

Visualizing Linear Models: An R Bag of Tricks Session 1: Getting Started

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SCS Short Course
March, 2021

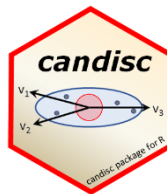
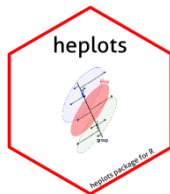
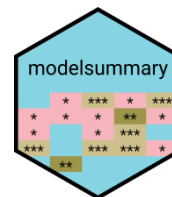
<https://friendly.github.io/VisMLM-course/>

Today's topics

- What you need for this course
- Why plot your data?
- Data plots
- Model (effect) plots
- Diagnostic plots

What you need

- R, version ≥ 3.6
 - Download from <https://cran.r-project.org/>
- RStudio IDE, highly recommended
 - <https://www.rstudio.com/products/rstudio/>
- R packages: see course web page
 - car
 - effects
 - heplots
 - candisc
 - visreg





Why plot your data?

Getting information from a table is like extracting sunlight from a cucumber. --- Farquhar & Farquhar, 1891

Information that is imperfectly acquired, is generally as imperfectly retained; and a man who has carefully investigated a printed table, finds, when done, that he has only a very faint and partial idea of what he has read; and that like a figure imprinted on sand, is soon totally erased and defaced.

*--- William Playfair, *The Commercial and Political Atlas* (p. 3), 1786*



Cucumbers

Table 7
Stevens et al. 2006, table 2: Determinants
of authoritarian aggression

Variable	Coefficient (Standard Error)
Constant	.41 (.93)
Countries	
Argentina	1.31 (.33)**B,M
Chile	.93 (.32)**B,M
Colombia	1.46 (.32)**B,M
Mexico	.07 (.32) ^{A,CH,CO,V}
Venezuela	.96 (.37)**B,M
Threat	
Retrospective egocentric economic perceptions	.20 (.13)
Prospective egocentric economic perceptions	.22 (.12) [#]
Retrospective sociotropic economic perceptions	-.21 (.12) [#]
Prospective sociotropic economic perceptions	-.32 (.12)*
Ideological distance from president	-.27 (.07)**
Ideology	
Ideology	.23 (.07)**
Individual Differences	
Age	.00 (.01)
Female	-.03 (.21)
Education	.13 (.14)
Academic Sector	.15 (.29)
Business Sector	.31 (.25)
Government Sector	-.10 (.27)
R^2	.15
Adjusted R^2	.12
N	500

Results of a one model for authoritarian aggression

The information is overwhelmed by footnotes & significance **stars**

**p < .01, *p < .05, #p < .10 (twotailed)

^ACoefficient is significantly different from Argentina's at p < .05;

^BCoefficient is significantly different from Brazil's at p < .05;

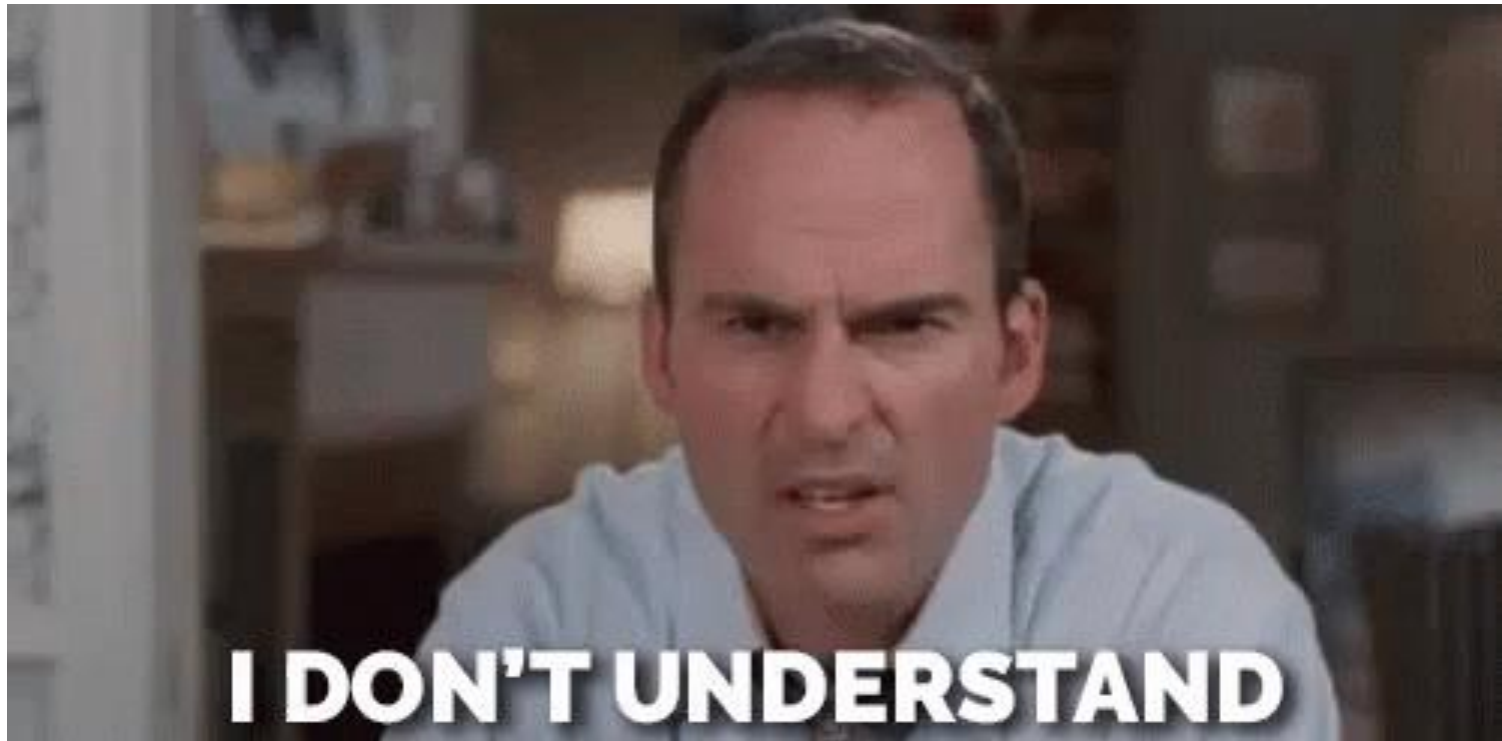
^{CH}Coefficient is significantly different from Chile's at p < .05;

^{CO}Coefficient is significantly different from Colombia's at p < .05;

^MCoefficient is significantly different from Mexico's at p < .05;

^VCoefficient is significantly different from Venezuela's at p < .05.

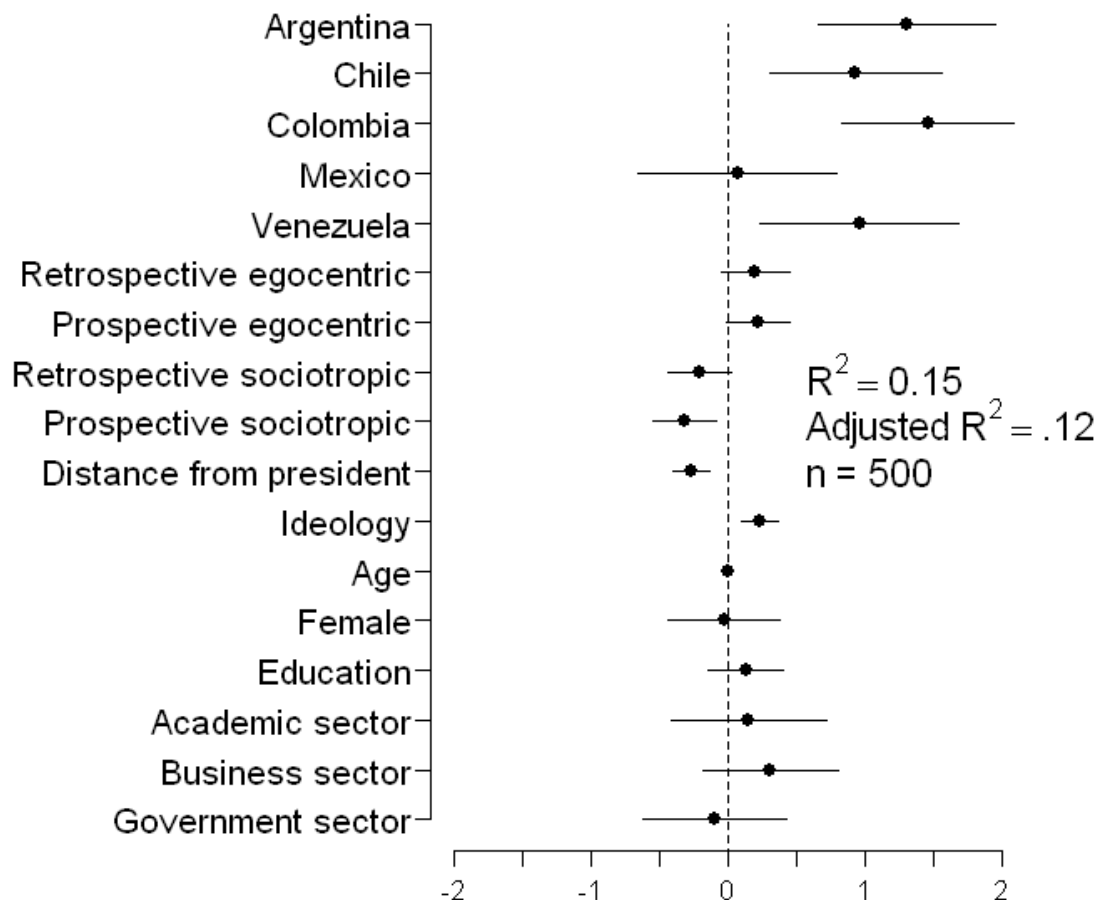
What's wrong with this picture?





Sunlight

`coefplot(model)`



Why didn't they say this in the first place?

NB: This is a presentation graph equivalent of the table

Shows coefficient with 95% CI

Run, don't walk toward the sunlight



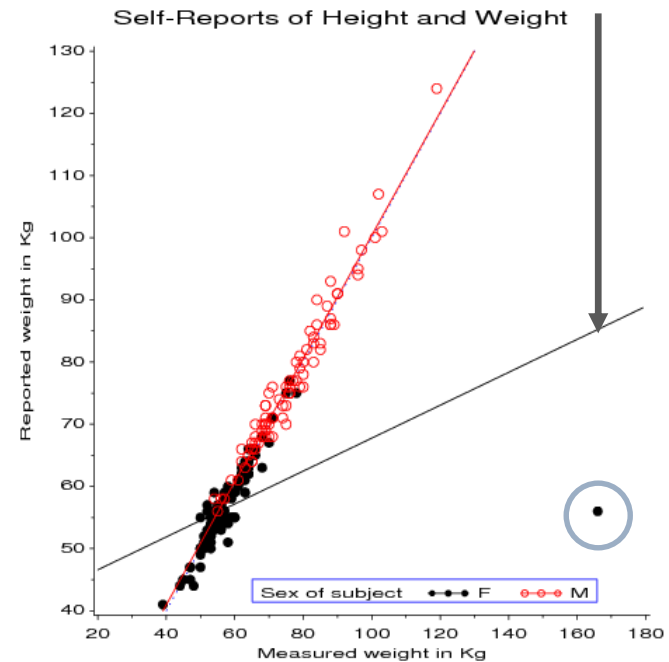
Graphs can give enlightenment



The greatest value of a picture is when it forces us to notice what we never expected to see.

-- John W. Tukey

Effect of one rotten point on regression



Dangers of numbers-only output

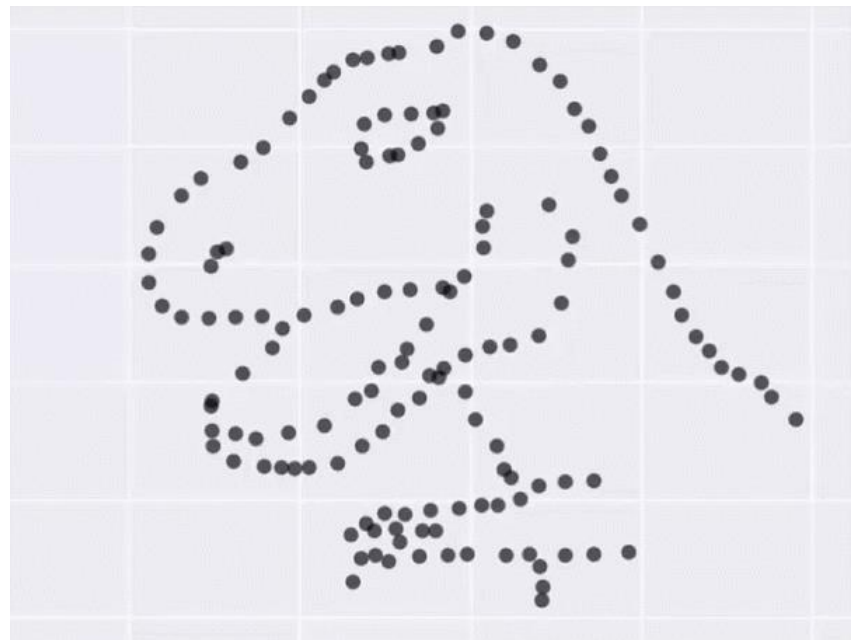
Student: You said to run descriptives and compute the correlation. What next?

Consultant: Did you plot your data?

```
X Mean: 54.26  
Y Mean: 47.83  
X SD   : 16.76  
Y SD   : 26.93  
Corr.  : -0.06
```

With **exactly** the same stats, the data could be *any* of these plots

See how this is done in R: <https://cran.r-project.org/web/packages/datasauRus/>



Sometimes, don't need numbers at all

COVID transmission risk ~ Occupancy * Ventilation * Activity * Mask? * Contact.time

A complex 5-way table,
whose message is clearly
shown w/o numbers

There are 1+ unusual cells
here. Can you see them?

