# Logistic Regression II

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Psych 6136

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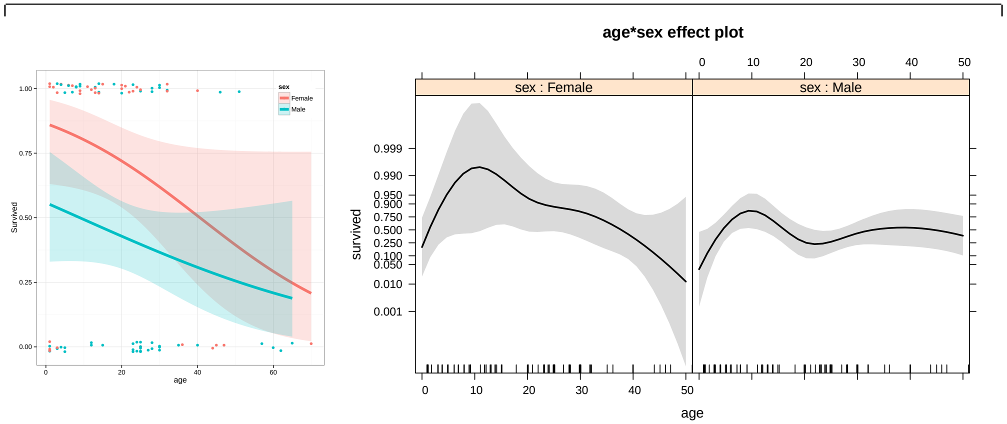


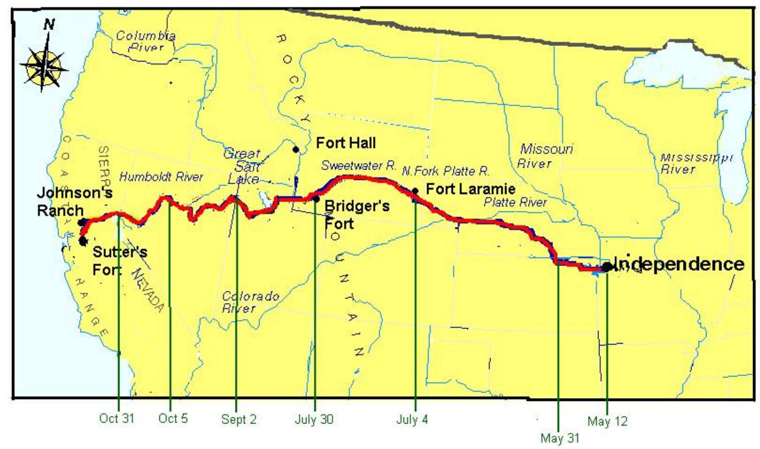



## Donner Party: A graphic tale of survival & influence

History:

- Apr–May, 1846: Donner/Reed families set out from Springfield, IL to CA
- Jul: Bridger's Fort, WY, 87 people, 23 wagons

TRAIL OF THE DONNER PARTY





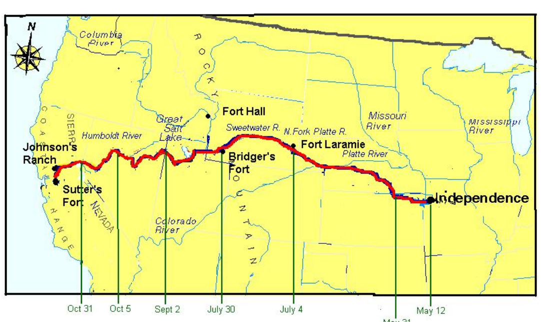



## Donner Party: A graphic tale of survival & influence

History:

- "Hasting's Cutoff", untried route through Salt Lake Desert, Wasatch Mtns. (90 people)
- Worst recorded winter: Oct 31 blizzard— Missed by 1 day, stranded at "Truckee Lake" (now Donner's Lake, Reno)
  - Rescue parties sent out ("Dire necessity", "Forelorn hope", ...)
  - Relief parties from CA: 42 survivors (Mar–Apr, '47)

TRAIL OF THE DONNER PARTY







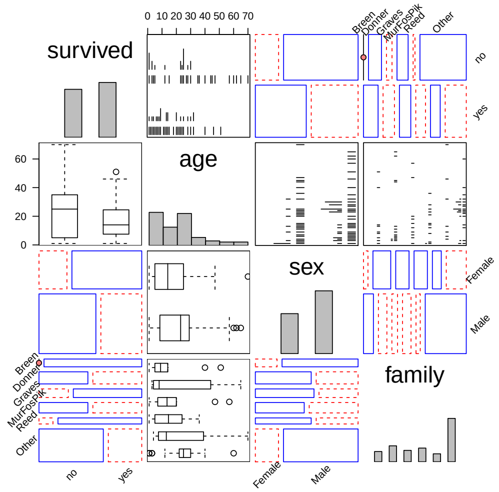

## Donner Party: Data

```
data("Donner", package="vcdExtra")
Donner$survived <- factor(Donner$survived, labels=c("no", "yes"))
```

```
library(car)
some(Donner, 12)

##                         family age    sex survived      death
## Breen, Peter             Breen   3   Male      yes       <NA>
## Donner, George          Donner  62   Male       no 1847-03-18
## Donner, Jacob           Donner  65   Male       no 1846-12-21
## Foster, Jeremiah     MurFosPik   1   Male       no 1847-03-13
## Graves, Jonathan        Graves   7   Male      yes       <NA>
## Graves, Mary Ann        Graves  20 Female      yes       <NA>
## Graves, Nancy           Graves   9 Female      yes       <NA>
## McCutchen, Harriet McCutchen   1 Female       no 1847-02-02
## Reed, James               Reed  46   Male      yes       <NA>
## Reed, Thomas Keyes        Reed   4   Male      yes       <NA>
## Reinhardt, Joseph        Other  30   Male       no 1846-12-21
## Wolfinger, Doris      FosdWolf  20 Female      yes       <NA>
```

# Overview: a `gpairs()` plot

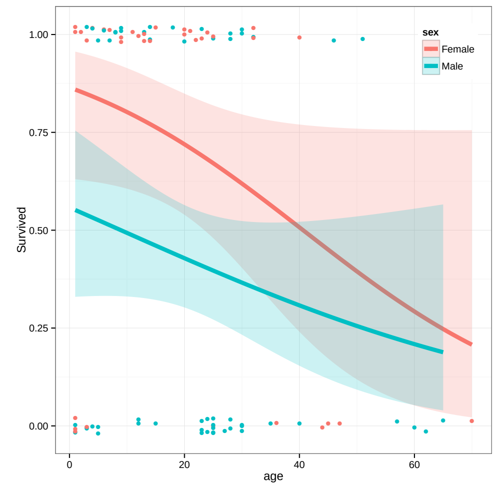
- Binary response: `survived`
- Categorical predictors: `sex`, `family`
- Quantitative predictor: `age`
- Q: Is the effect of age linear?
- Q: Are there interactions among predictors?
- This is a generalized pairs plot, with different plots for each pair






Model building Exploratory plots

# Exploratory plots




- Survival decreases with age for both men and women
- Women more likely to survive, particularly the young
- Data is thin at older ages



