Categorical Data Analysis: Course Overview

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Psych 6136

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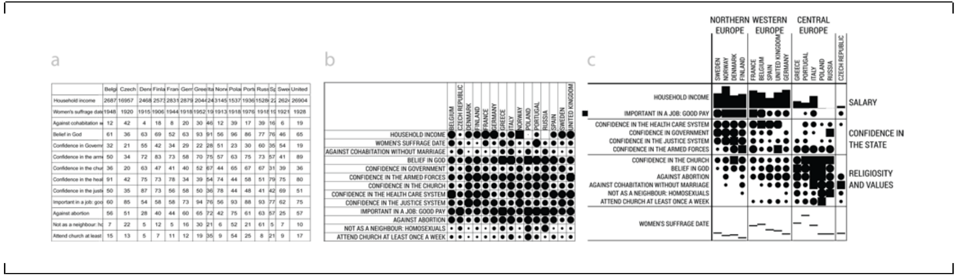


Course goals

This course is designed as a broad, applied introduction to the statistical analysis of categorical (or discrete) data, with an emphasis on:

Emphasis: visualization methods

- exploratory graphics: see patterns, trends, anomalies in your data
- model diagnostic methods: assess violations of assumptions
- model summary methods: provide an interpretable summary of your data

Emphasis: theory ⇒ practice

- Understand how to translate research questions into statistical hypotheses and models
- Understand the difference between simple, non-parametric approaches (e.g., χ^2 test for indpendence) and model-based methods (logistic regression, GLM)
- Framework for thinking about categorical data analysis in *visual* terms

Course outline

1. Exploratory and hypothesis testing methods

- Week 1: Overview; Introduction to R
- Week 2: One-way tables and goodness-of-fit test
- Week 3: Two-way tables: independence and association
- Week 4: Two-way tables: ordinal data and dependent samples
- Week 5: Three-way tables: different types of independence
- Week 6: Correspondence analysis

2. Model-based methods

- Week 7: Logistic regression I
- Week 8: Logistic regression II
- Week 9: Multinomial logistic regression models
- Week 10: Log-linear models
- Week 11: Loglinear models: Advanced topics
- Week 12: Generalized Linear Models: Poisson regression
- Week 13: Course summary & additional topics

Textbooks

Main texts:

- Friendly, M. and Meyer, D. (2015). *Visualizing Categorical Data with R*. To be published by Chapman & Hall. Chapters will be made available on the web (password protected). http://euclid.psych.yorku.ca/www/psy6136/
- Agresti, Alan (2007). *An Introduction to Categorical Data Analysis*. 2nd ed. John Wiley & Sons, Inc.: New York. ISBN: 978-0-471-22618-5. Available in the bookstore.

Supplementary readings:
For those who desire a more in-depth treatment of categorical data analysis:

- Agresti, Alan (2013). *Categorical Data Analysis*. 3rd ed. New York: John Wiley & Sons, Inc. New York. ISBN: 978-0-470-46363-5

What is categorical data?

A ***categorical variable*** is one for which the possible measured or assigned values consist of a discrete set of categories, which may be *ordered* or *unordered*.

Some typical examples are:

- `Gender`, with categories "Male", "Female".
- `Marital status`, with categories "Never married", "Married", "Separated", "Divorced", "Widowed".
- `Party preference`, with categories "NDP", "Liberal", "Conservative", "Green".
- `Treatment outcome`, with categories "no improvement", "some improvement", or "marked improvement".
- `Age`, with categories "0-9", "10-19", "20-29", "30-39", $\ldots$.
- `Number of children`, with categories $0, 1, 2, \ldots$.

Categorical data structures: 1-way tables

Simplest case: 1-way frequency distribution

- Unordered factor

Hair	Black	Brown	Red	Blond
	108	286	71	127

Hair color among 592 students

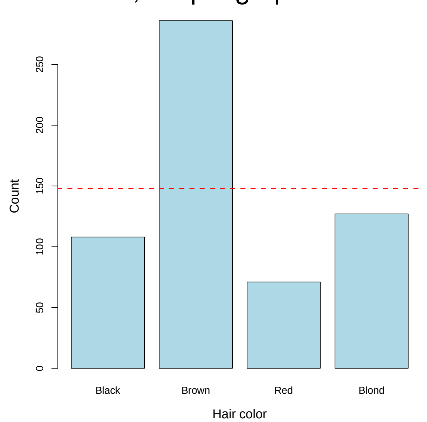
Party	BQ	Cons	Green	Liberal	NDP	Total
N	104	392	126	404	174	1200
%	8.7	32.6	10.5	33.7	14.5	100

