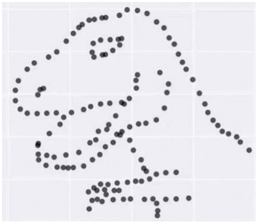

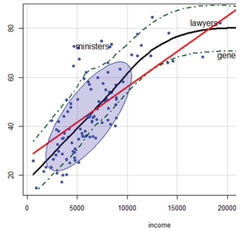

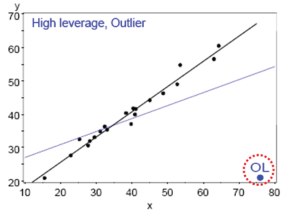

Visualizing Linear Models: An R Bag of Tricks Session 1: Getting Started

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SCS Short Course
Oct. 2020

Today's topics

- What you need for this course
- Why plot your data?
- Data plots
- Model (effect) plots
- Diagnostic plots

What you need

- R, version >=3.6
  - Download from https://cran.r-project.org/
- RStudio IDE, highly recommended
  - https://www.rstudio.com/products/rstudio/
- R packages: see course web page
  - car
  - effects
  - heplots
  - candisc

Why plot your data?

Getting information from a table is like extracting sunlight from a cucumber. --- Farquhar & Farquhar, 1891

Information that is imperfectly acquired, is generally as imperfectly retained; and a man who has carefully investigated a printed table, finds, when done, that he has only a very faint and partial idea of what he has read; and that like a figure imprinted on sand, is soon totally erased and defaced.
--- William Playfair, *The Commercial and Political Atlas* (p. 3), 1786

Cucumbers

Table 7
Stevens et al. 2006, table 2: Determinants of authoritarian aggression

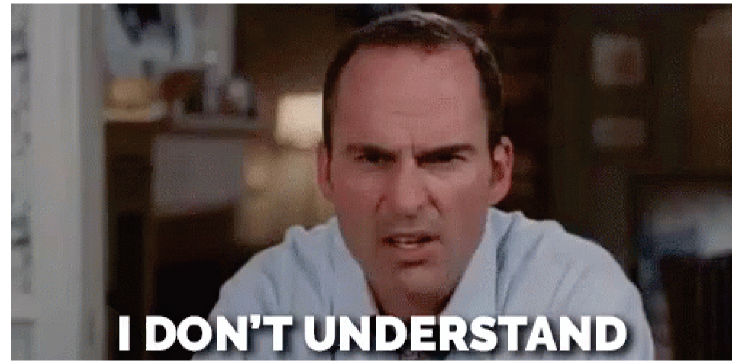
Variable	Coefficient (Standard Error)
Constant	.41 (.93)
Countries	
Argentina	1.31 (.33)**B,M
Chile	.93 (.32)**B,M
Colombia	1.46 (.32)**B,M
Mexico	.07 (.32)A,CH,CO,V
Venezuela	.96 (.37)**B,M
Threat	
Retrospective egocentric economic perceptions	.20 (.13)
Prospective egocentric economic perceptions	.22 (.12)#
Retrospective sociotropic economic perceptions	–.21 (.12)#
Prospective sociotropic economic perceptions	–.32 (.12)*
Ideological distance from president	–.27 (.07)**
Ideology	
Ideology	.23 (.07)**
Individual Differences	
Age	.00 (.01)
Female	–.03 (.21)
Education	.13 (.14)
Academic Sector	.15 (.29)
Business Sector	.31 (.25)
Government Sector	–.10 (.27)
R^2	.15
Adjusted R^2	.12
N	500

Results of a one model for authoritarian aggression

The information is overwhelmed by footnotes & significance **stars**

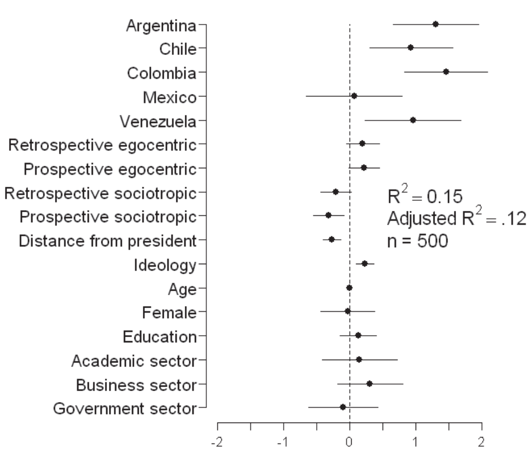
**p < .01, *p < .05, #p < .10 (twotailed)
ACoefficient is significantly different from Argentina's at p < .05;
BCoefficient is significantly different from Brazil's at p < .05;
CHCoefficient is significantly different from Chile's at p < .05;
COCoefficient is significantly different from Colombia's at p < .05;
MCoefficient is significantly different from Mexico's at p < .05;
VCoefficient is significantly different from Venezuela's at p < .05.

