

Visualizing Linear Models: An R Bag of Tricks Session 1: Getting Started

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SCS Short Course
Oct-Nov, 2021

<https://friendly.github.io/VisMLM-course/>

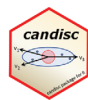
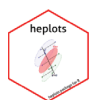
Today's topics

- What you need for this course
- Why plot your data?
- Data plots
- Model (effect) plots
- Diagnostic plots

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What you need

- R, version ≥ 3.6
 - Download from <https://cran.r-project.org/>
- RStudio IDE, highly recommended
 - <https://www.rstudio.com/products/rstudio/>
- R packages: see course web page
 - car
 - effects
 - heplots
 - candisc
 - visreg



R script to install packages:
<https://friendly.github.io/VisMLM-course/R/install-vismlm-pkgs.r>

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Why plot your data?

Getting information from a table is like extracting sunlight from a cucumber. --- Farquhar & Farquhar, 1891

Information that is imperfectly acquired, is generally as imperfectly retained; and a man who has carefully investigated a printed table, finds, when done, that he has only a very faint and partial idea of what he has read; and that like a figure imprinted on sand, is soon totally erased and defaced.

--- William Playfair, *The Commercial and Political Atlas* (p. 3), 1786

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Cucumbers

Table 7
Stevens et al. 2006, table 2: Determinants
of authoritarian aggression

Variable	Coefficient (Standard Error)
Constant	.41 (.93)
Countries	
Argentina	1.31 (.33)**B,M
Chile	.93 (.32)**B,M
Colombia	1.46 (.32)**B,M
Mexico	.07 (.32) ^{A,CH,CO,V}
Venezuela	.96 (.37)**B,M
Threat	
Retrospective egocentric economic perceptions	.20 (.13)
Prospective egocentric economic perceptions	.22 (.12)*
Retrospective sociotropic economic perceptions	-.21 (.12)*
Prospective sociotropic economic perceptions	-.32 (.12)*
Ideological distance from president	-.27 (.07)**
Ideology	
Ideology	.23 (.07)**
Individual Differences	
Age	.00 (.01)
Female	-.03 (.21)
Education	.13 (.14)
Academic Sector	.15 (.29)
Business Sector	.31 (.25)
Government Sector	-.10 (.27)
R ²	.15
Adjusted R ²	.12
N	500

Results of a one model for authoritarian
aggression

The information is overwhelmed by
footnotes & significance **stars**

**p < .01, *p < .05, ^Ap < .10 (twotailed)

^ACoefficient is significantly different from Argentina's at
p < .05;

^BCoefficient is significantly different from Brazil's at p < .05;

^{CH}Coefficient is significantly different from Chile's at p < .05;

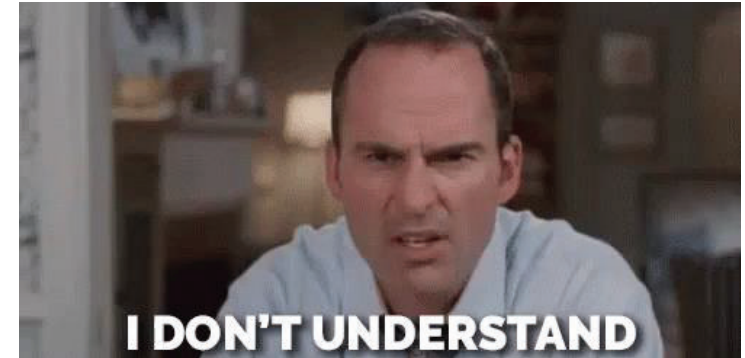
^{CO}Coefficient is significantly different from Colombia's at
p < .05;

^VCoefficient is significantly different from Mexico's at p < .05;

^VCoefficient is significantly different from Venezuela's at
p < .05.

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What's wrong with this picture?

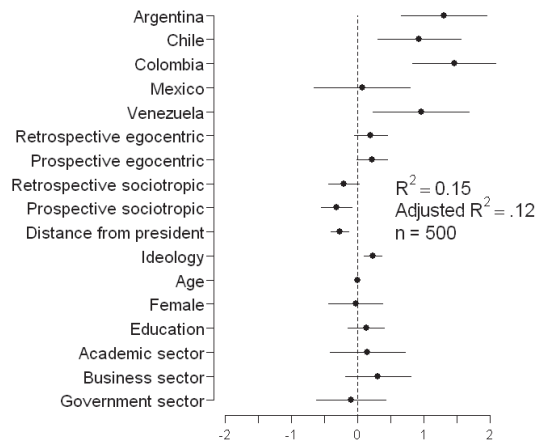


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Sunlight

coefplot(model)



Why didn't they say
this in the first place?

NB: This is a
presentation graph
equivalent of the
table

Shows **standardized
coefficient** with 95%
CI

Factors (Country,
sector) are shown
relative to the
baseline category

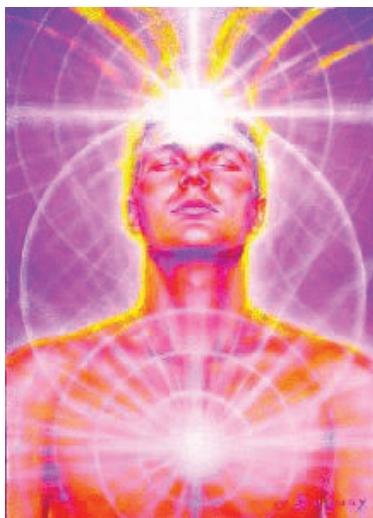
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Run, don't walk toward the sunlight



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Graphs can give enlightenment



The greatest value of a picture is when it forces us to notice what we never expected to see.
-- John W. Tukey

Effect of one rotten point on regression



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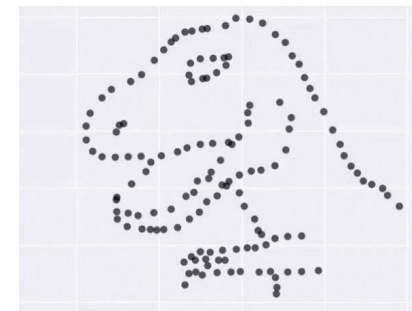
Dangers of numbers-only output

Student: You said to run descriptives and compute the correlation. What next?