Model building Exploratory plots

# Using ggplot2

Basic plot: survived vs. age, colored by sex, with jittered points

```
gg <- ggplot(Donner,
             aes(age, as.numeric(survived=="yes"), color = sex)) +
  ylab("Survived") +
  geom_point(position = position_jitter(height = 0.02, width = 0))
```

Add conditional linear logistic regressions with `stat_smooth(method="glm")`

```
gg + stat_smooth(method = "glm", family = binomial, formula = y ~ x,
                 alpha = 0.2, size=2, aes(fill = sex))
```



Model building Exploratory plots

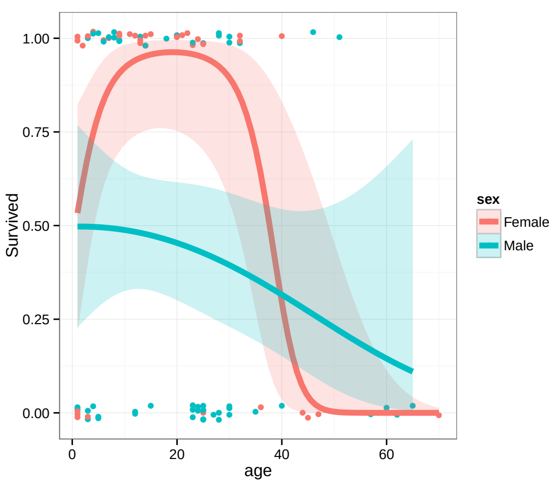
# Questions

- Is the relation of survival to age well expressed as a linear logistic regression model?
  - Allow a quadratic or higher power, using `poly(age,2)`, `poly(age,3)`,

$$\begin{aligned}\text{logit}(\pi_i) &= \alpha + \beta_1 x_i + \beta_2 x_i^2 \\ \text{logit}(\pi_i) &= \alpha + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 \\ &\ldots\end{aligned}$$

  - Use *natural spline* functions, `ns(age, df)`
  - Use non-parametric smooths, `loess(age, span, degree)`
- Is the relation the same for men and women? i.e., do we need an interaction of age and sex?
  - Allow an interaction of `sex * age` or `sex * f(age)`
  - Test goodness-of-fit relative to the main effects model

```
gg + stat_smooth(method = "glm", family = binomial,
                 formula = y ~ poly(x,2),
                 alpha = 0.2, size=2, aes(fill = sex))
```

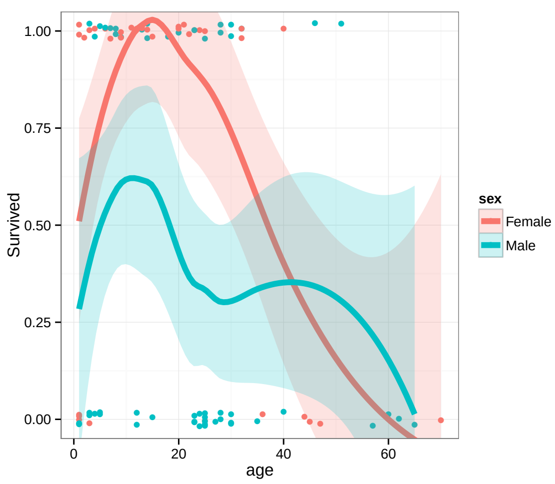



Fit separate quadratics for males and females





```
gg + stat_smooth(method = "loess", span=0.9,
                 alpha = 0.2, size=2,
                 aes(fill = sex)) + coord_cartesian(ylim=c(-.05,1.05))
```




Fit separate loess smooths for males and females





# Fitting models

Models with linear effect of age:

```
donner.mod1 <- glm(survived ~ age + sex,
                   data=Donner, family=binomial)
donner.mod2 <- glm(survived ~ age * sex,
                   data=Donner, family=binomial)
Anova(donner.mod2)

## Analysis of Deviance Table (Type II tests)
##
## Response: survived
##         LR Chisq Df Pr(>Chisq)
## age         5.52  1     0.0188 *
## sex         6.73  1     0.0095 **
## age:sex     0.40  1     0.5269
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```





# Fiting models

Models with quadratic effect of age:

```
donner.mod3 <- glm(survived ~ poly(age,2) + sex,
                   data=Donner, family=binomial)
donner.mod4 <- glm(survived ~ poly(age,2) * sex,
                   data=Donner, family=binomial)
Anova(donner.mod4)

## Analysis of Deviance Table (Type II tests)
##
## Response: survived
##                  LR Chisq Df Pr(>Chisq)
## poly(age, 2)         9.91  2     0.0070 **
## sex                  8.09  1     0.0044 **
## poly(age, 2):sex     8.93  2     0.0115 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

# Comparing models

```
library(vcdExtra)
LRstats(donner.mod1, donner.mod2, donner.mod3, donner.mod4)

## Likelihood summary table:
##             AIC BIC LR Chisq Df Pr(>Chisq)
## donner.mod1 117 125    111.1 87      0.042 *
## donner.mod2 119 129    110.7 86      0.038 *
## donner.mod3 115 125    106.7 86      0.064 .
## donner.mod4 110 125     97.8 84      0.144
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

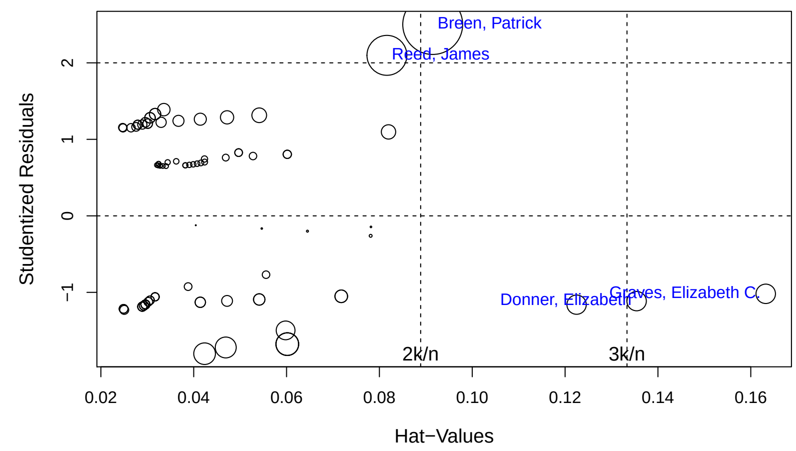
| | linear | non-linear | $\Delta\chi^2$ | *p*-value |
|---|---|---|---|---|
| additive | 111.128 | 106.731 | 4.396 | 0.036 |
| non-additive | 110.727 | 97.799 | 12.928 | 0.000 |
| $\Delta\chi^2$ | 0.400 | 8.932 | | |
| *p*-value | 0.527 | 0.003 | | |





# Who was influential?