What's wrong with this picture?



Sunlight

coefplot(model)



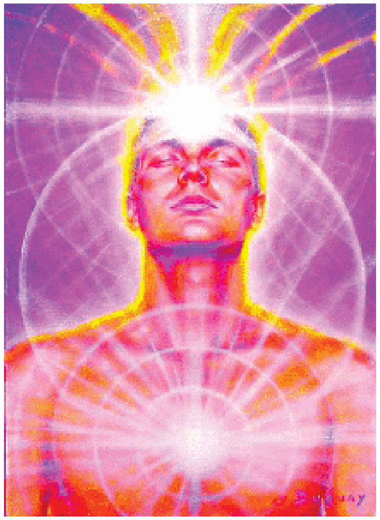
Why didn't they say this in the first place?

NB: This is a presentation graph equivalent of the table

Shows coefficient with 95% CI

Run, don't walk toward the sunlight

Graphs can give enlightenment



The greatest value of a picture is when it forces us to notice what we never expected to see.
-- John W. Tukey

Dangers of numbers-only output

Student: You said to run descriptives and compute the correlation. What next?

Consultant: Did you plot your data?

```
X Mean: 54.26
Y Mean: 47.83
X SD  : 16.76
Y SD  : 26.93
Corr. : -0.06
```

With exactly the same stats, the data could be *any* of these plots

See how this in done in R: https://cran.r-project.org/web/packages/datasauRus/

Sometimes, don't need numbers at all

COVID transmission risk ~ Occupancy * Ventilation * Activity * Mask? * Contact.time

A complex 5-way table, whose message is clearly shown w/o numbers

There are 1+ unusual cells here. Can you see them?

From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? *BMJ* 2020;370:m3223, *doi: https://doi.org/10.1136/bmj.m3223*

If you do need tables– make them pretty

Several R packages make it easier to construct tables

Flipper lengths (mm) of the famous penguins of Palmer Station, Antarctica.

		Female		Male	
Species	Distribution	Avg.	Std. Dev.	Avg.	Std. Dev.
		188	5.6	192	6.6
		192	5.8	200	6.0
		213	3.9	222	5.7

Artwork by @allison_horst

Linear models

- Model:

$$y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + \varepsilon_i$$

- Xs: quantitative predictors, factors, interactions, …
- Assumptions:
  - **Linearity**: Predictors (possibly transformed) are linearly related to the outcome, **y**. [This just means linear in the **parameters**.]
  - **Specification**: No important predictors have been omitted; only important ones included. [This is often key & overlooked.]
  - The "holy trinity":
    - **Independence**: the errors are uncorrelated
    - **Homogeneity of variance**: Var(ε_i) = σ^2 = constant
    - **Normality**: ε_i have a normal distribution

$$\varepsilon_i \sim_{iid} \mathrm{N}(0, \sigma^2)$$

Plots for linear models

- Data plots:
  - plot response (y) vs. predictors, with smooth summaries
  - scatterplot matrix --- all pairs
- Model (effect) plots
  - plot predicted response ($\hat{y}$) vs. predictors, controlling for variables not shown.
- Diagnostic plots

Occupational Prestige data

- Data on prestige of 102 occupations and
  - average education (years)
  - average income ($)
  - % women
  - type (Blue Collar, Professional, White Collar)

```
> head(Prestige)
                    education income women prestige census type
gov.administrators      13.11  12351 11.16     68.8   1113 prof
general.managers        12.26  25879  4.02     69.1   1130 prof
accountants             12.77   9271 15.70     63.4   1171 prof
purchasing.officers     11.42   8865  9.11     56.8   1175 prof
chemists                14.62   8403 11.68     73.5   2111 prof
physicists              15.64  11030  5.13     77.6   2113 prof
```


