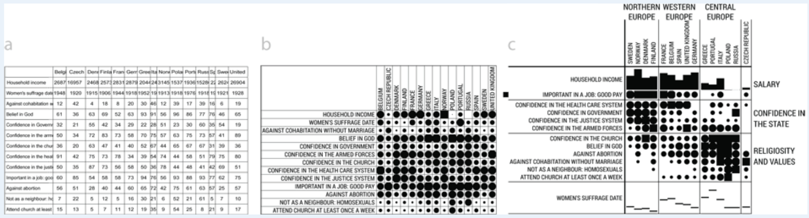


Categorical Data Analysis

Course overview

Psy 6136 Categorical Data Analysis

Michael Friendly

Psych 6136

http://friendly.github.io/psy6136

Course goals

This course is designed as a broad, applied introduction to the statistical analysis of categorical data, with an emphasis on:

Emphasis: visualization methods

- exploratory graphics: see patterns, trends, anomalies in your data
- model diagnostic methods: assess violations of assumptions
- model summary methods: provide an interpretable summary of your data

Emphasis: theory ⇒ practice

- Understand how to translate research questions into statistical hypotheses and models
- Understand the difference between simple, non-parametric approaches (e.g., χ^2 test for indpendence) and model-based methods (logistic regression, GLM)
- Framework for thinking about categorical data analysis in *visual* terms

---

Course outline

1. Exploratory and hypothesis testing methods

- Week 1: Overview; Introduction to R
- Week 2: One-way tables and goodness-of-fit test
- Week 3: Two-way tables: independence and association
- Week 4: Two-way tables: ordinal data and dependent samples
- Week 5: Three-way tables: different types of independence
- Week 6: Correspondence analysis

2. Model-based methods

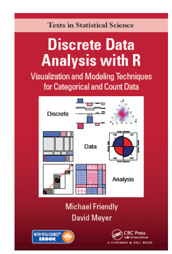
- Week 7: Logistic regression I
- Week 8: Logistic regression II
- Week 9: Multinomial logistic regression models
- Week 10: Log-linear models
- Week 11: Loglinear models: Advanced topics
- Week 12: Generalized Linear Models: Poisson regression
- Week 13: Course summary & additional topics

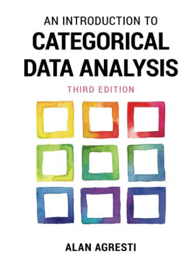
Textbooks

Main texts

- Friendly & Meyer (2016). *Discrete Data Analysis with R: Visualizing & Modeling Techniques for Categorical & Count Data*
  - 30% discount on Routledge web site (code: ADC22)
  - Draft chapters on http://euclid.psych.yorku.ca/www/psy6136
  - DDAR web site: https://ddar.datavis.ca
- Agresti (2007). *An Introduction to Categorical Data Analysis*, 3rd E. Wiley & Sons.

eBook available
PDF on course web site

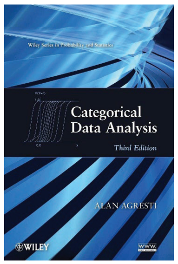


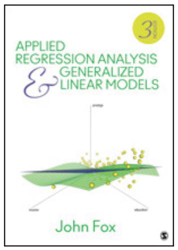


Textbooks

Supplementary readings

- Agresti (2013). *Categorical Data Analysis*, 3rd ed. [More mathematical, but the current Bible of CDA]
  - PDF available: https://bityl.co/FG9c
- Fox (2016). *Applied Regression Analysis and Generalized Linear Models*, 3rd ed.



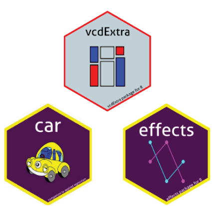


Expectations & grading

- I expect you will read chapters in *DDAR* & Agresti *Intro* each week
  - See Topic Schedule on course web site
  - R exercises: A few are listed as (ungraded) Assignments
  - Class discussion: Help make classes participatory
- Evaluation:
  - (2 x 40%) Two take-home projects: Analysis & research report, based on assignment problems or your own data
  - (20%)
    - Assignment portfolio: best work, enhanced
    - Research report on journal article(s) of theory / application of CDA
    - In-class presentation (~15 min) on application of general interest

What you need

- R, version >=3.6 [R 4.2 is current]
  - Download from https://cran.r-project.org/
- RStudio IDE, highly recommended
  - https://www.rstudio.com/products/rstudio/
- R packages: see course web page
  - vcd
  - vcdExtra
  - car
  - effects
  - …



R script to install packages: https://friendly.github.io/6136/R/install-vcd-pkgs.R

What is categorical data?