Consultant: Did you plot your data?

```
X Mean: 54.26
Y Mean: 47.83
X SD : 16.76
Y SD : 26.93
Corr. : -0.06
```

With **exactly** the same stats, the data could be **any** of these plots



See how this is done in R: <https://cran.r-project.org/web/packages/datasauRus/>

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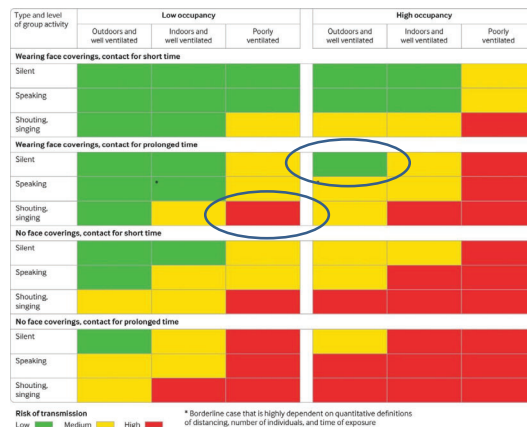
Sometimes, don't need numbers at all

COVID transmission risk ~ Occupancy * Ventilation * Activity * Mask? * Contact.time

A complex 5-way table, whose message is clearly shown w/o numbers

A semi-graphic table shows the **patterns** in the data

There are 1+ unusual cells here. Can you see them?



From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? *BMJ* 2020;370:m3223, doi: <https://doi.org/10.1136/bmj.m3223>

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If you do need tables– make them pretty







Several R packages make it easier to construct informative & pretty semi-graphic tables

Presentation graph

Perhaps too cute!

Distribution of variables shown

Flipper lengths (mm) of the famous penguins of Palmer Station, Antarctica.

Species	Distribution	Female		Male	
		Avg.	Std. Dev.	Avg.	Std. Dev.
 Adelie		188	5.6	192	6.6
 Chinstrap		192	5.8	200	6.0
 Gentoo		213	3.9	222	5.7

Artwork by @allison_horst

produced using modelsummary::datasummary,
<https://vincentarelbundock.github.io/modelsummary/articles/datasummary.html>

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Visual table ideas: Heatmap shading

Heatmap shading: Shade the **background** of each cell according to some criterion

The trends in the US and Canada are made obvious

NB: Table rows are sorted by Jan. value, lending coherence

Background shading ~ value:
US & Canada are made to stand out.

Tech note: use white text on a darker background

Unemployment rate in selected countries
January-August 2020, sorted by the unemployment rate in January.

country	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Japan	2.4%	2.4%	2.5%	2.6%	2.9%	2.8%	2.9%	3.0%
Netherlands	3.0%	2.9%	2.9%	3.4%	3.6%	4.3%	4.5%	4.6%
Germany	3.4%	3.6%	3.8%	4.0%	4.2%	4.3%	4.4%	4.4%
Mexico	3.6%	3.6%	3.2%	4.8%	4.3%	5.4%	5.2%	5.0%
US	3.6%	3.5%	4.4%	14.7%	13.3%	11.1%	10.2%	8.4%
South Korea	4.0%	3.3%	3.8%	3.8%	4.5%	4.3%	4.2%	3.2%
Denmark	4.9%	4.9%	4.8%	4.9%	5.5%	6.0%	6.3%	6.1%
Belgium	5.1%	5.0%	5.0%	5.1%	5.0%	5.0%	5.0%	5.1%
Australia	5.3%	5.1%	5.2%	6.4%	7.1%	7.4%	7.5%	6.8%
Canada	5.5%	5.6%	7.8%	13.0%	13.7%	12.3%	10.9%	10.2%
Finland	6.8%	6.9%	7.0%	7.3%	7.5%	7.8%	8.0%	8.1%

Source: OECD • Get the data • Created with Datarapper

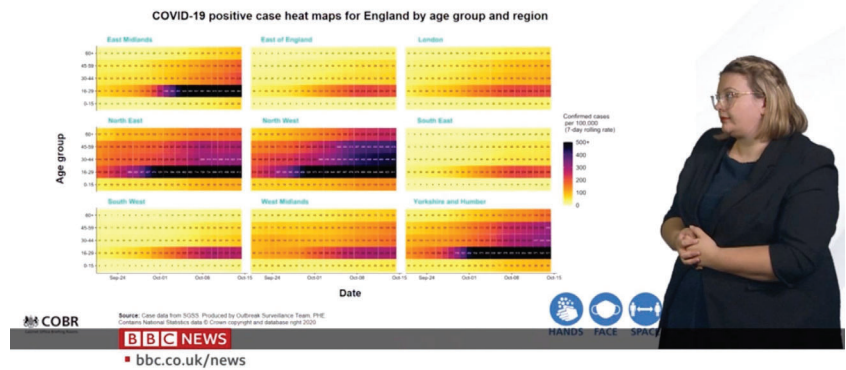
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Visual table ideas: Heatmap shading

As seen on TV ...

Covid rate ~ Age x Date x UK region

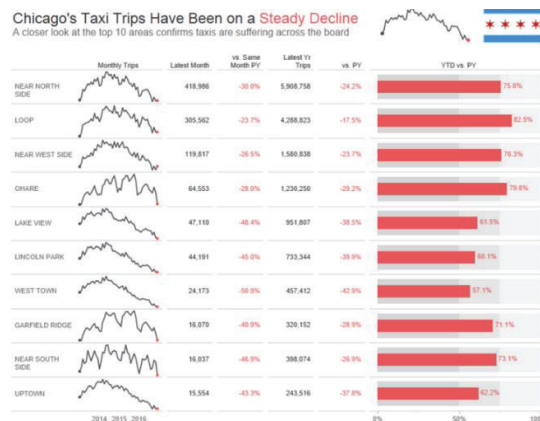
Better: incorporate geography, not just arrange regions alphabetically



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Visual table ideas: Sparklines

Sparklines: Mini graphics inserted into table cells or text



From: <https://www.pluralsight.com/guides/tableau-playbook-sparklines>

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Linear models

• Model:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i$$

• Xs: quantitative predictors, factors, interactions, ...

• Assumptions:

- **Linearity:** Predictors (possibly transformed) are linearly related to the outcome, y . [This just means linear in the **parameters**.]
- **Specification:** No important predictors have been omitted; only important ones included. [This is often key & overlooked.]
- The "holy trinity":
 - **Independence:** the errors are uncorrelated
 - **Homogeneity of variance:** $\text{Var}(\varepsilon_i) = \sigma^2 = \text{constant}$
 - **Normality:** ε_i have a normal distribution

$$\varepsilon_i \sim_{iid} N(0, \sigma^2)$$

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The General Linear Model

- “linear” models can include:
 - transformed predictors: \sqrt{age} , $\log(income)$
 - polynomial terms: age^2 , age^3 , $poly(age, n)$
 - categorical “factors”, coded as dummy (0/1) variables
 - treated (Yes/No), Gender (M/F/non-binary)
 - interactions: effects of x_1 vary over levels of x_2
 - treated \times age, treated \times sex, (2 way)
 - treated \times age \times sex (3 way)
- Linear model means **linear** in the parameters (β_i),

$$y = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 \log(income) + \beta_4 (sex="F") + \beta_5 age \times (sex="F") + \dots$$
- In R, all handled by `lm(y ~ ...)`

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Fitting linear models in R: lm()

