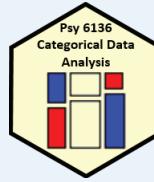


# Categorical Data Analysis

## Course overview



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Psych 6136  
<http://friendly.github.io/psy6136>



## Course outline

### 1. Exploratory and hypothesis testing methods

- Week 1: Overview; Introduction to R
- Week 2: One-way tables and goodness-of-fit test
- Week 3: Two-way tables: independence and association
- Week 4: Two-way tables: ordinal data and dependent samples
- Week 5: Three-way tables: different types of independence
- Week 6: Correspondence analysis

### 2. Model-based methods

- Week 7: Logistic regression I
- Week 8: Logistic regression II
- Week 9: Multinomial logistic regression models
- Week 10: Log-linear models
- Week 11: Loglinear models: Advanced topics
- Week 12: Generalized Linear Models: Poisson regression
- Week 13: Course summary & additional topics

## Course goals

This course is designed as a broad, applied introduction to the statistical analysis of categorical data, with an emphasis on:

### Emphasis: visualization methods

- exploratory graphics: see patterns, trends, anomalies in your data
- model diagnostic methods: assess violations of assumptions
- model summary methods: provide an interpretable summary of your data

### Emphasis: theory $\Rightarrow$ practice

- Understand how to translate research questions into statistical hypotheses and models
- Understand the difference between simple, non-parametric approaches (e.g.,  $\chi^2$  test for independence) and **model-based methods** (logistic regression, GLM)
- Framework for thinking about categorical data analysis in *visual* terms

## Course schedule

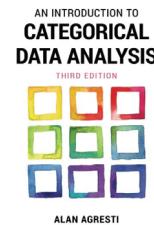
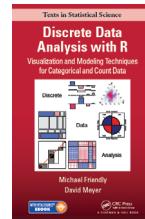
The [schedule](#) page provides links to slides, tutorials, readings & R scripts

Week	Topic	Readings	R	R files	knight
1	Overview <a href="#">[slides]</a> <a href="#">[4up]</a> <a href="#">[Working with R Studio]</a> <a href="#">[4up]</a>	DDAR: <a href="#">Ch1</a> , <a href="#">Ch2</a> Agresti: Ch1	<a href="#">R-intro.R</a> <a href="#">[knight]</a>		
2	Discrete distributions <a href="#">[slides]</a> <a href="#">[4up]</a>	DDAR: <a href="#">Ch3</a>	<a href="#">R-data.R</a> <a href="#">[knight]</a> <a href="#">binomial.R</a> <a href="#">[knight]</a>		
3	Two-Way Tables: Independence & Association <a href="#">[slides]</a> <a href="#">[4up]</a>	DDAR: <a href="#">Ch4</a> Agresti: Ch2	<a href="#">berk-4fold.R</a> <a href="#">[knight]</a> <a href="#">vision-sieve.R</a> <a href="#">[knight]</a>		
4	Two-Way Tables: Ordinal Data and Dependent Samples <a href="#">[Tutorial]</a> on two-way tables	DDAR: <a href="#">Ch4</a> Agresti: Ch2	<a href="#">msdiag-agree.R</a> <a href="#">[knight]</a> <a href="#">haireye-spineplot.R</a> <a href="#">[knight]</a>		
5	Loglinear Models and Mosaic Displays <a href="#">[slides]</a> <a href="#">[4up]</a> <a href="#">[Tutorial]</a> on loglin models; <a href="#">[Mosaic display animation]</a>	DDAR: <a href="#">Ch5</a> Agresti: 2.7, Ch. 7	<a href="#">berkeley-glm.R</a> <a href="#">[knight]</a> <a href="#">titanic-loglin.R</a> <a href="#">[knight]</a>		
6	Correspondence Analysis <a href="#">[slides]</a> <a href="#">[4up]</a> <a href="#">[Tutorial]</a> on CA,	DDAR: <a href="#">Ch6</a>	<a href="#">mental-ca.R</a> <a href="#">[knight]</a> <a href="#">mca-presex3.R</a> <a href="#">[knight]</a>		
7	Logistic Regression I <a href="#">[slides]</a> <a href="#">[4up]</a> <a href="#">[Logistic regression tutorial]</a>	DDAR: <a href="#">7.1-7.3</a> Agresti: 3.1-3.2; Ch 4	<a href="#">arthritis-logistic.R</a> <a href="#">[knight]</a> <a href="#">cowles-logistic.R</a> <a href="#">[knight]</a> <a href="#">Arrests-logistic.R</a> <a href="#">[knight]</a>		
8	Logistic Regression II <a href="#">[slides]</a> <a href="#">[4up]</a>	DDAR: <a href="#">7.3-7.4</a> Agresti: Ch 4-5	<a href="#">cowles-effect.R</a> <a href="#">[knight]</a> <a href="#">Arrests-effects.R</a> <a href="#">[knight]</a> <a href="#">berkeley-diag.R</a> <a href="#">[knight]</a>		
		...			

# Textbooks

## Main texts

- Friendly & Meyer (2016). *Discrete Data Analysis with R: Visualizing & Modeling Techniques for Categorical & Count Data*
  - 30% discount on [Routledge web site](#) (code: ADC22)
  - Draft chapters linked in [Schedule](#)
  - DDAR web site: <https://ddar.datavis.ca>
- Agresti (2007). *An Introduction to Categorical Data Analysis*, 3<sup>rd</sup> E. Wiley & Sons.
  - eBook available: <https://bit.ly/3Wzqv0n>

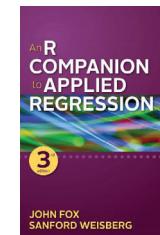
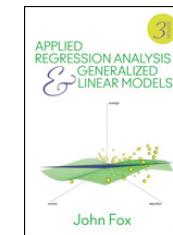
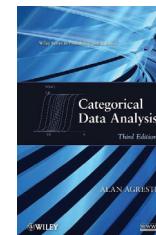


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# Textbooks

## Supplementary readings

- Agresti (2013). *Categorical Data Analysis*, 3<sup>rd</sup> ed. [More mathematical, but the current Bible of CDA]
  - PDF available: <https://bitly.co/FG9c>
- Fox (2016). *Applied Regression Analysis and Generalized Linear Models*, 3<sup>rd</sup> ed. Particularly: Part IV on Generalized Linear Models
- Fox & Weisberg (2018). *An R Companion to Applied Regression*. Also, [web site for the book](#).