Type and level of group activity	Low occupancy			High occupancy		
	Outdoors and well ventilated	Indoors and well ventilated	Poorly ventilated	Outdoors and well ventilated	Indoors and well ventilated	Poorly ventilated
Wearing face coverings, contact for short time						
Silent	Low	Low	Low	Low	Low	Medium
Speaking	Low	Low	Low	Low	Low	Medium
Shouting, singing	Low	Low	Medium	Medium	Medium	High
Wearing face coverings, contact for prolonged time						
Silent	Low	Low	Medium	Low	Medium	High
Speaking	Low	* Low	Medium	* Medium	Medium	High
Shouting, singing	Low	Medium	High	Medium	High	High
No face coverings, contact for short time						
Silent	Low	Low	Medium	Medium	Medium	High
Speaking	Low	Medium	Medium	Medium	High	High
Shouting, singing	Medium	Medium	High	High	High	High
No face coverings, contact for prolonged time						
Silent	Low	Medium	High	Medium	High	High
Speaking	Medium	Medium	High	High	High	High
Shouting, singing	Medium	High	High	High	High	High

Risk of transmission
 Low ■ Medium ■ High ■

* Borderline case that is highly dependent on quantitative definitions of distancing, number of individuals, and time of exposure

From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? *BMJ* 2020;370:m3223, doi: <https://doi.org/10.1136/bmj.m3223>

If you do need tables– make them pretty




Several R packages make it easier to construct informative & pretty semi-graphic tables

Presentation graph

Perhaps too cute!

Distribution of variables shown

Flipper lengths (mm) of the famous penguins of Palmer Station, Antarctica.

Species	Distribution	Female		Male	
		Avg.	Std. Dev.	Avg.	Std. Dev.
ADÉLIE!		188	5.6	192	6.6
CHINSTRAP!		192	5.8	200	6.0
GENTOO!		213	3.9	222	5.7

Artwork by @allison_horst

Visual table ideas: Heatmap shading

Heatmap shading: Shade the background of each cell according to some criterion

The trends in the US and Canada are made obvious

NB: Table rows are sorted by Jan. value, lending coherence

Unemployment rate in selected countries

January-August 2020, sorted by the unemployment rate in January.

country	Jan ▲	Feb	Mar	Apr	May	Jun	Jul	Aug
Japan	2.4%	2.4%	2.5%	2.6%	2.9%	2.8%	2.9%	3.0%
Netherlands	3.0%	2.9%	2.9%	3.4%	3.6%	4.3%	4.5%	4.6%
Germany	3.4%	3.6%	3.8%	4.0%	4.2%	4.3%	4.4%	4.4%
Mexico	3.6%	3.6%	3.2%	4.8%	4.3%	5.4%	5.2%	5.0%
US	3.6%	3.5%	4.4%	14.7%	13.3%	11.1%	10.2%	8.4%
South Korea	4.0%	3.3%	3.8%	3.8%	4.5%	4.3%	4.2%	3.2%
Denmark	4.9%	4.9%	4.8%	4.9%	5.5%	6.0%	6.3%	6.1%
Belgium	5.1%	5.0%	5.0%	5.1%	5.0%	5.0%	5.0%	5.1%
Australia	5.3%	5.1%	5.2%	6.4%	7.1%	7.4%	7.5%	6.8%
Canada	5.5%	5.6%	7.8%	13.0%	13.7%	12.3%	10.9%	10.2%
Finland	6.8%	6.9%	7.0%	7.3%	7.5%	7.8%	8.0%	8.1%

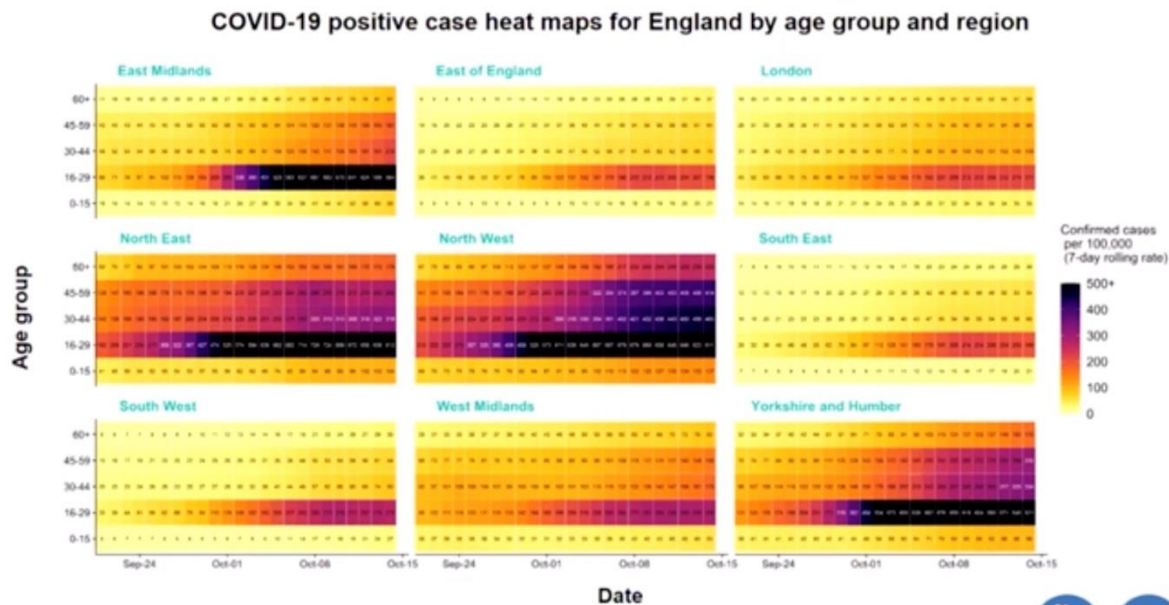
Source: [OECD](#) • [Get the data](#) • Created with [Datawrapper](#)

Visual table ideas: Heatmap shading

As seen on TV ...

Covid rate \sim Age x Date x UK region

Better: incorporate geography, not just arrange regions alphabetically



COBR
Cabinet Office Briefing Rooms

Source: Case data from SGSS. Produced by Outbreak Surveillance Team, PHE.
Contains National Statistics data © Crown copyright and database right 2020

BBC NEWS

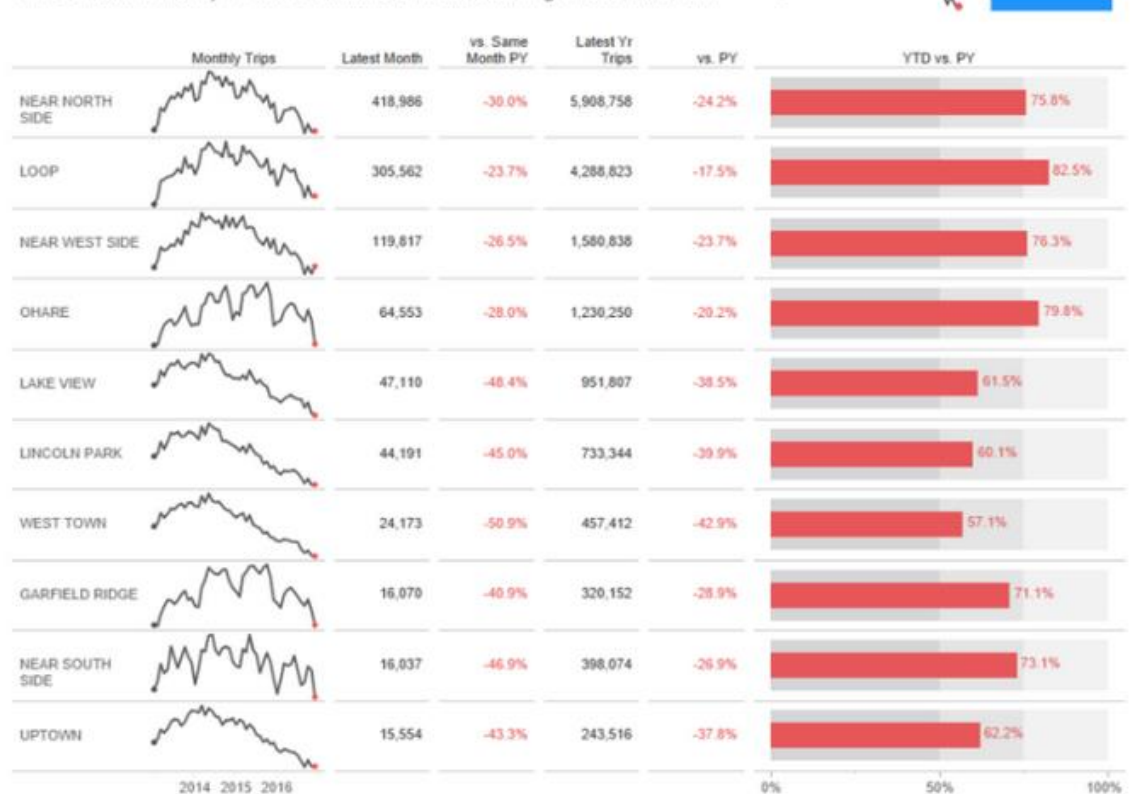
bbc.co.uk/news



Visual table ideas: Sparklines

Sparklines: Mini graphics inserted into table cells or text

Chicago's Taxi Trips Have Been on a **Steady Decline**
A closer look at the top 10 areas confirms taxis are suffering across the board



From: <https://www.pluralsight.com/guides/tableau-playbook-sparklines>

Linear models

- Model:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i$$

- Xs: quantitative predictors, factors, interactions, ...
- Assumptions:
 - **Linearity**: Predictors (possibly transformed) are linearly related to the outcome, y . [This just means linear in the **parameters**.]
 - **Specification**: No important predictors have been omitted; only important ones included. [This is often key & overlooked.]
 - The “holy trinity”:
 - **Independence**: the errors are uncorrelated
 - **Homogeneity of variance**: $\text{Var}(\varepsilon_i) = \sigma^2 = \text{constant}$
 - **Normality**: ε_i have a normal distribution

$$\left. \begin{array}{l} \text{Independence} \\ \text{Homogeneity of variance} \\ \text{Normality} \end{array} \right\} \varepsilon_i \sim_{iid} \mathcal{N}(0, \sigma^2)$$

The General Linear Model

- “linear” models can include:
 - transformed predictors: \sqrt{age} , $\log(income)$
 - polynomial terms: age^2 , age^3 , $poly(age, n)$
 - categorical “factors”, coded as dummy (0/1) variables
 - treated (Yes/No), Gender (M/F/non-binary)
 - interactions: effects of x_1 vary over levels of x_2
 - treated \times age, treated \times sex, (2 way)
 - treated \times age \times sex (3 way)
- Linear model means linear in the parameters (β_i),
$$y = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 \log(income) + \beta_4 (sex = \text{"F"}) + \beta_5 age \times (sex = \text{"F"}) + \epsilon$$
- In R, all handled by `lm(y ~ ...)`

Fitting linear models in R: `lm()`

- In R, `lm()` for everything
 - Regression models (`X1, ...` **quantitative**)

```
lm(y ~ X1, data=dat)           # simple linear regression
lm(y ~ X1+X2+X3, data=dat)      # multiple linear regression
lm(y ~ (X1+X2+X3)^2, data=dat)  # all two-way interactions
lm(log(y) ~ poly(X,3), data=dat) # arbitrary transformations
```

- ANOVA/ANCOVA models (`A, B, ...` **factors**)

```
lm(y ~ A)                      # one way ANOVA
lm(y ~ A*B)                    # two way: A + B + A:B
lm(y ~ X + A)                  # one way ANCOVA
lm(y ~ (A+B+C)^2)              # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

Fitting linear models in R: `lm()`

- Multivariate models: `lm()` with 2+ y vars
 - Multivariate regression

```
lm(cbind(y1, y2) ~ x1 + x2 + x3)           # std MMreg: all linear
lm(cbind(y1, y2) ~ poly(x1,2) + poly(x2,2)) # response surface
```

- MANOVA/MANCOVA models

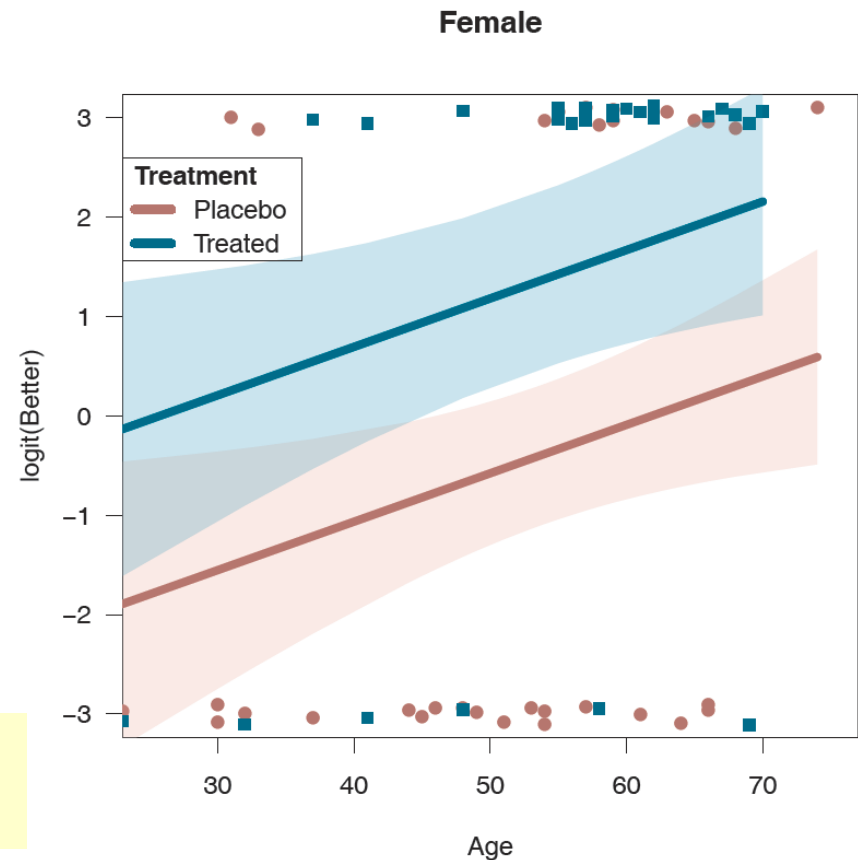
```
lm(cbind(y1, y2, y3) ~ A * B)           # 2-way MANOVA: A + B + A:B
lm(cbind(y1, y2, y3) ~ X + A)           # MANCOVA (equal slopes)
lm(cbind(y1, y2) ~ X + A + X:A)         # heterogeneous slopes
```

Generalized Linear Models: glm()

Transformations of y & other error distributions

- $y \in (0/1)$: lived/died; success/fail; ...
- logit (log odds) model:
 - $\text{logit}(y) = \log \frac{\Pr(y=1)}{\Pr(y=0)}$
 - linear logit model:
 $\text{logit}(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots$