A **categorical variable** is one for which the possible measured or assigned values consist of a discrete set of categories, which may be *ordered* or *unordered*. Some typical examples are:

- `Gender`, with categories {"male", "female", "trans"}
- `Marital status`: { "Never married", "Married", "Separated", "Divorced", "Widowed" }
- `Party preference`: {"NDP", "Liberal", "Conservative", "Green"}
- `Treatment improvement`: {"none", "some", "marked"}
- `Age`: {"0-9", "10-19", "20-29", "30-39", … }.
- `Number of children`: $0, 1, 2, 3, \ldots$.

Questions:

- Which of these are ordered (ordinal)?
- Which could be treated as numeric? How?
- Which have missing categories, sometimes ignored, or treated as "Other"

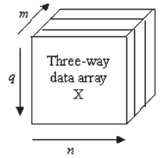
Categorical data: Structures

Categorical (frequency) data appears in various forms

- Tables: often the result of table() or xtabs()
  - 1-way
  - 2-way – 2 × 2, r × c
  - 3-way
- Matrices: matrix(), with row & col names
- Arrays: array(), with dimnames()
- Data frames
  - Case form (individual observations)
  - Frequency form

Gender compared to handedness

	Handed		
	Left	Right	
Female	7	46	53
Male	5	63	68
	12	109	121



1-way tables

- Unordered factors

	Black	Brown	Red	Blond
n	108	286	71	127
%	0.18	0.48	0.12	0.21

Hair color of 592 students

	BQ	Cons	Green	Liberal	NDP
n	104	392	126	404	174
%	0.087	0.33	0.1	0.34	0.14

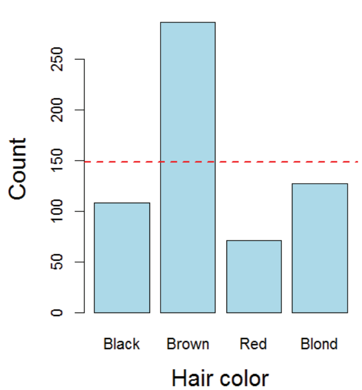
Voting intentions in Harris-Decima poll, 8/21/08

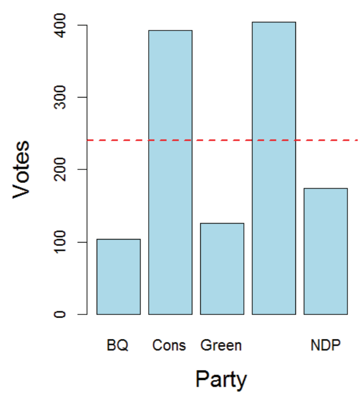
Questions:

- Are all hair colors equally likely?
- Aside from Brown hair, are others equally likely?
- Is there a diff in voting intentions for Liberal vs. Conservative

1-way tables

- Even here, simple graphs are more informative than tables





But these don't really answer the questions. Why?

1-way tables

- Ordered, quantitative factors
  - Number of sons in Saxony families with 12 children

```
> data(Saxony, package="vcd")
> Saxony
nMales
   0    1    2    3    4    5    6    7    8    9   10   11   12
   3   24  104  286  670 1033 1343 1112  829  478  181   45    7
```

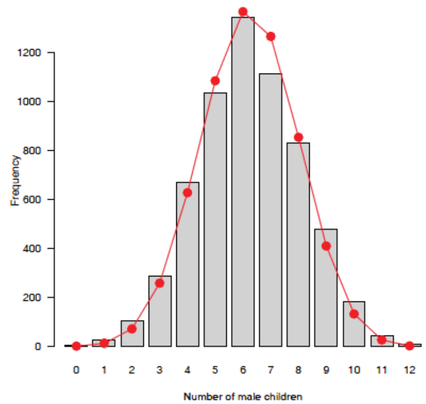
Questions:

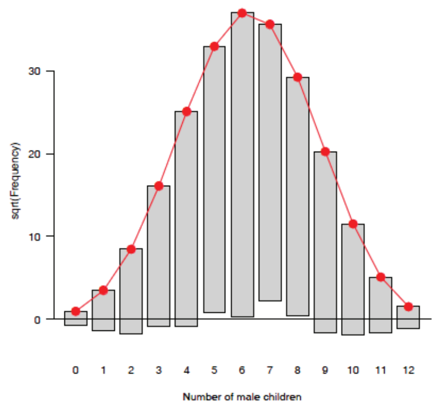
- What is the form of this distribution?
- Is it useful to think of this as a binomial distribution?
- If so, is Pr(male) = 0.5 reasonable to describe the data?
- How could families have > 10 children?