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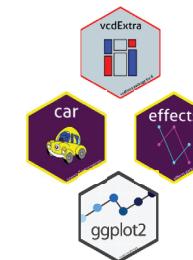
# Expectations & grading

- I expect you will read chapters in *DDAR* & Agresti *Intro* each week
  - See [Topic Schedule](#) on course web site
  - R exercises & tutorials: Please work on these
  - R [Assignments](#) : Ungraded, but please submit them when assigned
  - Class discussion: Help make classes participatory
- Evaluation:
  - (2 x 40%) Two take-home projects: Analysis & research report, based on assignment problems or your own data
  - (20%)
    - Assignment portfolio: best work, enhanced
    - Research report on journal article(s) of theory / application of CDA
    - In-class presentation (~15 min) on application of general interest

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# The R you need

- R, version >=3.6 [R 4.2 is current]
  - Download from <https://cran.r-project.org/>
- RStudio IDE, highly recommended
  - <https://www.rstudio.com/products/rstudio/>
- R packages:
  - vcd
  - vcdExtra
  - car
  - effects
  - ggplot2
  - ...



R script to install packages:  
<https://friendly.github.io/6136/R/install-vcd-pkgs.R>

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## Categorical data analysis: History

- Categorical data analysis is a relatively recent arrival
  - 1888 – Galton introduces the concept of correlation
  - 1908 – Student's t-distribution for the mean of small samples
  - 1931 – L. L. Thrustone: Multiple factor analysis
  - 1935 – R. A. Fisher's Design of Experiments – ANOVA
  - ... (time passes)
- 1972 – Nelder & Wedderburn develop the central ideas of **generalized linear models** (logistic & poison regression)
- 1973 – J-P. Benzecri: Correspondence analysis (analysis des données)
- 1974 – Bishop , Fienberg, Holland introduce the **loglinear model** for discrete data, ANOVA for log(Freq)
- 1984 – Leo Goodman enhanced loglinear models for complex data: RC models, mobility tables, panel data, ...

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## What is categorical data?

A **categorical variable** is one for which the possible measured or assigned values consist of a **discrete set of categories**, which may be *ordered* or *unordered*. Some typical examples are:

- Gender, with categories {"male", "female", "trans"}
- Marital status: { "Never married", "Married", "Separated", "Divorced", "Widowed" }
- Party preference: {"NDP", "Liberal", "Conservative", "Green"}
- Treatment improvement: {"none", "some", "marked"}
- Age: {"0-9", "10-19", "20-29", "30-39", ... }.
- Number of children: 0, 1, 2, 3, ... .

Questions:

- Which of these are **ordered** (ordinal)?
- Which could be treated as **numeric**? How?
- Which have **missing categories**, sometimes ignored, or treated as "Other"

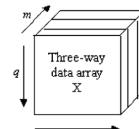
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## Categorical data: Structures

Categorical (frequency) data appears in various forms

- Tables:** often the result of **table()** or **xtabs()**
  - 1-way
  - 2-way –  $2 \times 2$ ,  $r \times c$
  - 3-way
- Matrices:** **matrix()**, with row & col names
- Arrays:** **array()**, with **dimnames()**
- Data frames**
  - Case form (individual observations)
  - Frequency form

Gender compared to handedness		
	Handed	
	Left	Right
Female	7	46
Male	5	63
	12	109
	121	



Hair	Eye	Freq
1 Black	Brown	68
2 Brown	Brown	119
3 Red	Brown	26
4 Blond	Brown	7
5 Black	Blue	20
6 Brown	Blue	84
7 Red	Blue	17
8 Blond	Blue	94

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## 1-way tables

- Unordered factors**

	Black	Brown	Red	Blond
n	108	286	71	127
%	0.18	0.48	0.12	0.21

Hair color of 592 students

	BQ	Cons	Green	Liberal	NDP
n	104	392	126	404	174
%	0.087	0.33	0.1	0.34	0.14

Voting intentions in Harris-Decima poll, 8/21/08

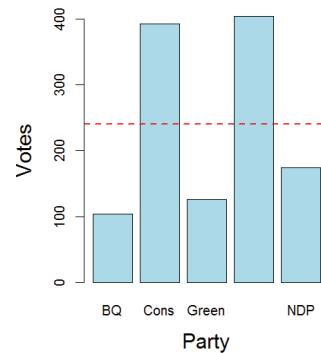
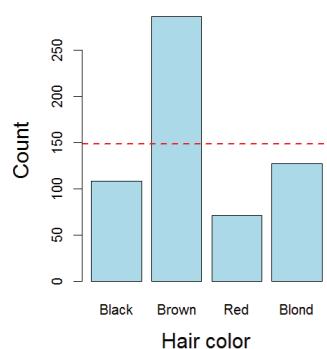
Questions:

- Are all hair colors equally likely?
- Aside from Brown hair, are others equally likely?
- Is there a diff in voting intentions for Liberal vs. Conservative

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## 1-way tables

- Even here, simple graphs are more informative than tables



But these don't really answer the questions. Why?

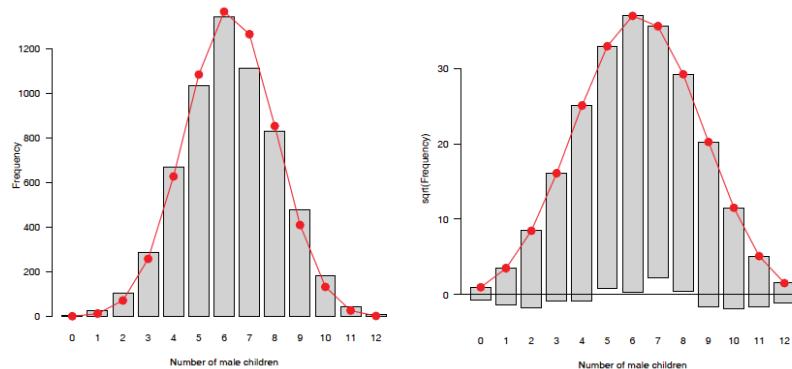
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## 1-way tables: graphs

For a particular distribution in mind:

- Plot the data together with the fitted frequencies
- Better still: **hanging rootogram**: freq on sqrt scale; hang bars from fitted values



## 1-way tables

- Ordered**, quantitative factors

- Number of sons in Saxony families with 12 children

```
> data(Saxony, package="vcd")
```

```
> Saxony
```

```
nMales
```

0	1	2	3	4	5	6	7	8	9	10	11	12
3	24	104	286	670	1033	1343	1112	829	478	181	45	7

Questions:

- What is the **form** of this distribution?
- Is it useful to think of this as a **binomial distribution**?
- If so, is  $\Pr(\text{male}) = 0.5$  reasonable to describe the data?
- How could families have > 10 children?