```
library(car)
res <- influencePlot(donner.mod3, id.col="blue", scale=8, id.n=2)
```








# Why are they influential?

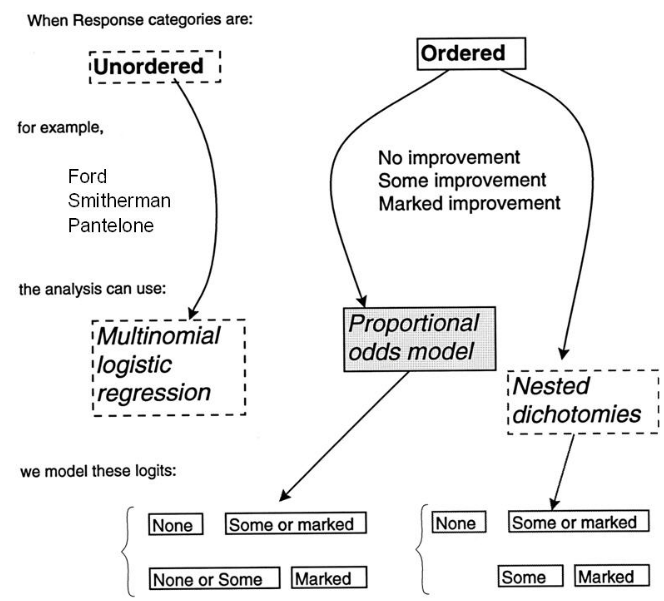
```
idx <- which(rownames(Donner) %in% rownames(res))
# show data together with diagnostics
cbind(Donner[idx,2:4], res)

##                     age    sex survived StudRes     Hat   CookD
## Breen, Patrick       51   Male      yes   2.501 0.09148 0.32354
## Donner, Elizabeth    45 Female       no  -1.114 0.13541 0.03409
## Graves, Elizabeth C. 47 Female       no  -1.019 0.16322 0.03418
## Reed, James          46   Male      yes   2.098 0.08162 0.14364
```

- Patrick Breen, James Reed: Older men who survived
- Elizabeth Donner, Elizabeth Graves: Older women who died
- Moral lessons of this story:
  - Don't try to cross the Donner Pass in late October; if you do, bring lots of food
  - Plots of fitted models show *only* what is included in the model
  - Discrete data often need smoothing (or non-linear terms) to see the pattern
  - Always examine model diagnostics — preferably graphic





# Polytomous responses: Overview



# Polytomous responses: Overview

- $m$ categories $\rightarrow (m-1)$ comparisons (logits)
  - One part of the model for each logit
  - Similar to ANOVA where an $m$-level factor $\rightarrow (m-1)$ contrasts (df)
- **Response categories *unordered***, e.g., vote NDP, Liberal, Green, Tory
  - Multinomial logistic regression
    - Fits $m-1$ logistic models for logits of category $i = 1, 2, \ldots m-1$ vs. category $m$
    - e.g.,

| | | | |
|---|---|---|---|
| NDP | | | Tory |
| | Liberal | | Tory |
| | | Green | Tory |

    - This is the most general approach
    - R: `multinom()` function in nnet
  - Can also use nested dichotomies





# Polytomous responses: Overview

- **Response categories *ordered***, e.g., None, Some, Marked improvement
  - Proportional odds model
    - Uses adjacent-category logits

| | |
|---|---|
| None | Some or Marked |
| None or Some | Marked |

    - Assumes slopes are equal for all $m-1$ logits; only intercepts vary
    - R: `polr()` in MASS
  - Nested dichotomies

| | |
|---|---|
| None | Some or Marked |
| Some | Marked |

    - Model each logit separately
    - $G^2$ s are additive $\rightarrow$ combined model

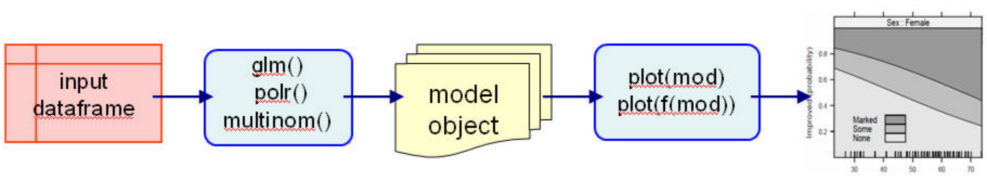




# Fitting and graphing: Overview

R:

- Model objects contain all necessary information for plotting
- Basic diagnostic plots with `plot(model)`
- Fitted values with `predict()`; customize with `points()`, `lines()`, etc.
- Effect plots most general








# Ordinal response: Proportional odds model

Arthritis treatment data:

| Sex | Treatment | Improvement None | Some | Marked | Total |
|---|---|---|---|---|---|
| F | Active | 6 | 5 | 16 | 27 |
| F | Placebo | 19 | 7 | 6 | 32 |
| M | Active | 7 | 2 | 5 | 14 |
| M | Placebo | 10 | 0 | 1 | 11 |

- Model logits for adjacent category cutpoints:

$$\text{logit}\,(\theta_{ij1}) = \log \frac{\pi_{ij1}}{\pi_{ij2} + \pi_{ij3}} = \text{logit ( None vs. [Some or Marked] )}$$

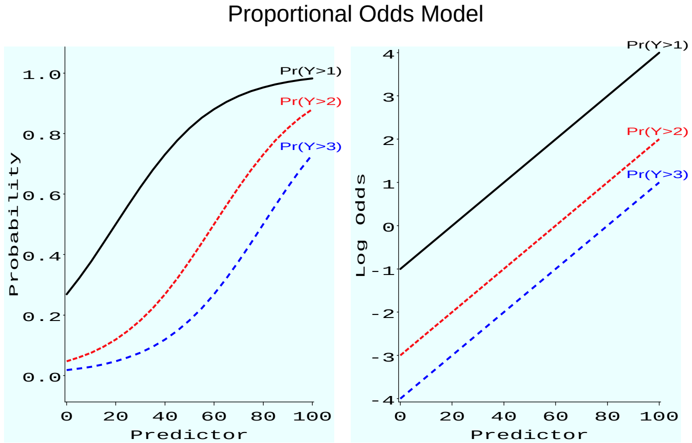
$$\text{logit}\,(\theta_{ij2}) = \log \frac{\pi_{ij1} + \pi_{ij2}}{\pi_{ij3}} = \text{logit ( [None or Some] vs. Marked)}$$

- Consider a logistic regression model for each logit:

$$\text{logit}(\theta_{ij1}) = \alpha_1 + \boldsymbol{x}_{ij}'\boldsymbol{\beta}_1 \qquad \text{None vs. Some/Marked}$$

$$\text{logit}(\theta_{ij2}) = \alpha_2 + \boldsymbol{x}_{ij}'\boldsymbol{\beta}_2 \qquad \text{None/Some vs. Marked}$$

- Proportional odds assumption: regression functions are parallel on the logit scale i.e., $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$.







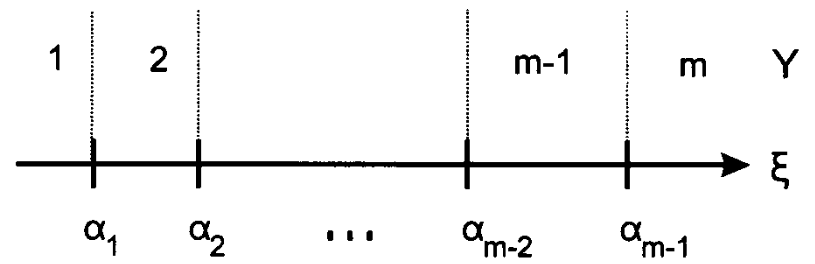

# Proportional odds: Latent variable interpretation

A simple motivation for the proportional odds model:

- Imagine a continuous, but *unobserved* response, $\xi$, a linear function of predictors

$$\xi_i = \boldsymbol{\beta}^\mathsf{T}\boldsymbol{x}_i + \epsilon_i$$

- The *observed* response, Y, is discrete, according to some *unknown* thresholds, $\alpha_1 < \alpha_2, < \cdots < \alpha_{m-1}$
- That is, the response, $Y = i$ if $\alpha_i \leq \xi_i < \alpha_{i+1}$
- Thus, intercepts in the proportional odds model $\sim$ thresholds on $\xi$

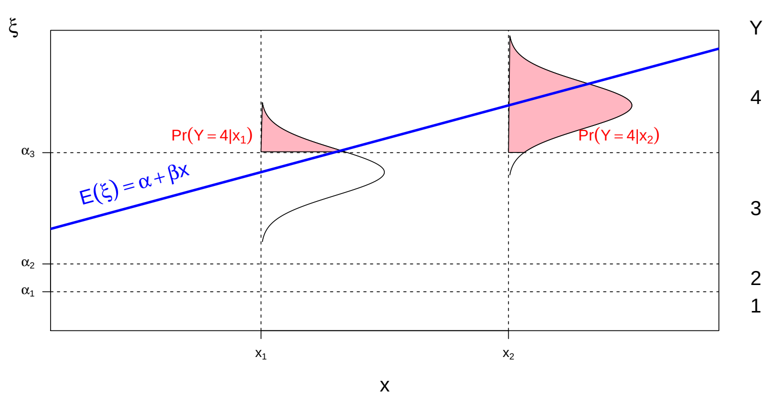







# Proportional odds: Latent variable interpretation

We can visualize the relation of the latent variable $\xi$ to the observed response $Y$, for two values, $x_1$ and $x_2$, of a single predictor, $X$ as shown below:

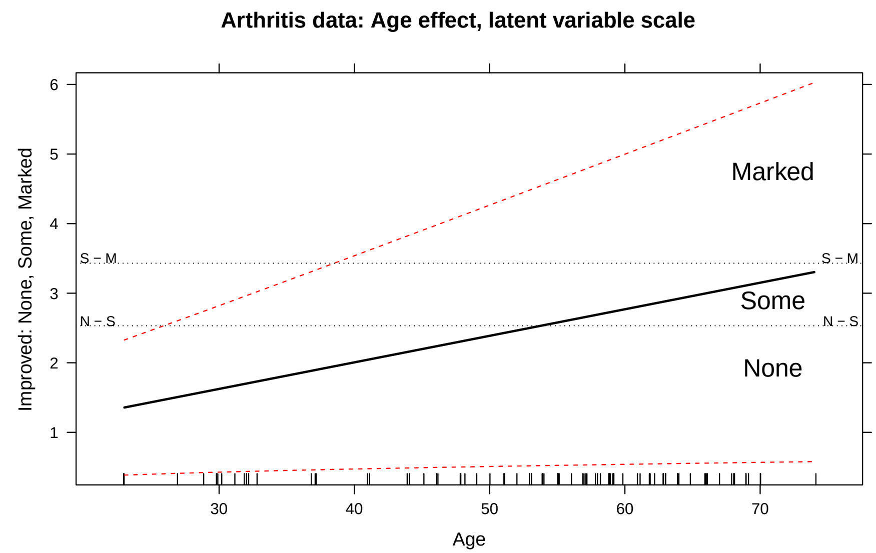







# Proportional odds: Latent variable interpretation

For the Arthritis data, the relation of improvement to age is shown below (using the effects package)



# Proportional odds models in R

- **Fitting:** **polr()** in MASS package

The response, `Improved` has been defined as an *ordered* factor

