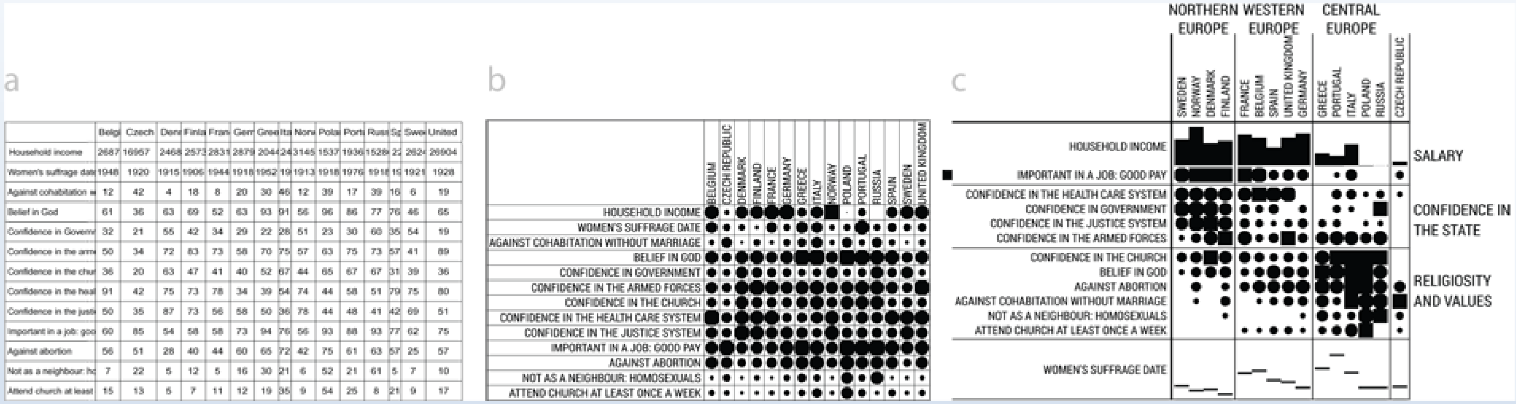

Categorical Data Analysis

Course overview

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Psych 6136

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Course goals

This course is designed as a broad, applied introduction to the statistical analysis of categorical data, with an emphasis on:

Emphasis: visualization methods

- exploratory graphics: see patterns, trends, anomalies in your data
- model diagnostic methods: assess violations of assumptions
- model summary methods: provide an interpretable summary of your data

Emphasis: theory $\Rightarrow$ practice

- Understand how to translate research questions into statistical hypotheses and models
- Understand the difference between simple, non-parametric approaches (e.g., χ^2 test for indpendence) and model-based methods (logistic regression, GLM)
- Framework for thinking about categorical data analysis in *visual* terms

Course outline

1. Exploratory and hypothesis testing methods

- Week 1: Overview; Introduction to R
- Week 2: One-way tables and goodness-of-fit test
- Week 3: Two-way tables: independence and association
- Week 4: Two-way tables: ordinal data and dependent samples
- Week 5: Three-way tables: different types of independence
- Week 6: Correspondence analysis

2. Model-based methods

- Week 7: Logistic regression I
- Week 8: Logistic regression II
- Week 9: Multinomial logistic regression models
- Week 10: Log-linear models
- Week 11: Loglinear models: Advanced topics
- Week 12: Generalized Linear Models: Poisson regression
- Week 13: Course summary & additional topics

Course schedule

The schedule page provides links to slides, tutorials, readings & R scripts

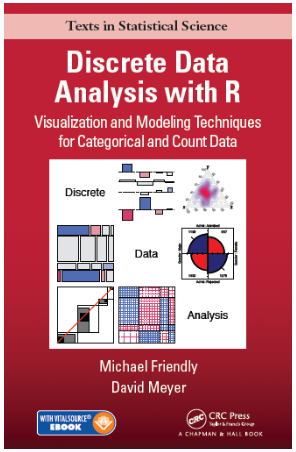
Week	Topic	Readings	R files
1	Overview [slides] [4up] [Working with R Studio] [4up]	DDAR: Ch1, Ch2; Agresti: Ch1	R-into.R [knitr]
2	Discrete distributions [slides] [4up]	DDAR: Ch3	R-data.R [knitr] binomial.R [knitr]
3	Two-Way Tables: Independence & Association [slides] [4up]	DDAR: Ch4; Agresti: Ch2	berk-4fold.R [knitr] vision-sieve.R [knitr]
4	Two-Way Tables: Ordinal Data and Dependent Samples [Tutorial] on two-way tables	DDAR: Ch4; Agresti: Ch2	msdiag-agree.R [knitr] haireye-spineplot.R [knitr]
5	Loglinear Models and Mosaic Displays [slides] [4up] [Tutorial] on loglin models; [Mosaic display animation]	DDAR: Ch5; Agresti: 2.7, Ch. 7	berkeley-glm.R [knitr] titanic-loglin.R [knitr]
6	Correspondence Analysis [slides] [4up] [Tutorial] on CA;	DDAR: Ch6	mental-ca.R [knitr] mca-presex3.R [knitr]
7	Logistic Regression I [slides] [4up] [Logistic regression tutorial]	DDAR: 7.1-7.3; Agresti: 3.1-3.2; Ch 4	arthritis-logistic.R [knitr] cowles-logistic.R [knitr] Arrests-logistic.R [knitr]
8	Logistic Regression II [slides] [4up]	DDAR: 7.3-7.4; Agresti: Ch 4-5	cowles-effect.R [knitr] Arrests-effects.R [knitr] berkeley-diag.R [knitr]