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## 2-way tables: 2 x 2 x ...

- Two-way

Admit	Gender	
	Male	Female
Admitted	1198	557
Rejected	1493	1278

Admission to  
graduate programs  
at UC Berkeley

- Three-way, stratified by another factor

... by Department

Admit	Gender	Dept					
		A	B	C	D	E	F
Admitted	Male	512	353	120	138	53	22
	Female	89	17	202	131	94	24
Rejected	Male	313	207	205	279	138	351
	Female	19	8	391	244	299	317

Questions:

- Is admission associated with gender?
- Does admission rate vary with department?

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## Larger tables: $r \times c \times \dots$

```
> margin.table(HairEyeColor, 1:2)
  Eye
  Hair  Brown  Blue Hazel Green
  Black   68    20    15     5
  Brown   119   84    54    29
  Red     26    17    14    14
  Blond    7    94    10    16
```

```
> ftable(Eye ~ Sex + Hair, data=HairEyeColor)
  Eye Brown Blue Hazel Green
Sex   Hair
Male  Black     32   11    10    3
      Brown    53   50    25   15
      Red      10   10     7    7
      Blond     3   30     5    8
Female Black    36    9     5    2
      Brown   66   34    29   14
      Red     16    7     7    7
      Blond    4   64     5    8
```

2-way

Actually, this is a 2-way  
margin of a 3-way table

3-way (& higher) can  
be “flattened” for a  
more convenient  
display

formula notation:  
row vars ~ col vars

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## Table form

- Table form is convenient for display, but information is **implicit**

- a table has dimensions, `dim()` and `dimnames()`
- the “observations” are the cells in the tables
- the “variables” are the dimensions of the table (factors)
- the cell value is the count or frequency

```
> dim(haireye)
[1] 4 4
> dimnames(haireye)
$Hair
[1] "Black" "Brown" "Red" "Blond"
$Eye
[1] "Brown" "Blue" "Hazel" "Green"
```

```
> names(dimnames(haireye)) # factor names
[1] "Hair" "Eye"
> prod(dim(haireye)) # of cells
[1] 16
> sum(haireye) # total count
[1] 592
```

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## Datasets: frequency form

- Another common format is a dataset in **frequency form**

```
> as.data.frame(haireye)
  Hair Eye Freq
1 Black Brown   68
2 Brown Brown  119
3 Red Brown   26
4 Blond Brown    7
5 Black Blue   20
6 Brown Blue   84
7 Red Blue    17
8 Blond Blue   94
9 Black Hazel   15
10 Brown Hazel  54
11 Red Hazel   14
12 Blond Hazel  10
13 Black Green   5
14 Brown Green   29
15 Red Green    14
16 Blond Green   16
```

- Create: `as.data.frame(table)`
- One row for each cell
- Columns: factors + `Freq` or `count`

Questions:

- What are the dimensions of the table?
- What is the total frequency?

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## Datasets: case form

- Raw data often arrives in **case form**

```
> expand.dft(as.data.frame(haireye)) |>
  as_tibble() |>
  mutate(age = round(runif(n =
  sum(haireye), min=17, max=29)))
# A tibble: 592 x 3
  Hair   Eye   age
  <chr> <chr> <dbl>
1 Black Brown   19
2 Black Brown   19
3 Black Brown   27
4 Black Brown   23
5 Black Brown   19
6 Black Brown   29
7 Black Brown   25
8 Black Brown   29
9 Black Brown   17
10 Black Brown  23
# ... with 582 more rows
```

- One obs. per case
- # rows = sum of counts
- `vcdExtra::expand.dft()` expands to frequency form
- case form is required if there are continuous variables
- case form is **tidy**
- not all CDA functions play well with tibbles

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## Converting data forms

R functions for CDA sometimes accept only tables (matrices), or data frames, in either case for frequency form.

You may have to convert your data from one form to another

From this ↓	To this ↓	To this ↓	To this ↓
	Case form	Freq form	Table form
<b>Case form</b>	---	Z <- xtabs(~ A + B) as.data.frame(Z)	table(A, B)
<b>Freq form</b>	expand.dft(X)	---	xtabs(Freq ~ A + B)
<b>Table form</b>	expand.dft(X)	as.data.frame(X)	---

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## Categorical data analysis: Methods

Methods for categorical data analysis fall into two main categories

### Non-parametric, randomization-based methods

- Make minimal assumptions
- Useful for **hypothesis-testing**:
  - Are men more likely to be admitted than women?
  - Are hair color and eye color associated?
  - Does the binomial distribution fit these data?
- Mostly for **two-way** tables (possibly stratified)
- R:
  - Pearson Chi-square: `chisq.test()`
  - Fisher's exact test (for small expected frequencies): `fisher.test()`
  - Mantel-Haenszel tests (ordered categories: test for *linear* association): `CMHtest()`
- SAS: PROC FREQ — can do all the above
- SPSS: Crosstabs

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## Categorical data analysis: Methods

### Model-based methods

- Must assume random sample (possibly stratified)
- Useful for **estimation** purposes: Size of effects (std. errors, confidence intervals)
- More suitable for **multi-way** tables
- Greater flexibility; fitting specialized models
  - Symmetry, quasi-symmetry, structured associations for square tables
  - Models for ordinal variables
- R: `glm()` family, Packages: `car`, `gnm`, `vcg`, ...
  - estimate standard errors, covariances for model parameters
  - confidence intervals for parameters, predicted Pr{response}
- SAS: PROC LOGISTIC, CATMOD, GENMOD , INSIGHT (Fit YX), ...
- SPSS: Hiloglinear, Loglinear, Generalized linear models

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## Models: Response vs. Association

### Response models

- Sometimes, one variable is a natural discrete response.
  - Q: How does the response relate to explanatory variables?
    - Admit ~ Gender + Dept
    - Party ~ Age + Education + Urban
- ⇒ Logit models, logistic regression, generalized linear models

### Association models

- Sometimes, the main interest is just association among variables
  - Q: Which variables are associated, and how?
    - Berkeley data: [Admit Gender]? [Admit Dept]? [Gender Dept]
    - Hair-eye data: [Hair Eye]? [Hair Sex]? [Eye, Sex]
- ⇒ Loglinear models