- In R, `lm()` for everything
 - Regression models (X_1, \dots **quantitative**)

```
lm(y ~ X1, data=dat)           # simple linear regression
lm(y ~ X1+X2+X3, data=dat)     # multiple linear regression
lm(y ~ (X1+X2+X3)^2, data=dat) # all two-way interactions
lm(log(y) ~ poly(X,3), data=dat) # arbitrary transformations
```

- ANOVA/ANCOVA models (A, B, \dots **factors**)

```
lm(y ~ A)                      # one way ANOVA
lm(y ~ A*B)                    # two way: A + B + A:B
lm(y ~ X + A)                  # one way ANCOVA
lm(y ~ (A+B+C)^2)              # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

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Fitting linear models in R: lm()

- Multivariate models: `lm()` with 2+ y vars

- Multivariate regression

```
lm(cbind(y1, y2) ~ X1 + X2 + X3) # std MMreg: all linear
lm(cbind(y1, y2) ~ poly(X1,2) + poly(X2,2)) # response surface
```

- MANOVA/MANCOVA models

```
lm(cbind(y1, y2, y3) ~ A * B) # 2-way MANOVA: A + B + A:B
lm(cbind(y1, y2, y3) ~ X + A) # MANCOVA (equal slopes)
lm(cbind(y1, y2) ~ X + A + X:A) # heterogeneous slopes
```

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Generalized Linear Models: glm()

Transformations of y & other error distributions

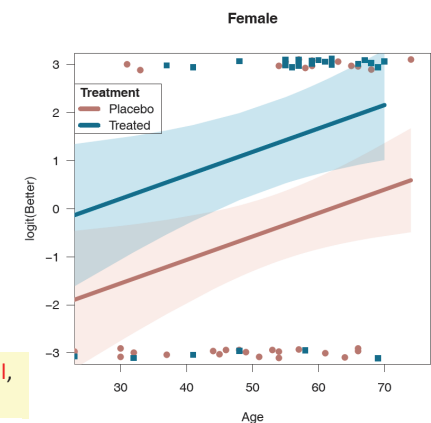
- $y \in (0/1)$: lived/died; success/fail; ...

- logit (log odds) model:

- $\text{logit}(y) = \log \frac{\Pr(y=1)}{\Pr(y=0)}$
- linear logit model:

$$\text{logit}(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots$$

```
glm(better ~ age + treat, family=binomial, data=Arthritis)
```



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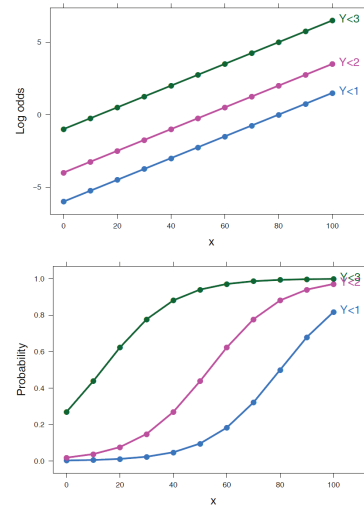
Generalized Linear Models

Ordinal responses

- Improved \in ("None" < "Some" < "Marked")
- Models: Proportional odds, generalized logits, ...

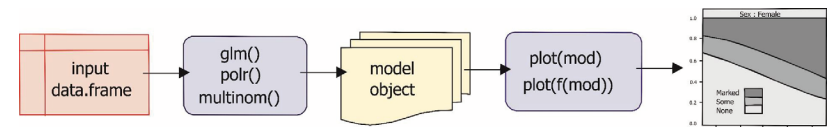
```
library(MASS)
polr(Improved ~ Sex + Treat + Age,
     data=Arthritis)
```

```
library(nnet)
multinom(Improved ~ Sex + Treat + Age,
         data=Arthritis)
```



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Model-based methods: Overview

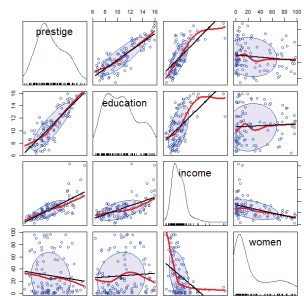
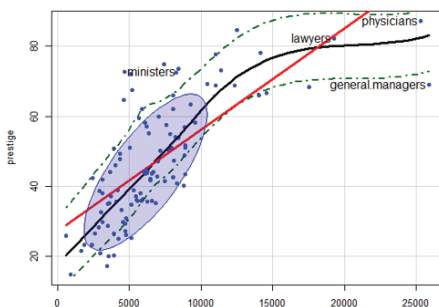


- models in R are specified by a symbolic model formula, applied to a data.frame
 - mod<-lm(prestige ~ income + educ, data=Prestige)
 - mod<-glm(better ~ age + sex + treat, data=Arthritis, family=binomial)
 - mod<-MASS:polr(improved ~ age + sex + treat, data=Arthritis)
- result (mod) is a "model object", of class "lm", "glm", ...
- method functions:
 - plot(mod), plot(f(mod)), ...
 - summary(mod), coef(mod), predict(mod), ...

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Plots for linear models

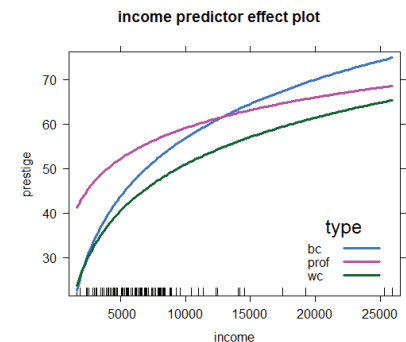
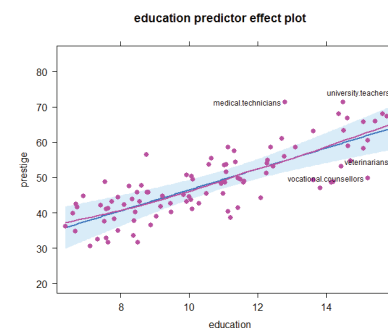
- Data plots:
 - plot response (y) vs. predictors, with smooth summaries
 - scatterplot matrix --- all pairs



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Plots for linear models

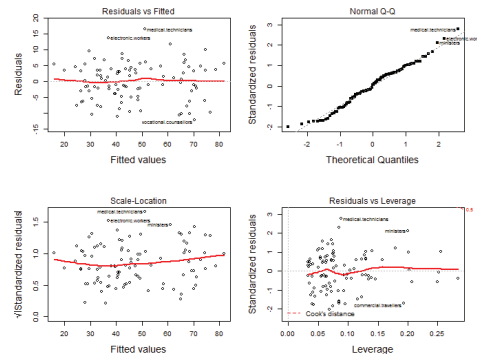
- Model (effect) plots
 - plot predicted response (\hat{y}) vs. predictors, **controlling** for variables not shown.



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Plots for linear models

- Diagnostic plots
 - N QQ plot: normality of residuals? outliers?
 - Influence plots: leverage & outliers
 - Spread-level plots (non-constant variance?)



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R packages

- car
 - Enhanced scatterplots
 - Diagnostic plots
- effects
 - Plot fitted effects of one predictor, controlling all others
- visreg
 - similar to effect plots, simpler syntax
- Both effects & visreg handle nearly all formula-based models
 - lm(), glm(), gam(), rlm(), nlme(), ...



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Occupational Prestige data

- Data on prestige of 102 occupations and
 - average education (years)
 - average income (\$)
 - % women
 - type (Blue Collar, Professional, White Collar)