```
data(Arthritis, package="vcd")
head(Arthritis$Improved)

## [1] Some   None   None   Marked Marked Marked
## Levels: None < Some < Marked
```

Fitting:

```
library(MASS)          # for polr()
library(car)           # for Anova()

arth.polr <- polr(Improved ~ Sex + Treatment + Age,
                  data=Arthritis)
summary(arth.polr)
Anova(arth.polr)       # Type II tests
```



The **summary()** function gives standard statistical results:

```
> summary(arth.polr)
```

```
Call:
polr(formula = Improved ~ Sex + Treatment + Age, data = Arthritis)

Coefficients:
                   Value  Std. Error  t value
SexMale         -1.25168     0.54636  -2.2909
TreatmentTreated 1.74529     0.47589   3.6674
Age              0.03816     0.01842   2.0722

Intercepts:
            Value   Std. Error t value
None|Some    2.5319  1.0571     2.3952
Some|Marked  3.4309  1.0912     3.1442

Residual Deviance: 145.4579
AIC: 155.4579
```

The car::**Anova()** function gives hypothesis tests for model terms:

```
> Anova(arth.polr)      # Type II tests
```

```
Anova Table (Type II tests)

Response: Improved
          LR Chisq Df Pr(>Chisq)
Sex         5.6880  1  0.0170812 *
Treatment  14.7095  1  0.0001254 ***
Age         4.5715  1  0.0325081 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- **anova()** gives Type I (sequential) tests — not usually useful
- Type II (partial) tests control for the effects of all other terms



# Testing the proportional odds assumption

- The PO model is valid only when the slopes are equal for all predictors
- This can be tested by comparing this model to the generalized logit NPO model

$$\text{PO}: \quad L_j = \alpha_j + \mathbf{x}^\mathsf{T}\beta \qquad j = 1, \ldots, m-1 \tag{1}$$

$$\text{NPO}: \quad L_j = \alpha_j + \mathbf{x}^\mathsf{T}\beta_j \qquad j = 1, \ldots, m-1 \tag{2}$$

- A likelihood ratio test requires fitting both models calculating $\Delta G^2 = G^2_{\text{NPO}} - G^2_{\text{PO}}$ with $p$ df.
- This can be done using **vglm()** in the VGAM package
- The rms package provides a visual assessment, plotting the conditional mean $E(X \mid Y)$ of a given predictor, $X$, at each level of the ordered response $Y$.
- If the response behaves ordinally in relation to $X$, these means should be strictly increasing or decreasing with $Y$.

# Testing the proportional odds assumption

In VGAM, the PO model is fit using `family = cumulative(parallel=TRUE)`

```
library(VGAM)
arth.po <- vglm(Improved ~ Sex + Treatment + Age, data=Arthritis,
                family = cumulative(parallel=TRUE))
```

The more general NPO model can be fit using `parallel=FALSE`.

```
arth.npo <- vglm(Improved ~ Sex + Treatment + Age, data=Arthritis,
                 family = cumulative(parallel=FALSE))
```

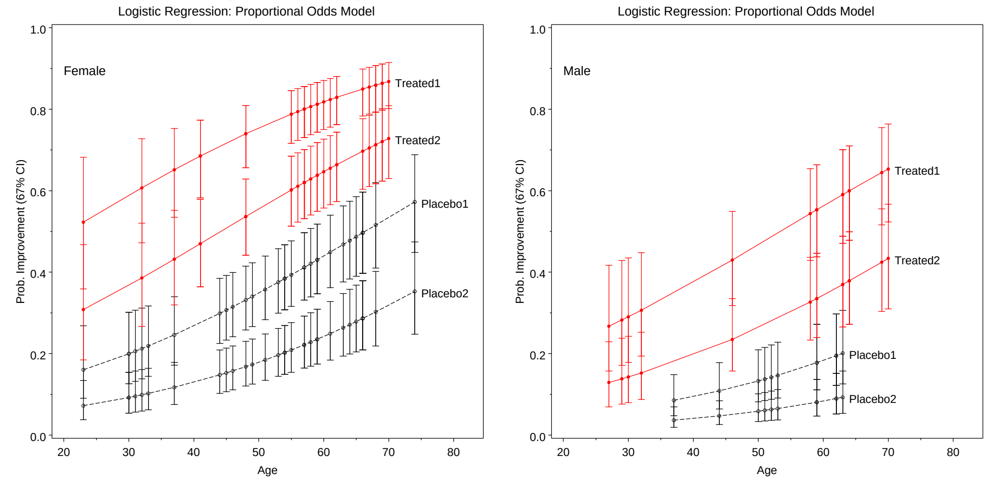
The LR test says the PO model is OK:

```
VGAM::lrtest(arth.npo, arth.po)

## Likelihood ratio test
##
## Model 1: Improved ~ Sex + Treatment + Age
## Model 2: Improved ~ Sex + Treatment + Age
##   #Df LogLik Df Chisq Pr(>Chisq)
## 1 160  -71.8
## 2 163  -72.7  3  1.88        0.6
```