. . .

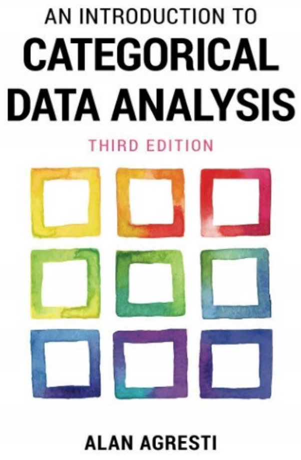
Textbooks

Main texts

- Friendly & Meyer (2016). *Discrete Data Analysis with R: Visualizing & Modeling Techniques for Categorical & Count Data*
    - 30% discount on Routledge web site (code: ADC22)
    - Draft chapters linked in Schedule
    - DDAR web site: https://ddar.datavis.ca



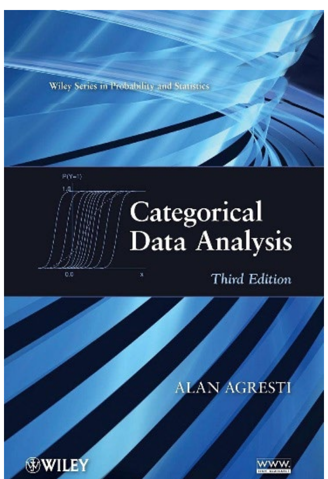
- Agresti (2007). *An Introduction to Categorical Data Analysis*, 3rd E. Wiley & Sons.
    - eBook available: https://bit.ly/3Wzqv0n
    - Or, via York Bookstore

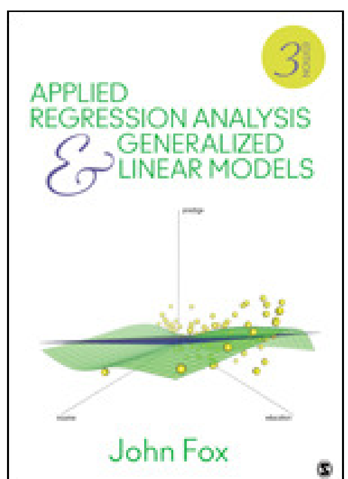


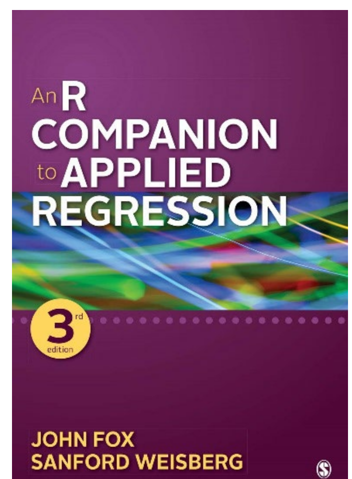
Textbooks

Supplementary readings

- Agresti (2013). *Categorical Data Analysis*, 3rd ed. [More mathematical, but the current Bible of CDA]
  - PDF available: https://bityl.co/FG9c
- Fox (2016). *Applied Regression Analysis and Generalized Linear Models*, 3rd ed. Particularly: Part IV on Generalized Linear Models
- Fox & Weisberg (2018). *An R Companion to Applied Regression.* Also, [web site for the book](#).







Expectations & grading

- I expect you will read chapters in *DDAR* & Agresti *Intro* each week
  - See Topic Schedule on course web site
  - R exercises & tutorials: Please work on these
  - R Assignments : Ungraded, but please submit them when assigned
  - Class discussion: Help make classes participatory
- Evaluation: (tentative: subject to change)
  - (2 x 40%) Two take-home projects: Analysis & research report, based on assignment problems or your own data
  - (20%)
    - Assignment portfolio: best work, enhanced
    - Research report on journal article(s) of theory / application of CDA
    - In-class presentation (~15 min) on application of general interest

The R you need

- R, version >=3.6 [R 4.2 is current]
  - Download from https://cran.r-project.org/
- RStudio IDE, highly recommended
  - https://www.rstudio.com/products/rstudio/
- R packages:
  - vcd
  - vcdExtra
  - car
  - effects
  - ggplot2
  - ...