```
> car::some(Prestige, 6)
      education income women prestige census type
architects    15.44  14163   2.69    78.1  2141 prof
physicians     15.96  25308  10.56    87.2  3111 prof
commercial.artists 11.09   6197  21.03    57.2  3314 prof
tellers.cashiers  10.64  2448  91.76    42.3  4133 wc
bakers          7.54   4199  33.30    38.9  8213 bc
aircraft.workers  8.78   6573   5.78    43.7  8515 bc
```

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Follow along

The R script ([prestige-ex.R](#)) for this example is linked on the course page. Download and open in R Studio to follow along.

- Examples:
 - Prestige data [prestige-ex.R](#) || [prestige-ex.html](#)
 - Penguin data [penguins-lm-ex.R](#) || [penguins-lm-ex.html](#)

The script was run with 'knitr' (ctrl+shift+K) in R Studio to create the HTML output ([prestige-ex.html](#))

The **Code** button there allows you to download the R code and comments



(These show a simple way to turn R scripts into finished documents)

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Informative scatterplots

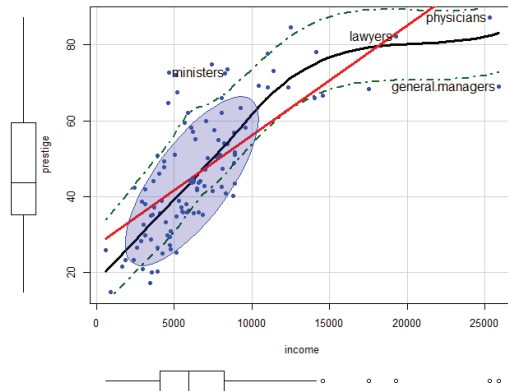
Scatterplots are most useful when enhanced with annotations & statistical summaries

Data ellipse and regression line show the linear model, $\text{prestige} \sim \text{income}$

Point labels show possible outliers

Smoothed (loess) curve and CI show the trend

Boxplots show marginal distributions



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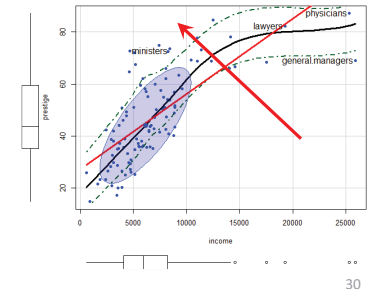
Informative scatterplots

`car::scatterplot()` provides all these enhancements

```
scatterplot(prestige ~ income, data=Prestige,
  pch = 16,
  regLine = list(col = "red", lwd=3),
  smooth = list(smoother=loessLine,
    lty.smooth = 1, col.smooth = "black",
    lwd.smooth=3, col.var = "darkgreen"),
  ellipse = list(levels = 0.68),
  id = list(n=4, col="black", cex=1.2))
```

Skewed distribution of income & non-linear relation suggest need for a transformation

Arrow rule: move on the scale of powers in direction of the bulge
e.g.: $x \rightarrow \sqrt{x}$ or $\log(x)$



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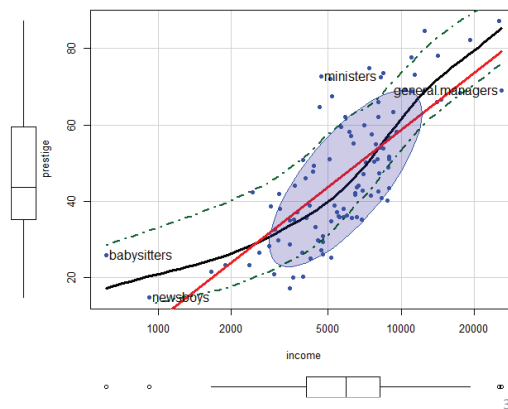
Try log(income)

```
scatterplot(prestige ~ income, data=Prestige,
  log = "x", # plot on log scale
  pch = 16,
  regLine = list(col = "red", lwd=3),
  ...)
```

Income now ~ symmetric

Relation closer to linear

$\log(\text{income})$: interpret as effect of a multiple



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Stratify by type?

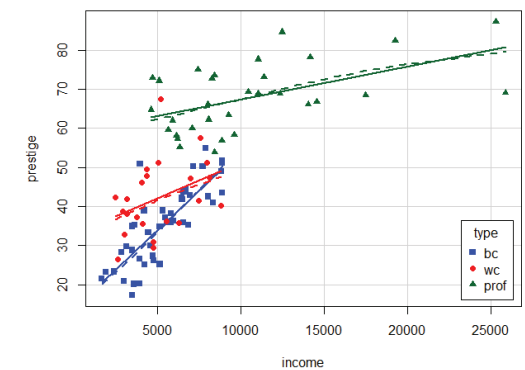
```
scatterplot(prestige ~ income | type, data=Prestige,
  col = c("blue", "red", "darkgreen"),
  pch = 15:17,
  legend = list(coords="bottomright",
    smooth=list(smoother=loessLine, var=FALSE, span=1, lwd=4))
```

Formula: $\text{type} \rightarrow \text{"given type"}$

Different slopes: interaction of $\text{income} * \text{type}$

Provides another explanation of the non-linear relation

This may be a new finding!



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Scatterplot matrix

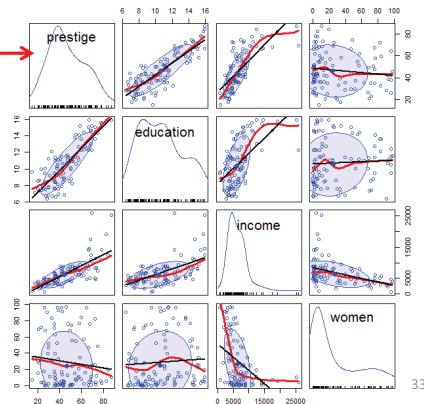
```
scatterplotMatrix(~ prestige + education + income + women,
  data=Prestige,
  regLine = list(method=lm, lty=1, lwd=2, col="black"),
  smooth=list(smooth=loessLine, spread=FALSE,
    lty.smooth=1, lwd.smooth=3, col.smooth="red"),
  ellipse=list(levels=0.68, fill.alpha=0.1))
```

prestige vs. all predictors

diagonal: univariate distributions

- income: + skewed
- %women: bimodal

off-diagonal: relations among predictors



Fit a simple model