Full-model plot of predicted probabilities:




- Intercept1: [Marked , Some] vs. [None]
- Intercept2: [Marked] vs. [Some, None]
- On logit scale, these would be parallel lines
- Effects of age, treatment, sex similar to what we saw before

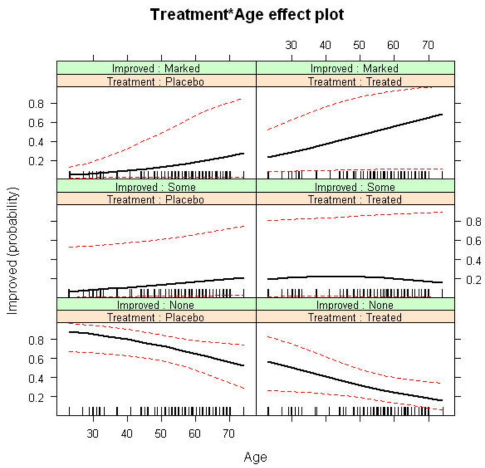




# Proportional odds models in R: Plotting

- **Plotting:** `plot(effect())` in effects package

```
> library(effects)
> plot(effect("Treatment:Age", arth.polr))
```




- The default plot shows all details
- But, is harder to compare across treatment and response levels

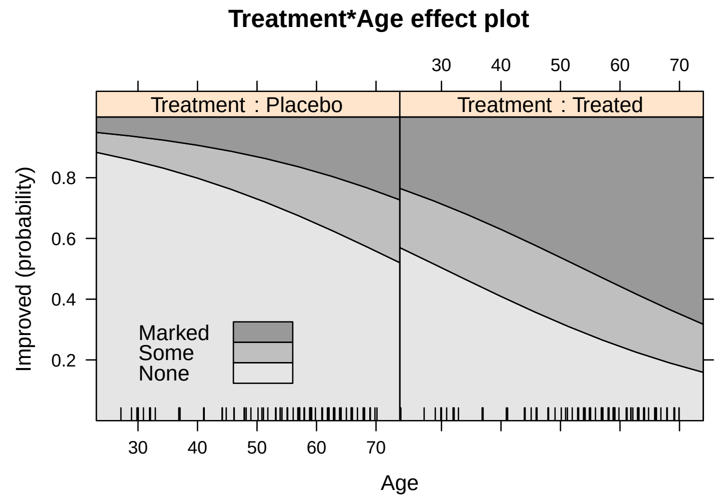




# Proportional odds models in R: Plotting

Making visual comparisons easier:

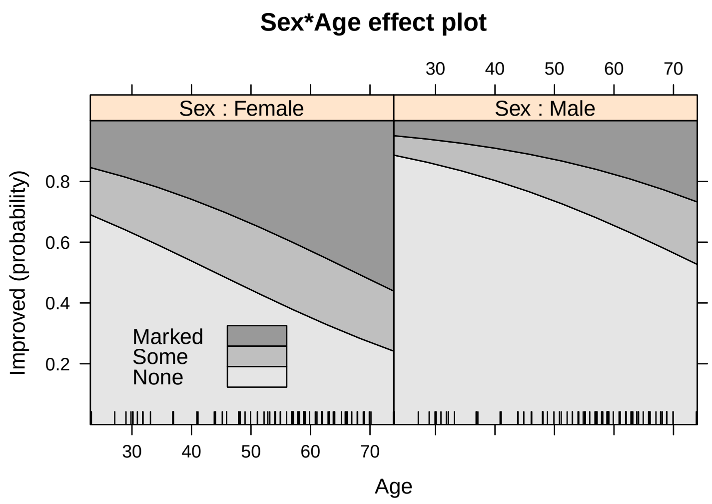
```
> plot(effect("Treatment:Age", arth.polr), style='stacked')
```



# Proportional odds models in R: Plotting

Making visual comparisons easier:

```
> plot(effect("Sex:Age", arth.polr), style='stacked')
```



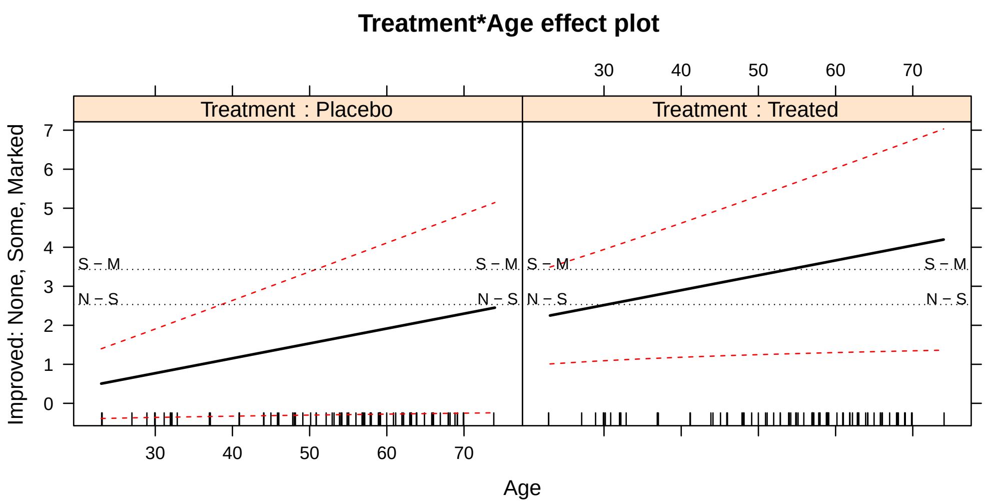





# Proportional odds models in R: Plotting

These plots are even simpler on the logit scale, using `latent=TRUE` to show the cutpoints between response categories

```
> plot(effect("Treatment:Age", arth.polr, latent=TRUE))
```