R script to install packages:
https://friendly.github.io/6136/R/install-vcd-pkgs.R

Categorical data analysis: History

- Categorical data analysis is a relatively recent arrival
  - 1888 – Galton introduces the concept of correlation
  - 1908 – Student's t-distribution for the mean of small samples
  - 1931 – L. L. Thrustone: Multiple factor analysis
  - 1935 – R. A. Fisher's Design of Experiments – ANOVA
  - . . . (time passes)

  - 1972 – Nelder & Wedderburn develop the central ideas of generalized linear models (logistic & poison regression)
  - 1973 – J-P. Benzecri: Correspondence analysis (analysis des donnés)
  - 1974 – Bishop , Fienberg, Holland introduce the loglinear model for discrete data, ANOVA for log(Freq)
  - 1984 – Leo Goodman enhanced loglinear models for complex data: RC models, mobility tables, panel data, …

What is categorical data?

A **categorical variable** is one for which the possible measured or assigned values consist of a discrete set of categories, which may be *ordered* or *unordered*.
Some typical examples are:

- `Gender`, with categories {"male", "female", "trans"}
- `Marital status`: { "Never married", "Married", "Separated", "Divorced", "Widowed" }
- `Party preference`: {"NDP", "Liberal", "Conservative", "Green"}
- `Treatment improvement`: {"none", "some", "marked"}
- `Age`: {"0-9", "10-19", "20-29", "30-39", ... }.
- `Number of children`: $0, 1, 2, 3, \ldots$.

Questions:

- Which of these are ordered (ordinal)?
- Which could be treated as numeric? How?
- Which have missing categories, sometimes ignored, or treated as "Other"

Categorical data: Structures

Categorical (frequency) data appears in various forms

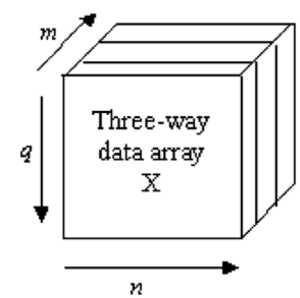
- Tables: often the result of `table()` or `xtabs()`
  - 1-way
  - 2-way – 2 × 2, r × c
  - 3-way

Gender compared to handedness

	Handed		
	Left	Right	
Female	7	46	53
Male	5	63	68
	12	109	121

margins

- Matrices: `matrix()`, with row & col names
- Arrays: `array()`, with `dimnames()`



- Data frames
  - Case form (individual observations)
  - Frequency form

	Hair	Eye	Freq
1	Black	Brown	68
2	Brown	Brown	119
3	Red	Brown	26
4	Blond	Brown	7
5	Black	Blue	20
6	Brown	Blue	84
7	Red	Blue	17
8	Blond	Blue	94

1-way tables

- Unordered factors

	Black	Brown	Red	Blond
n	108	286	71	127
%	0.18	0.48	0.12	0.21

Hair color of 592 students

	BQ	Cons	Green	Liberal	NDP
n	104	392	126	404	174
%	0.087	0.33	0.1	0.34	0.14

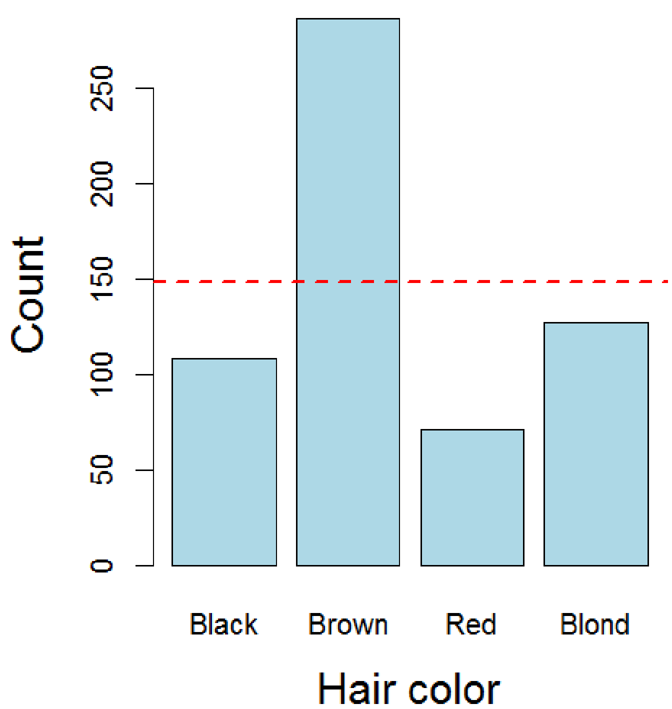
Voting intentions in Harris-Decima poll, 8/21/08