```
> mod0 <- lm(prestige ~ education + income + women,
+ data=Prestige)
> summary(mod0)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-6.7943342	3.2390886	-2.098	0.0385 *
education	4.1866373	0.3887013	10.771	< 2e-16 ***
income	0.0013136	0.0002778	4.729	7.58e-06 ***
women	-0.0089052	0.0304071	-0.293	0.7702

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.7982 Adjusted R-squared: 0.792
F-statistic: 129.2 on 3 and 98 DF, p-value: < 2.2e-16

Fits very well

But this ignores:

- nonlinear relation with income: should use log(income)
- occupation type
- possible interaction of income*type

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Fit a more complex model

```
> mod1 <- lm(prestige ~ education + women +
+ log(income)*type, data=Prestige)
> summary(mod1)
```

add interaction of log income by type

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-152.20589	23.24988	-6.547	3.54e-09 ***
education	2.92817	0.58828	4.978	3.08e-06 ***
women	0.08829	0.03234	2.730	0.00761 **
log(income)	18.98191	2.82853	6.711	1.67e-09 ***
typeprof	85.26415	30.45819	2.799	0.00626 **
typewc	29.41334	36.50749	0.806	0.42255
log(income):typeprof	-9.01239	3.41020	-2.643	0.00970 **
log(income):typewc	-3.83343	4.26034	-0.900	0.37063

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.8751 Adjusted R-squared: 0.8654
F-statistic: 90.07 on 7 and 90 DF, p-value: < 2.2e-16

Fits even better!

But how to understand?

Coefs for type compare mean "wc" and "prof" to "bc"

Coefs for log(income)*type compare "wc" and "prof" slopes with that of "bc"

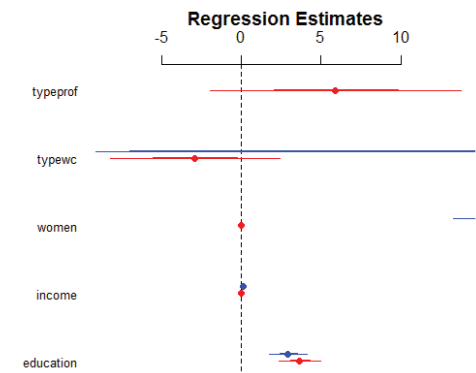
Coefficient plots

Plots of coefficients with CI often more informative than tables

```
arm::coefplot(mod0, col.pts="red", cex.pts=1.5)
```

```
arm::coefplot(mod1, add=TRUE, col.pts="blue", cex.pts=1.5)
```

This plots raw coefficients, and the Xs are on different scales, so effect of income doesn't appear significant.



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Model (effect) plots

- We'd like to see plots of the predicted value (\hat{y}) of the response against predictors (x_j)
 - Ordinary plot of y vs. x_j doesn't allow for other correlations
 - Must **control** (adjust) for other predictors (x_{-j}) not shown in a given plot
- Effect plots
 - Variables not shown (x_{-j}) are averaged over.
 - Slopes of lines reflect the **partial** coefficient in the model
 - Partial residuals can be shown also

For details, see `vignette("predictor-effects-gallery", package="effects")`

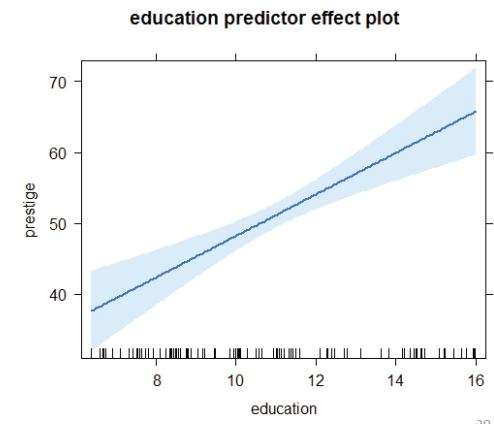
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Model (effect) plots: education

```
library("effects")
mod1.e1 <- predictorEffect("education", mod1)
plot(mod1.e1)
```

This graph shows the **partial** slope for education, controlling for all others

For each ↑ year in education, fitted prestige ↑2.93 points, (other predictors held fixed)

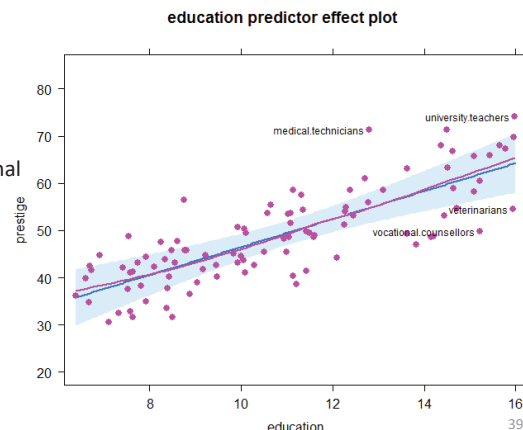


Model (effect) plots

```
mod1.e1a <- predictorEffect("education", mod1, residuals=TRUE)
plot(mod1.e1a,
     residuals.pch=16, id=list(n=4, col="black"))
```

Partial residuals show the residual of prestige controlling for other predictors

Unusual points here would signal undue influence



Model (effect) plots: women

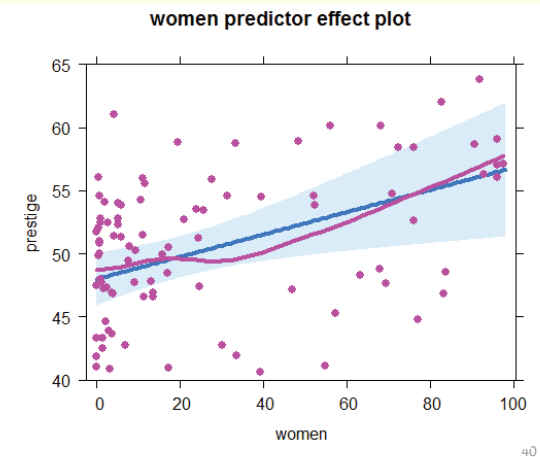
```
mod1.e2 <- predictorEffect("women", mod1, residuals=TRUE)
plot(mod1.e2, ylim=c(40, 65), lwd=4,
     residuals.pch=16)
```

Surprise!

Prestige of occupations ↑ with % women (controlling for other variables)

Another 10% women ↑ prestige by 0.88 points

How to interpret this?