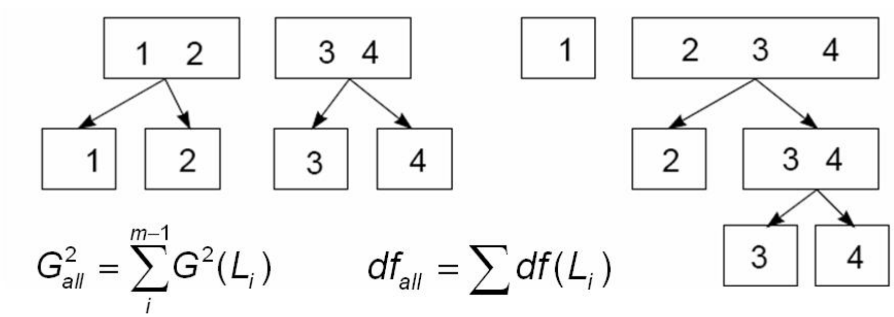







# Polytomous response: Nested dichotomies

- $m$ categories $\rightarrow$ $(m-1)$ comparisons (logits)
- If these are formulated as $(m-1)$ nested dichotomies:
  - Each dichotomy can be fit using the familiar binary-response logistic model,
  - the $m-1$ models will be statistically independent ($G^2$ statistics will be additive)
  - (Need some extra work to summarize these as a single, combined model)
- This allows the slopes to differ for each logit




$$G^2_{all} = \sum_i^{m-1} G^2(L_i) \qquad df_{all} = \sum df(L_i)$$

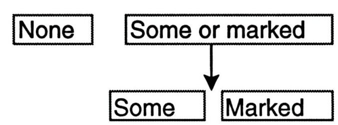




# Nested dichotomies: Examples

$m = 3$

Arthritis treatment

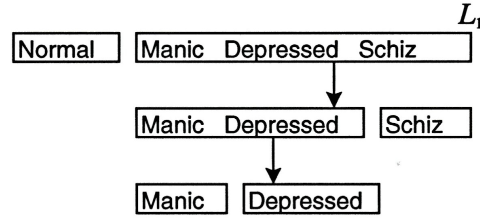



$$L_1 = log \frac{\pi_1}{\pi_2 + \pi_3}$$

$$L_2 = log \frac{\pi_2}{\pi_3}$$

$m = 4$

Psychiatric diagnosis




$$L_1 = log \frac{\pi_1}{\pi_2 + \pi_3 + \pi_4}$$

$$L_2 = log \frac{\pi_4}{\pi_2 + \pi_3}$$

$$L_3 = log \frac{\pi_2}{\pi_3}$$

# Example: Women's Labour-Force Participation

Data: *Social Change in Canada Project*, York ISR, car::`Womenlf` data

- **Response:** not working outside the home (n=155), working part-time (n=42) or working full-time (n=66)
- Model as two nested dichotomies:
  - Working (n=106) vs. NotWorking (n=155)
  - Working full-time (n=66) vs. working part-time (n=42).

| | | |
|---|---|---|
| $L_1$: | not working | part-time, full-time |
| $L_2$: | | part-time \| full-time |

- **Predictors:**
  - Children? — 1 or more minor-aged children
  - Husband's Income — in \$1000s
  - Region of Canada (not considered here)





# Nested dichotomoies: Combined tests

- Nested dichotomies $\rightarrow$ $\chi^2$ tests and df for the separate logits are independent
- $\rightarrow$ add, to give tests for the full *m*-level response (manually)

```
                       Global tests of BETA=0
                                                              Prob
Test                 Response         ChiSq        DF        ChiSq

Likelihood Ratio     working        36.4184         2       <.0001
                     fulltime       39.8468         2       <.0001
                     ALL            76.2652         4       <.0001
```

Wald tests for each coefficient:

```
              Wald tests of maximum likelihood estimates
                                                        Prob
     Variable       Response       WaldChiSq     DF      ChiSq

     Intercept      working          12.1164      1     0.0005
                    fulltime         20.5536      1     <.0001
                    ALL              32.6700      2     <.0001

     children       working          29.0650      1     <.0001
                    fulltime         24.0134      1     <.0001
                    ALL              53.0784      2     <.0001

     husinc         working           4.5750      1     0.0324
                    fulltime          7.5062      1     0.0061
                    ALL              12.0813      2     0.0024
```





# Nested dichotomies: recoding

In R, first create new variables, `working` and `fulltime`, using the `recode()` function in the car:

```
> library(car)    # for data and Anova()
> data(Womenlf)
> Womenlf <- within(Womenlf,{
+   working <- recode(partic, " 'not.work' = 'no'; else = 'yes' ")
+   fulltime <- recode (partic,
+     " 'fulltime' = 'yes'; 'parttime' = 'no'; 'not.work' = NA")})
> some(Womenlf)
```

```
       partic hincome children   region fulltime working
31   not.work      13  present  Ontario     <NA>      no
34   not.work       9   absent  Ontario     <NA>      no
55   parttime       9  present Atlantic       no     yes
86   fulltime      27   absent       BC      yes     yes
96   not.work      17  present  Ontario     <NA>      no
141  not.work      14  present  Ontario     <NA>      no
180  fulltime      13   absent       BC      yes     yes
189  fulltime       9  present Atlantic      yes     yes
234  fulltime       5   absent   Quebec      yes     yes
240  not.work      13  present   Quebec     <NA>      no
```





# Nested dichotomies: fitting

Then, fit models for each dichotomy:

```
> contrasts(children)<- 'contr.treatment'
> mod.working <- glm(working ~ hincome + children, family=binomial, data=
> mod.fulltime <- glm(fulltime ~ hincome + children, family=binomial, dat
```

Some output from `summary(mod.working)`:

```
Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)       1.33583    0.38376   3.481   0.0005 ***
hincome          -0.04231    0.01978  -2.139   0.0324 *
childrenpresent -1.57565    0.29226  -5.391    7e-08 ***
```

Some output from `summary(mod.fulltime)`:

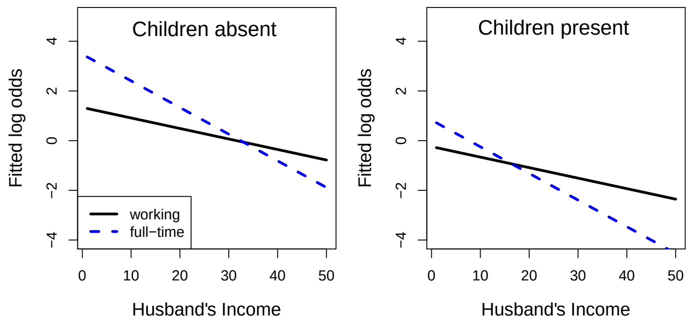
```
Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
(Intercept)       3.47777    0.76711   4.534 5.80e-06 ***
hincome          -0.10727    0.03915  -2.740  0.00615 **
childrenpresent -2.65146    0.54108  -4.900 9.57e-07 ***
```

## Nested dichotomies: interpretation

Write out the predictions for the two logits, and compare coefficients:

$$\log\left(\frac{\Pr(\text{working})}{\Pr(\text{not working})}\right) = 1.336 - 0.042\,\text{H\$} - 1.576\,\text{kids}$$
$$\log\left(\frac{\Pr(\text{fulltime})}{\Pr(\text{parttime})}\right) = 3.478 - 0.107\,\text{H\$} - 2.652\,\text{kids}$$

Better yet, plot the predicted log odds for these equations:








## Nested dichotomies: plotting

For plotting, calculate the predicted probabilities (or logits) over a grid of combinations of the predictors in each sub-model, using the **predict()** function.

`type='response'` gives these on the probability scale, whereas `type='link'` (the default) gives these on the logit scale.

```
> pred <- expand.grid(hincome=1:45, children=c('absent', 'present'))
> # get fitted values for both sub-models
> p.work     <- predict(mod.working, pred, type='response')
> p.fulltime <- predict(mod.fulltime, pred, type='response')
```

The fitted value for the fulltime dichotomy is conditional on working outside the home; multiplying by the probability of working gives the unconditional probability.