This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

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# Models: Response vs. Association

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    - Admit ~ Gender + Dept
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This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

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# Response models

Analysis methods for categorical outcome (response) variables have close parallels with those for quantitative outcomes

	Quantitative outcome	Categorical outcome
Continuous predictor	Regression: lm(y ~ x1 + x2)	Logistic regression: glm() Loglinear model: loglm() Ordered: prop. odds model: polr()
Categorical predictor	ANOVA: lm(y ~ A + B) Ordered: polynomial contrasts	$\chi^2$ tests: chisq.test() Ordered: CMH tests, CMHtest() Loglinear model: loglm()
Both	ANCOVA: lm(y ~ A + B + X)	Logistic regression: glm() Loglinear model: loglm()

All use similar model formulas:

```
lm(y ~ A)                      # one way ANOVA  
lm(y ~ A*B)                     # two way: A + B + A:B  
lm(y ~ X + A)                   # one-way ANCOVA  
lm(y ~ (A+B+C)^2)               # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

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# Response models

For **quantitative** outcomes, lm() for everything, formula notation

```
lm(y ~ A)                      # one way ANOVA  
lm(y ~ A*B)                     # two way: A + B + A:B  
lm(y ~ X + A)                   # one-way ANCOVA  
lm(y ~ (A+B+C)^2)               # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

For **categorical** outcomes, different modeling functions for different outcome types

```
glm(binary ~ X + A, family="binomial")    # logistic regression  
glm(Freq ~ X + A, family="poisson")        # poisson regression  
MASS::polr(multicat ~ X + A)              # ordinal regression  
nnet::multinom(multicat ~ X + A)           # multinomial regression  
loglin(table, margins)                    # loglinear model  
MASS::loglm(Freq ~ .)                     # loglinear model, . = A+B+C+ ...  
MASS::loglm(Freq ~ .^2)                   # + all two-way associations
```

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# Data display: Tables vs. Graphs

If I can't picture it, I can't understand it.

Albert Einstein

Getting information from a table is like extracting sunlight from a cucumber.  
Farquhar & Farquhar, 1891

## Tables vs. Graphs

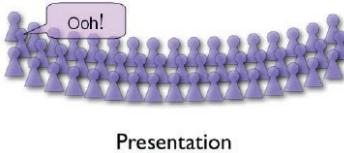
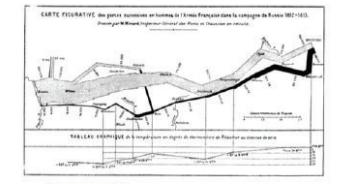
- Tables are best suited for *look-up* and calculation—
  - read off exact numbers
  - show additional calculations (e.g., % change)
- Graphs are better for:
  - showing *patterns, trends, anomalies*,
  - making *comparisons*
  - seeing the *unexpected*!
- Visual presentation as *communication*:
  - what do you want to say or show?
  - ⇒ design graphs and tables to 'speak to the eyes'

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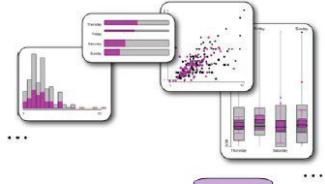
## Graphical methods: Communication goals

Different graphs for different audiences

- **Presentation:** A carefully crafted graph to appeal to a wide audience
- **Exploration, analysis:** Possibly many related graphs, different perspectives, narrow audience (often: just you!)



Presentation

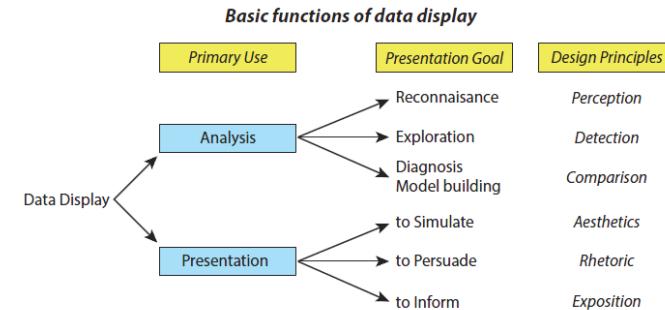


Exploration

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## Graphical methods: Presentation goals

- Different presentation goals appeal to different design principles

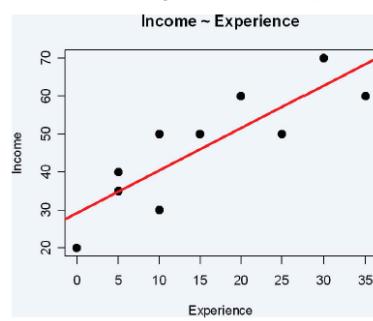


Think: What do I want to communicate? For what purpose?

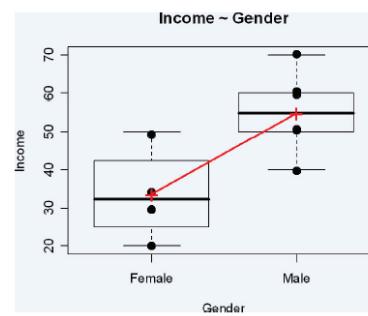
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## Graphical methods: Quantitative data

Quantitative data (amounts) are naturally displayed in terms of **magnitude ~ position along a scale**



Scatterplot of Income vs. Experience

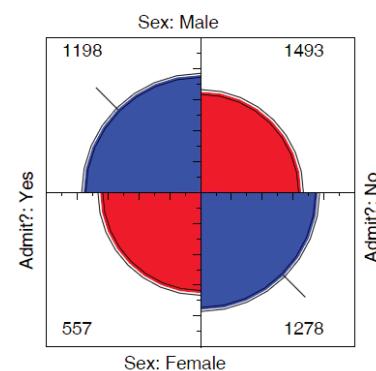


Boxplot of Income by Gender

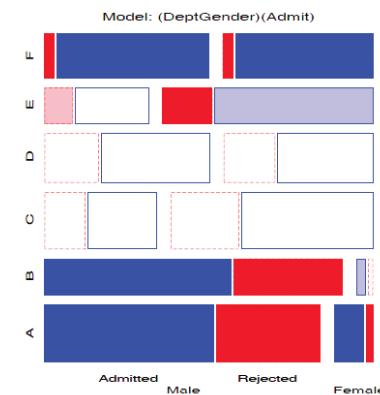
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## Graphical methods: Categorical data

Frequency data (counts) are more naturally displayed in terms of **count ~ area** (Friendly, 1995)



Fourfold display for  $2 \times 2$  table



Mosaic plot for  $3 \times 2$  table

Friendly, M. (1995). *Conceptual and visual models for categorical data*. American Statistician, 49: 153-160.