1-way tables: graphs

For a particular distribution in mind:

- Plot the data together with the fitted frequencies
- Better still: **hanging rootogram**: freq on sqrt scale; hang bars from fitted values





2-way tables: 2 x 2 x …

- Two-way

Admit \ Gender	Male	Female
Admitted	1198	557
Rejected	1493	1278

Admission to graduate programs at UC Berkeley

- Three-way, stratified by another factor

… by Department

Admit	Gender \ Dept	A	B	C	D	E	F
Admitted	Male	512	353	120	138	53	22
	Female	89	17	202	131	94	24
Rejected	Male	313	207	205	279	138	351
	Female	19	8	391	244	299	317

Questions:

- Is admission associated with gender?
- Does admission rate vary with department?

Larger tables

```
> margin.table(HairEyeColor, 1:2)
       Eye
Hair     Brown Blue Hazel Green
  Black     68   20    15     5
  Brown    119   84    54    29
  Red       26   17    14    14
  Blond      7   94    10    16
```

2-way
Actually, this is a 2D margin of a 3-way table

```
> ftable(Eye ~ Sex + Hair, data=HairEyeColor)
             Eye Brown Blue Hazel Green
Sex    Hair
Male   Black        32   11    10     3
       Brown        53   50    25    15
       Red          10   10     7     7
       Blond         3   30     5     8
Female Black        36    9     5     2
       Brown        66   34    29    14
       Red          16    7     7     7
       Blond         4   64     5     8
```

3-way (& higher) can be “flattened” for a more convenient display

formula notation:
row vars ~ col vars

Table form

- Table form is convenient for display, but information is implicit
  - a table has dimensions, dim() and dimnames()
  - the “observations” are the cells in the tables
  - the “variables” are the dimensions of the table (factors)
  - the cell value is the count or frequency

```
> dim(haireye)
[1] 4 4
> dimnames(haireye)
$Hair
[1] "Black" "Brown" "Red"   "Blond"

$Eye
[1] "Brown" "Blue"  "Hazel" "Green"
```

```
> names(dimnames(haireye)) # factor names
[1] "Hair" "Eye"
> prod(dim(haireye))         # of cells
[1] 16
> sum(haireye)               # total count
[1] 592
```

Datasets: frequency form

- Another common format is a dataset in frequency form

```
> as.data.frame(haireye)
    Hair   Eye Freq
1  Black Brown   68
2  Brown Brown  119
3    Red Brown   26
4  Blond Brown    7
5  Black  Blue   20
6  Brown  Blue   84
7    Red  Blue   17
8  Blond  Blue   94
9  Black Hazel   15
10 Brown Hazel   54
11   Red Hazel   14
12 Blond Hazel   10
13 Black Green    5
14 Brown Green   29
15   Red Green   14
16 Blond Green   16
```

- Use as.data.frame(table)
- One row for each cell
- Columns: factors + Freq or count

Datasets: case form

- Raw data often arrives in case form

```
> expand.dft(as.data.frame(haireye)) |>
+   as_tibble() |>
+   mutate(age = round( runif( n =
        sum(haireye), min=17, max=29)))
# A tibble: 592 x 3
   Hair  Eye     age
   <chr> <chr> <dbl>
 1 Black Brown    19
 2 Black Brown    19
 3 Black Brown    27
 4 Black Brown    23
 5 Black Brown    19
 6 Black Brown    29
 7 Black Brown    25
 8 Black Brown    29
 9 Black Brown    17
10 Black Brown    23
# ... with 582 more rows
```

- One obs. per case
- # rows = sum of counts
- vcdExtra::expand.dft() expands frequency form
- case form is required if there are continuous variables
- case form is tidy
- not all CDA functions play well with tibbles

Categorical data analysis: Methods

Methods for categorical data analysis fall into two main categories

Non-parametric, randomization-based methods

- Make minimal assumptions
- Useful for hypothesis-testing:
  - Are men more likely to be admitted than women?
  - Are hair color and eye color associated?
  - Does the binomial distribution fit these data?
- Mostly for two-way tables (possibly stratified)
- R:
  - Pearson Chi-square: `chisq.test()`
  - Fisher's exact test (for small expected frequencies): `fisher.test()`
  - Mantel-Haenszel tests (ordered categories: test for *linear* association): `CMHtest()`
- SAS: PROC `FREQ` — can do all the above
- SPSS: Crosstabs

Categorical data analysis: Methods

Model-based methods

- Must assume random sample (possibly stratified)
- Useful for estimation purposes: Size of effects (std. errors, confidence intervals)
- More suitable for multi-way tables
- Greater flexibility; fitting specialized models
  - Symmetry, quasi-symmetry, structured associations for square tables
  - Models for ordinal variables
- R: `glm()` family, Packages: car, gnm, vcd, ...
  - estimate standard errors, covariances for model parameters
  - confidence intervals for parameters, predicted Pr{response}
- SAS: PROC `LOGISTIC`, `CATMOD`, `GENMOD` , `INSIGHT` (Fit YX), ...
- SPSS: Hiloglinear, Loglinear, Generalized linear models