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Model (effect) plots: income

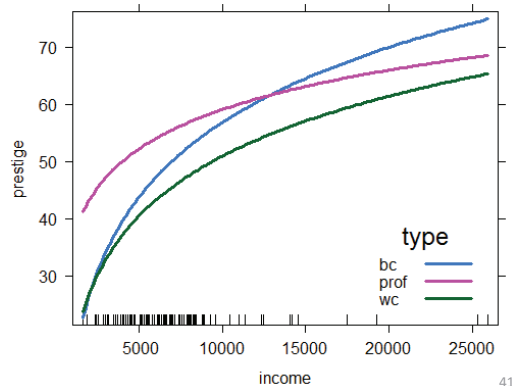
```
plot(predictorEffect("income", mod1),
     lines=list(multiline=TRUE, lwd=3),
     key.args = list(x=.7, y=.35))
```

income predictor effect plot

Income interacts with type in the model

The plot is curved because $\log(\text{income})$ is in the model

Curvature reflects marginal effect of income for each occupation type



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visreg plots: Air quality data

Daily air quality measurements in New York, May - Sep 1973

How does Ozone concentration vary with solar radiation, wind speed & temperature?

```
> head(airquality)
  Ozone Solar.R Wind Temp Month Day
1    41    190   7.4   67     5   1
2    36    118   8.0   72     5   2
3    12    149  12.6   74     5   3
4    18    313  11.5   62     5   4
5    NA     NA  14.3   56     5   5
6    28     NA  14.9   66     5   6
```

see: <https://pbreheny.github.io/visreg/> for examples & details

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Air quality: main effects model

```
> fit1 <- lm(Ozone ~ Solar.R + wind + Temp, data=airquality)
> summary(fit1)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-64.3421	23.0547	-2.79	0.0062 **
Solar.R	0.0598	0.0232	2.58	0.0112 *
wind	-3.3336	0.6544	-5.09	1.5e-06 ***
Temp	1.6521	0.2535	6.52	2.4e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

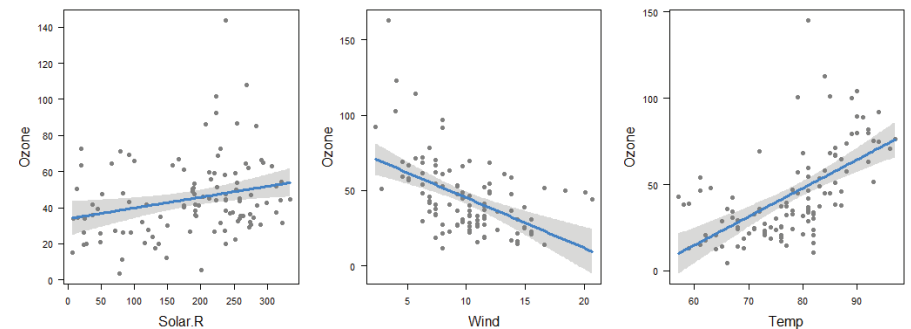
Residual standard error: 21.18 on 107 degrees of freedom
(42 observations deleted due to missingness)
Multiple R-squared: 0.6059, Adjusted R-squared: 0.5948
F-statistic: 54.83 on 3 and 107 DF, p-value: < 2.2e-16

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visreg conditional plots

```
visreg(fit1, "Solar.R")
visreg(fit1, "wind")
visreg(fit1, "Temp")
```

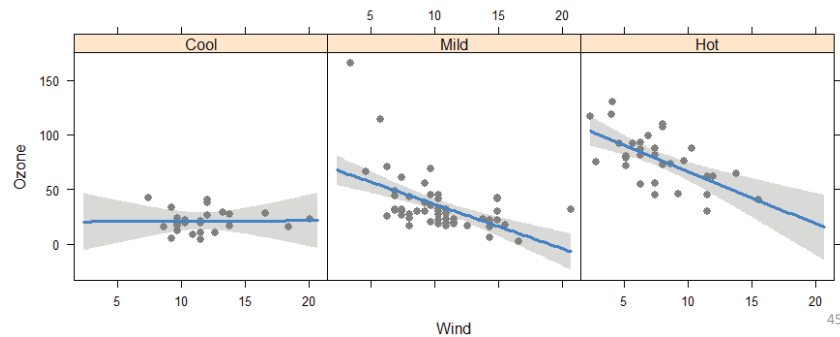
model summary =
predicted values (line) +
confidence band (uncertainty) +
partial residuals (objections)



Factor variables & interactions

```
# cut Temp into three ordered levels of equal range
airquality$Heat <- cut(airquality$Temp, 3,
  labels=c("Cool", "Mild", "Hot"))

# fit model with interaction of Wind * Heat
fit2 <- lm(Ozone ~ Solar.R + Wind*Heat, data=airquality)
visreg(fit2, "Wind", by="Heat", layout=c(3,1), points=list(cex=1))
```



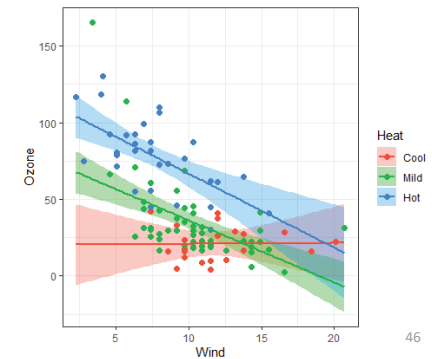
Factor variables & interactions

```
visreg(fit2, "Wind", by="Heat",
  overlay=TRUE,
  gg=TRUE,
  points=list(size=2)) +
  theme_bw()
```

`overlay=TRUE` → superpose panels
`gg=TRUE` → uses ggplot

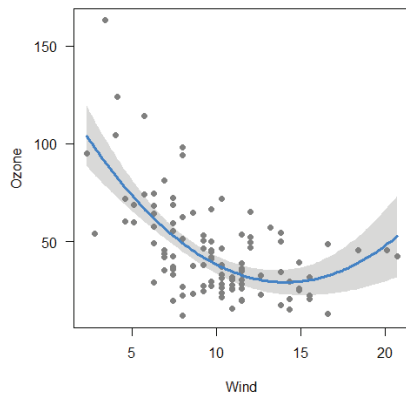
This allows slope for Wind to vary with Heat e.g., Wind has no effect when Cool

This model still assumes **linear** effects of Heat & Wind

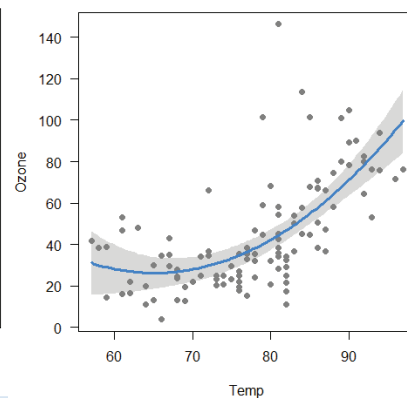


Non-linear effects

```
fit <- lm(Ozone ~ Solar.R + poly(Wind,2) +
  Temp, data=airquality)
visreg(fit, "Wind")
```



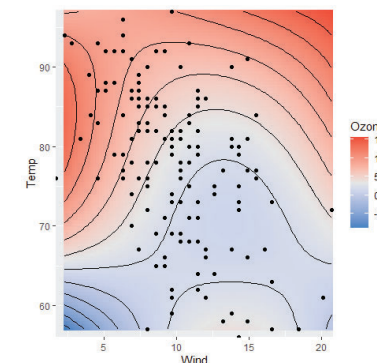
```
fit <- lm(Ozone ~ Solar.R + Wind +
  poly(Temp,2), data=airquality)
visreg(fit, "Temp")
```



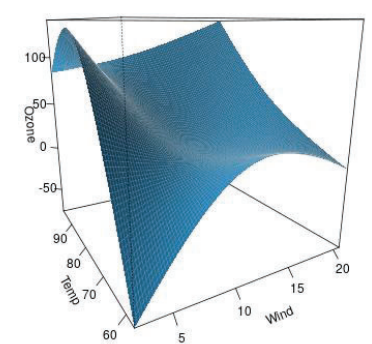
Response surface models (visreg2d)