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## Effective data display

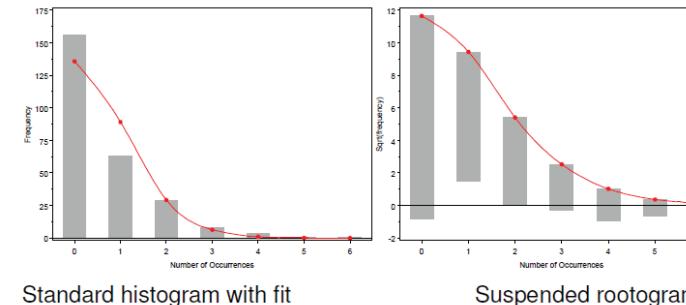
- Make the data stand out
  - Fill the data region (axes, ranges)
  - Use **visually distinct** symbols (shape, color) for different groups
  - Avoid **chart junk**, heavy grid lines that detract from the data
- Facilitate comparison
  - Emphasize the important comparisons **visually**
  - **Side-by-side** easier than in separate panels
  - “data” vs. a “standard” easier against a **horizontal** line
  - Show **uncertainty** where possible
- Effect ordering
  - For **variables** and **unordered** factors, arrange them according to the **effects** to be seen

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## Facilitate comparison

**Comparisons**— Make visual comparisons easy

- Visual grouping— connect with lines, make key comparisons contiguous
- Baselines— compare **data** to **model** against a line, preferably horizontal
- Frequencies often better plotted on a square-root scale

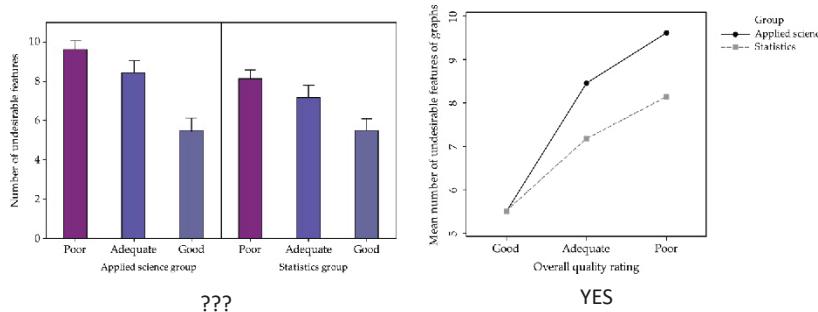


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## Make comparisons *direct*

- Use **points** not bars (and don't dynamite them with ineffective error bars!)
- Connect similar circumstances to be compared by **lines**
- **Same panel** comparisons easier than different panels

Is there evidence of an interaction here?



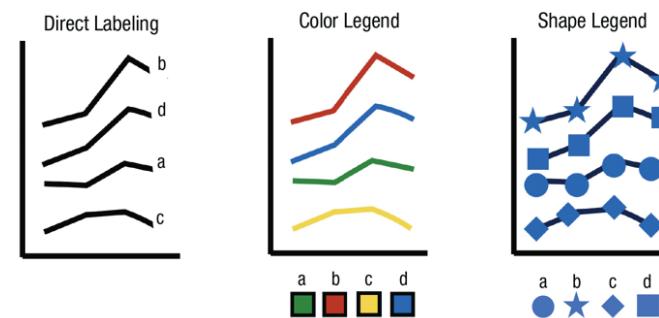
Published in Ian Gordon, Sue Finch: *Journal of Computational and Graphical Statistics* 2015, 24, 1210-1229.  
DOI: 10.1080/10618600.2014.989324  
Copyright © 2015 American Statistical Association, Institute of Mathematical Statistics, and Interface Foundation of North America

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## Direct labels vs. legends

**Direct labels** for points, lines and regions are usually easier and faster than **legends**

- Give the names of the four groups shown in the line graph at left in top-to-bottom order.  
(Answer: b, d, a, c.)
- Now do so for the graphs using **color** or **shape** legends
- You need to look back and forth between the graph and legend



Source: Franconeri et al. DOI:[10.1177/15291006211051956](https://doi.org/10.1177/15291006211051956)

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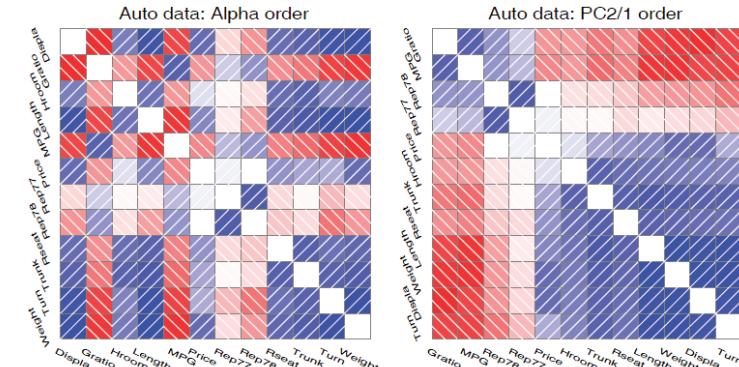
## Effect ordering

- Information presentation is always **ordered**
  - in **time** or sequence (a talk or written paper)
  - in **space** (table or graph)
  - Constraints of time & space are dominant— can conceal or reveal the important message
- Effect ordering for data display**
  - Sort the data by the **effects to be seen**
  - Order the data to **facilitate the task** at hand
    - lookup** – find a value
    - comparison** – which is greater?
    - detection** – find patterns, trends, anomalies

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## Effect Ordering: Correlations

- Effect ordering** (Friendly and Kwan, 2003)— In tables and graphs, sort unordered factors according to the effects you want to see/show.



Friendly & Kwan (2003). *Corrrgrams: Exploratory displays for correlation matrices*. *American Statistician*, 54(4): 316-324.