Models: Response vs. Association

Response models

- Sometimes, one variable is a natural discrete response.
- Q: How does the response relate to explanatory variables?
  - Admit ~ Gender + Dept
  - Party ~ Age + Education + Urban

⇒ Logit models, logististic regression, generalized linear models

Association models

- Sometimes, the main interest is just association among variables
- Q: Which variables are associated, and how?
  - Berkeley data: [Admit Gender]? [Admit Dept]? [Gender Dept]
  - Hair-eye data: [Hair Eye]? [Hair Sex]? [Eye, Sex]

⇒ Loglinear models

This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

Models: Response vs. Association

Response models

- Sometimes, one variable is a natural discrete response.
- Q: How does the response relate to explanatory variables?
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⇒ Logit models, logististic regression, generalized linear models

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- Q: Which variables are associated, and how?
  - Berkeley data: [Admit Gender]? [Admit Dept]? [Gender Dept]
  - Hair-eye data: [Hair Eye]? [Hair Sex]? [Eye, Sex]

⇒ Loglinear models

This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

Response models

Analysis methods for categorical outcome (response) variables have close parallels with those for quantitative outcomes

	Quantitative outcome	Categorical outcome
Continuous predictor	Regression: lm(y ~ x1 + x2)	Logistic regression: glm() Loglinear model: loglm() Ordered: prop. odds model: polr()
Categorical predictor	ANOVA: lm(y ~ A + B) Ordered: polynomial contrasts	χ^2 tests: chisq.test() Ordered: CMH tests, CMHtest() Loglinear model: loglm()
Both	ANCOVA: lm(y ~ A + B + x)	Logistic regression: glm() Loglinear model: loglm()

All use similar model formulas:

```
lm(y ~ A)                  # one way ANOVA
lm(y ~ A*B)                # two way: A + B + A:B
lm(y ~ X + A)              # one-way ANCOVA
lm(y ~ (A+B+C)^2)          # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```


Response models

For quantitative outcomes, lm() for everything, formula notation

```
lm(y ~ A)                  # one way ANOVA
lm(y ~ A*B)                # two way: A + B + A:B
lm(y ~ X + A)              # one-way ANCOVA
lm(y ~ (A+B+C)^2)          # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

For categorical outcomes, different modeling functions for different outcome types

```
glm(binary ~ X + A, family="binomial")   # logistic regression
glm(Freq ~ X + A, family="poisson")      # poisson regression
MASS::polr(multicat ~ X + A)             # ordinal regression
nnet::multinom(multicat ~ X + A)         # multinomial regression
loglin(table, margins)                   # loglinear model
MASS::loglm(Freq ~ .)                    # loglinear model, . = A+B+C+ …
MASS::loglm(Freq ~ .^2)                  #  + all two-way associations
```

Data display: Tables vs. Graphs

If I can't picture it, I can't understand it. Albert Einstein

Getting information from a table is like extracting sunlight from a cucumber. Farquhar & Farquhar, 1891

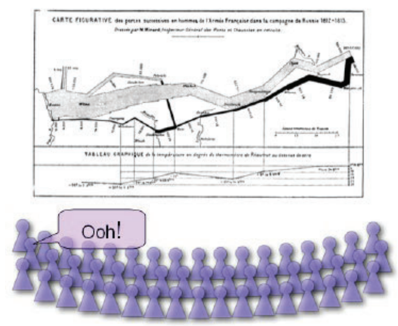
Tables vs. Graphs

- Tables are best suited for *look-up* and calculation—
  - read off exact numbers
  - show additional calculations (e.g., % change)
- Graphs are better for:
  - showing *patterns, trends, anomalies*,
  - making *comparisons*
  - seeing the *unexpected*!
- Visual presentation as *communication*:
  - what do you want to say or show?
  - $\Longrightarrow$ design graphs and tables to 'speak to the eyes'

Graphical methods: Communication goals

Different graphs for different audiences

- **Presentation**: A carefully crafted graph to appeal to a wide audience
- **Exploration, analysis**: Possibly many related graphs, different perspectives, narrow audience (often: just you!)

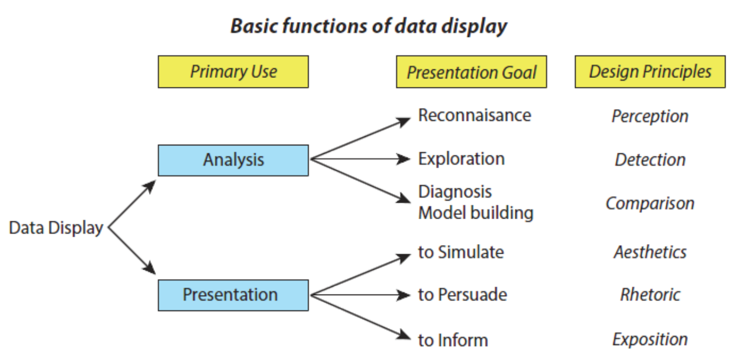


Presentation

Exploration

Graphical methods: Presentation goals

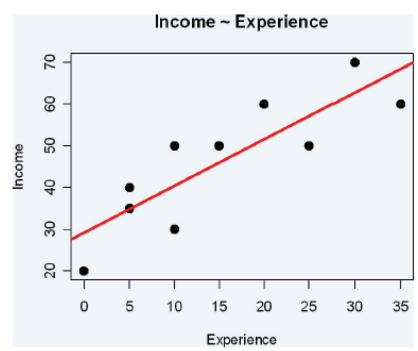
- Different presentation goals appeal to different design principles