```
glm(better ~ age + treat, family=binomial,  
    data=Arthritis)
```



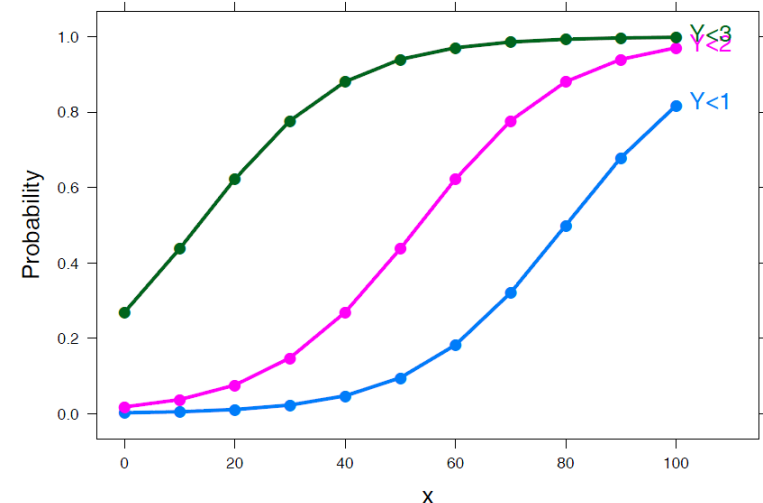
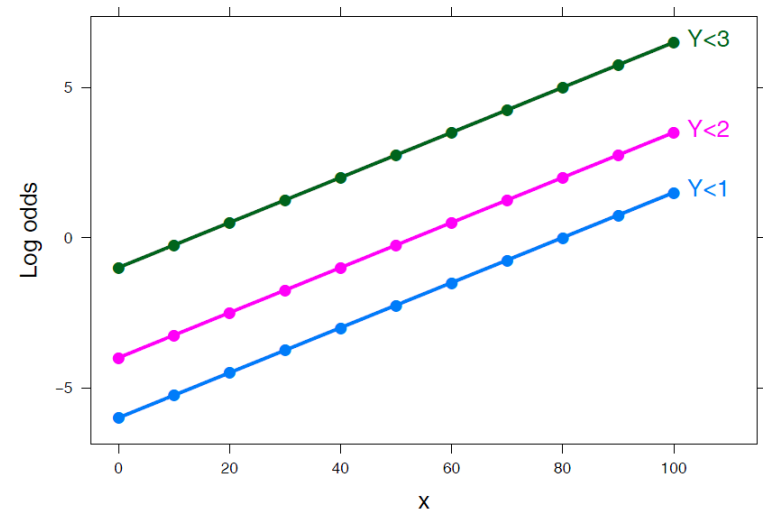
Generalized Linear Models

Ordinal responses

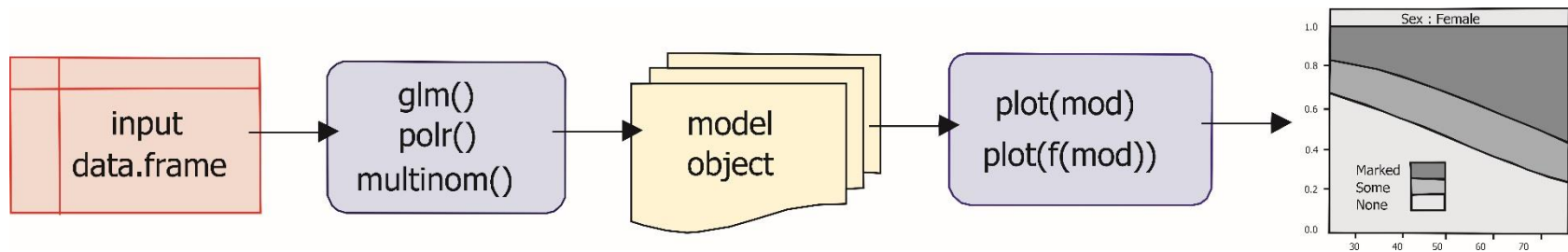
- Improved \in (“None” < “Some” < “Marked”)
- Models: Proportional odds, generalized logits, ...

```
library(MASS)
polr(Improved ~ Sex + Treat + Age,
     data=Arthritis)

library(nnet)
multinom(Improved ~ Sex + Treat + Age,
         data=Arthritis)
```



Model-based methods: Overview



- models in R are specified by a symbolic model formula, applied to a data.frame
 - `mod<-lm`
 - `mod<-glm(better ~ age + sex + treat, data=Arthritis, family=binomial)`
 - `mod<-MASS:polr(improved ~ age + sex + treat, data=Arthritis)`
- result (`mod`) is a “model object”, of class “`lm`”, “`glm`”, ...
- method functions:
 - `plot(mod)`, `plot(f(mod))`, ...
 - `summary(mod)`, `coef(mod)`, `predict(mod)`, ...

Plots for linear models

- Data plots:
 - plot response (y) vs. predictors, with smooth summaries
 - scatterplot matrix --- all pairs
- Model (effect) plots
 - plot predicted response (\hat{y}) vs. predictors, **controlling** for variables not shown.
- Diagnostic plots
 - Influence plots: leverage & outliers
 - Spread-level plots (non-constant variance?)

R packages

- **car**
 - Enhanced scatterplots
 - Diagnostic plots
- **effects**
 - Plot fitted effects of one predictor, controlling all others
- **visreg**
 - similar to effect plots, simpler syntax
- Both **effects** & **visreg** handle nearly all formula-based models
 - `lm()`, `glm()`, `gam()`, `rlm`, `nlme()`, ...

Occupational Prestige data

- Data on prestige of 102 occupations and
 - average education (years)
 - average income (\$)
 - % women
 - type (Blue Collar, Professional, White Collar)

```
> car::some(Prestige, 6)
```

	education	income	women	prestige	census	type
architects	15.44	14163	2.69	78.1	2141	prof
physicians	15.96	25308	10.56	87.2	3111	prof
commercial.artists	11.09	6197	21.03	57.2	3314	prof
tellers.cashiers	10.64	2448	91.76	42.3	4133	wc
bakers	7.54	4199	33.30	38.9	8213	bc
aircraft.workers	8.78	6573	5.78	43.7	8515	bc

Informative scatterplots

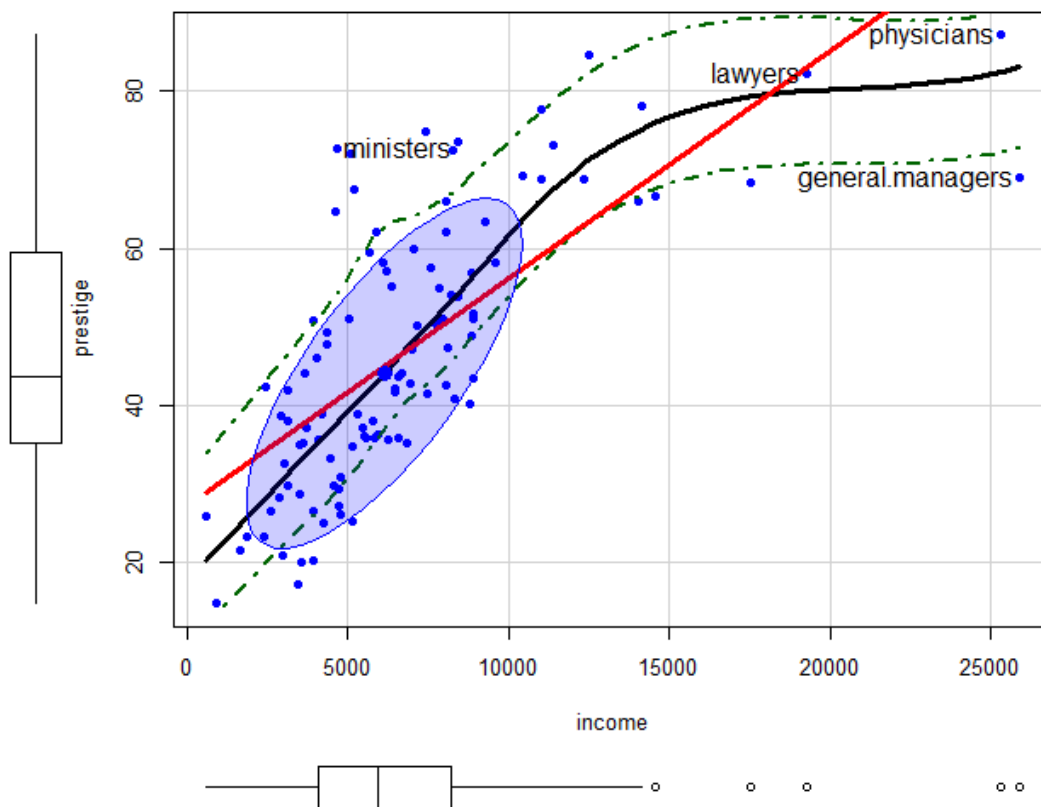
Scatterplots are most useful when enhanced with annotations & statistical summaries

Data ellipse and regression line show the linear model, $\text{prestige} \sim \text{income}$

Point labels show possible outliers

Smoothed (loess) curve and CI show the trend

Boxplots show marginal distributions



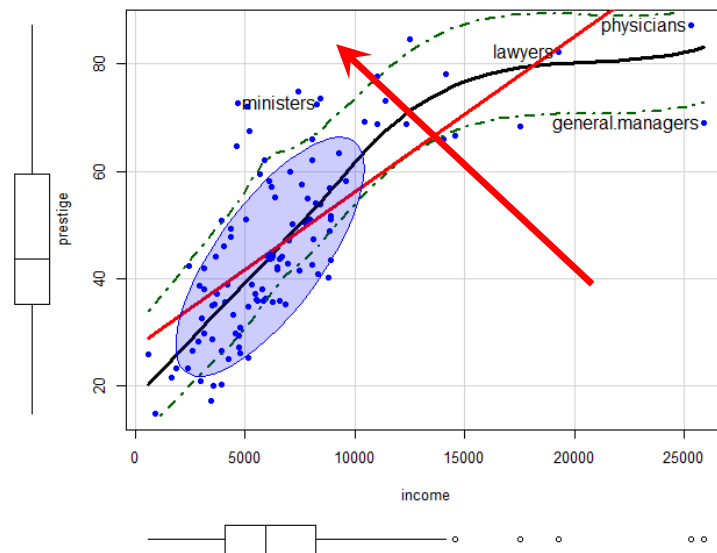
Informative scatterplots

`car::scatterplot()` provides all these enhancements

```
scatterplot(prestige ~ income, data=Prestige,  
            pch = 16,  
            regLine = list(col = "red", lwd=3),  
            smooth = list(smoother=loessLine,  
                          lty.smooth = 1, col.smooth = "black",  
                          lwd.smooth=3, col.var = "darkgreen"),  
            ellipse = list(levels = 0.68),  
            id = list(n=4, col="black", cex=1.2))
```

Skewed distribution of income & non-linear relation suggest need for a transformation

Arrow rule: move on the scale of powers in direction of the bulge
e.g.: $x \rightarrow \sqrt{x}$ or $\log(x)$



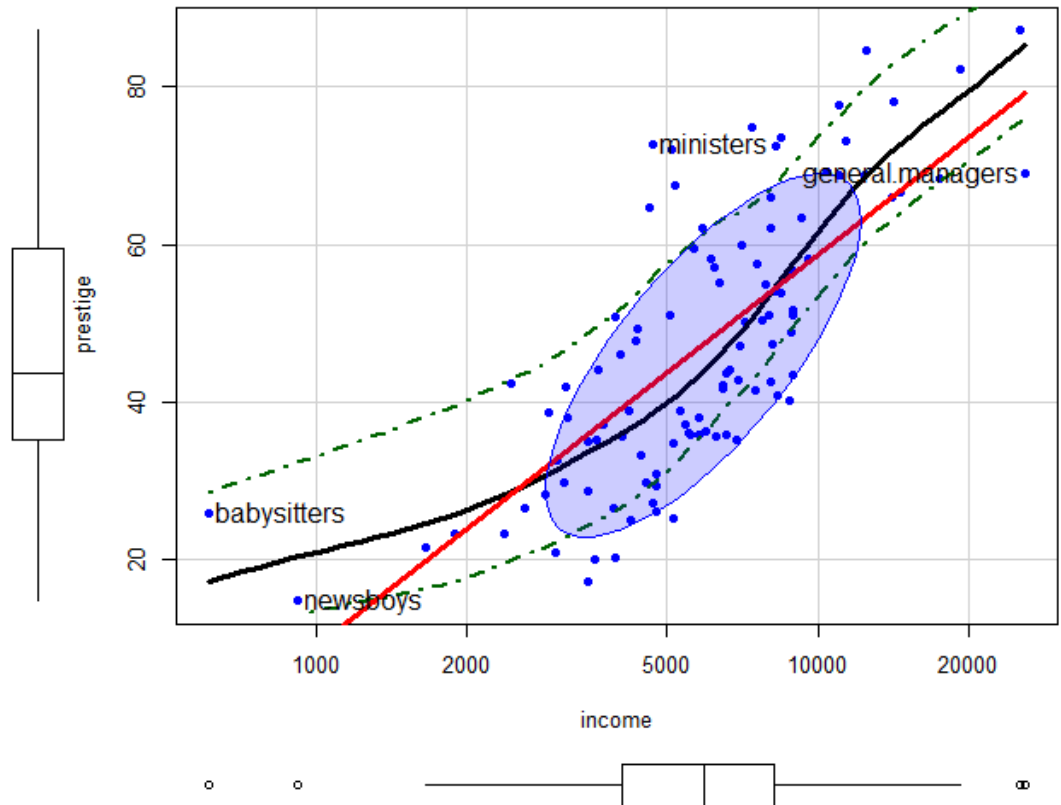
Try log(income)