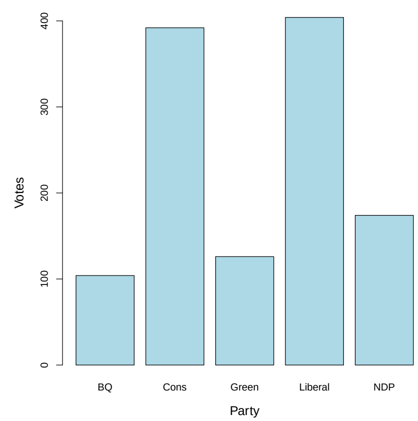
Voting intention in Harris-Decima poll, 8/21/08

- Questions:
  - Are all hair colors equally likely?
  - Do blondes have more fun?
  - Is there a difference in voting intentions between Liberal and Conservative?

Categorical data structures: 1-way tables

Even here, simple graphs are better than tables





But these don't really provide answers to the questions. Why?

Categorical data structures

Simplest case: 1-way frequency distribution

- Ordered, quantitative factor

nMales												
0	1	2	3	4	5	6	7	8	9	10	11	12
3	24	104	286	670	1033	1343	1112	829	478	181	45	7

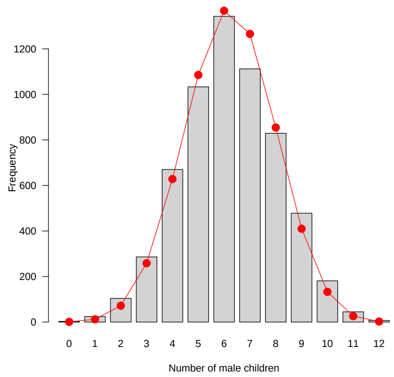
# of sons in Saxony families with 12 children

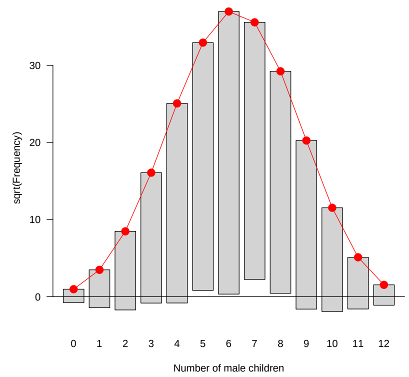
- Questions:
  - What is the *form* of this distribution?
  - Is it useful to think of this as a ***binomial distribution***?
  - If so, is $\Pr(male) = .5$ reasonable?
  - How could so many families have 12 children?

Categorical data structures: 1-way tables

When a particular distribution is in mind,

- better to plot the data together with the fitted frequencies
- better still: a ***hanging rootogram***– plot frequencies on sqrt scale, and hang the bars from the fitted values.





Categorical data structures: 2x2 tables

Contingency tables ($2 \times 2 \times \ldots$)

- Two-way

Admit	Gender Male	Female
Admitted	1198	557
Rejected	1493	1278

Admission to graduate programs at UC Berkeley

- Three-way, stratified by another factor

… by Department

Admit	Gender	Dept A	B	C	D	E	F
Admitted	Male	512	353	120	138	53	22
	Female	89	17	202	131	94	24
Rejected	Male	313	207	205	279	138	351
	Female	19	8	391	244	299	317

Questions:

- Is admission associated with gender?
- Does admission rate vary with department?