```
# Fit quadratics in both Wind & Temp and interaction Wind * Temp
fitp <- lm(Ozone ~ Solar.R + poly(Wind,2) * poly(Temp,2), data=airquality)
```

```
visreg2d(fitp, "Wind", "Temp", plot.type="gg") +
  geom_contour(aes(z=z), color="black")
```



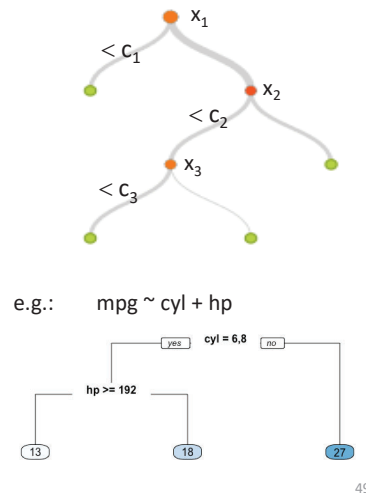
```
visreg2d(fitp, "Wind", "Temp", plot.type="persp")
```



Regression trees

Regression trees are a non-parametric alternative to linear models

- Essential ideas:
 - Find predictor and split value which minimizes SSE
 - fitted value in each subgroup = mean
 - repeat, recursively, splitting by next best predictor
- Large literature
 - cost, complexity tradeoff
 - pruning methods
 - boosting, cross-validation
 - tree averaging



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Prestige data: rpart tree

```

> library(rpart)      # calculating regression trees
> library(rpart.plot) # plotting regression trees

> rmod <- rpart(prestige ~ education + income + women + type,
  data=Prestige,
  method = "anova")

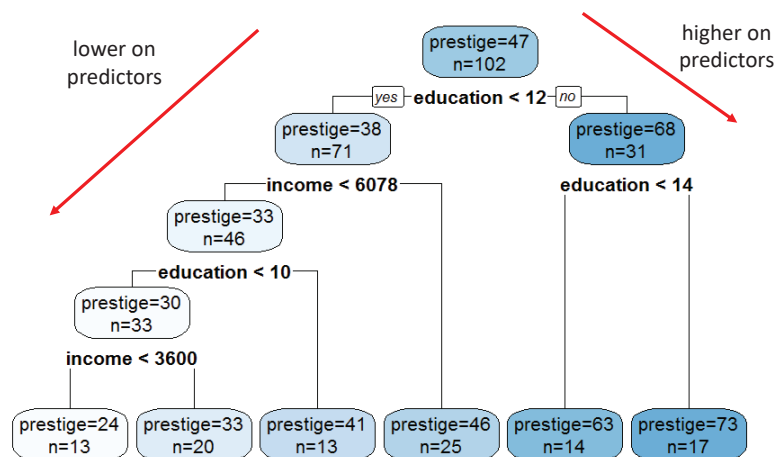
> rpart.rules(rmod)    # print prediction rules
prestige
24 when education < 10      & income < 3600
33 when education < 10      & income is 3600 to 6078
41 when education is 10 to 12 & income < 6078
46 when education < 12      & income >= 6078
63 when education is 12 to 14
73 when education >= 14
    
```

Only education & income are involved in this simple model.
Other controls allow setting classification details

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Prestige data: rpart tree

