
mode:gender
Model 3: result == "Died" ~ age + mode + gender + age:mode + age:gender +
   mode:gender + age:mode:gender
 Resid. Df Resid. Dev Df Deviance Pr(>Chi)
                64599
2
                 64384 19
                               214
                                     <2e-16 ***
                 64371 12
         40
                                1.3
                                       0.37
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> LRstats(acc.mod1, acc.mod2, acc.mod3)
Likelihood summary table:
           AIC BIC LR Chisq Df Pr(>Chisq)
acc.mod1 64617 64638
                        64599 71
                                     <2e-16 ***
acc.mod2 64440 64507
                        64384 52
                                     <2e-16 ***
acc.mod3 64451 64547
                        64371 40
                                     <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(d) Interpret the results of the analysis using effect plots for the two-way model, separately for each of the model terms. Describe verbally the nature of the age\*gender effect. Which mode of transportation leads to greatest risk of death?

risk of death?
★ Controlling for mode, males and females have similar profiles, with the lowest probability of death in the 10–19 group, and highest in the 50+ age group.

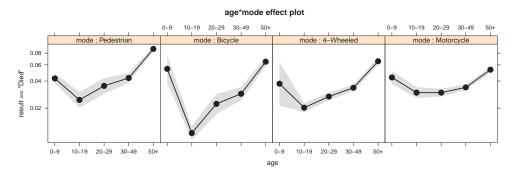
```
> plot(Effect(c("age", "gender"), acc.mod2), ci.style="bands")
```





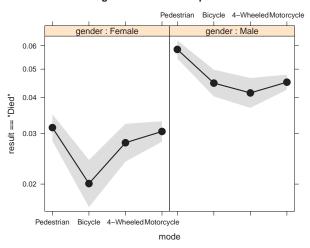
There are roughly similar profiles across age, with bicycle accidents showing the greatest differences among age groups.

> plot(Effect(c("age", "mode"), acc.mod2), ci.style="bands", layout=c(4,1))



```
> plot(Effect(c("gender", "mode"), acc.mod2), ci.style="bands")
```

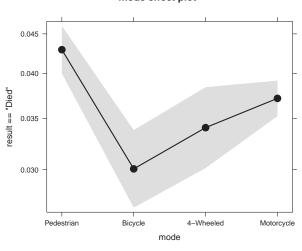
#### gender\*mode effect plot



Overall, accidents involving motorcycles have the greatest probability of death, controlling for all other factors.

> plot(Effect("mode", acc.mod2), ci.style="bands")

#### mode effect plot



### **Chapter 8** Models for Polytomous Responses

Exercise 8.1 For the women's labor force participation data (Womenlf), the response variable, partic, can be treated as ordinal by using

```
> Womenlf$partic <- ordered(Womenlf$partic,
+ levels=c('not.work', 'parttime', 'fulltime'))</pre>
```

Use the methods in Section 8.1 to test whether the proportional odds model holds for these data.



Exercise 8.2 The data set *housing* in the MASS package gives a  $3 \times 3 \times 4 \times 2$  table in frequency form relating (a) satisfaction (Sat) of residents with their housing (High, Medium, Low), (b) perceived degree of influence (Infl) they have on the management of the property (High, Medium, Low), (c) Type of rental (Tower, Atrium, Apartment, Terrace), and (d) contact (Cont) residents have with other residents (Low, High). Consider satisfaction as the ordinal response variable.

(a) Fit the proportional odds model with additive (main) effects of housing type, influence in management, and contact with neighbors to this data. (Hint: Using polr(), with the data in frequency form, you need to use the weights argument to supply the Freq variable.)



(b) Investigate whether any of the two-factor interactions among Infl, Type, and Cont add substantially to goodness of fit of this model. (Hint: use stepAIC(), with the scope formula ~ .^2 and direction="forward".)



(c) For your chosen model from the previous step, use the methods of Section 8.1.5 to plot the probabilities of the categories of satisfaction.



(d) Write a brief summary of these analyses, interpreting *how* satisfaction with housing depends on the predictor variables.



**Exercise 8.3** The data TV on television viewing was analyzed using correspondence analysis in Example 6.4, ignoring the variable Time, and extended in Exercise 6.9. Treating Network as a three-level response variable, fit a generalized logit model (Section 8.3) to explain the variation in viewing in relation to Day and Time. The TV data is a three-way table, so you will need to convert it to a frequency data frame first.

```
> data("TV", package="vcdExtra")
> TV.df <- as.data.frame.table(TV)</pre>
```

(a) Fit the main-effects model, Network ~ Day + Time, with multinom(). Note that you will have to supply the weights argument because each row of TV. df represents the number of viewers in the Freq variable.



(b) Prepare an effects plot for the fitted probabilities in this model.