Categorical data structures: Larger tables

Contingency tables (larger)

- Two-way

Hair	Eye Brown	Blue	Hazel	Green
Black	68	20	15	5
Brown	119	84	54	29
Red	26	17	14	14
Blond	7	94	10	16

- Three-way

Sex	Hair	Eye Brown	Blue	Hazel	Green
Male	Black	32	11	10	3
	Brown	53	50	25	15
	Red	10	10	7	7
	Blond	3	30	5	8
Female	Black	36	9	5	2
	Brown	66	34	29	14
	Red	16	7	7	7
	Blond	4	64	5	8

Table and case-form

- The previous examples were shown in table form
  - # observations = # cells in the table
  - variables: factors + COUNT
- Each has an equivalent representation in case form
  - # observations = total COUNT
  - variables: factors
- Case form is required if there are continuous variables

Sex	Hair	Eye Brown	Blue	Hazel	Green
Male	Black	32	11	10	3
	Brown	53	50	25	15
	Red	10	10	7	7
	Blond	3	30	5	8
Female	Black	36	9	5	2
	Brown	66	34	29	14
	Red	16	7	7	7
	Blond	4	64	5	8

Categorical data: Analysis methods

Methods of analysis for categorical data fall into two main categories:

Non-parametric, randomization-based methods

- Make minimal assumptions
- Useful for hypothesis-testing:
  - Are men more likely to be admitted than women?
  - Are hair color and eye color associated?
  - Does the binomial distribution fit these data?
- Mostly for two-way tables (possibly stratified)
- R:
  - Pearson Chi-square: `chisq.test()`
  - Fisher's exact test (for small expected frequencies): `fisher.test()`
  - Mantel-Haenszel tests (ordered categories: test for *linear* association): `CMHtest()`
- SAS: PROC `FREQ` — can do all the above
- SPSS: Crosstabs

Categorical data: Analysis methods

Model-based methods

- Must assume random sample (possibly stratified)
- Useful for estimation purposes: Size of effects (std. errors, confidence intervals)
- More suitable for multi-way tables
- Greater flexibility; fitting specialized models
  - Symmetry, quasi-symmetry, structured associations for square tables
  - Models for ordinal variables
- R: `glm()` family, Packages: car, gnm, vcd, ...
  - estimate standard errors, covariances for model parameters
  - confidence intervals for parameters, predicted Pr{response}
- SAS: PROC `LOGISTIC`, `CATMOD`, `GENMOD` , `INSIGHT` (Fit YX), ...
- SPSS: Hiloglinear, Loglinear, Generalized linear models

Categorical data: Response vs. Association models

Response models

- Sometimes, one variable is a natural discrete response.
- Q: How does the response relate to explanatory variables?
  - Admit ∼ Gender + Dept
  - Party ∼ Age + Education + Urban

⇒ Logit models, logististic regression, generalized linear models

Association models

- Sometimes, the main interest is just association among variables
- Q: Which variables are associated, and how?
  - Berkeley data: [Admit Gender]? [Admit Dept]? [Gender Dept]
  - Hair-eye data: [Hair Eye]? [Hair Sex]? [Eye, Sex]

⇒ Loglinear models

This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

Graphical methods: Tables and Graphs

If I can't picture it, I can't understand it. Albert Einstein

Getting information from a table is like extracting sunlight from a cucumber. Farquhar & Farquhar, 1891

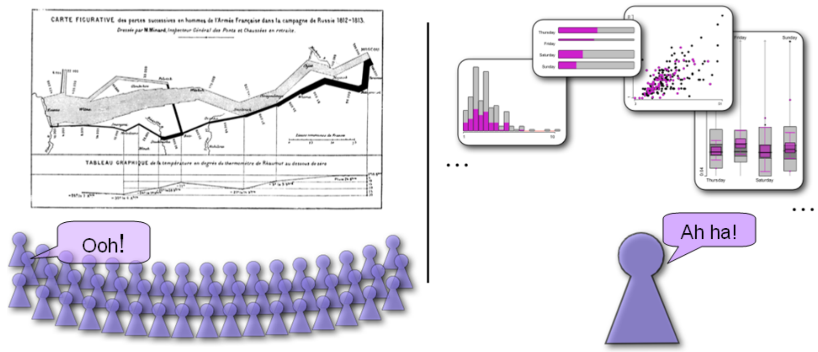
Tables vs. Graphs

- Tables are best suited for *look-up* and calculation—
  - read off exact numbers
  - show additional calculations (e.g., % change)
- Graphs are better for:
  - showing *patterns, trends, anomalies*,
  - making *comparisons*
  - seeing the *unexpected*!
- Visual presentation as *communication*:
  - what do you want to say or show?
  - ⟹ design graphs and tables to 'speak to the eyes'

Graphical methods: Communication goals

Different audiences require different graphs:

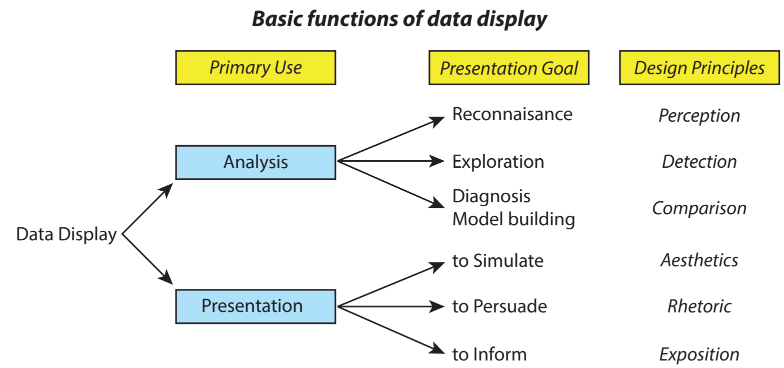
- **Presentation**: A single, carefully crafted graph to appeal to a wide audience
- **Exploration, analysis**: Many related graphics from different perspectives, for a narrow audience (often: you!)



Presentation Exploration

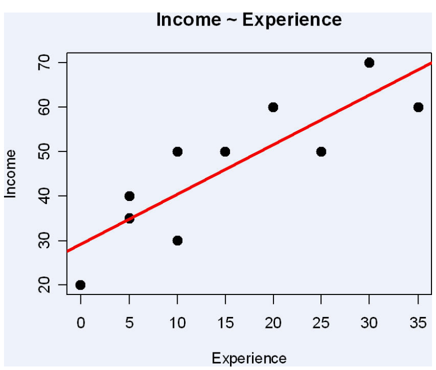
Graphical methods: Presentation goals

Different presentation goals appeal to different design principles

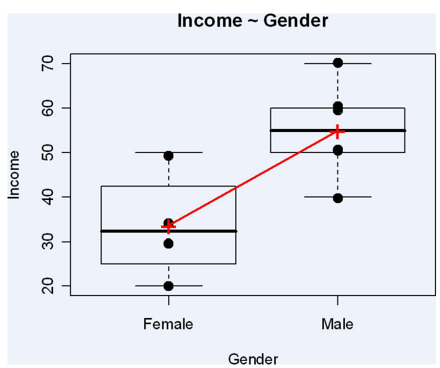


Graphical methods: Quantitative data

Quantitative data (amounts) are naturally displayed in terms of **magnitude** $\sim$ **position along a scale**



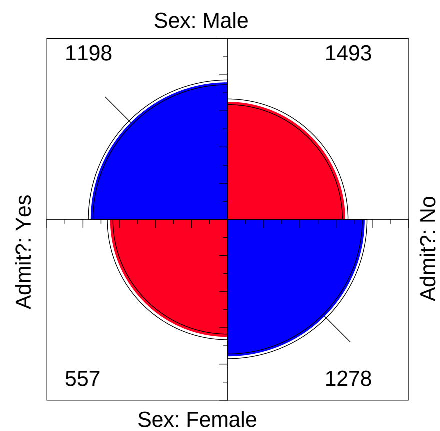
Scatterplot of Income vs. Experience



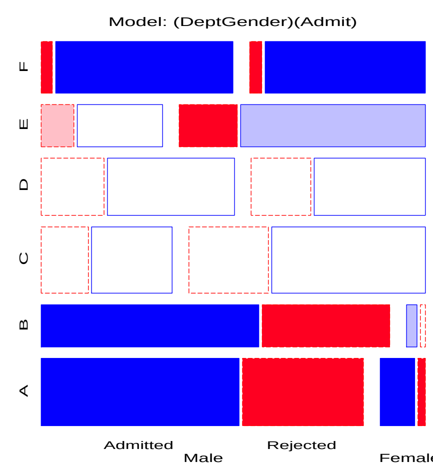
Boxplot of Income by Gender

Graphical methods: Categorical data

Frequency data (counts) are more naturally displayed in terms of **count** $\sim$ **area** (Friendly, 1995)