Informative scatterplots

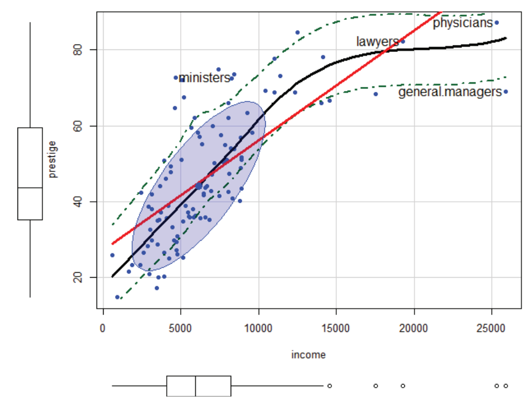
Scatterplots are most useful when enhanced with annotations & statistical summaries

Boxplots show marginal distributions

Data ellipse and regression line show the linear model, prestige ~ income

Point labels show possible outliers

Smoothed (loess) curve and CI show the trend



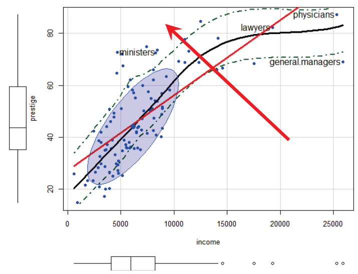
Informative scatterplots

`car::scatterplot()` provides all of these enhancements

```
scatterplot(prestige ~ income, data=Prestige,
            pch = 16,
            regLine = list(col = "red", lwd=3),
            smooth = list(smoother=loessLine,
                          lty.smooth = 1, col.smooth = "black",
                          lwd.smooth=3, col.var = "darkgreen"),
            ellipse = list(levels = 0.68),
            id = list(n=4, col="black", cex=1.2))
```

Skewed distribution of income & non-linear relation suggest need for a transformation

Arrow rule: move on the scale of powers in direction of the bulge

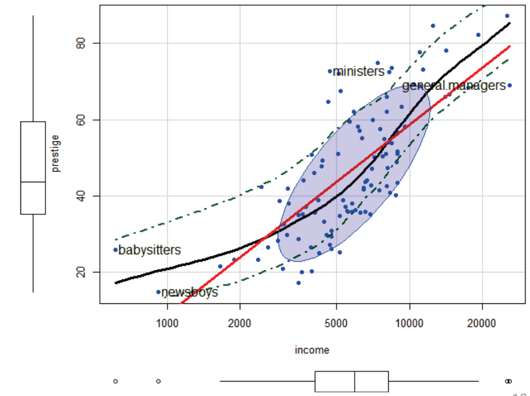


Try log(income)

```
scatterplot(prestige ~ income, data=Prestige,
            log = "x",
            pch = 16,
            regLine = list(col = "red", lwd=3),
            … )
```

Income now ~ symmetric

Relation closer to linear



Stratify by type?

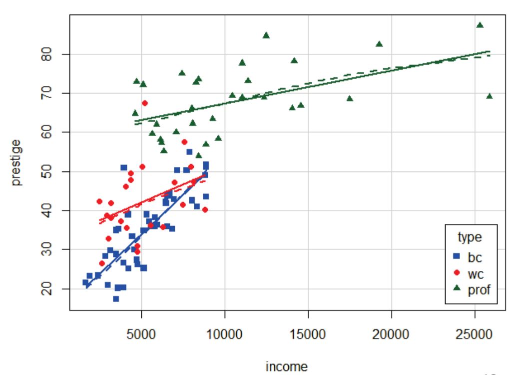
```
scatterplot(prestige ~ income | type, data=Prestige,
      col = c("blue", "red", "darkgreen"),
      pch = 15:17,
      legend = list(coords="bottomright"),
      smooth=list(smoother=loessLine, var=FALSE, span=1, lwd=4))
```

Formula: | type → "given type"

Different slopes: interaction of income * type

Provides another explanation of the non-linear relation

This is a new finding!