+

(c) Interpret these results in comparison to the correspondence analysis in Example 6.4.

\*

**Exercise 8.4** \* Refer to Exercise 5.10 for a description of the *Vietnam* data set in vcdExtra. The goal here is to fit models for the polytomous response variable in relation to year and sex.

(a) Fit the proportional odds model to these data, allowing an interaction of year and sex.



(b) Is there evidence that the proportional odds assumption does not hold for this data set? Use the methods described in Section 8.1 to assess this.

\*

(c) Fit the multinomial logistic model, also allowing an interaction. Use car::Anova() to assess the model terms.



(d) Produce an effect plot for this model and describe the nature of the interaction.



(e) Fit the simpler multinomial model in which there is no effect of year for females and the effect of year is linear for males (on the logit scale). Test whether this model is significantly worse than the general multinomial model with interaction.



## **Chapter 9** Loglinear and Logit Models for Contingency Tables

**Exercise 9.1** Consider the data set DaytonSurvey (described in Example 2.6), giving results of a survey of use of alcohol (A), cigarettes (C), and marijuana (M) among high school seniors. For this exercise, ignore the variables sex and race, by working with the marginal table Dayton. ACM, a  $2 \times 2 \times 2$  table in frequency data frame form.

(a) Use loglm() to fit the model of mutual independence, [A][C][M].



(b) Prepare mosaic display(s) for associations among these variables. Give a verbal description of the association between cigarette and alcohol use.



(c) Use fourfold() to produce fourfold plots for each pair of variables, AC, AM, and CM, stratified by the remaining one. Describe these associations verbally.



**Exercise 9.2** Continue the analysis of the *DaytonSurvey* data by fitting the following models:

(a) Joint independence, [AC][M]



(b) Conditional independence, [AM][CM]



(c) Homogeneous association, [AC][AM][CM]



(d) Prepare a table giving the goodness-of-fit tests for these models, as well as the model of mutual independence, [A][C][M], and the saturated model, [ACM]. *Hint*: anova() and LRstats() are useful here. Which model appears to give the most reasonable fit?



Exercise 9.3 The data set Caesar in VCdExtra gives a  $3 \times 2^3$  frequency table classifying 251 women who gave birth by Caesarian section by Infection (three levels: none, Type 1, Type2) and Risk, whether Antibiotics were used, and whether the Caesarian section was Planned or not. Infection is a natural response variable, but the table has quite a few zeros.

(a) Use structable () and mosaic () to see the locations of the zero cells in this table.



(b) Use loglm () to fit the baseline model [I][ RAP]. Is there any problem due to zero cells indicated in the output?

(c) For the purpose of this excercise, treat all the zero cells as *sampling zeros* by adding 0.5 to all cells, e.g., Caesar1 <- Caesar + 0.5. Refit the baseline model.



(d) Now fit a "main effects" model [IR][ IA][ IP][ RAP] that allows associations of Infection with each of the predictors.

 $\star$ 

Exercise 9.4 The <code>Detergent</code> in <code>vcdExtra</code> gives a  $2^3 \times 3$  table classifying a sample of 1,008 consumers according to their preference for (a) expressed <code>Preference</code> for Brand "X" or Brand "M" in a blind trial, (b) <code>Temperature</code> of laundry water used, (c) previous use (<code>M\_user</code>) of detergent Brand "M," and (d) the softness (<code>Water\_softness</code>) of the laundry water used.

(a) Make some mosaic displays to visualize the associations among the table variables. Try using different orderings of the table variables to make associations related to Preference more apparent.



(b) Use a doubledecker () plot to visualize how Preference relates to the other factors.



(c) Use loglm() to fit the baseline model [P][TMW] for Preference as the response variable. Use a mosaic display to visualize the lack of fit for this model.



# **Chapter 10 Extending Loglinear Models**

**Exercise 10.1** Example 10.5 presented an analysis of the data on visual acuity for the subset of women in the *VisualAcuity* data. Carry out a parallel analysis of the models fit there for the men in this data set, given by:

```
> data("VisualAcuity", package="vcd")
> men <- subset(VisualAcuity, gender=="male", select=-gender)</pre>
```



Exercise 10.2 Table 10.1 gives a  $4 \times 4$  table of opinions about premarital sex and whether methods of birth control should be made available to teenagers aged 14–16, from the 1991 General Social Survey (Agresti, 2013, Table 10.3). Both variables are ordinal, and their grades are represented by the case of the row and column labels.

Table 10.1: Opinions about premarital sex and availability of teenage birth control. Source: Agresti (2013, Table 10.3).

Premarital sex	•	Birth cont	rol	
Tromarian sex	DISAGREE	disagree	agree	AGREE
WRONG	81	68	60	38
Wrong	24	26	29	14
wrong	18	41	74	42
OK	36	57	161	157

(a) Fit the independence model to these data using loglm() or glm().