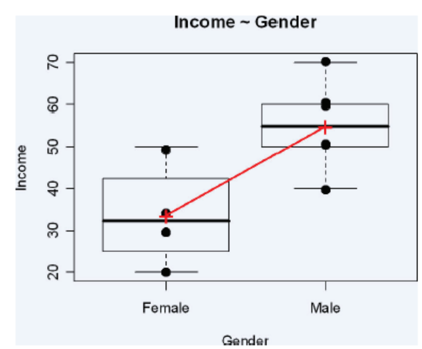
Think: What do I want to communicate? For what purpose?

Graphical methods: Quantitative data

Quantitative data (amounts) are naturally displayed in terms of **magnitude $\sim$ position along a scale**



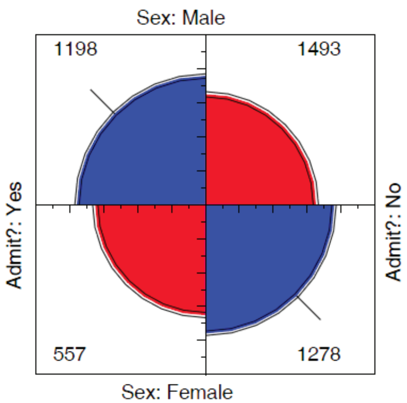
Scatterplot of Income vs. Experience



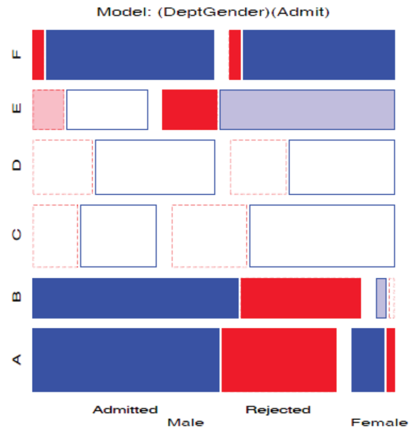
Boxplot of Income by Gender

Graphical methods: Categorical data

Frequency data (counts) are more naturally displayed in terms of **count** $\sim$ **area** (Friendly, 1995)



Fourfold display for 2×2 table

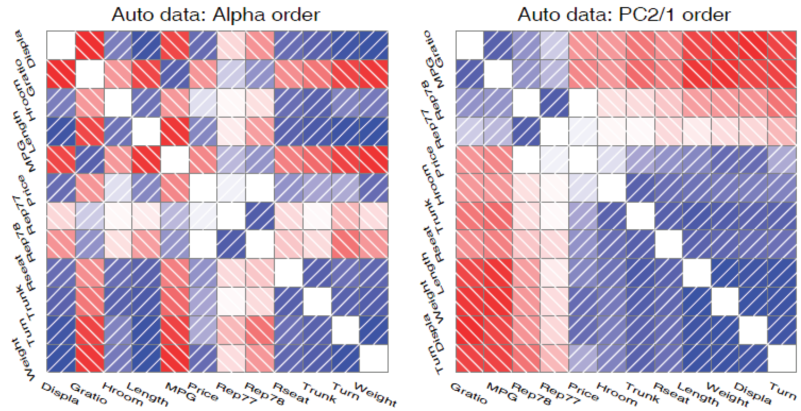


Mosaic plot for 3-way table

Friendly, M. (1995). Conceptual and visual models for categorical data. *American Statistician*, **49**: 153-160.

Principles of graphical display

- **Effect ordering** (Friendly and Kwan, 2003)— In tables and graphs, sort unordered factors according to the effects you want to see/show.



Friendly & Kwan (2003). Corrgrams: Exploratory displays for correlation matrices. *American Statistician*, **54**(4): 316-324.