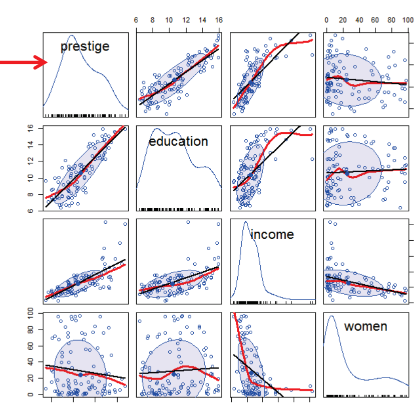
Scatterplot matrix

```
scatterplotMatrix(~ prestige + education + income + women ,
        data=Prestige,
         regLine = list(method=lm, lty=1, lwd=2, col="black"),
         smooth=list(smoother=loessLine, spread=FALSE,
                  lty.smooth=1, lwd.smooth=3, col.smooth="red"),
         ellipse=list(levels=0.68, fill.alpha=0.1))
```

prestige vs. all predictors

diagonal: univariate distributions
- income: + skewed
- %women: bimodal

off-diagonal: relations among predictors



Fit a model

```
> mod1 <- lm(prestige ~ education + poly(women, 2) +
+                        log(income)*type, data=Prestige)
> summary(mod1)

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          -137.500     23.522   -5.85  8.2e-08 ***
education               2.959      0.582    5.09  2.0e-06 ***
poly(women, 2)1        28.339     10.190    2.78   0.0066 **
poly(women, 2)2        12.566      7.095    1.77   0.0800 .
log(income)            17.514      2.916    6.01  4.1e-08 ***
typeprof               74.276     30.736    2.42   0.0177 *
typewc                  0.969     39.495    0.02   0.9805
log(income):typeprof   -7.698      3.451   -2.23   0.0282 *
log(income):typewc     -0.466      4.620   -0.10   0.9199
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’
1

Multiple R-squared:  0.879,   Adjusted R-squared:  0.868
F-statistic: 81.1 on 8 and 89 DF,  p-value: <2e-16
```

- allow women^2 term
- interaction of log(income) and type

Fits very well!

Model (effect) plots

- We'd like to see plots of the predicted value ($\hat{y}$) of the response against predictors
  - But must control for other predictors not shown in a given plot
  - Variables not shown in a given plot are averaged over.
  - Slopes of lines reflect the partial coefficient in the model
  - Partial residuals can be shown also

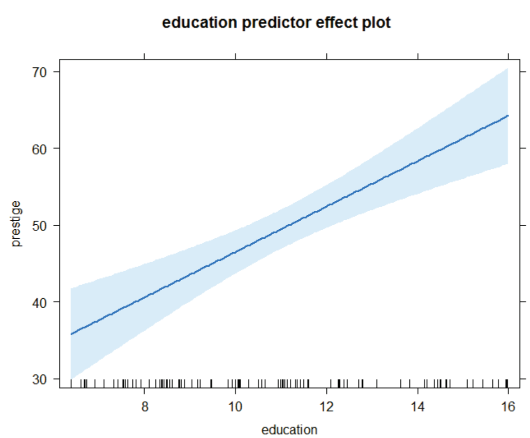
For details, see `vignette("predictor-effects-gallery", package="effects")`

Model (effect) plots: education

```
library("effects")
mod1.e1 <- predictorEffect("education", mod1)
plot(mod1.e1)
```

This graph shows the partial slope for education.

For each ↑ year in education, fitted prestige ↑2.96 points, (other predictors held fixed)

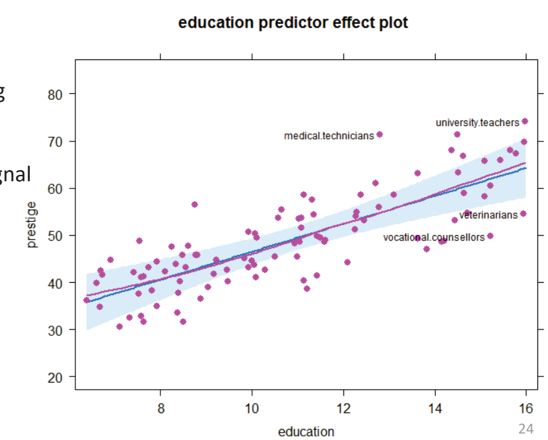


Model (effect) plots

```
mod1.e1a <- predictorEffect("education", mod1, residuals=TRUE)
plot(mod1.e1a,
     residuals.pch=16, id=list(n=4, col="black"))
```