(b) Make a mosaic display showing departure from independence and describe verbally the pattern of association.



(c) Treating the categories as equally spaced, fit the  $L \times L$  model of uniform association, as in Section 10.1. Test the difference against the independence model with a likelihood-ratio test.



(d) Fit the RC(1) model with gnm (), and test the difference of this against the model of uniform association.



(e) Write a brief summary of these results, including plots useful for explaining the relationships in this data set.



Exercise 10.3 For the data on attitudes toward birth control in Table 10.1.

(a) Calculate and plot the observed local log odds ratios.



(b) Also fit the R, C, and R+C models.



(c) Use the method described in Section 10.1.2 to visualize the structure of fitted local log odds ratios implied by each of these models, together with the RC(1) model.



**Exercise 10.4** The data set gss8590 in logmult gives a  $4\times5\times4$  table of education levels and occupational categories for the four combinations of gender and race from the General Social Surveys, 1985–1990, as reported by Wong (2001, Table 2). Wong (2010, Table 2.3B) later used the subset pertaining to women to illustrate RC(2) models. This data is created below as Women.tab, correcting an inconsistency to conform with the 2010 table.

```
*
```

```
> data("gss8590", package="logmult")
> Women.tab <- margin.table(gss8590[,,c("White Women", "Black Women")], 1:2)
> Women.tab[2,4] <- 49
> colnames(Women.tab)[5] <- "Farm"</pre>
```

(a) Fit the independence model, and also the RC(1) and RC(2) models using rc() with marginal weights, as illustrated in Example 10.4. Summarize these statistical tests in a table.



(b) Plot the solution for the RC(2) model with 68% confidence ellipses. What verbal labels would you use for the two dimensions?



(c) Is there any indication that a simpler model, using integer scores for the row (Education) or column (Occupation) categories, or both, might suffice? If so, fit the analogous column effects, row effects, or  $L \times L$  model, and compare with the models fit in part (a).



### **Chapter 11 Generalized Linear Models for Count Data**

**Exercise 11.1** Poole (1989) studied the mating behavior of elephants over 8 years in Amboseli National Park, Kenya. A focal aspect of the study concerned the mating success of males in relation to age, since larger males tend to be more successful in mating. Her data were used by Ramsey and Schafer (2002, Chapter 22) as a case study, and are contained in the Sleuth2 (Ramsey et al., 2012) package (Ramsey et al., 2012) as *case2201*.

For convenience, rename this to elephants, and study the relation between Age (at the beginning of the study) and number of successful Matings for the 41 adult male elephants observed over the course of this study, ranging in age from 27–52.

(a) Create some exploratory plots of Matings against Age in the styles illustrated in this chapter. To do this successfully, you will have to account for the fact that Matings has a range of only 0–9, and use some smoothing methods to show the trend.



(b) Repeat (a) above, but now plotting log (Matings+1) against Age to approximate a Poisson regression with a log link and avoid problems with the zero counts.

 $\star$ 

- (c) Fit a linear Poisson regression model for Matings against Age. Interpret the fitted model *verbally* from a graph of predicted number of matings and/or from the model coefficients. (*Hint*: Using Age-27 will make the intercept directly interpretable.)
  - \*
- (d) Check for nonlinearity in the relationship by using the term poly (Age, 2) in a new model. What do you conclude?
  - \*
- (e) Assess whether there is any evidence of overdispersion in these data by fitting analogous quasi-Poisson and negative-binomial models.
  - \*

**Exercise 11.2** The data set *quine* in MASS gives data on absenteeism from schools in rural New South Wales, Australia. 146 children were classified by ethnic background (Eth), age (Age, a factor), Sex, and Learner status (Lrn), and the number of days absent (Days) from school in a particular school year was recorded.

- (a) Fit the all main-effects model in the Poisson family and examine the tests of these effects using summary() and Car::Anova(). Are there any terms that should be dropped according to these tests?
  - $\star$
- (b) Re-fit this model as a quasi-Poisson model. Is there evidence of overdispersion? Test for overdispersion formally, using dispersiontest() from AER (Kleiber and Zeileis, 2015).
  - $\star$
- (c) Carry out the same significance tests and explain why the results differ from those for the Poisson model.
  - \*

**Exercise 11.3** The data set *AirCrash* in vcdExtra was analyzed in Exercise 5.2 and Exercise 6.3 in relation to the Phase of the flight and Cause of the crash. Additional variables include the number of Fatalities and Year. How does Fatalities depend on the other variables?