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## Tabular displays: Main effect ordering

- Tables are often presented with rows/cols ordered **alphabetically**
  - good for lookup
  - bad for seeing patterns, trends, anomalies

Table 1: Average Barley Yields (rounded), Means by Site and Variety

Variety	Site					Mean	
	Crookston	Duluth	Grand Rapids	Morris	University Farm		
Glabron	32	28	22	32	40	46	33.3
Manchuria	36	26	28	31	27	41	31.5
No. 457	40	28	26	36	35	50	35.8
No. 462	40	25	22	39	31	55	35.4
No. 475	38	30	17	33	27	44	31.8
Peatland	33	32	31	37	30	42	34.2
Svansota	31	24	23	30	31	43	30.4
Trebi	44	32	25	45	33	57	39.4
Velvet	37	24	28	32	33	44	33.1
Wisconsin No. 38	43	30	28	38	39	58	39.4
<b>Mean</b>	37.4	28.0	24.9	35.4	32.7	48.1	34.4

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## Tabular displays: Main effect ordering

- Better: sort rows/cols by **means/medians**
- Shade cells according to **residual** from additive model

Table 2: Average Barley Yields, sorted by Mean, shaded by residual from the model Yield = Variety + Site

Variety	Site					Mean	
	Grand Rapids	Duluth	University Farm	Morris	Crookston		
Svansota	23	24	31	30	31	43	30.4
Manchuria	28	26	27	31	36	41	31.5
No. 475	17	30	27	33	38	44	31.8
Velvet	28	24	33	32	37	44	33.1
Glabron	22	28	40	32	32	46	33.3
Peatland	31	32	30	37	33	42	34.2
No. 462	22	25	31	39	40	55	35.4
No. 457	26	28	35	36	40	50	35.8
Wisconsin No. 38	28	30	39	38	43	58	39.4
<b>Mean</b>	24.9	28.0	32.7	35.4	37.4	48.1	34.4

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# Effect ordering: Frequency tables

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Alpha ordered

Eye color	Hair color			
	Blond	Black	Brown	Red
Blue	94	20	17	84
Brown	7	68	26	119
Green	10	15	14	54
Hazel	16	5	14	29

Model:	<i>Independence</i> : [Hair][Eye] $\chi^2(9) = 138.29$
Color coding:	<-4 <-2 <-1 0 >1 >2 >4
n in each cell:	n < expected n > expected

There is an association, but it is hard to see the general pattern

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# Effect ordering: Frequency tables

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Effect ordered

Eye color	Hair color			
	Black	Brown	Red	Blond
Brown	68	119	26	7
Hazel	15	54	14	10
Green	5	29	14	16
Blue	20	84	17	94

Model:	<i>Independence</i> : [Hair][Eye] $\chi^2(9) = 138.29$
Color coding:	<-4 <-2 <-1 0 >1 >2 >4
n in each cell:	n < expected n > expected

The pattern is clearer when the eye colors are **permuted**: light hair goes with light eyes & vice-versa

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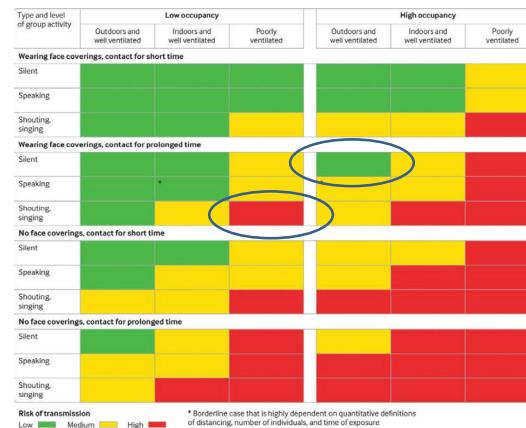
# Sometimes, don't need numbers at all

COVID transmission risk ~ Occupancy \* Ventilation \* Activity \* Mask? \* Contact.time

A complex 5-way table, whose message is clearly shown w/o numbers

A semi-graphic table shows the **patterns** in the data

There are 1+ unusual cells here. Can you see them?



From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? BMJ 2020;370:m3223, doi: <https://doi.org/10.1136/bmj.m3223>

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# Visual table ideas: Heatmap shading

**Heatmap shading:** Shade the **background** of each cell according to some criterion

## Unemployment rate in selected countries

January-August 2020, sorted by the unemployment rate in January.

country	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Japan	2.4%	2.4%	2.5%	2.6%	2.9%	2.8%	2.9%	3.0%
Netherlands	3.0%	2.9%	2.9%	3.4%	3.6%	4.3%	4.5%	4.6%
Germany	3.4%	3.6%	3.8%	4.0%	4.2%	4.3%	4.4%	4.4%
Mexico	3.6%	3.6%	3.2%	4.8%	4.3%	5.4%	5.2%	5.0%
US	3.6%	3.5%	4.4%	14.7%	13.3%	11.1%	10.2%	8.4%
South Korea	4.0%	3.3%	3.8%	3.8%	4.5%	4.3%	4.2%	3.2%
Denmark	4.9%	4.9%	4.8%	4.9%	5.5%	6.0%	6.3%	6.1%
Belgium	5.1%	5.0%	5.0%	5.1%	5.0%	5.0%	5.0%	5.1%
Australia	5.3%	5.1%	5.2%	6.4%	7.1%	7.4%	7.5%	6.8%
Canada	5.5%	5.6%	7.8%	13.0%	13.7%	12.3%	10.9%	10.2%
Finland	6.8%	6.9%	7.0%	7.3%	7.5%	7.8%	8.0%	8.1%

Source: OECD • Get the data • Created with Datawrapper

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# Bertifier: Turning tables into graphs

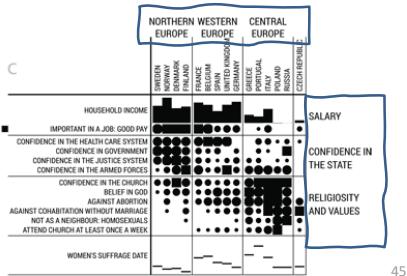
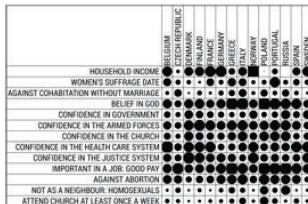
a attitudes & attributes