Partial residuals show the residual of prestige controlling for other predictors

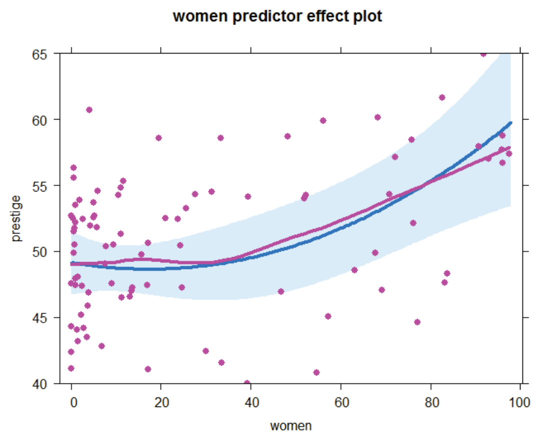
Unusual points here would signal undue influence



Model (effect) plots: women

```
mod1.e2 <- predictorEffect("women", mod1, residuals=TRUE)
plot(mod1.e2, ylim=c(40, 65), lwd=4,
     residuals.pch=16)
```

Surprise!
Prestige of occupations ↑ with % women (controlling for other variables

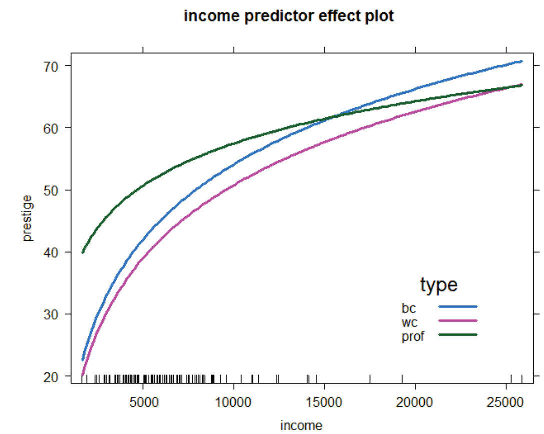


Model (effect) plots: income

```
plot(predictorEffect("income", mod1),
     lines=list(multiline=TRUE, lwd=3),
     key.args = list(x=.7, y=.35))
```

Income interacts with type in the model

The plot is curved because log(income) is in the model



Diagnostic plots

- The linear model, $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ assumes:
  - Residuals, ε_i are normally distributed, $\varepsilon_i \sim N(0,\sigma^2)$
  - (Normality not required for **X**s)
  - Constant variance, $\mathrm{Var}(\varepsilon_i) = \sigma^2$
  - Observations y_i are statistically independent
- Violations → inferences may not be valid
- A variety of plots can diagnose all these problems
- Other methods (boxCox, boxTidwell) diagnose the need for transformations of **y** or **X**s.

The "regression quartet"

In R, plotting a `lm` model object → the "regression quartet" of plots

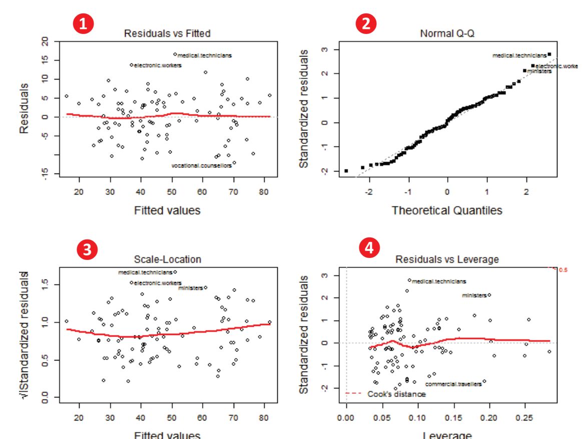
```
plot(mod1, lwd=2, cex.lab=1.4)
```

1 Residuals: should be flat vs. fitted values

2 Q-Q plot: should follow the 45° line

3 Scale-location: should be flat if constant variance

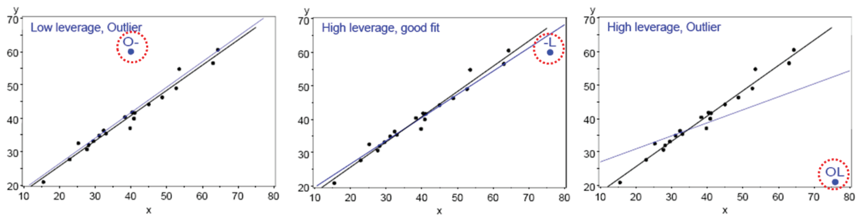
4 Resids vs. leverage: can show influential observations