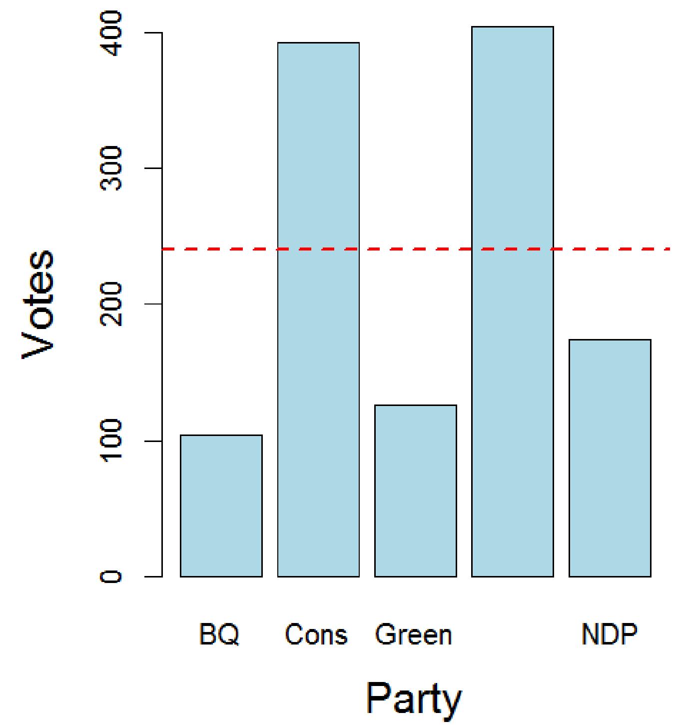
Questions:

- Are all hair colors equally likely?
- Aside from Brown hair, are others equally likely?
- Is there a diff in voting intentions for Liberal vs. Conservative

1-way tables

- Even here, simple graphs are more informative than tables





But these don’t really answer the questions. Why?

1-way tables

- Ordered, quantitative factors
  - Number of sons in Saxony families with 12 children

```
> data(Saxony, package="vcd")
> Saxony
nMales
   0     1     2     3     4     5     6     7     8     9    10    11    12
   3    24   104   286   670  1033  1343  1112   829   478   181    45     7
```

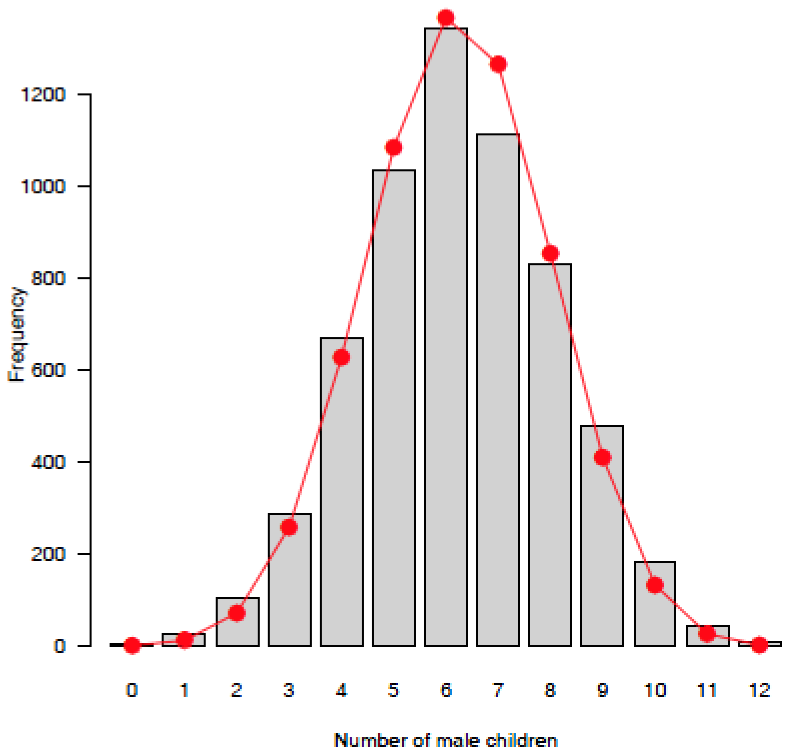
Questions:

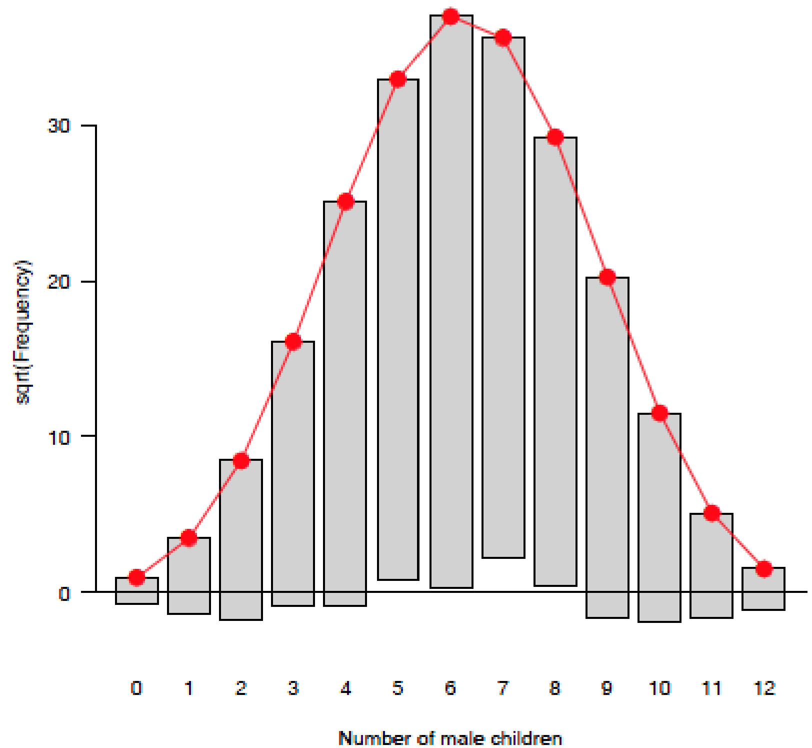
- What is the form of this distribution?
- Is it useful to think of this as a binomial distribution?
- If so, is Pr(male) = 0.5 reasonable to describe the data?
- How could families have > 10 children?

1-way tables: graphs

For a particular distribution in mind:

- Plot the data together with the fitted frequencies
- Better still: **hanging rootogram**: freq on sqrt scale; hang bars from fitted values





2-way tables: 2 × 2 × …

- Two-way

Admit	Gender: Male	Female
Admitted	1198	557
Rejected	1493	1278

Admission to graduate programs at UC Berkeley

- Three-way, stratified by another factor

… by Department

Admit	Gender	Dept: A	B	C	D	E	F
Admitted	Male	512	353	120	138	53	22
	Female	89	17	202	131	94	24
Rejected	Male	313	207	205	279	138	351
	Female	19	8	391	244	299	317

Questions:

- Is admission associated with gender?
- Does admission rate vary with department?

Larger tables: $r \times c \times \ldots$

```
> margin.table(HairEyeColor, 1:2)
         Eye
Hair      Brown Blue Hazel Green
  Black      68   20    15     5
  Brown     119   84    54    29
  Red        26   17    14    14
  Blond       7   94    10    16
```

2-way
Actually, this is a 2-way margin of a 3-way table

```
> ftable(Eye ~ Sex + Hair, data=HairEyeColor)
             Eye Brown Blue Hazel Green
Sex    Hair
Male   Black        32   11    10     3
       Brown        53   50    25    15
       Red          10   10     7     7
       Blond         3   30     5     8
Female Black        36    9     5     2
       Brown        66   34    29    14
       Red          16    7     7     7
       Blond         4   64     5     8
```

3-way (& higher) can be “flattened” for a more convenient display

formula notation:
row vars ~ col vars

Table form

- Table form is convenient for display, but information is implicit
  - a table has dimensions, **`dim()`** and **`dimnames()`**
  - the "observations" are the cells in the tables
  - the "variables" are the dimensions of the table (factors)
  - the cell value is the count or frequency

```
> dim(haireye)
[1] 4 4
> dimnames(haireye)
$Hair
[1] "Black" "Brown" "Red"   "Blond"

$Eye
[1] "Brown" "Blue"  "Hazel" "Green"
```

```
> names(dimnames(haireye)) # factor names
[1] "Hair" "Eye"
> prod(dim(haireye))            # of cells
[1] 16
> sum(haireye)                  # total count
[1] 592
```

Datasets: frequency form

- Another common format is a dataset in frequency form

```
> as.data.frame(haireye)
    Hair   Eye Freq
1  Black Brown   68
2  Brown Brown  119
3    Red Brown   26
4  Blond Brown    7
5  Black  Blue   20
6  Brown  Blue   84
7    Red  Blue   17
8  Blond  Blue   94
9  Black Hazel   15
10 Brown Hazel   54
11   Red Hazel   14
12 Blond Hazel   10
13 Black Green    5
14 Brown Green   29
15   Red Green   14
16 Blond Green   16
```

- Create: **`as.data.frame(table)`**
- One row for each cell
- Columns: factors + **`Freq`** or **`count`**

Questions:

- What are the dimensions of the table?
- What is the total frequency?