Tabular displays

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Alpha ordered

Eye color	Hair color			
	Blond	Black	Brown	Red
Blue	94	20	17	84
Brown	7	68	26	119
Green	10	15	14	54
Hazel	16	5	14	29

Model:	*Independence*: [Hair][Eye] χ^2 (9)= 138.29						
Color coding:	$<$-4	$<$-2	$<$-1	0	$>$1	$>$2	$>$4
n in each cell:	$n <$ expected				$n >$ expected		

There is an association, but it is hard to see the general pattern

Tabular displays

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Effect ordered

Eye color	Hair color			
	Black	Brown	Red	Blond
Brown	68	119	26	7
Hazel	15	54	14	10
Green	5	29	14	16
Blue	20	84	17	94

Model:	*Independence*: [Hair][Eye] χ^2 (9)= 138.29						
Color coding:	$<$-4	$<$-2	$<$-1	0	$>$1	$>$2	$>$4
n in each cell:	$n <$ expected				$n >$ expected		

The pattern is clearer when the eye colors are permuted: light hair goes with light eyes & vice-versa

Sometimes, don't need numbers at all

COVID transmission risk ~ Occupancy * Ventilation * Activity * Mask? * Contact.time

A complex 5-way table, whose message is clearly shown w/o numbers

A semi-graphic table shows the patterns in the data

There are 1+ unusual cells here. Can you see them?

From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? *BMJ* 2020;370:m3223, *doi: https://doi.org/10.1136/bmj.m3223*

Visual table ideas: Heatmap shading

Heatmap shading: Shade the background of each cell according to some criterion

The trends in the US and Canada are made obvious

NB: Table rows are sorted by Jan. value, lending coherence

Background shading ~ value:
US & Canada are made to stand out.

Tech note: use white text on a darker background

Unemployment rate in selected countries

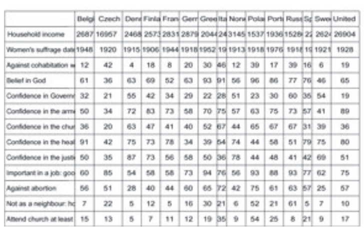
January-August 2020, sorted by the unemployment rate in January.

country	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug
Japan	2.4%	2.4%	2.5%	2.6%	2.9%	2.8%	2.9%	3.0%
Netherlands	3.0%	2.9%	2.9%	3.4%	3.6%	4.3%	4.5%	4.6%
Germany	3.4%	3.6%	3.8%	4.0%	4.2%	4.3%	4.4%	4.4%
Mexico	3.6%	3.6%	3.2%	4.8%	4.3%	5.4%	5.2%	5.0%
US	**3.6%**	**3.5%**	**4.4%**	**14.7%**	**13.3%**	**11.1%**	**10.2%**	**8.4%**
South Korea	4.0%	3.3%	3.8%	3.8%	4.5%	4.3%	4.2%	3.2%
Denmark	4.9%	4.9%	4.8%	4.9%	5.5%	6.0%	6.3%	6.1%
Belgium	5.1%	5.0%	5.0%	5.1%	5.0%	5.0%	5.0%	5.1%
Australia	5.3%	5.1%	5.2%	6.4%	7.1%	7.4%	7.5%	6.8%
Canada	**5.5%**	**5.6%**	**7.8%**	**13.0%**	**13.7%**	**12.3%**	**10.9%**	**10.2%**
Finland	6.8%	6.9%	7.0%	7.3%	7.5%	7.8%	8.0%	8.1%

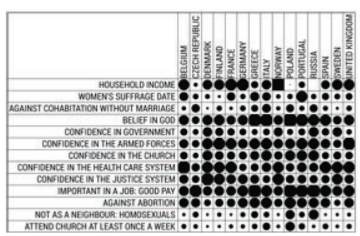
Source: OECD • Get the data • Created with Datawrapper

Bertifier: Turning tables into graphs

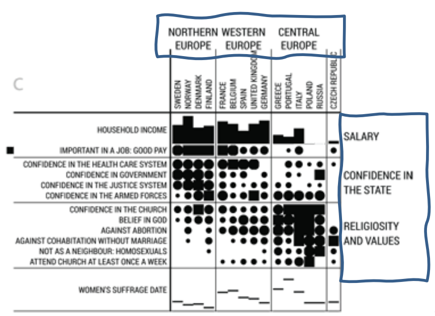
a attitudes & attributes



b encode values by size & shape



c

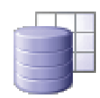


(a) Table: attitudes and attributes by country
(b) Visual: encode values by size, shape
(c) Sort & group by themes, country regions