```
rpart.plot(rmod, prefix="prestige=")
```



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Diagnostic plots

- The linear model, $y = X\beta + \epsilon$ assumes:
 - Residuals, ϵ_i are normally distributed, $\epsilon_i \sim N(0, \sigma^2)$
 - (Normality **not** required for X s)
 - Constant variance, $\text{Var}(\epsilon_i) = \sigma^2$
 - Observations y_i are statistically independent
- Violations \rightarrow inferences may not be valid
- A variety of plots can diagnose all these problems
- Other methods (boxCox, boxTidwell) diagnose the need for transformations of y or X s.

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The “regression quartet”

In R, plotting a `lm` model object → the “regression quartet” of plots

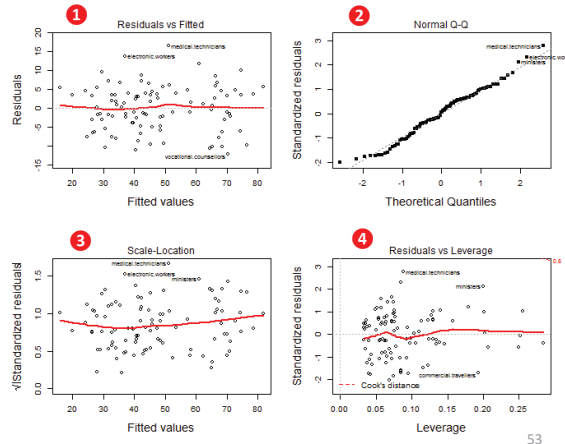
`plot(mod1, lwd=2, cex.lab=1.4)`

1 Residuals: should be flat vs. fitted values ✓

2 Q-Q plot: should follow the 45° line ✓

3 Scale-location: should be flat if constant variance ✓

4 Resids vs. leverage: can show influential observations

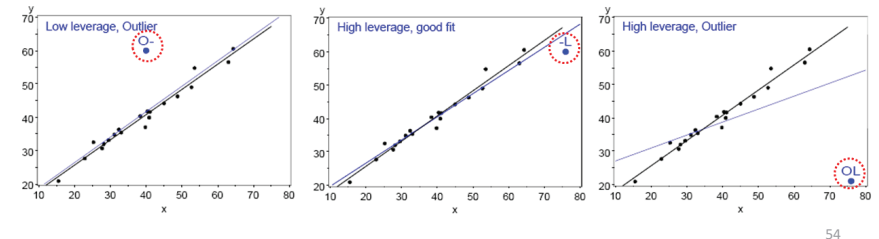


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Unusual data: Leverage & Influence

- “Unusual” observations can have dramatic effects on least-squares estimates in linear models
- Three archetypal cases:
 - Typical X (low leverage), bad fit -- Not much harm
 - Unusual X (high leverage), good fit -- Not much harm
 - Unusual X (high leverage), bad fit -- **BAD, BAD, BAD**
- Influential observations: unusual in *both* X & Y
- Heuristic formula:

$$\text{Influence} = \text{X leverage} \times \text{Y residual}$$



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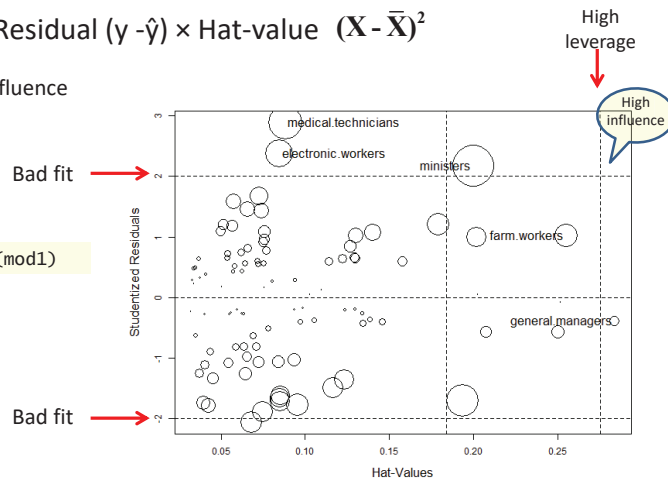
Influence plots

Influence (Cook’s D) measures impact of individual obs. on coefficients, fitted values

$$\text{Influence} \sim \text{Residual } (y - \hat{y}) \times \text{Hat-value } (\mathbf{X} - \bar{\mathbf{X}})^2$$

Bubble size ~ influence

`influencePlot(mod1)`



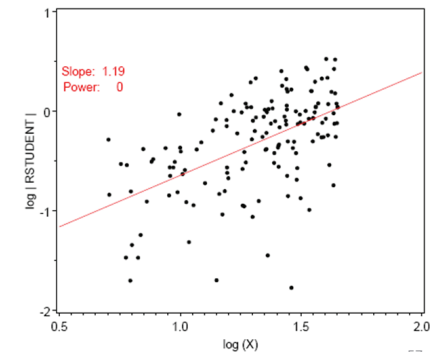
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Spread-level plots

- To diagnose non-constant variance, plot:
 - $\log |\text{Std. residual}|$ vs. $\log(x)$
 - $\log(\text{IQR})$ vs $\log(\text{median})$ [for grouped data]
- If \approx linear w/ slope b , transform $y \rightarrow y^{(1-b)}$

Artificial data, generated so $\sigma \sim x$

- $b \approx 1 \rightarrow \text{power} = 0$
- \rightarrow analyze $\log(y)$



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Spread-level plot: baseball data

Data on salary and batter performance from 1987 season

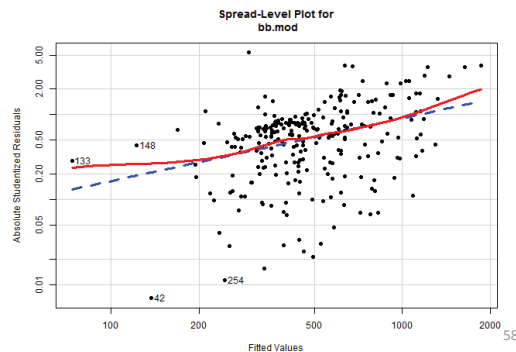
```
data("Baseball", package="vcd")
bb.mod <- lm(sal87 ~ years + hits + runs + homeruns, data=Baseball)
spreadLevelPlot(bb.mod, pch=16, lwd=3,
               id=list(n=2))
```

Suggested power transformation: 0.2609

slope = .74 → p = .26

i.e., $y \rightarrow \log(y)$ or $y^{1/4}$

NB: both axes plotted on log scale



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Box Cox transformation

- Box & Cox proposed to transform y to a power, $y \rightarrow y^{(\lambda)}$ to minimize the residual SS (or maximize the likelihood)

- Makes $y^{(\lambda)}$ more nearly normal
- Makes $y^{(\lambda)}$ more nearly linear in with X

Formula for $y^{(\lambda)}$

- $y^{(0)} : \log_e(y)$
- $\lambda < 0$: flip sign to keep same order

$$y_i^{(\lambda)} = \begin{cases} \frac{y_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0, \\ \ln(y_i) & \text{if } \lambda = 0, \end{cases}$$

Power(p)	Transformation	Name
2	Y^2	Square
1	Y (No transformation)	Original Data
$\frac{1}{2}$	\sqrt{Y}	Square root
0	$\log Y$ or $\log_{10}(Y)$	Logarithm
$-\frac{1}{2}$	$-1/\sqrt{Y}$	Reciprocal Root
-1	$-1/Y$	Reciprocal
-2	$-1/Y^2$	Reciprocal Square

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Example: Cars93 data

How does gas mileage (MPG.city) depend on vehicle weight?

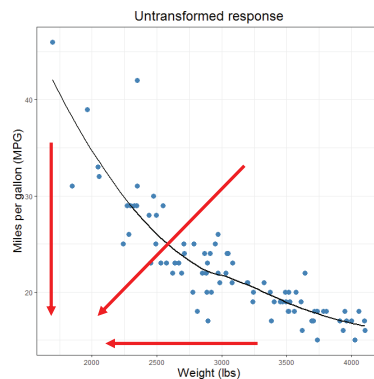
```
> cars.mod <- lm(MPG.city ~ Weight, Cars93)
> coef(cars.mod)
(Intercept)      Weight
  47.04835      -0.00803
```

Relationship clearly non-linear

Tukey arrow rule: transform Y (or X) as arrow thru the curve bulges

$y \rightarrow \sqrt{y}, \log(y), 1/y$

$x \rightarrow \sqrt{x}, \log(x), 1/x$

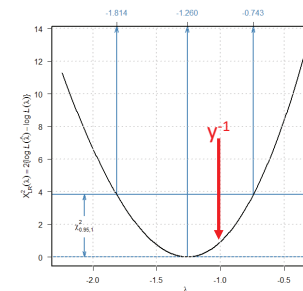


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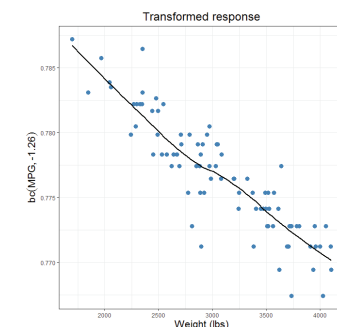
MASSextra package

```
> library(MASSextra)
> box_cox(cars.mod) # plot log likelihood vs. lambda
> lambda(cars.mod)
[1] -1.26
```

The plot of $-\log(L) \sim \text{RSS}$ shows the minimum & CI



plot(bc(MPG.city, lambda(cars.mod)))



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Summary

- Tables are for look-up; graphs can give insight
- “Linear” models include so much more than ANOVA & regression
- Data plots are more effective when enhanced
 - data ellipses → strength & precision of correlation
 - regression lines and smoothed curves
 - point identification → noteworthy observations
- Effect plots show informative views of models
 - Visualize conditional effects, holding others constant
- Diagnostic plots can reveal influential observations and need for transformations.