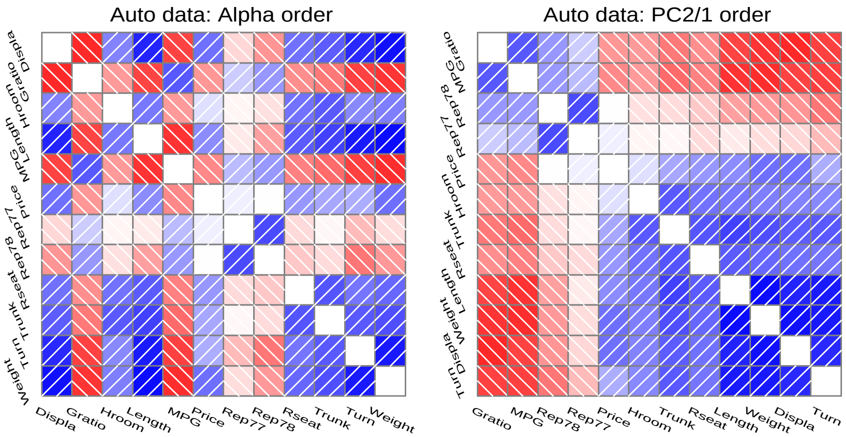


Fourfold display for 2×2 table



Mosaic plot for 3-way table

- **Principles of Graphical Displays**
  - **Effect ordering** (Friendly and Kwan, 2003)— In tables and graphs, sort unordered factors according to the effects you want to see/show.



"Corrgrams: Exploratory displays for correlation matrices" (Friendly, 2002)

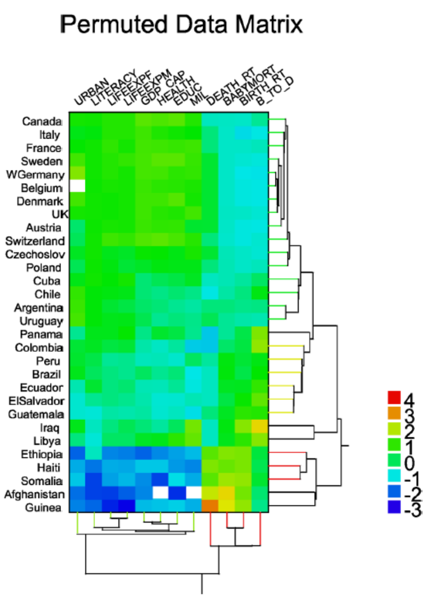
- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Effect ordered

Eye color	Hair color: Black	Brown	Red	Blond
Brown	68	119	26	7
Hazel	15	54	14	10
Green	5	29	14	16
Blue	20	84	17	94

Model:	*Independence*: [Hair][Eye] χ^2 (9)= 138.29						
Color coding:	<-4	<-2	<-1	0	>1	>2	>4
n in each cell:	$n <$ expected				$n >$ expected		

Clustered heat map: Showing patterns in tables

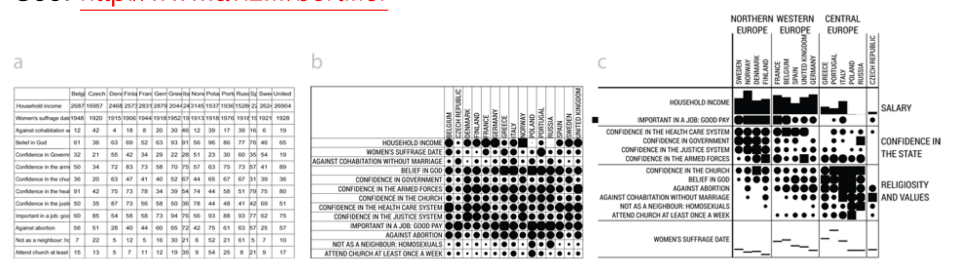


The clustered heat map is one method for making large tables more visually understandable.