Bertifier: Bertin's reorderable matrix
See: http://www.aviz.fr/bertifier

Data, pictures, models & stories

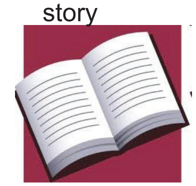
Goal: Tell a credible story about some real data problem



data

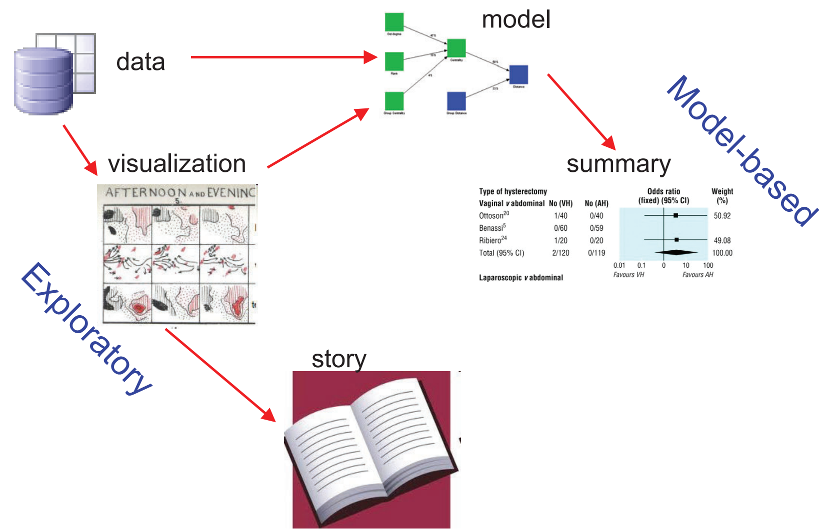
Gender bias
Measles vaccination
Global warming
…

story



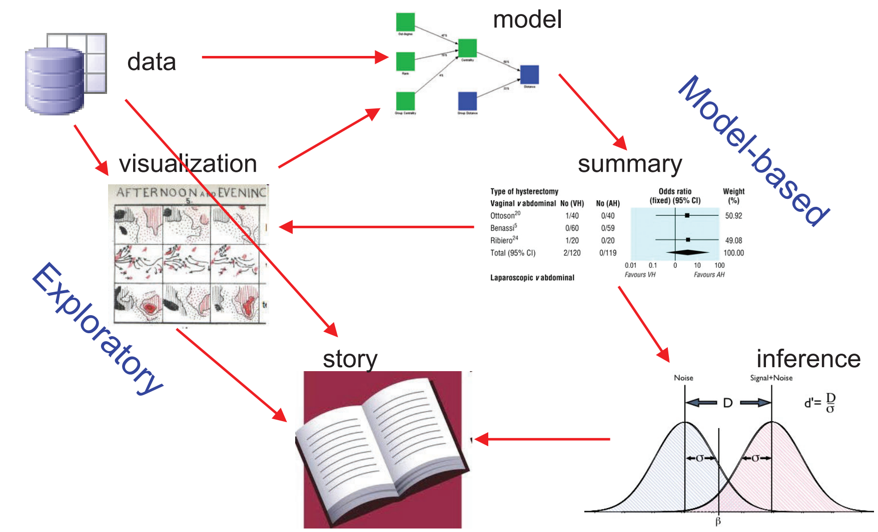
Data, pictures, models & stories

Two paths to enlightenment

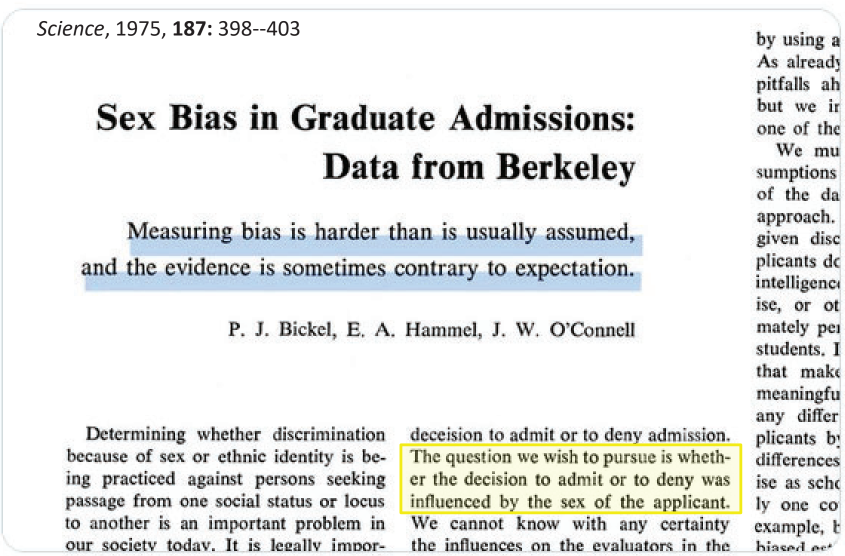


Data, pictures, models & stories

Now, tell the story!



Gender Bias at UC Berkeley?



Science, 1975, **187**: 398--403

Sex Bias in Graduate Admissions: Data from Berkeley

Measuring bias is harder than is usually assumed, and the evidence is sometimes contrary to expectation.