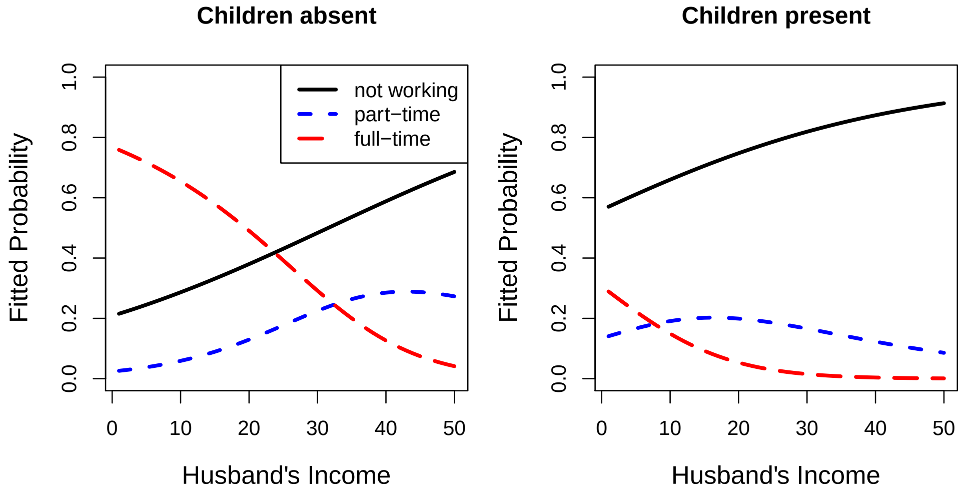
```
> p.full <- p.work * p.fulltime
> p.part <- p.work * (1 - p.fulltime)
> p.not <- 1 - p.work
```





## Nested dichotomies in R: plotting

The plot below was produced using the basic R functions **plot()**, **lines()** and **legend()**. See the file `wlf-nested.R` on the course web page for details.








## Polytomous response: Generalized Logits

- Models the probabilities of the $m$ response categories as $m-1$ logits comparing each of the first $m-1$ categories to the last (reference) category.
- Logits for any pair of categories can be calculated from the $m-1$ fitted ones.
- With $k$ predictors, $x_1, x_2, \ldots, x_k$, for $j = 1, 2, \ldots, m-1$,

$$L_{jm} \equiv \log\left(\frac{\pi_{ij}}{\pi_{im}}\right) = \beta_{0j} + \beta_{1j}\,x_{i1} + \beta_{2j}\,x_{i2} + \cdots + \beta_{kj}\,x_{ik} = \boldsymbol{\beta}_j^{\mathsf{T}} \boldsymbol{x}_i$$

  - One set of fitted coefficients, $\boldsymbol{\beta}_j$ for each response category except the last.
  - Each coefficient, $\beta_{hj}$, gives the effect on the log odds of a unit change in the predictor $x_h$ that an observation belongs to category $j$ vs. category $m$.
- Probabilities in response caegories are calculated as:

$$\pi_{ij} = \frac{\exp(\boldsymbol{\beta}_j^{\mathsf{T}} \boldsymbol{x}_i)}{\sum_{j=1}^{m-1} \exp(\boldsymbol{\beta}_j^{\mathsf{T}} \boldsymbol{x}_i)}\ ,\ j = 1, \ldots, m-1\,; \qquad \pi_{im} = 1 - \sum_{j=1}^{m-1} \pi_{ij}$$

# Generalized logit models: Fitting

- In R, the generalized logit model can be fit using the **multinom()** function in the nnet
- For interpretation, it is useful to reorder the levels of `partic` so that `not.work` is the baseline level.

```
Womenlf$partic <- ordered(Womenlf$partic,
    levels=c('not.work', 'parttime', 'fulltime'))
library(nnet)
mod.multinom <- multinom(partic ~ hincome + children, data=Womenl
summary(mod.multinom, Wald=TRUE)
Anova(mod.multinom)
```

The **Anova()** tests are similar to what we got from summing these tests from the two nested dichotomies:

```
Analysis of Deviance Table (Type II tests)

Response: partic
         LR Chisq Df Pr(>Chisq)
hincome      15.2  2    0.00051 ***
children     63.6  2    1.6e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

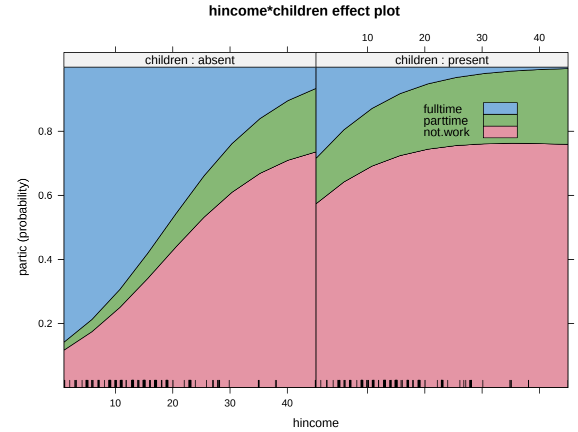




# Generalized logit models: Plotting

- As before, it is much easier to interpret a model from a plot than from coefficients, but this is particularly true for polytomous response models
- `style="stacked"` shows cumulative probabilities

```
library(effects)
plot(effect("hincome*children", mod.multinom), style="stacked")
```



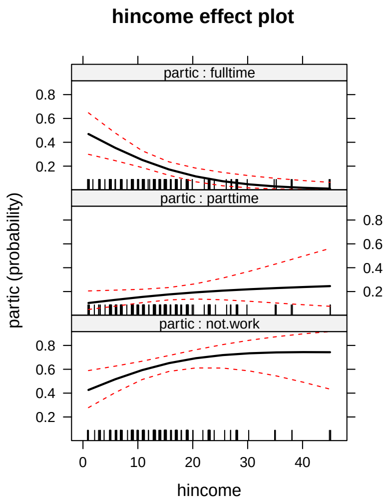


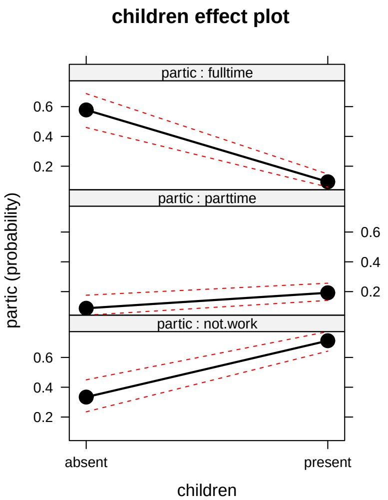



# Generalized logit models: Plotting

- You can also view the effects of husband's income and children separately in this main effects model with `plot(allEffects))`.