Unusual data: Leverage & Influence

- "Unusual" observations can have dramatic effects on least-squares estimates in linear models
- Three archetypal cases:
  - Typical X (low leverage), bad fit -- Not much harm
  - Unusual X (high leverage), good fit -- Not much harm
  - Unusual X (high leverage), bad fit -- BAD, BAD, BAD
- Influential observations: unusual in *both* X & Y
- Heuristic formula:

Influence = X leverage x Y residual



Influence plots

Influence (Cook's D) measures impact of individual obs. on coefficients, fitted values

Influence ~ Residual (y -ŷ) × Hat-value $(\mathbf{X} - \bar{\mathbf{X}})^2$

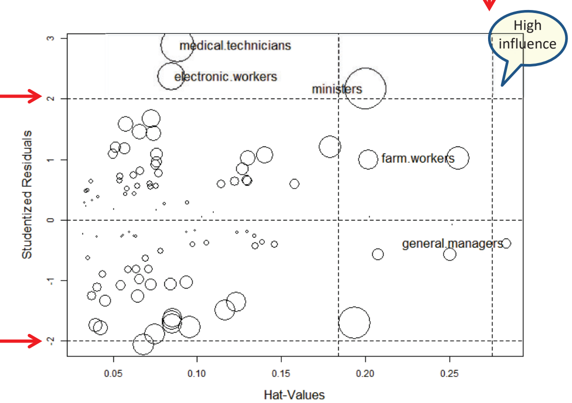
High leverage

Bubble size ~ influence

Bad fit

```
influencePlot(mod1)
```

Bad fit

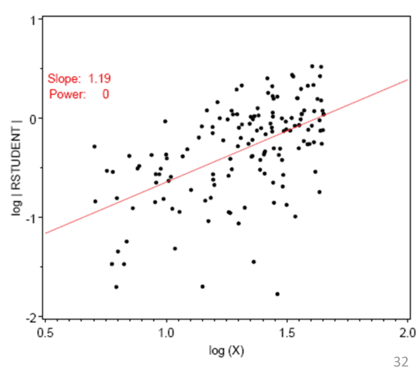


Spread-level plots

- To diagnose non-constant variance, plot:
  - log |Std. residual| vs. log (x)
  - log (IQR) vs log (median) [for grouped data]
- If ≈ linear w/ slope b, transform y → y $^{(1-b)}$

Artificial data, generated so σ ~ x

- b ≈ 1 → power = 0
- → analyze log(y)



Spread-level plot: baseball data

Data on salary and batter performance from 1987 season

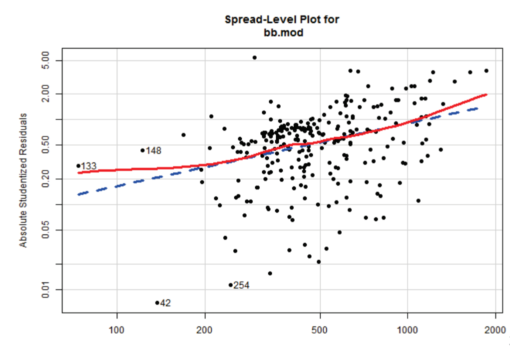
```
data("Baseball", package="vcd")
bb.mod <- lm(sal87 ~ years + hits + runs + homeruns, data=Baseball)
spreadLevelPlot(bb.mod, pch=16, lwd=3,
                id=list(n=2))

## Suggested power transformation: 0.2609
```

slope = .74 → p = .26

i.e., y → log(y) or $y^{1/4}$

NB: both axes plotted on log scale



Summary

- Tables are for look-up; graphs can give insight
- Data plots are more effective when enhanced
  - data ellipses → strength & precision of correlation
  - regression lines and smoothed curves
  - point identification → noteworthy observations
- Effect plots show informative views of models
- Diagnostic plots can reveal influential observations and need for transformations.