	Belgi	Czech	Denn	Finn	Franc	Germ	Gree	Ita	Non	Pola	Portu	Rusia	Sc	Swe	United
Household income	2687	16907	2406	2571	2833	2078	2042	1455	1537	1936	1520	21	2620	2004	
Women's suffrage date	1948	1920	1915	1906	1944	1818	1952	1910	1913	1918	1976	1914	1911	1921	1928
Against cohabitation w.	12	42	4	18	8	20	30	46	12	39	17	39	16	6	19
Belief in God	61	36	63	69	92	63	91	96	86	77	76	46	65		
Confidence in Government	32	21	55	42	34	29	22	28	61	23	30	60	35	54	19
Confidence in the army	59	34	72	83	73	58	70	75	67	63	75	73	57	41	89
Confidence in the health	36	20	63	47	41	40	52	44	65	67	67	31	39	36	
Confidence in the just	91	42	75	73	78	34	39	64	74	44	58	51	79	75	80
Confidence in the polis	50	35	87	73	56	58	50	36	78	44	48	41	69	51	
Important in a job: god	69	85	54	58	58	73	94	76	66	93	77	62	75		
Against abortion	56	28	40	44	60	65	72	42	75	61	63	57	25	57	
Not as a neighbour	7	22	5	12	5	16	30	21	6	52	21	61	5	7	10
Attend church at least	15	13	5	7	11	12	19	35	9	54	25	8	21	9	17

- (a) Table: attitudes and attributes by country
- (b) Visual: encode values by size, shape
- (c) Sort & group by themes, country regions

Bertifier: Bertin's reorderable matrix  
See: <http://www.aviz.fr/bertifier>

b encode values by size & shape



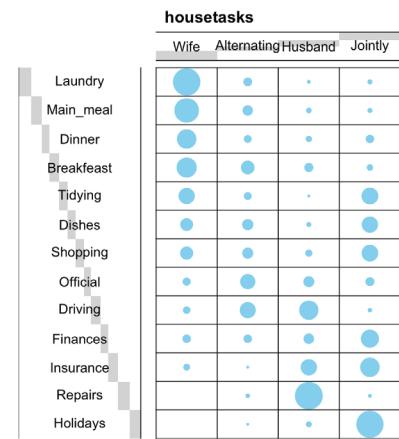
# Example: Household tasks

Who does what in households?

Size of symbols in a balloon plot shows the frequencies

	Who_does_it?	Alternating	Husband	Jointly	Wife
Breakfast		36	15	7	82
Dinner		11	7	13	77
Dishes		24	4	53	32
Driving		51	75	3	10
Finances		13	21	66	13
Holidays		1	6	153	0
Insurance		1	53	77	8
Laundry		14	2	4	156
Main_meal		20	5	4	124
Official		46	23	15	12
Repairs		3	160	2	0
Shopping		23	9	55	33
Tidying		11	1	57	53

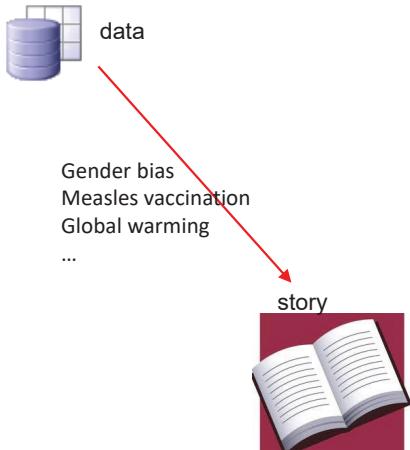
Rows and columns were permuted to show the relationship more clearly



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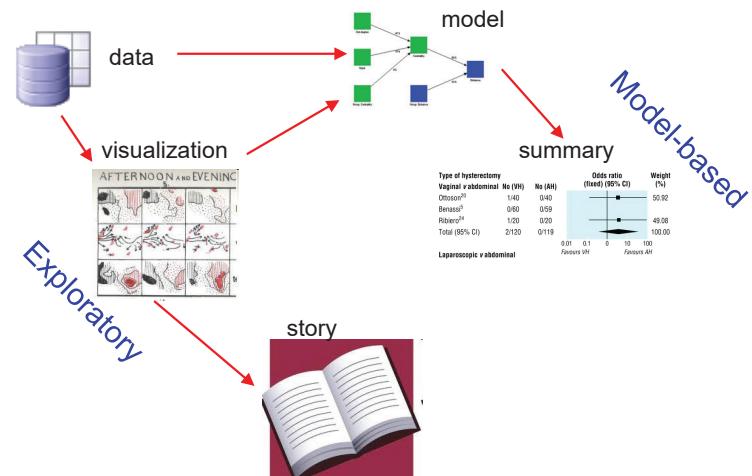
## Data, pictures, models & stories

Goal: Tell a credible story about some real data problem



## Data, pictures, models & stories

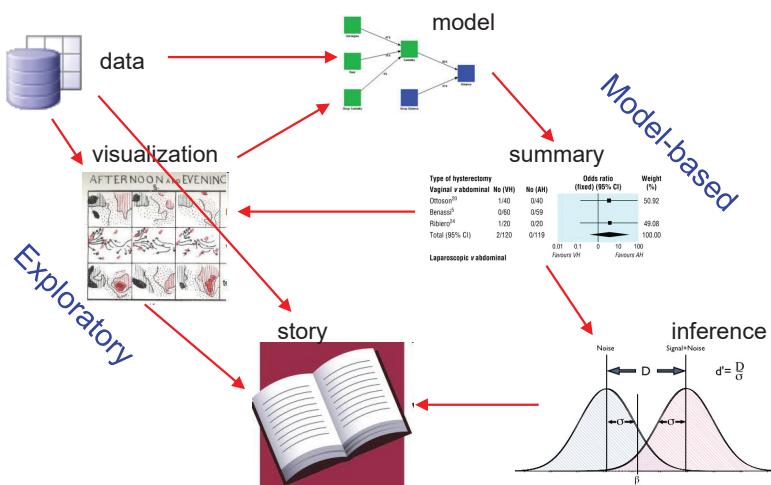
Two paths to enlightenment



Type of hysterectomy	Vaginal v abdominal	No (VH)	No (AH)	Odds ratio (fixed) (95% CI)	Weight
Ottosen <sup>[27]</sup>	1/40	0/40	0/40	0.00	0.00
Berssen <sup>[28]</sup>	1/20	0/20	0/20	0.00	0.00
Total (95% CI)	2/60	0/60	0/60	0.00	0.00
Laparoscopic v abdominal					
				50.92	0.119

# Data, pictures, models & stories

Now, tell the story!