- (a) Use the methods of this chapter to make some exploratory plots relating fatalities to each of the predictors.
  - $\star$
- (b) Fit a main effects poisson regression model for Fatalities, and make effects plots to visualize the model. Which phases and causes result in the largest number of fatalities?
  - $\star$
- (c) A linear effect of Year might not be appropriate for these data. Try using a natural spline term, ns (Year, df) to achieve a better, more adequate model.
  - \*
- (d) Use a model-building tool like add1 () or MASS::stepAIC() to investigate whether there are important two-way interactions among the factors and your chosen effect for Year.
  - $\star$
- (e) Visualize and interpret your final model and write a brief summary to answer the question posed.
  - $\star$

Exercise 11.4 Male double-crested cormorants use advertising behavior to attract females for breeding. The *Cormorants* data set in vcdExtra gives some results from a study by Meagan Mc Rae (2015) on counts of advertising males observed two or three times a week at six stations in a tree-nesting colony for an entire breeding season. The number of advertising birds was counted and these observations were classified by characteristics of the trees and nests. The goal was to determine how this behavior varies temporally over the season and spatially over observation stations, as well as with characteristics of nesting sites. The response variable is count and other predictors are shown below. See help (Cormorants, package="vcdExtra") for further details.

```
> data("Cormorants", package = "vcdExtra")
> car::some(Cormorants)
                                             density tree_health count
      category week station
                               nest height
39
                                                 few
                    B1 partial
                                      high
                                                           dead
112
           Pre
                         C1
                                                 few
                                                         healthy
                         C2 partial
                                       low
                                                 few
                                                            dead
           Pre
                                 no
                                      high
                                                            dead
185 Incubation
                         B1 partial
                                       high
                                                 few
                                                            dead
                                                         healthy
188 Incubation
```

241	Incubation		C2	no	high mode	few	dead dead	1 1
257	Incubation	6	В2	no	high	few	healthy	3
317	Incubation	9	B2	no	high mode	erate	dead	1

(a) Using the methods illustrated in this chapter, make some exploratory plots of the number of advertising birds against week in the breeding season, perhaps stratified by another predictor, like tree height, nest condition, or observation station. To see anything reasonable, you should plot count on a log (or square root) scale, jitter the points, and add smoothed curves. The variable category breaks the weeks into portions of the breeding season, so adding vertical lines separating those will be helpful for interpretation.

\*

(b) Fit a main-effects Poisson GLM to these data and test the terms using Anova () from the car package.

 $\star$ 

(c) Interpret this model using an effects plot.

\*

(d) Investigate whether the effect of week should be treated as linear in the model. You could try using a polynomial term like poly (week, degree) or perhaps better, using a natural spline term like ns (week, df) from the splines package.

\*

(e) Test this model for overdispersion, using either a quasipoisson family or dispersiontest () in AER.



**Exercise 11.5** For the *CodParasites* data, recode the area variable as an ordered factor as suggested in footnote 13. Test the hypotheses that prevalence and intensity of cod parasites is linearly related to area.



**Exercise 11.6** In Example 11.10, we ignored other potential predictors in the *CodParasites* data: depth, weight, length, sex, stage, and age. Use some of the graphical methods shown in this case study to assess whether any of these are related to prevalence and intensity.



**Exercise 11.7** The analysis of the *PhdPubs* data in the examples in this chapter were purposely left incomplete, going only as far as the negative binomial model.

(a) Fit the zero-inflated and hurdle models to this data set, considering whether the count component should be Poisson or negative-binomial, and whether the zero model should use all predictors or only a subset. Describe your conclusions from this analysis in a few sentences.

 $\star$ 

(b) Using the methods illustrated in this chapter, create some graphs summarizing the predicted counts and probabilities of zero counts for one of these models.



(c) For your chosen model, use some of the diagnostic plots of residuals and other measures shown in Section 11.6 to determine if your model solves any of the problems noted in Example 11.17 and Example 11.18, and whether there are any problems that remain.



**Exercise 11.8** In Example 11.19 we used a simple analysis of  $\log(y+1)$  for the multivariate responses in the *NMES1988* data using a classical MLM (Eqn. (11.16)) as a rough approximation of a multivariate Poisson model. The HE plot in Figure 11.40 was given as a visual summary, but did not show the data. Examine why the MLM is not appropriate statistically for these data, as follows:

(a) Calculate residuals for the model nmes.mlm using

```
> resids <- residuals(nmes.mlm, type="deviance")</pre>
```

- (b) Make univariate density plots of these residuals to show their univariate distributions. These should be approximately normal under the MLM. What do you conclude?
  - $\star$
- (c) Make some bivariate plots of these residuals. Under the MLM, each should be bivariate normal with elliptical contours and linear regressions. Add 2D density contours (kde2d(), or geom\_density2d() in ggplot2) and some smoothed curve. What do you conclude?

 $\star$ 

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