```
scatterplot
```

Income now ~ symmetric

Relation closer to linear

log(income): interpret as
effect of a multiple



Stratify by type?

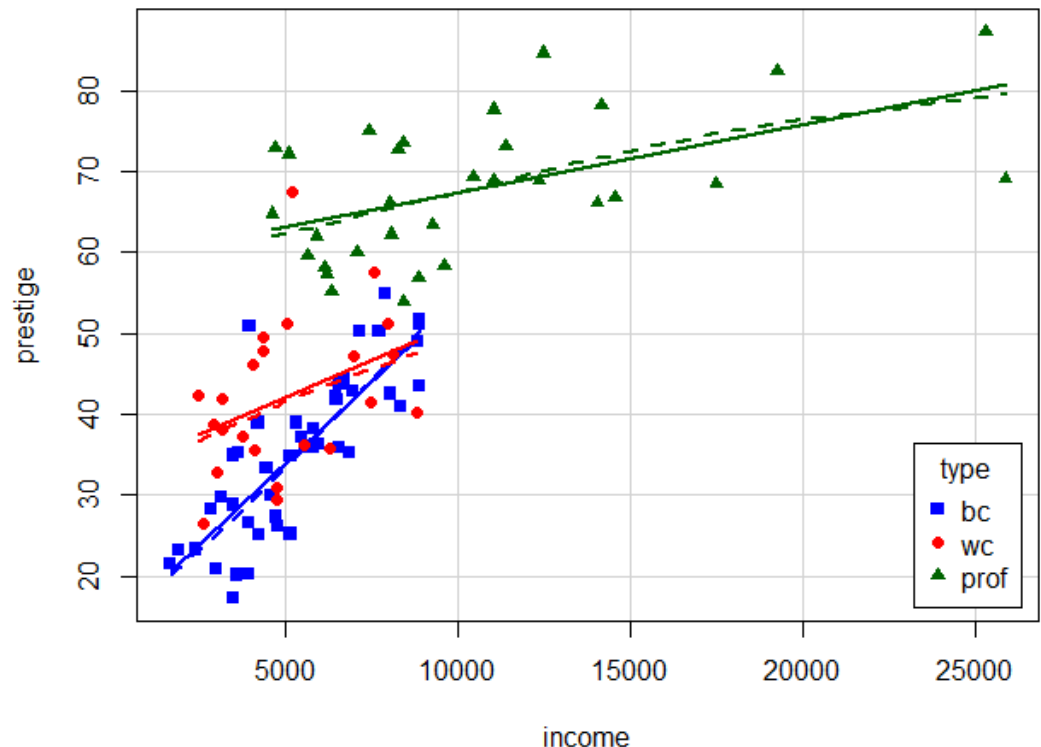
```
scatterplot(prestige ~ income | type, data=Prestige,  
  col = c("blue", "red", "darkgreen"),  
  pch = 15:17,  
  legend = list(coords="bottomright"),  
  smooth=list(smoother=loessLine, var=FALSE, span=1, lwd=4))
```

Formula: | **type** → “given
type”

Different slopes: **interaction** of
income * type

Provides another explanation
of the non-linear relation

This may be a new finding!



Scatterplot matrix

```
scatterplotMatrix(~ prestige + education + income + women ,  
  data=Prestige,  
  regLine = list(method=lm, lty=1, lwd=2, col="black"),  
  smooth=list(smooth=loessLine, spread=FALSE,  
    lty.smooth=1, lwd.smooth=3, col.smooth="red"),  
  ellipse=list(levels=0.68, fill.alpha=0.1))
```

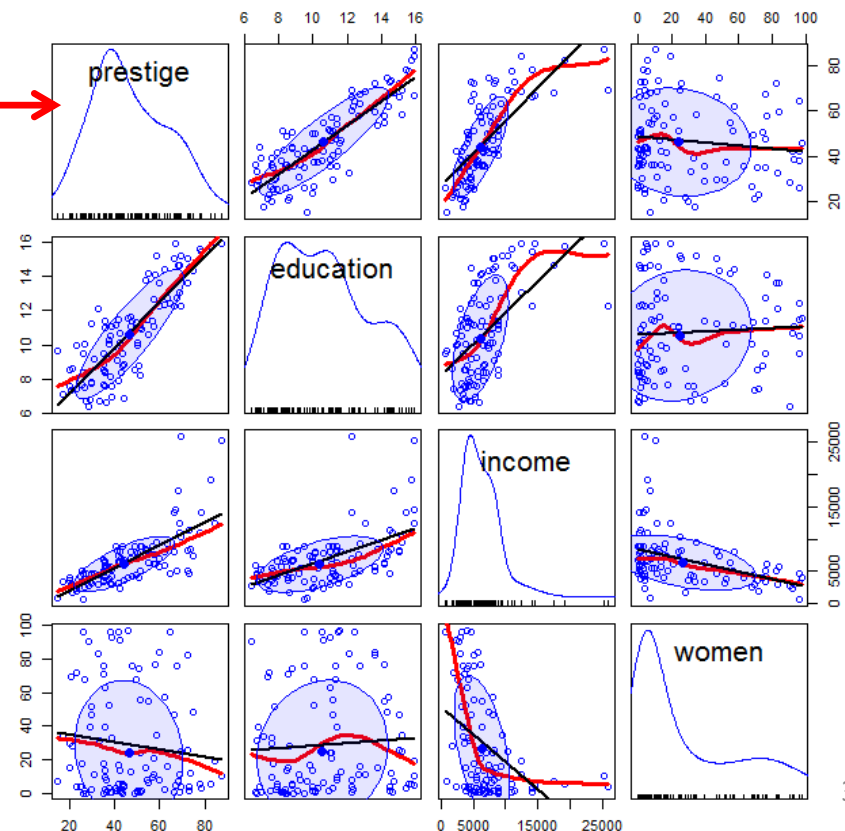
prestige vs. all predictors



diagonal: univariate distributions

- income: + skewed
- %women: bimodal

off-diagonal: relations among predictors



Fit a model

```
> mod1 <- lm(prestige ~ education + poly(women, 2) +  
+ log(income)*type, data=Prestige)  
> summary(mod1)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-137.500	23.522	-5.85	8.2e-08	***
education	2.959	0.582	5.09	2.0e-06	***
poly(women, 2)1	28.339	10.190	2.78	0.0066	**
poly(women, 2)2	12.566	7.095	1.77	0.0800	.
log(income)	17.514	2.916	6.01	4.1e-08	***
typeprof	74.276	30.736	2.42	0.0177	*
typewc	0.969	39.495	0.02	0.9805	
log(income):typeprof	-7.698	3.451	-2.23	0.0282	*
log(income):typewc	-0.466	4.620	-0.10	0.9199	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.879, Adjusted R-squared: 0.868
F-statistic: 81.1 on 8 and 89 DF, p-value: <2e-16

- allow women² term
- interaction of log(income) and type

Fits very well!

Model (effect) plots

- We'd like to see plots of the predicted value (\hat{y}) of the response against predictors (x_j)
 - Ordinary plot of y vs. x_j doesn't allow for other correlations
 - → Must **control** for other predictors (x_{-j}) not shown in a given plot
 - Variables not shown (x_{-j}) are averaged over.
 - Slopes of lines reflect the partial coefficient in the model
 - Partial residuals can be shown also

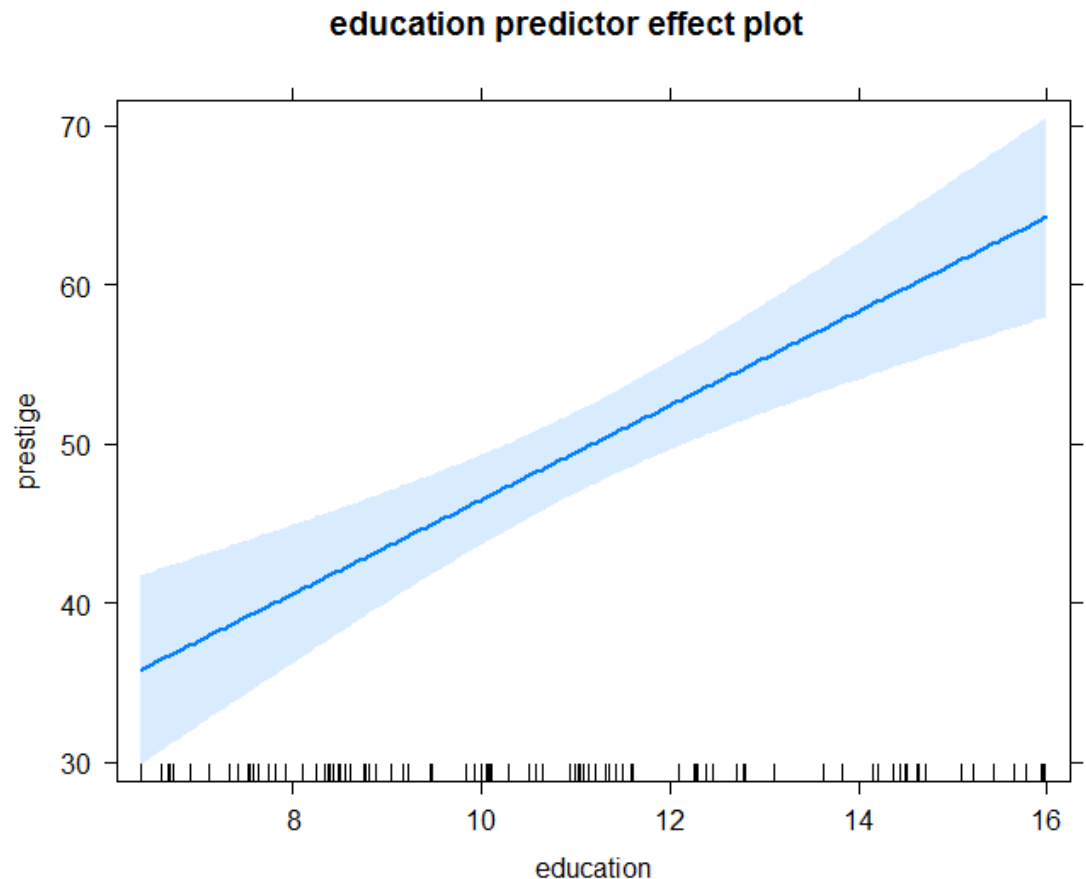
For details, see `vignette("predictor-effects-gallery", package="effects")`

Model (effect) plots: education

```
library("effects")  
mod1.e1 <- predictorEffect("education", mod1)  
plot(mod1.e1)
```

This graph shows the partial slope for education

For each \uparrow year in education, fitted prestige $\uparrow 2.96$ points, (other predictors held fixed)

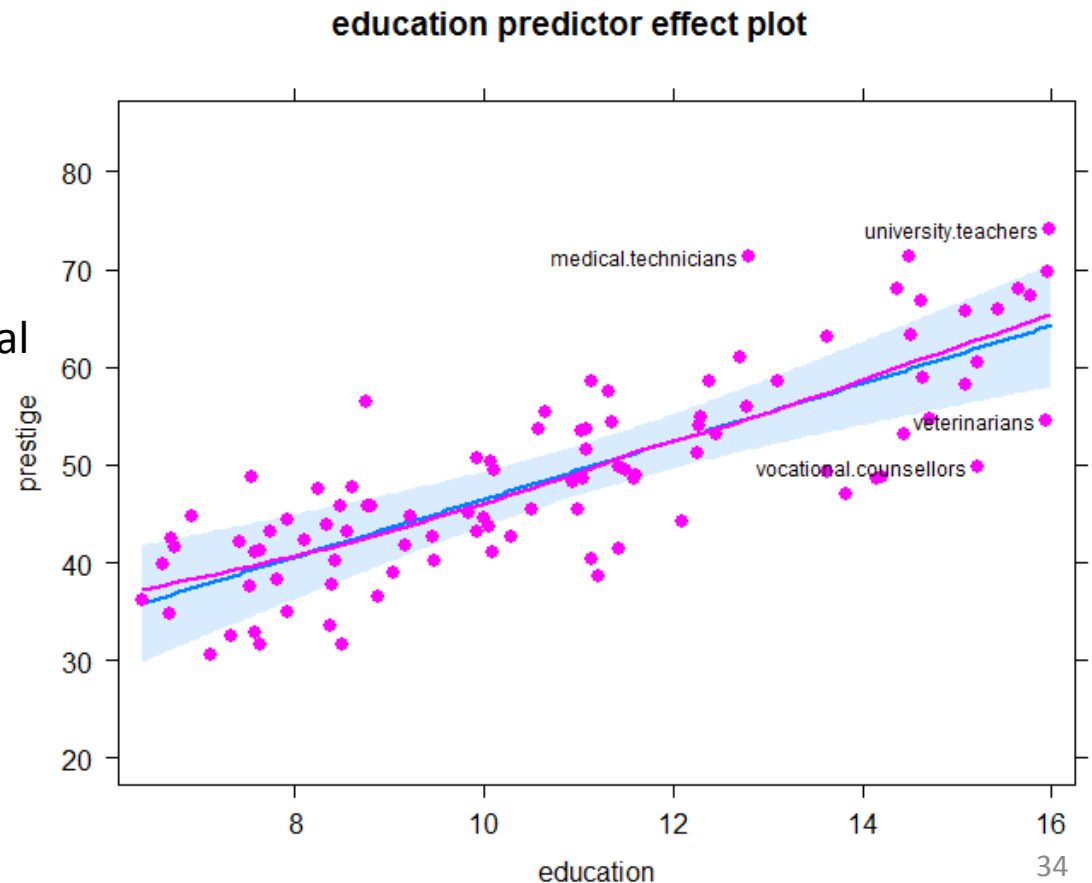


Model (effect) plots

```
mod1.e1a <- predictorEffect("education", mod1, residuals=TRUE)
plot(mod1.e1a,
     residuals.pch=16, id=list(n=4, col="black"))
```