P. J. Bickel, E. A. Hammel, J. W. O'Connell

Determining whether discrimination because of sex or ethnic identity is being practiced against persons seeking passage from one social status or locus to another is an important problem in our society today. It is legally impor-

deceision to admit or to deny admission. The question we wish to pursue is whether the decision to admit or to deny was influenced by the sex of the applicant. We cannot know with any certainty the influences on the evaluators in the

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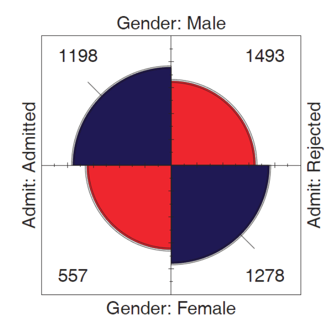
2 × 2 Frequency Tables: Fourfold displays

Table: Admissions to Berkeley graduate programs

	Admitted	Rejected	Total	% Admit	Odds(Admit)
Males	1198	1493	2691	44.52	0.802
Females	557	1278	1835	30.35	0.437
Total	1755	2771	4526	38.78	0.633

odds ratio (θ) = 1.84



Males nearly twice as likely to be admitted

- Is this a "significant" association?
- Is it evidence for gender bias?
- How to measure strength of association?
- How to visualize?

Fourfold display:

- quarter circles, area ~ frequency
- ratio of areas: odds ratio (θ)
- confidence bands: overlap iff θ ≈ 1
- visualize significance!

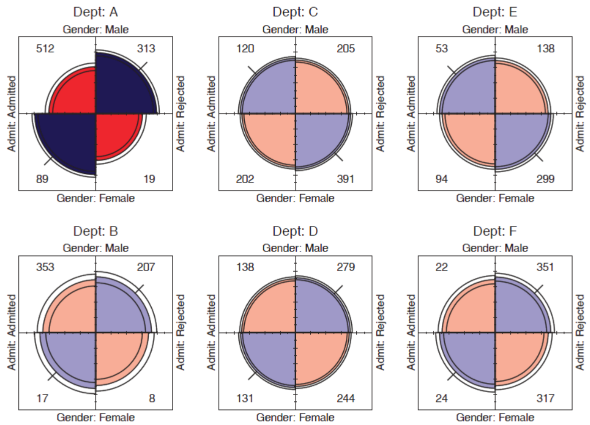
2 × 2 × k Stratified tables

The data arose from 6 graduate departments

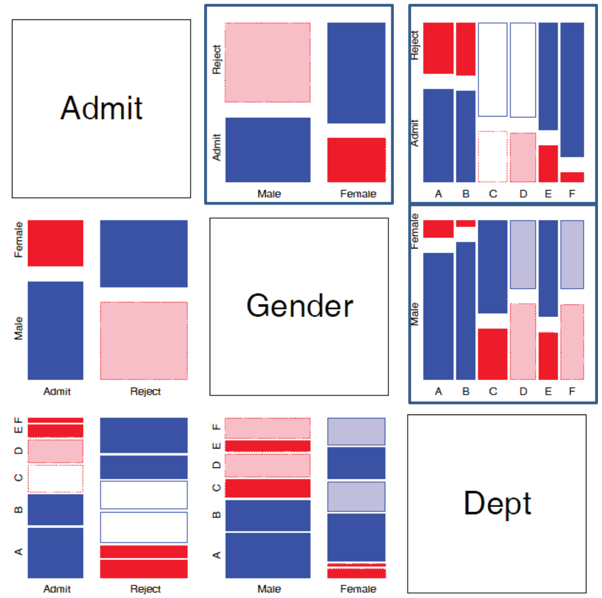
No difference between males & females, except in Dept A where women more likely to be admitted!

Design:

- small multiples
- encode direction by color
- encode signif. by shading



Mosaic matrices



Scatterplot matrix analog for categorical data

All pairwise views
Small multiples → comparison

The answer: Simpson's Paradox

- Depts A, B were easiest
- Applicants to A, B mostly male
- ∴ Males more likely to be admitted overall

Graphical methods for categorical data

In general, these share similar ideas & scope with methods for quantitative data

Exploratory methods

- Minimal assumptions (like non-parametric methods)
- Show the *data*, not just *summaries*
- But can add summaries: smoothed curve(s), trend lines, ...
- Help detect *patterns, trends, anomalies*, suggest hypotheses

Plots for model-based methods

- Residual plots - departures from model, omitted terms, ...
- Effect plots - estimated probabilities of response or log odds
- Diagnostic plots - influence, violation of assumptions

Summary

- Categorical data involves some new ideas
  - Discrete variables: unordered or ordered
  - Counts, frequencies
- New / different data structures & functions
  - tables – 1-way, 2-way, 3-way, ... table(), xtabs()
  - similar in matrices or arrays matrix(), array()
  - datasets:
    - frequency form
    - case form
- Graphical methods: often use area ~ Freq
- Models: Most are ≈ natural extensions of lm()