## $2 \times 2$ Frequency Tables: Fourfold displays

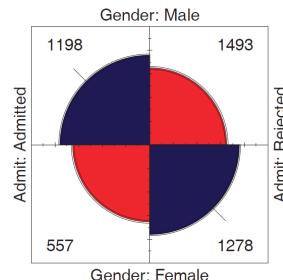
Table: Admissions to Berkeley graduate programs

	Admitted	Rejected	Total	% Admit	Odds(Admit)
Males	1198	1493	2691	44.52	0.802
Females	557	1278	1835	30.35	0.437
Total	1755	2771	4526	38.78	0.633

odds ratio ( $\theta$ ) = 1.84

Males nearly twice as likely to be admitted

- Is this a “significant” association?
- Is it evidence for gender bias?
- How to measure strength of association?
- How to visualize?



Fourfold display:

- quarter circles, area ~ frequency
- ratio of areas: odds ratio ( $\theta$ )
- confidence bands: overlap iff  $\theta \approx 1$
- visualize signif. by shading

## Gender Bias at UC Berkeley?

Science, 1975, 187: 398–403

### Sex Bias in Graduate Admissions: Data from Berkeley

Measuring bias is harder than is usually assumed, and the evidence is sometimes contrary to expectation.

P. J. Bickel, E. A. Hammel, J. W. O’Connell

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deceision to admit or to deny admission. The question we wish to pursue is whether the decision to admit or to deny was influenced by the sex of the applicant. We cannot know with any certainty the influences on the evaluators in the

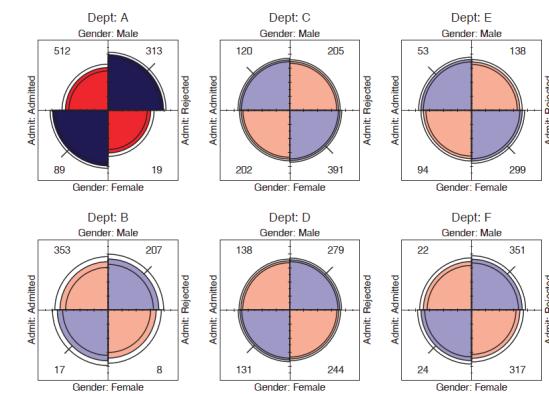
## $2 \times 2 \times k$ Stratified tables

The data arose from 6 graduate departments

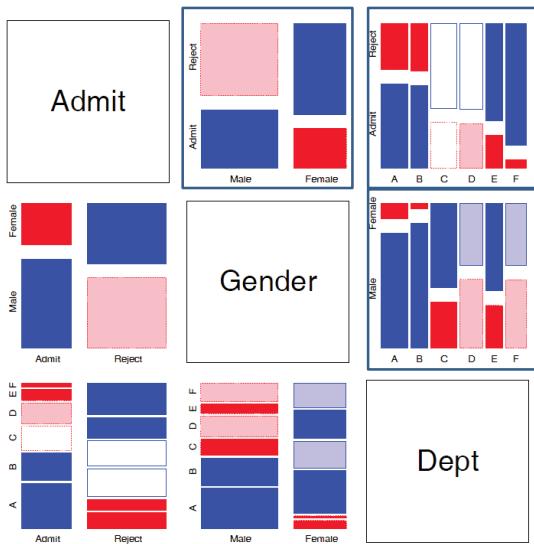
No difference between males & females, except in Dept A where women more likely to be admitted!

Design:

- small multiples
- encode direction by color
- encode signif. by shading



# Mosaic matrices



Scatterplot matrix analog for categorical data

All pairwise views  
Small multiples → comparison

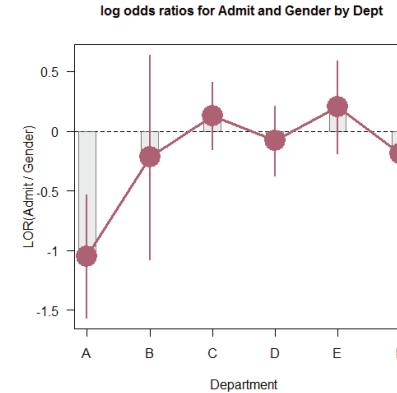
The answer: **Simpson's Paradox**

- Depts A, B were easiest
- Applicants to A, B mostly male
- ∴ Males more likely to be admitted **overall**

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# Measures & models

If the focus is on the association between **gender** and **admission** for each department the **odds ratio**:  $\text{odds}(\text{Admit}|\text{Male}) / \text{odds}(\text{Admit}|\text{Female})$  is a good summary



$\text{odds} = \text{Pr}(\text{Admit}) / \text{Pr}(\text{Reject})$   
 $\text{OR} = \text{odds}(\text{Admit}|\text{M}) / \text{odds}(\text{Admit}|\text{F})$

$\text{OR} = 1 \rightarrow \text{M/F equally likely admitted}$

$\text{LOR} = \log(\text{OR}) : 0 \rightarrow \text{M/F equally likely admitted}$

Std errors & CIs provide individual signif tests

Models provide a comprehensive summary

Now, we can tell the story !

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# Graphical methods for categorical data

These share similar ideas & scope with methods for quantitative data

## Exploratory methods

- Minimal assumptions (like non-parametric methods)
- Show the *data*, not just *summaries*
- But can add summaries: smoothed curve(s), trend lines, ...
- Help detect *patterns, trends, anomalies*, suggest hypotheses

## Plots for model-based methods

- Residual plots - departures from model, omitted terms, ...
- Effect plots - estimated probabilities of response or log odds
- Diagnostic plots - influence, violation of assumptions

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# Plots: Data, Model, Data+Model

- **Data plots:** well-known. Help to answer:
  - What do the data look like?
  - Are there unusual features? (outliers, non-linear relations)
  - What kinds of summaries would be useful?
- **Model plots**
  - What does the model look like? (plot predicted values)
  - How does the model change when parameters change? (plot competing models)
  - How does the model change when the data is changed? (influence plots)
- **Data+Model plots**
  - How well does model fit the data? (focus on residuals)
  - Does model fit uniformly good/bad, or just in some regions?
  - Model uncertainty: show confidence regions
  - Data support: where is data too thin to make a difference?

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## Summary

- Categorical data involves some new ideas
  - Discrete variables: `unordered` or `ordered`
  - Counts, frequencies as outcomes
- New / different data structures & functions
  - tables – 1-way, 2-way, 3-way, ... `table()`, `xtabs()`
  - similar in matrices or arrays `matrix()`, `array()`
  - datasets:
    - frequency form
    - case form
- Graphical methods: often use area ~ Freq
  - Consider: graphical comparisons, effect order
- Models: Most are  $\approx$  natural extensions of `lm()`