Partial residuals show the residual of prestige controlling for other predictors

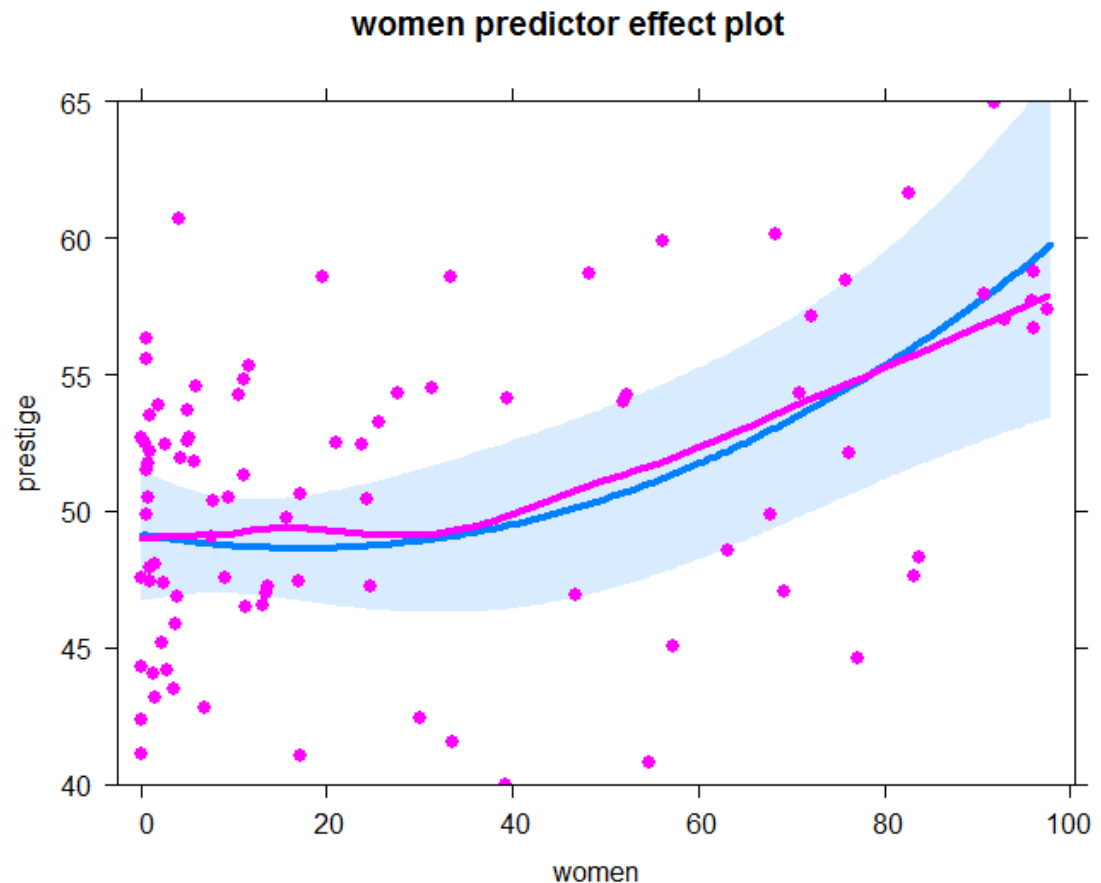
Unusual points here would signal undue influence



Model (effect) plots: women

```
mod1.e2 <- predictorEffect("women", mod1, residuals=TRUE)
plot(mod1.e2, ylim=c(40, 65), lwd=4,
     residuals.pch=16)
```

Surprise!
Prestige of occupations ↑
with % women (controlling
for other variables)

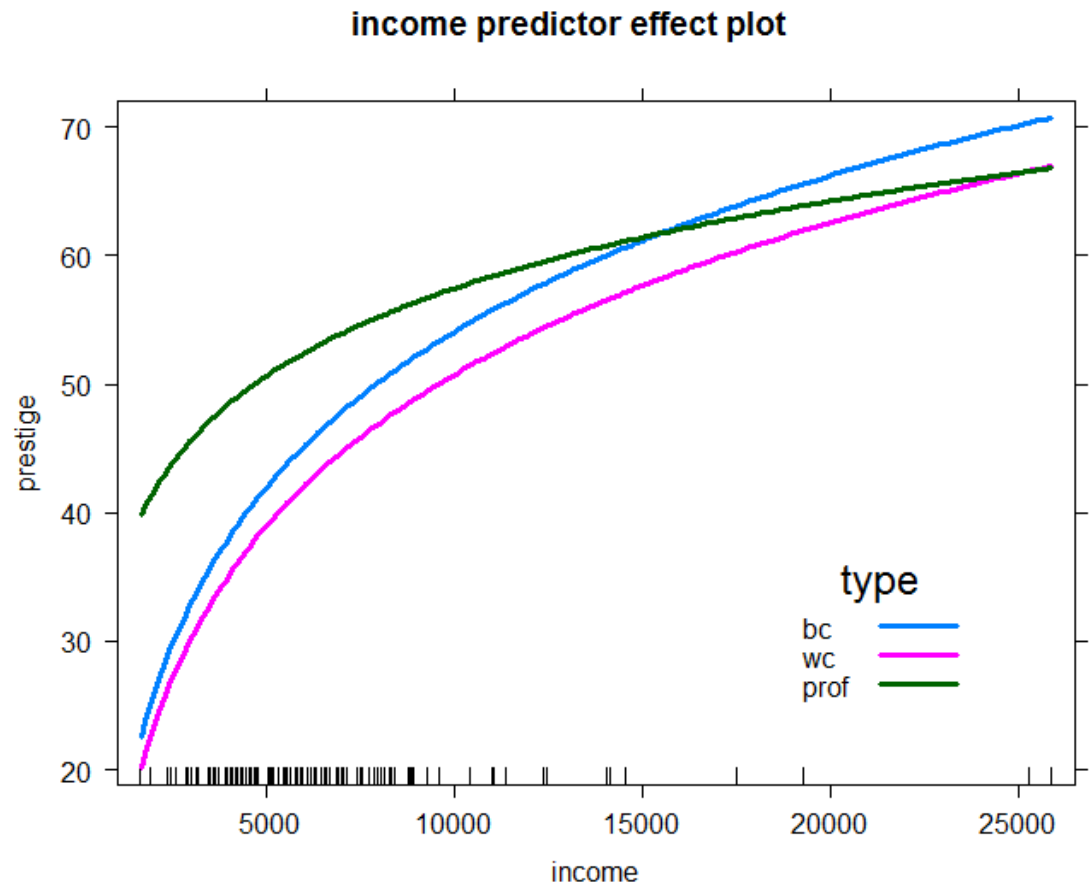


Model (effect) plots: income

```
plot(predictorEffect("income", mod1),  
      lines=list(multiline=TRUE, lwd=3),  
      key.args = list(x=.7, y=.35))
```

Income interacts with type in the model

The plot is curved because $\log(\text{income})$ is in the model



visreg plots: Air quality data

Daily air quality measurements in New York, May - Sep 1973

How does Ozone concentration vary with solar radiation, wind speed & temperature?

```
> head(airquality)
  Ozone Solar.R wind Temp Month Day
1    41    190  7.4   67     5   1
2    36    118  8.0   72     5   2
3    12    149 12.6   74     5   3
4    18    313 11.5   62     5   4
5    NA     NA 14.3   56     5   5
6    28     NA 14.9   66     5   6
```

see: <https://pbreheny.github.io/visreg/> for examples & details

Air quality: main effects model

```
> fit1 <- lm(Ozone ~ Solar.R + Wind + Temp, data=airquality)
> summary(fit1)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	-64.3421	23.0547	-2.79	0.0062	**
Solar.R	0.0598	0.0232	2.58	0.0112	*
Wind	-3.3336	0.6544	-5.09	1.5e-06	***
Temp	1.6521	0.2535	6.52	2.4e-09	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.18 on 107 degrees of freedom

(42 observations deleted due to missingness)

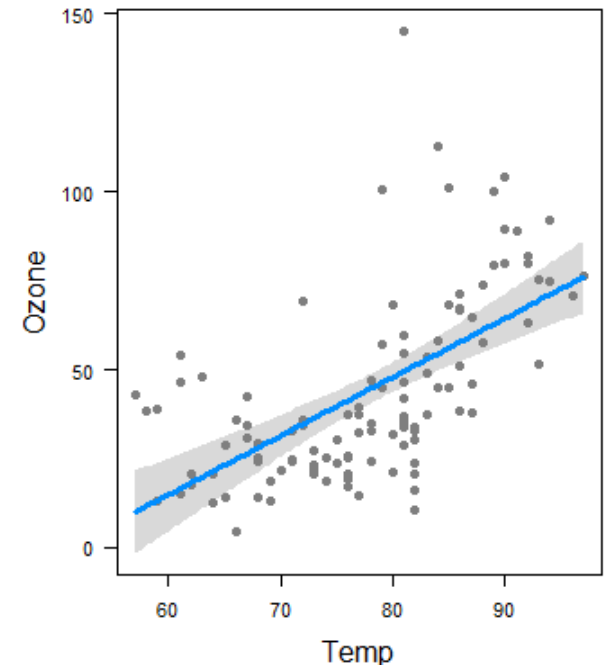
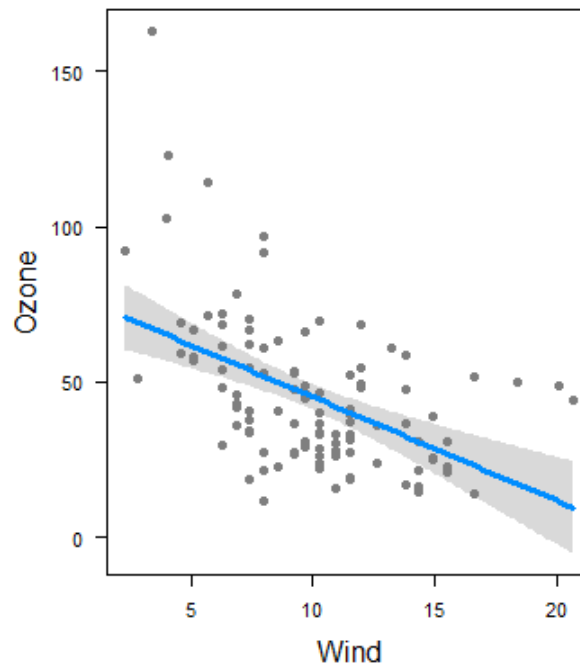
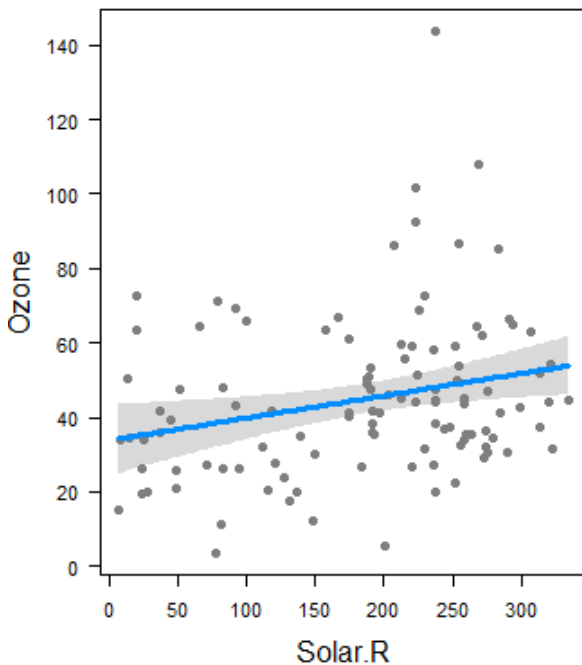
Multiple R-squared: 0.6059, Adjusted R-squared: 0.5948

F-statistic: 54.83 on 3 and 107 DF, p-value: < 2.2e-16

visreg conditional plots