```
plot(allEffects(mod.multinom), ask=FALSE)
```








# Political knowledge & party choice in Britain

Example from Fox & Andersen (2006): Data from 1997 British Election Panel Survey (BEPS)

- **Response**: Party choice— Liberal democrat, Labour, Conservative
- **Predictors**
  - Europe: 11-point scale of attitude toward European integration (high="Eurosceptic")
  - Political knowledge: knowledge of party platforms on European integration ("low"=0–3="high")
  - Others: Age, Gender, perception of economic conditions, evaluation of party leaders (Blair, Hague, Kennedy)– 1:5 scale
- **Model**:
  - Main effects of Age, Gender, economic conditions (national, household)
  - Main effects of evaluation of party leaders
  - Interaction of attitude toward European integration with political knowledge

# BEPS data: Fitting

Fit using **multinom()** in the nnet package

```
library(effects)   # data, plots
library(car)       # for Anova()
library(nnet)      # for multinom()
multinom.mod <- multinom(vote ~ age + gender + economic.cond.national +
    economic.cond.household + Blair + Hague + Kennedy +
    Europe*political.knowledge, data=BEPS)
Anova(multinom.mod)
```

```
Anova Table (Type II tests)

Response: vote
                           LR Chisq Df Pr(>Chisq)
age                            13.9  2    0.00097 ***
gender                          0.5  2    0.79726
economic.cond.national         30.6  2    2.3e-07 ***
economic.cond.household         5.7  2    0.05926 .
Blair                         135.4  2    < 2e-16 ***
Hague                         166.8  2    < 2e-16 ***
Kennedy                        68.9  2    1.1e-15 ***
Europe                         78.0  2    < 2e-16 ***
political.knowledge            55.6  2    8.6e-13 ***
Europe:political.knowledge     50.8  2    9.3e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```





# BEPS data: Interpretation?

How to understand the *nature* of these effects on party choice?

```
> summary(multinom.mod)
```

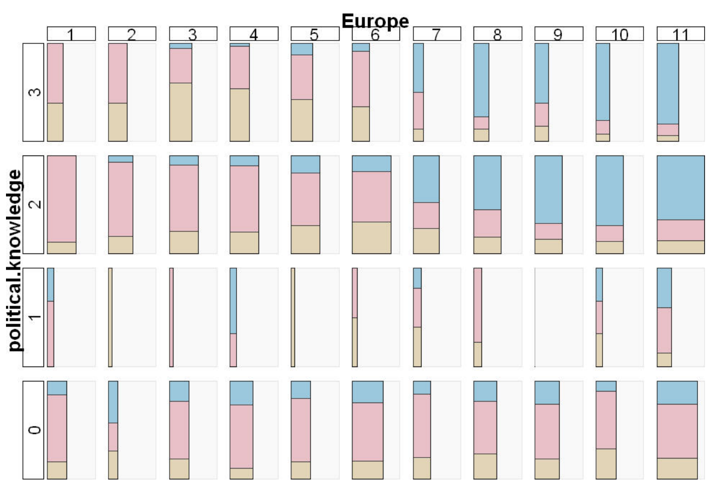
```
Call:
multinom(formula = vote ~ age + gender + economic.cond.national +
    economic.cond.household + Blair + Hague + Kennedy + Europe *
    political.knowledge, data = BEPS)

Coefficients:
                 (Intercept)      age gendermale economic.cond.national
Labour               -0.8734 -0.01980     0.1126                 0.5220
Liberal Democrat     -0.7185 -0.01460     0.0914                 0.1451
                 economic.cond.household  Blair   Hague Kennedy    Europe
Labour                          0.178632 0.8236 -0.8684  0.2396 -0.001706
Liberal Democrat                0.007725 0.2779 -0.7808  0.6557  0.068412
                 political.knowledge Europe:political.knowledge
Labour                        0.6583                    -0.1589
Liberal Democrat              1.1602                    -0.1829

Std. Errors:
                 (Intercept)      age gendermale economic.cond.national
Labour                0.6908 0.005364     0.1694                 0.1065
Liberal Democrat      0.7344 0.005643     0.1780                 0.1100
...

Residual Deviance: 2233
AIC: 2277
```





# BEPS data: Initial look, relative multiple barcharts

How does party choice— Liberal democrat, Labour, Conservative vary with political knowledge and Europe attitude (high="Eurosceptic")?

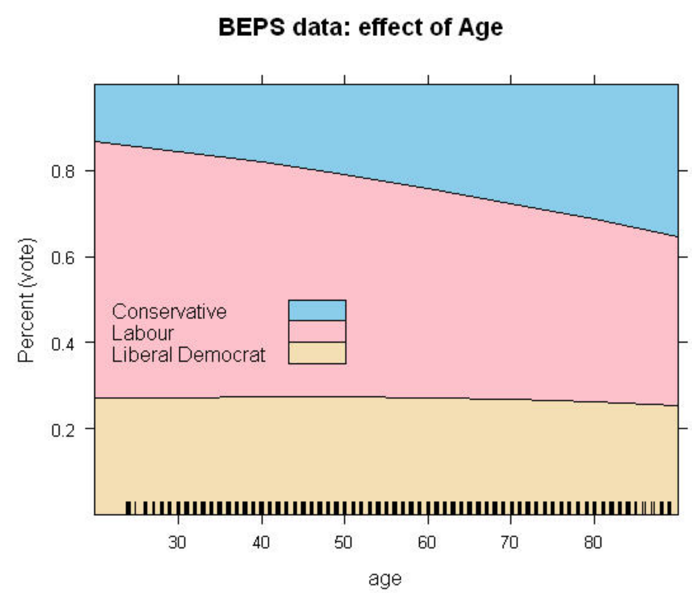






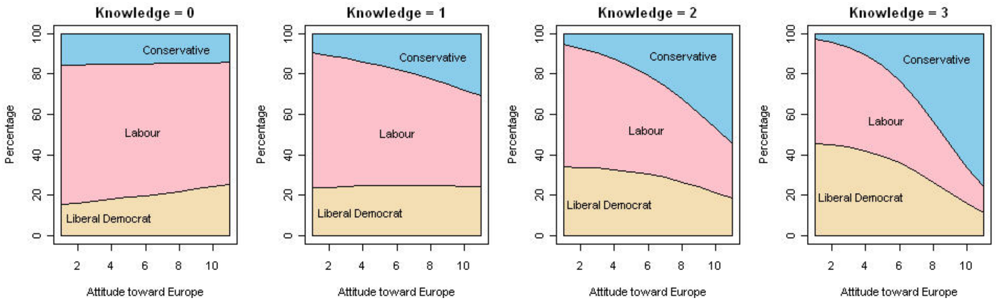

# BEPS data: Effect plots to the rescue!

Age effect: Older more likely to vote Conservative



# BEPS data: Effect plots to the rescue!

Attitude toward European integration $\times$ political knowledge effect:




- Low knowledge: little relation between attitude and party choice
- As knowledge increases: more Eurosceptic views more likely to support Conservatives
- $\Rightarrow$ detailed understanding of complex models depends strongly on visualization!





# Summary

- **Polytomous responses**
  - *m* response categories $\rightarrow$ $(m-1)$ comparisons (logits)
  - Different models for *ordered* vs. *unordered* categories
- **Proportional odds model**
  - Simplest approach for *ordered* categories: Same slopes for all logits
  - Requires proportional odds asumption to be met
  - R: MASS::`polr()`; VGAM::`vglm()`
- **Nested dichotomies**
  - Applies to ordered or unordered categories
  - Fit $m-1$ *separate* independent models $\rightarrow$ Additive $\chi^2$ values
  - R: only need `glm()`
- **Generalized (multinomial) logistic regression**
  - Fit $m-1$ logits as a *single* model
  - Results usually comparable to nested dichotomies
  - R: nnet::`multinom()`