- Social statistics from UN survey
- Rows and columns are sorted, using cluster analysis
- Standardized data values are encoded using color

Bertifier: Turning tables into graphics

Bertifier: A web app implementing Bertin's idea of the *reorderable matrix*.
See: http://www.aviz.fr/bertifier



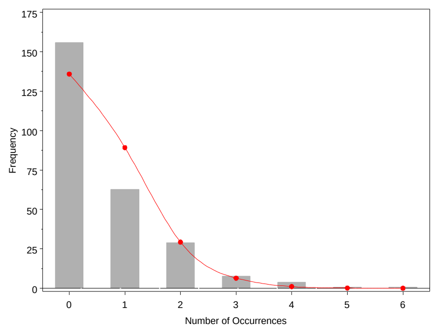
1. A table: Attitudes and attributes by country
2. Values encoded by size and shape
3. Sorted and grouped by themes and country regions

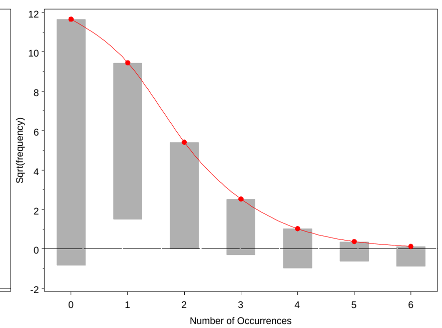
Watch: Youtube video of Bertifier

Visual comparisons

Comparisons— Make visual comparisons easy

- Visual grouping— connect with lines, make key comparisons contiguous
- Baselines— compare *data* to *model* against a line, preferably horizontal
- Frequencies often better plotted on a square-root scale

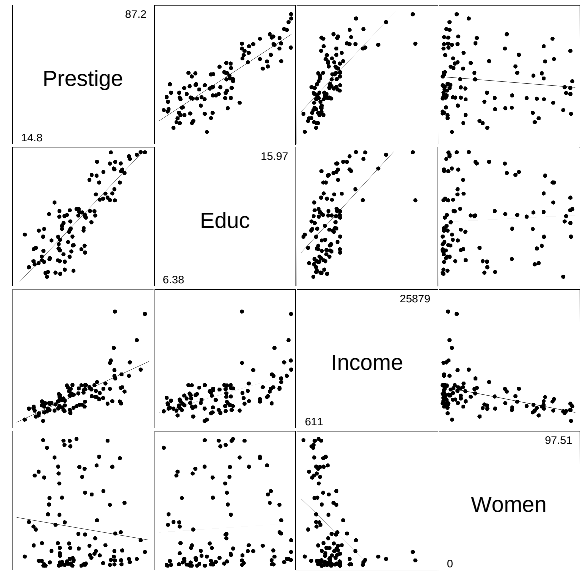




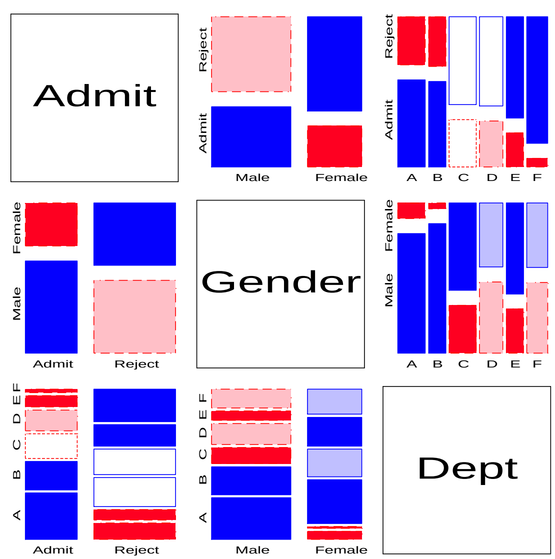
Standard histogram with fit

Suspended rootogram

- **Small multiples**— combine stratified graphs into coherent displays (Tufte, 1983)
  - e.g., scatterplot matrix for quantitative data: all pairwise scatterplots



- e.g., mosaic matrix for quantitative data: all pairwise mosaic plots



Graphical methods: Categorical data

Exploratory methods

- Minimal assumptions (like non-parametric methods)
- Show the *data*, not just *summaries*
- But can add summaries: smoothed curve(s), trend lines, ...
- Help detect *patterns, trends, anomalies*, suggest hypotheses

Plots for model-based methods

- Residual plots - departures from model, omitted terms, ...
- Effect plots - estimated probabilities of response or log odds
- Diagnostic plots - influence, violation of assumptions

References I

Friendly, M. Conceptual and visual models for categorical data. *The American Statistician*, 49:153–160, 1995.

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Friendly, M. and Kwan, E. Effect ordering for data displays. *Computational Statistics and Data Analysis*, 43(4):509–539, 2003.

Tufte, E. R. *The Visual Display of Quantitative Information*. Graphics Press, Cheshire, CT, 1983.