```
op <- par(mfrow=c(1,3), cex.lab=1.5)
visreg(fit1, "solar.R")
visreg(fit1, "wind")
visreg(fit1, "Temp")
par(op)
```

model summary =
predicted values (line) +
confidence band (uncertainty) +
partial residuals (objections)



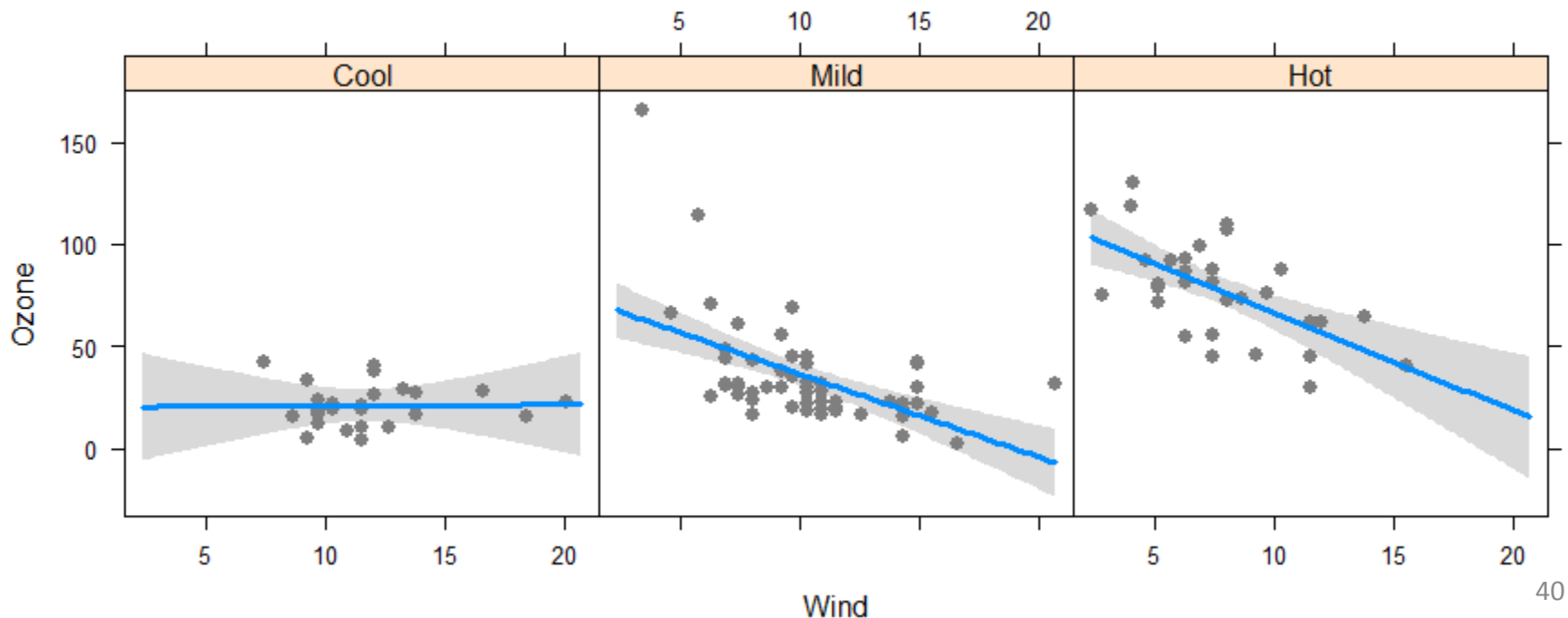
Factor variables & interactions

cut Temp into three ordered levels of equal range

```
airquality$Heat <- cut(airquality$Temp, 3,  
  labels=c("Cool","Mild","Hot"))
```

fit model with interaction of **Wind * Heat**

```
fit2 <- lm(Ozone ~ Solar.R + Wind*Heat, data=airquality)  
visreg(fit2, "Wind", by="Heat", layout=c(3,1), points=list(cex=1))
```



Factor variables & interactions

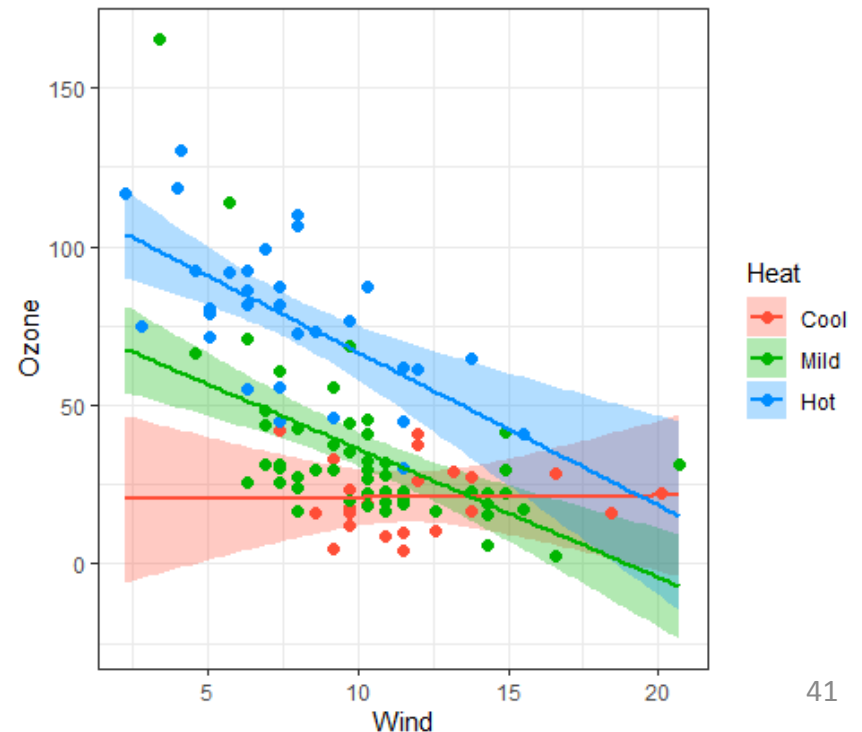
```
visreg(fit2, "wind", by="Heat",  
      overlay=TRUE,  
      gg=TRUE,  
      points=list(size=2)) +  
theme_bw()
```

`overlay=TRUE` → superpose panels

`gg=TRUE` → uses ggplot

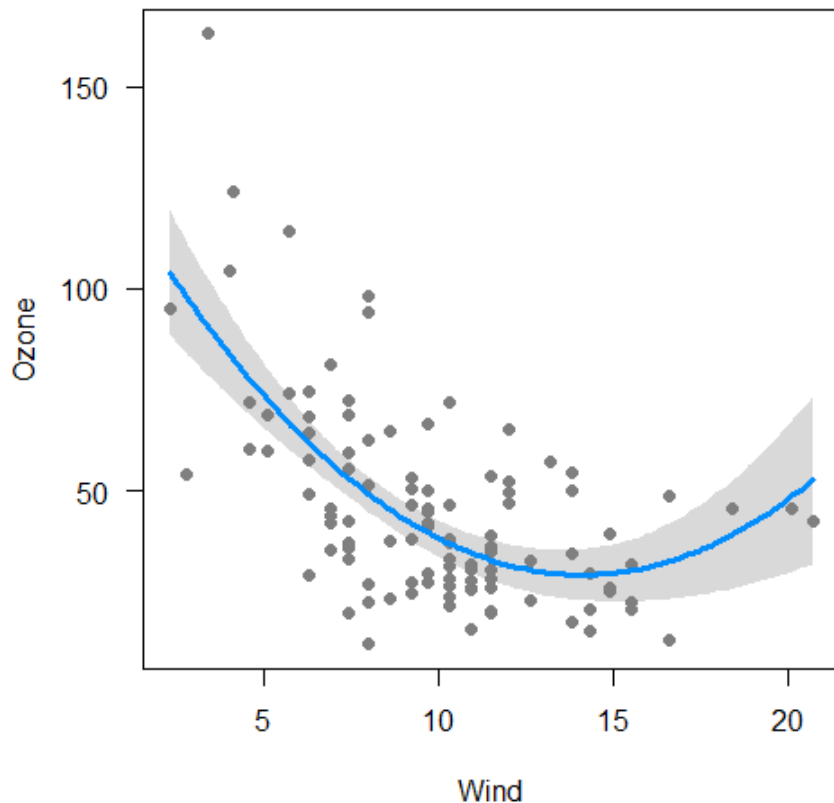
This allow slope for Wind to vary with Heat e.g., Wind has no effect when Cool

This model still assumes **linear** effects of Heat & Wind

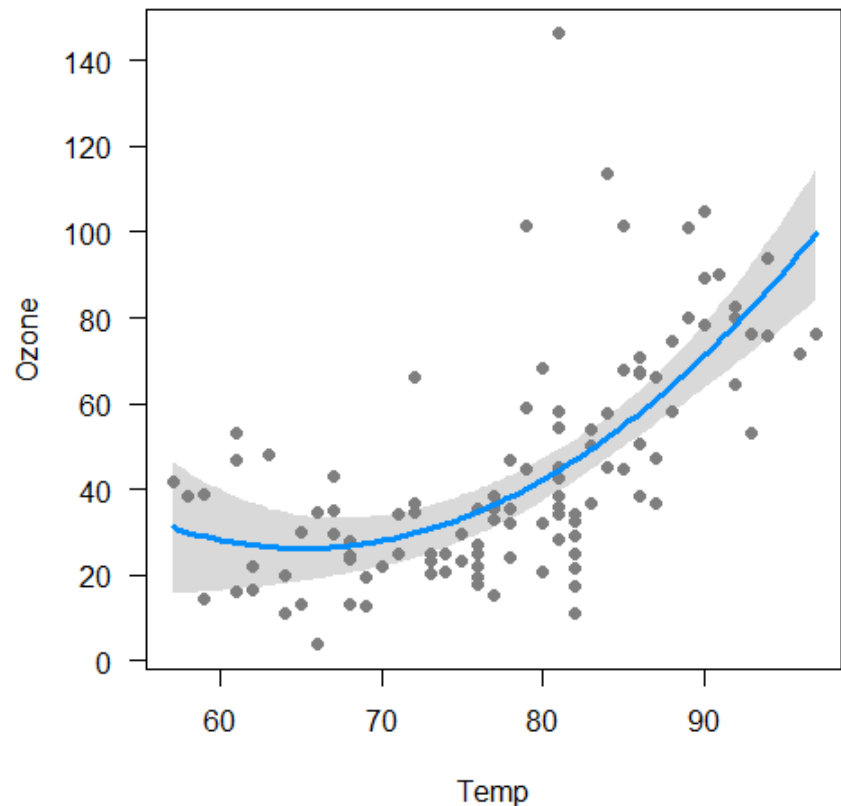


Non-linear effects

```
fit <- lm(Ozone ~ Solar.R + poly(Wind,2) +  
         Temp, data=airquality)  
visreg(fit, "Wind")
```



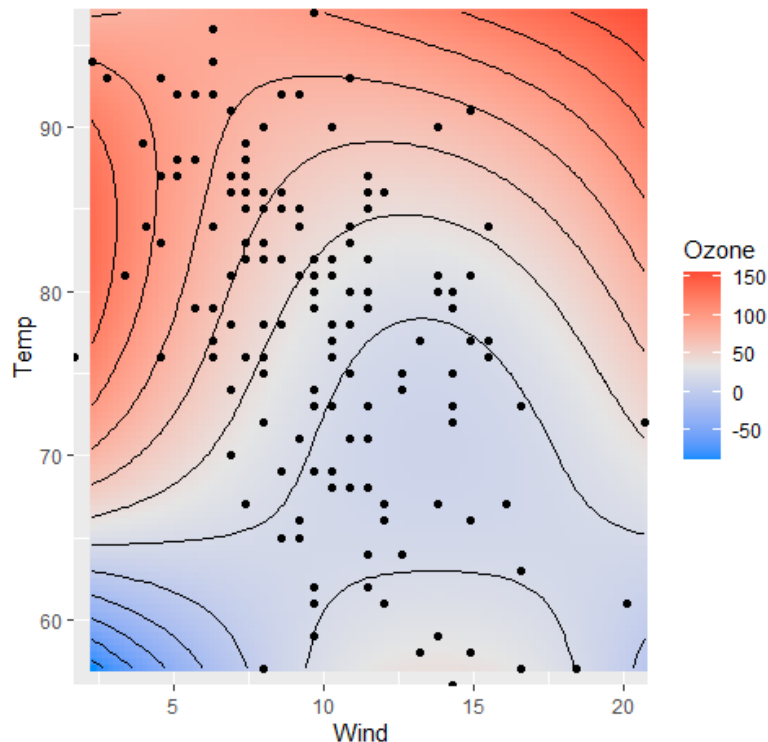
```
fit <- lm(Ozone ~ Solar.R + Wind +  
         poly(Temp,2), data=airquality)  
visreg(fit, "Temp")
```



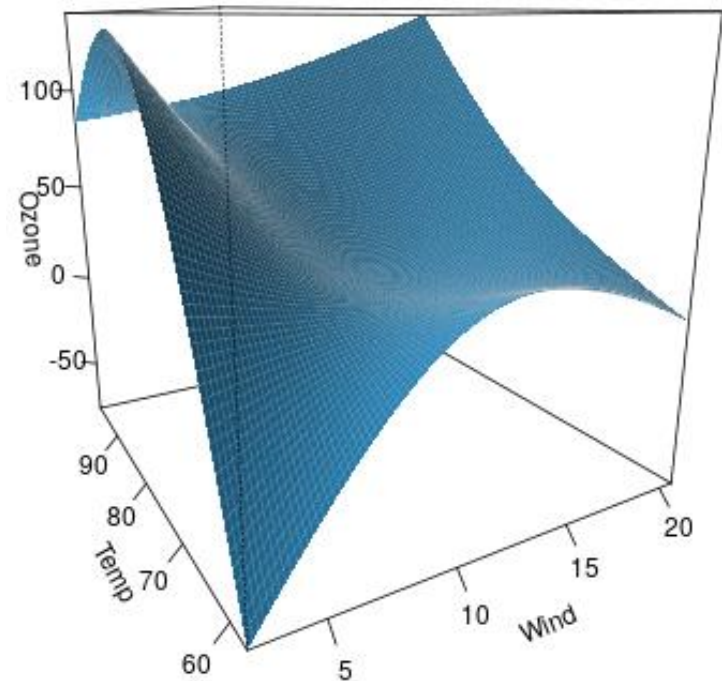
Response surface models (visreg2d)

```
# Fit quadratics in both Wind & Temp and interaction Wind * Temp  
fitp <- lm(Ozone ~ Solar.R + poly(Wind,2) * poly(Temp,2), data=airquality)
```

```
visreg2d(fitp, "Wind", "Temp", plot.type="gg") +  
  geom_contour(aes(z=z), color="black")
```