Datasets: case form

- Raw data often arrives in case form

```
> expand.dft(as.data.frame(haireye)) |>
    as_tibble() |>
    mutate(age = round( runif( n =
       sum(haireye), min=17, max=29)))
# A tibble: 592 x 3
   Hair  Eye     age
   <chr> <chr> <dbl>
 1 Black Brown    19
 2 Black Brown    19
 3 Black Brown    27
 4 Black Brown    23
 5 Black Brown    19
 6 Black Brown    29
 7 Black Brown    25
 8 Black Brown    29
 9 Black Brown    17
10 Black Brown    23
# ... with 582 more rows
```

- One obs. per case
- # rows = sum of counts

- `vcdExtra::expand.dft()` expands to frequency form

- case form is required if there are continuous variables

- case form is tidy
- not all CDA functions play well with tibbles

Converting data forms

R functions for CDA sometimes accept only tables (matrices), or data frames, in either case for frequency form.
You may have to convert your data from one form to another

From this ↓	To this ↓	To this ↓	To this ↓
	Case form	**Freq form**	**Table form**
Case form	---	Z <- xtabs(~ A+ B) as.data.frame(Z)	table(A, B)
Freq form	expand.dft(X)	---	xtabs(Freq ~ A + B)
Table form	expand.dft(X)	as.data.frame(X)	---

Categorical data analysis: Methods

Methods for categorical data analysis fall into two main categories

Non-parametric, randomization-based methods

- Make minimal assumptions
- Useful for hypothesis-testing:
  - Are men more likely to be admitted than women?
  - Are hair color and eye color associated?
  - Does the binomial distribution fit these data?
- Mostly for two-way tables (possibly stratified)
- R:
  - Pearson Chi-square: `chisq.test()`
  - Fisher's exact test (for small expected frequencies): `fisher.test()`
  - Mantel-Haenszel tests (ordered categories: test for *linear* association): `CMHtest()`
- SAS: PROC `FREQ` — can do all the above
- SPSS: Crosstabs

Categorical data analysis: Methods

Model-based methods

- Must assume random sample (possibly stratified)
- Useful for estimation purposes: Size of effects (std. errors, confidence intervals)
- More suitable for multi-way tables
- Greater flexibility; fitting specialized models
  - Symmetry, quasi-symmetry, structured associations for square tables
  - Models for ordinal variables
- R: `glm()` family, Packages: car, gnm, vcd, ...
  - estimate standard errors, covariances for model parameters
  - confidence intervals for parameters, predicted Pr{response}
- SAS: PROC `LOGISTIC`, `CATMOD`, `GENMOD` , `INSIGHT` (Fit YX), ...
- SPSS: Hiloglinear, Loglinear, Generalized linear models

Models: Response vs. Association

Response models

- Sometimes, one variable is a natural discrete response.
- Q: How does the response relate to explanatory variables?
  - Admit $\sim$ Gender + Dept
  - Party $\sim$ Age + Education + Urban

$\Rightarrow$ Logit models, logististic regression, generalized linear models

Association models

- Sometimes, the main interest is just association among variables
- Q: Which variables are associated, and how?
  - Berkeley data: [Admit Gender]? [Admit Dept]? [Gender Dept]
  - Hair-eye data: [Hair Eye]? [Hair Sex]? [Eye, Sex]

$\Rightarrow$ Loglinear models

This is similar to the distinction between regression/ANOVA vs. correlation and factor analysis

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$\Rightarrow$ Loglinear models

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Response models

Analysis methods for categorical outcome (response) variables have close parallels with those for quantitative outcomes

	Quantitative outcome	Categorical outcome
Continuous predictor	Regression: lm(y ~ x1 + x2)	Logistic regression: glm() Loglinear model: loglm() Ordered: prop. odds model: polr()
Categorical predictor	ANOVA: lm(y ~ A + B) Ordered: polynomial contrasts	χ^2 tests: chisq.test() Ordered: CMH tests, CMHtest() Loglinear model: loglm()
Both	ANCOVA: lm(y ~ A + B + X)	Logistic regression: glm() Loglinear model: loglm()

All use similar model formulas:

```
lm(y ~ A)                    # one way ANOVA
lm(y ~ A*B)                  # two way: A + B + A:B
lm(y ~ X + A)                # one-way ANCOVA
lm(y ~ (A+B+C)^2)            # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

Response models

For quantitative outcomes, lm() for everything, formula notation

```
lm(y ~ A)                    # one way ANOVA
lm(y ~ A*B)                  # two way: A + B + A:B
lm(y ~ X + A)                # one-way ANCOVA
lm(y ~ (A+B+C)^2)            # 3-way ANOVA: A, B, C, A:B, A:C, B:C
```

For categorical outcomes, different modeling functions for different outcome types