```
visreg2d(fitp, "Wind", "Temp", plot.type="persp" )
```



Regression trees

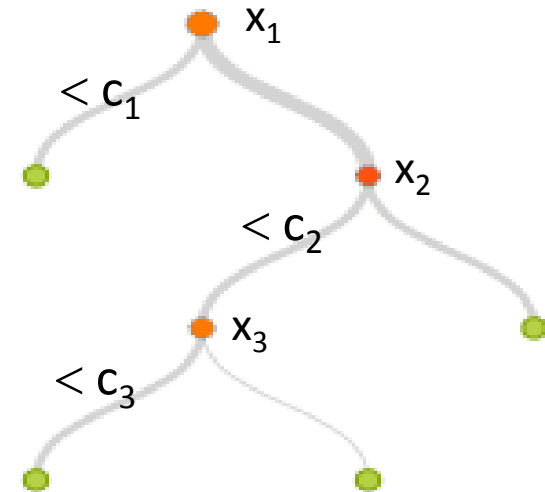
Regression trees are a non-parametric alternative to linear models

- Essential ideas:

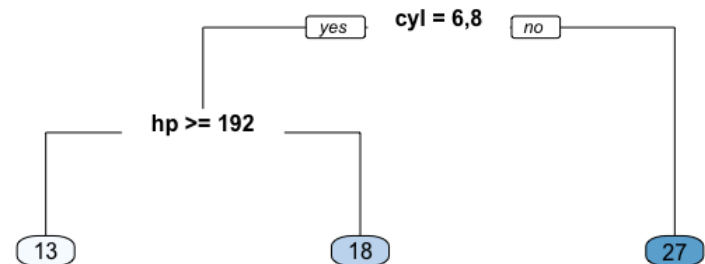
- Find predictor and split value which minimizes SSE
- fitted value in each subgroup = mean
- repeat, recursively, splitting by next best predictor

- Large literature

- cost, complexity tradeoff
- pruning methods
- boosting, cross-validation
- tree averaging



e.g.: $\text{mpg} \sim \text{cyl} + \text{hp}$



Prestige data: rpart tree

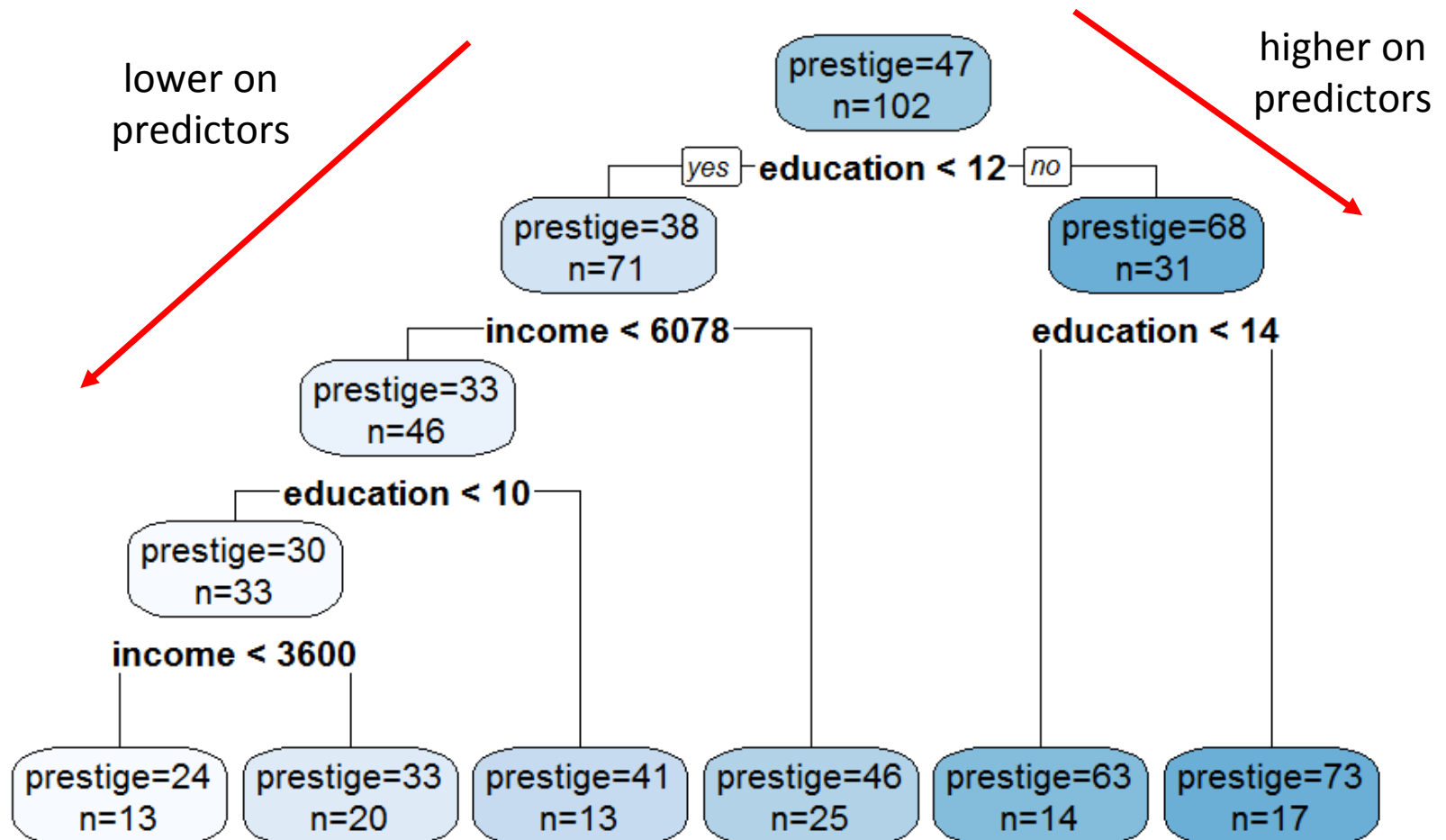
```
> library(rpart)           # calculating regression trees
> library(rpart.plot)      # plotting regression trees

> rmod <- rpart(prestige ~ education + income + women + type,
               data=Prestige,
               method = "anova")

> rpart.rules(rmod)        # print prediction rules
prestige
  24 when education < 10      & income < 3600
  33 when education < 10      & income is 3600 to 6078
  41 when education is 10 to 12 & income < 6078
  46 when education < 12      & income >= 6078
  63 when education is 12 to 14
  73 when education >= 14
```

Prestige data: rpart tree

```
rpart.plot(rmod, prefix="prestige=")
```



Diagnostic plots

- The linear model, $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ assumes:
 - Residuals, ε_i are normally distributed, $\varepsilon_i \sim N(0, \sigma^2)$
 - (Normality not required for \mathbf{X} s)
 - Constant variance, $\text{Var}(\varepsilon_i) = \sigma^2$
 - Observations y_i are statistically independent
- Violations \rightarrow inferences may not be valid
- A variety of plots can diagnose all these problems
- Other methods (boxCox, boxTidwell) diagnose the need for transformations of \mathbf{y} or \mathbf{X} s.

The “regression quartet”

In R, plotting a `lm` model object → the “regression quartet” of plots

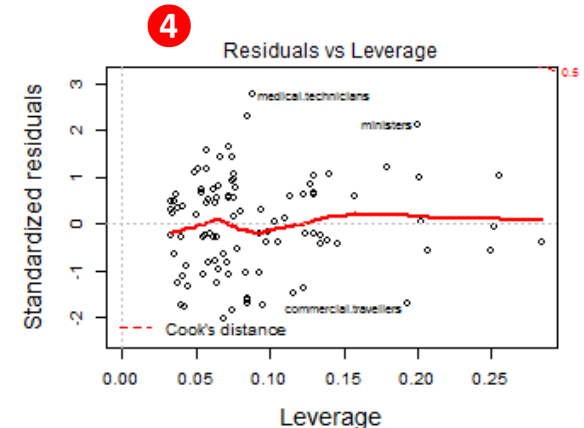
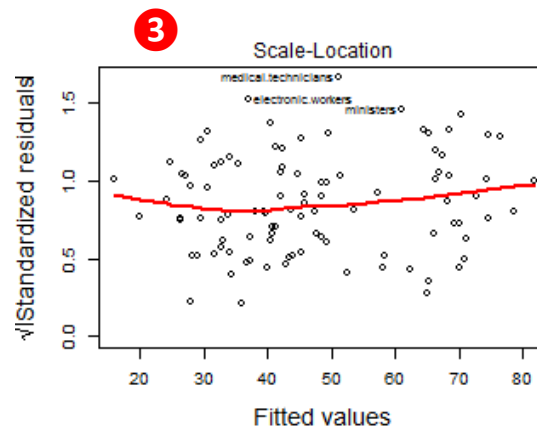
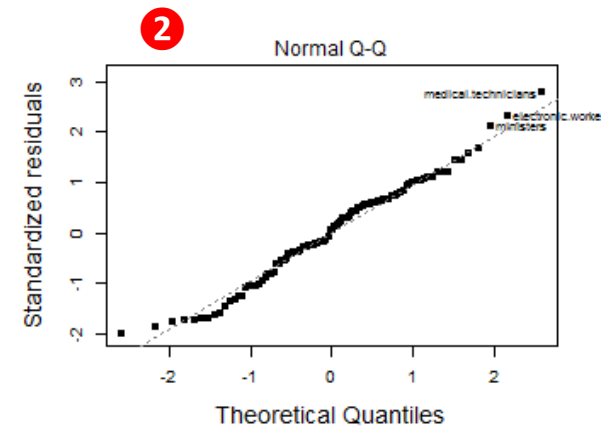
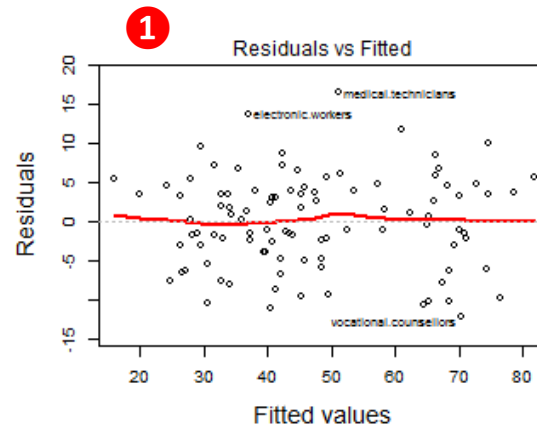
```
plot(mod1, lwd=2, cex.lab=1.4)
```

❶ Residuals: should be flat vs. fitted values

❷ Q-Q plot: should follow the 45° line

❸ Scale-location: should be flat if constant variance

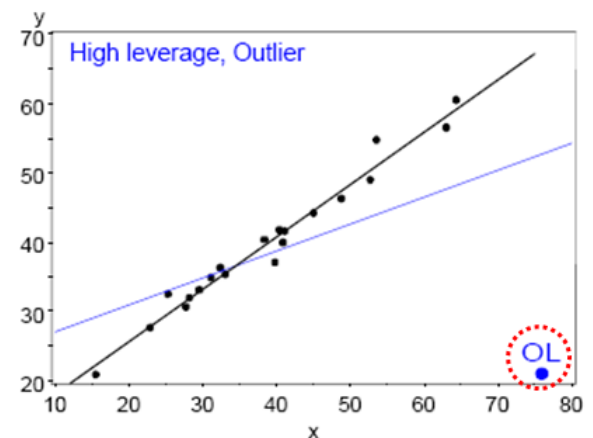
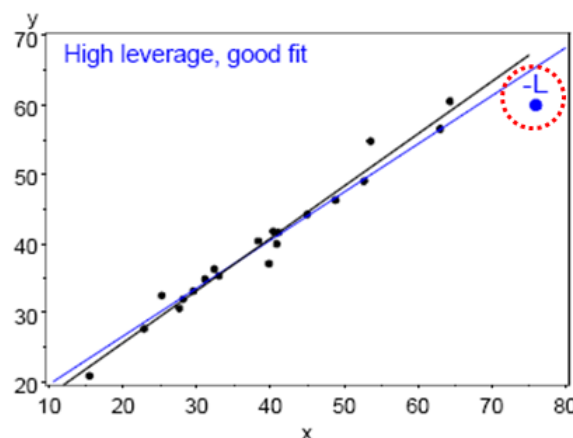
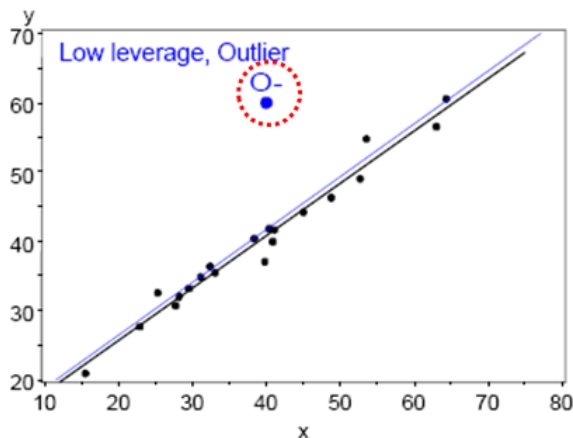
❹ Resids vs. leverage: can show influential observations



Unusual data: Leverage & Influence

- “Unusual” observations can have dramatic effects on least-squares estimates in linear models
- Three archetypal cases:
 - Typical X (low leverage), bad fit -- Not much harm
 - Unusual X (high leverage), good fit -- Not much harm
 - Unusual X (high leverage), bad fit -- **BAD, BAD, BAD**
- Influential observations: unusual in *both* X & Y
- Heuristic formula:

$$\text{Influence} = X \text{ leverage} \times Y \text{ residual}$$



Influence plots

Influence (Cook's D) measures impact of individual obs. on coefficients, fitted values

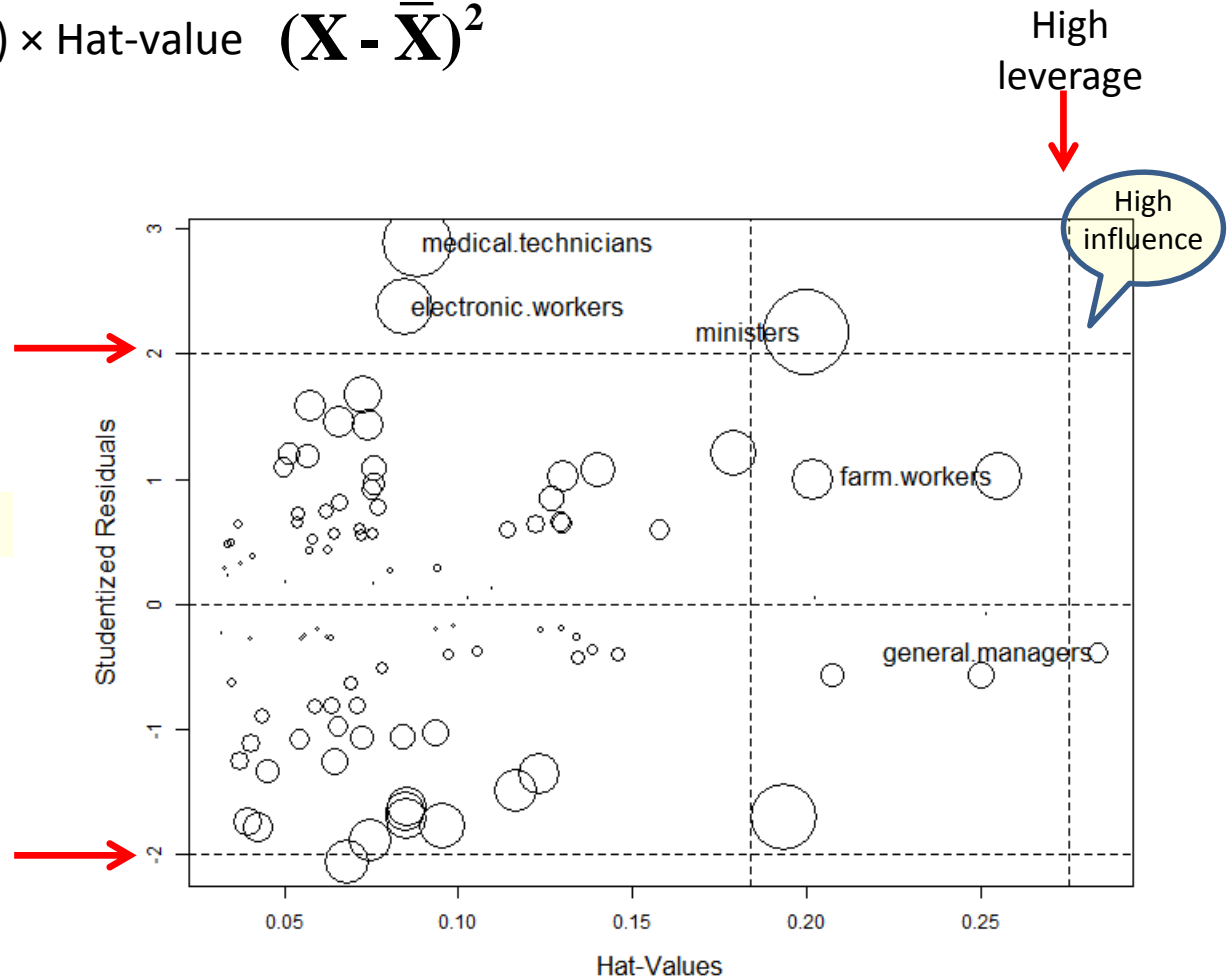
Influence \sim Residual $(y - \hat{y}) \times \text{Hat-value}$ $(\mathbf{X} - \bar{\mathbf{X}})^2$

Bubble size \sim influence

`influencePlot(mod1)`

Bad fit

Bad fit

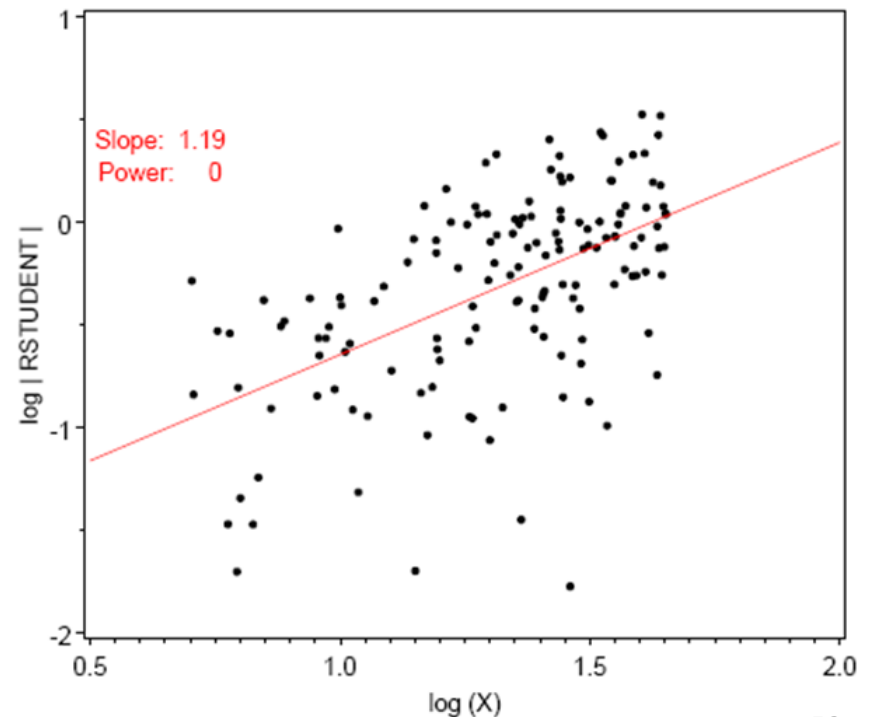


Spread-level plots

- To diagnose non-constant variance, plot:
 - $\log |\text{Std. residual}|$ vs. $\log(x)$
 - $\log(\text{IQR})$ vs $\log(\text{median})$ [for grouped data]
- If \approx linear w/ slope b , transform $y \rightarrow y^{(1-b)}$

Artificial data, generated so $\sigma \sim x$

- $b \approx 1 \rightarrow \text{power} = 0$
- \rightarrow analyze $\log(y)$



Spread-level plot: baseball data

Data on salary and batter performance from 1987 season

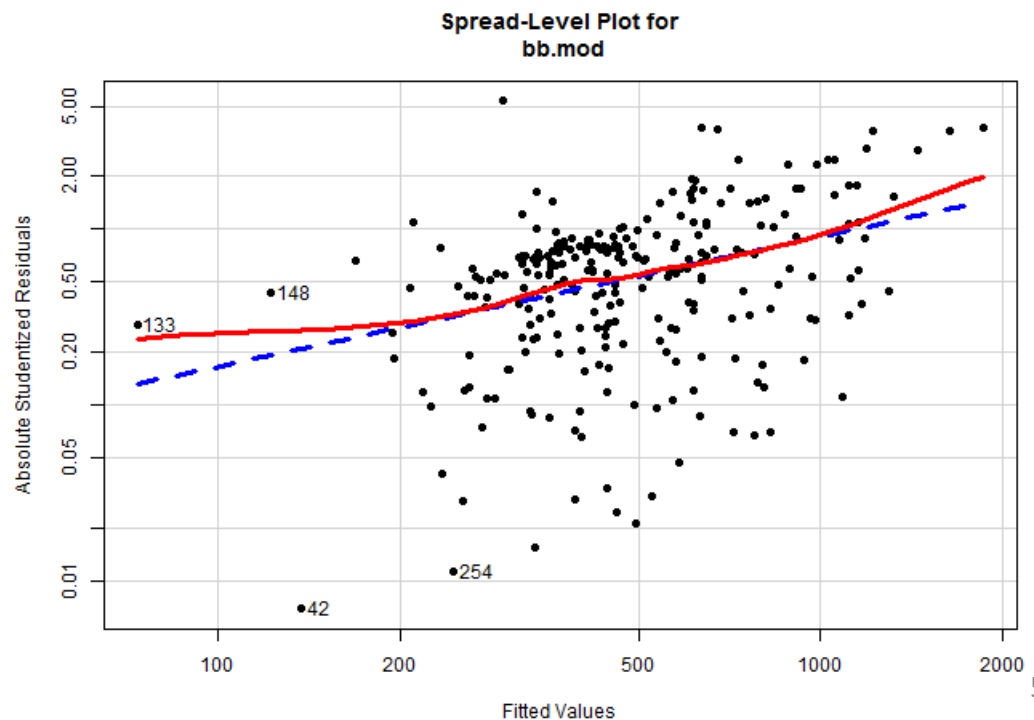
```
data("Baseball", package="vcd")
bb.mod <- lm(sal87 ~ years + hits + runs + homeruns, data=Baseball)
spreadLevelPlot(bb.mod, pch=16, lwd=3,
                id=list(n=2))
```

Suggested power transformation: 0.2609

slope = .74 \rightarrow p = .26

i.e., $y \rightarrow \log(y)$ or $y^{1/4}$

NB: both axes plotted on log scale



Box Cox transformation

- Box & Cox proposed to transform y to a power, $y \rightarrow y^{(\lambda)}$ to minimize the residual SS (or maximize the likelihood)
 - Makes $y^{(\lambda)}$ more nearly normal
 - Makes $y^{(\lambda)}$ more nearly linear in with X

Formula for $y^{(\lambda)}$

- $y^{(0)} : \log_e(y)$
- $\lambda < 0$: flip sign to keep same order

$$y_i^{(\lambda)} = \begin{cases} \frac{y_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0, \\ \ln(y_i) & \text{if } \lambda = 0, \end{cases}$$

Power(p)	Transformation	Name
2	Y^2	Square
1	Y (No transformation)	Original Data
$\frac{1}{2}$	\sqrt{Y}	Square root
"0"	$\log Y$ or $\log_{10}(Y)$	Logarithm
$-\frac{1}{2}$	$-1 / \sqrt{Y}$	Reciprocal Root
-1	$-1 / Y$	Reciprocal
-2	$-1 / Y^2$	Reciprocal Square

Example: Cars93 data

How does gas mileage (MPG.city) depend on vehicle weight?

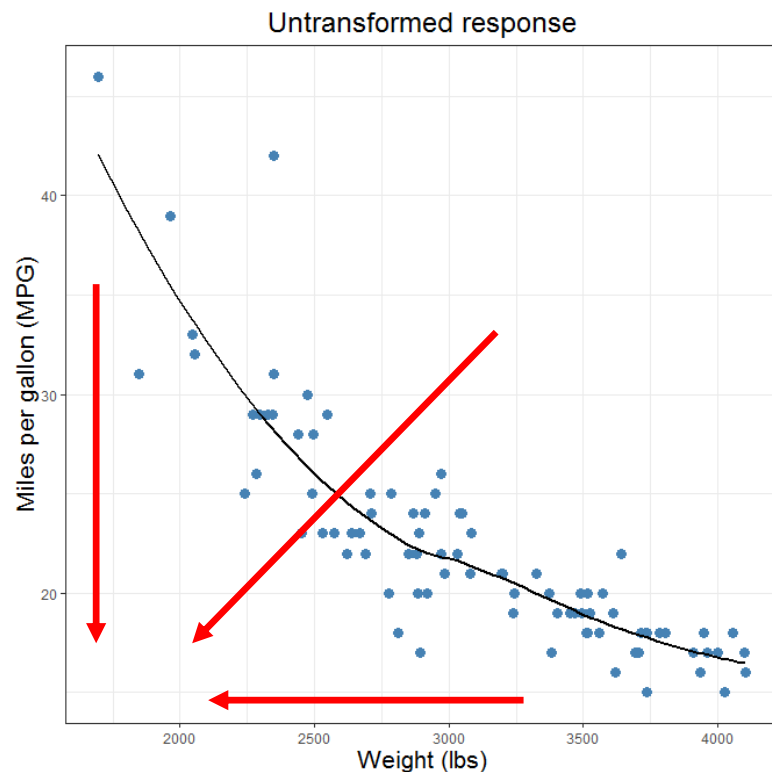
```
> cars.mod <- lm(MPG.city ~ Weight, Cars93)
> coef(cars.mod)
(Intercept)      weight
  47.04835      -0.00803
```

Relationship clearly non-linear

Tukey arrow rule: transform Y (or X)
as arrow thru the curve bulges

$y \rightarrow \sqrt{y}, \log(y), 1/y$

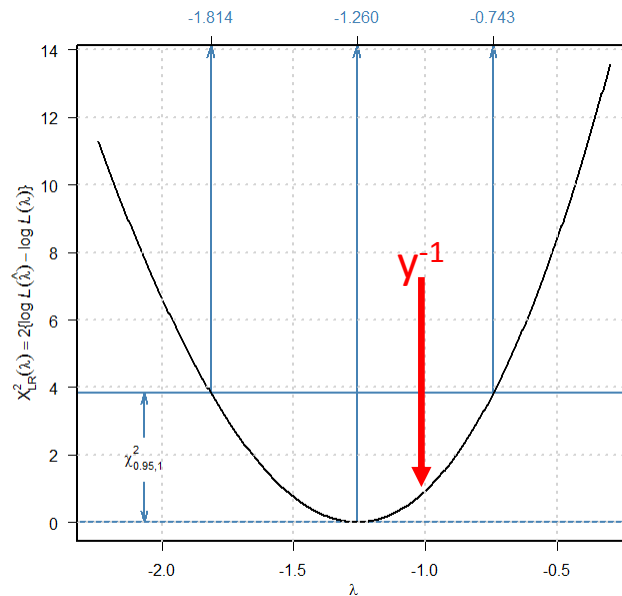
$x \rightarrow \sqrt{x}, \log(x), 1/x$



MASExtra package

```
> library(MASExtra)
> box_cox(cars.mod)    # plot log likelihood vs. lambda
> lamba(cars.mod)
[1] -1.26
```

The plot of $-\log(L) \sim \text{RSS}$ shows the minimum & CI



`plot(bc(MPG.city, lamba(cars.mod)))`



Summary

- Tables are for look-up; graphs can give insight
- “Linear” models include so much more than ANOVA & regression
- Data plots are more effective when enhanced
 - data ellipses → strength & precision of correlation
 - regression lines and smoothed curves
 - point identification → noteworthy observations
- Effect plots show informative views of models
 - Visualize conditional effects, holding others constant
- Diagnostic plots can reveal influential observations and need for transformations.