```
glm(binary ~ X + A, family="binomial")   # logistic regression
glm(Freq ~ X + A, family="poisson")      # poisson regression
MASS::polr(multicat ~ X + A)             # ordinal regression
nnet::multinom(multicat ~ X + A)         # multinomial regression
loglin(table, margins)                   # loglinear model
MASS::loglm(Freq ~ .)                    # loglinear model, . = A+B+C+ …
MASS::loglm(Freq ~ .^2)                  #  + all two-way associations
```

Data display: Tables vs. Graphs

If I can't picture it, I can't understand it. Albert Einstein

Getting information from a table is like extracting sunlight from a cucumber. Farquhar & Farquhar, 1891

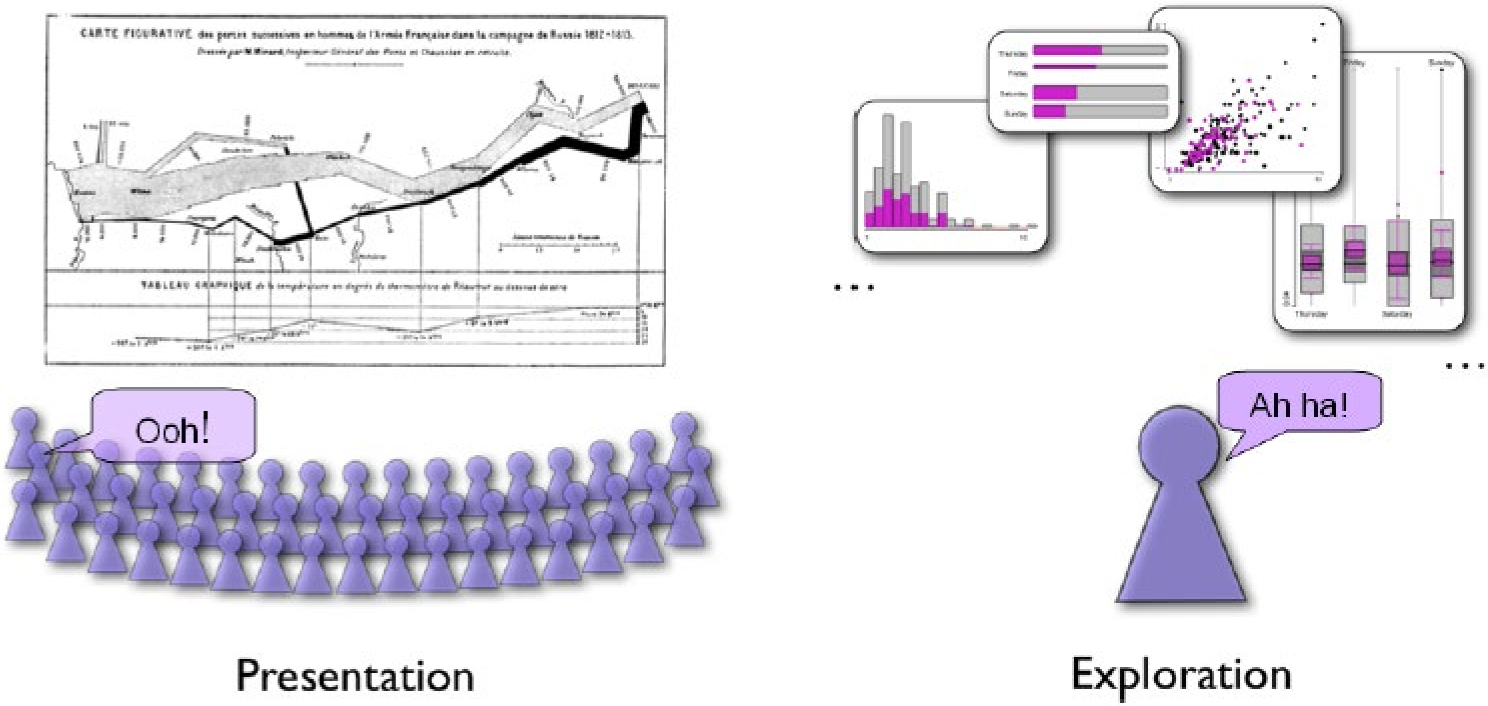
Tables vs. Graphs

- Tables are best suited for *look-up* and calculation—
  - read off exact numbers
  - show additional calculations (e.g., % change)
- Graphs are better for:
  - showing *patterns, trends, anomalies*,
  - making *comparisons*
  - seeing the *unexpected*!
- Visual presentation as *communication*:
  - what do you want to say or show?
  - $\Longrightarrow$ design graphs and tables to 'speak to the eyes'

Graphical methods: Communication goals

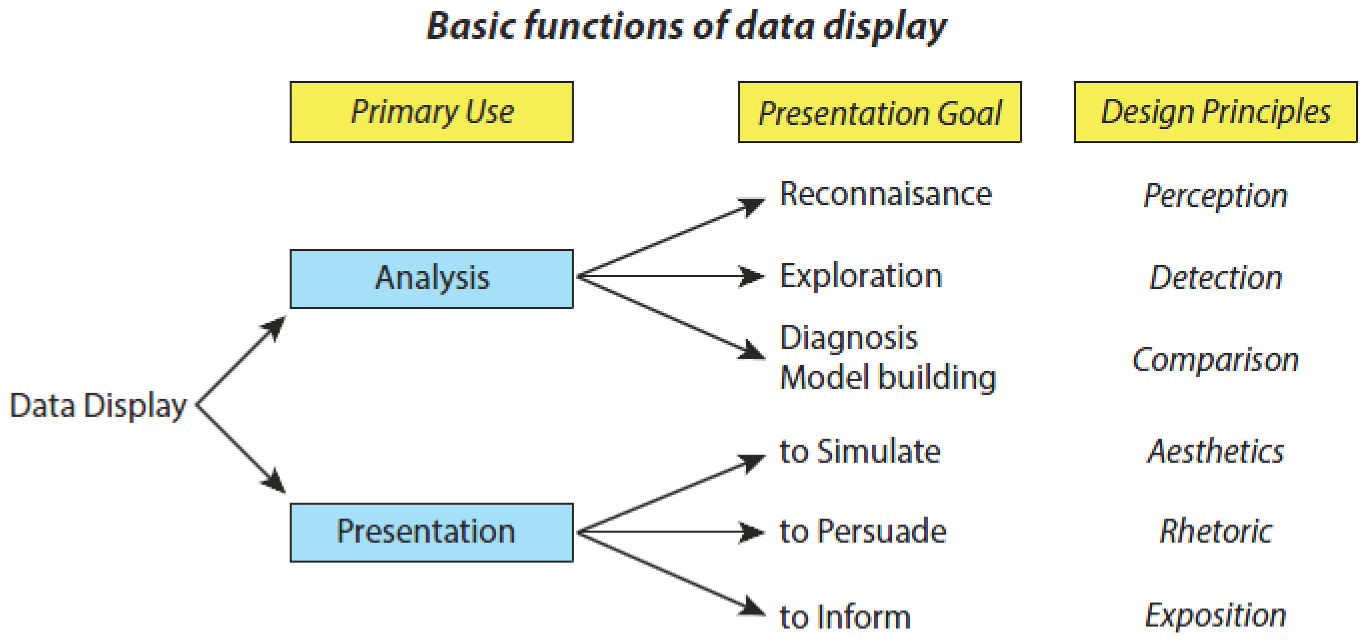
Different graphs for different audiences

- **Presentation**: A carefully crafted graph to appeal to a wide audience
- **Exploration, analysis**: Possibly many related graphs, different perspectives, narrow audience (often: just you!)



Graphical methods: Presentation goals

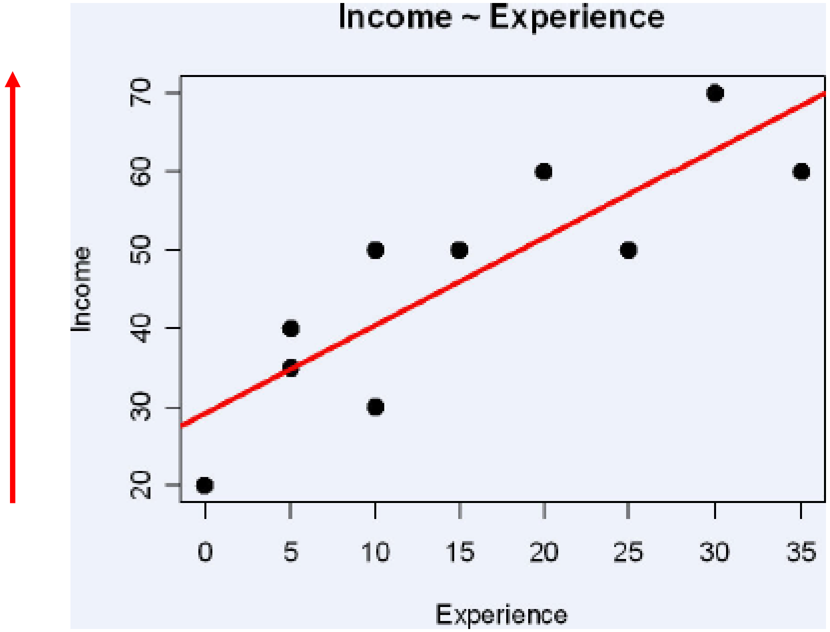
- Different presentation goals appeal to different design principles



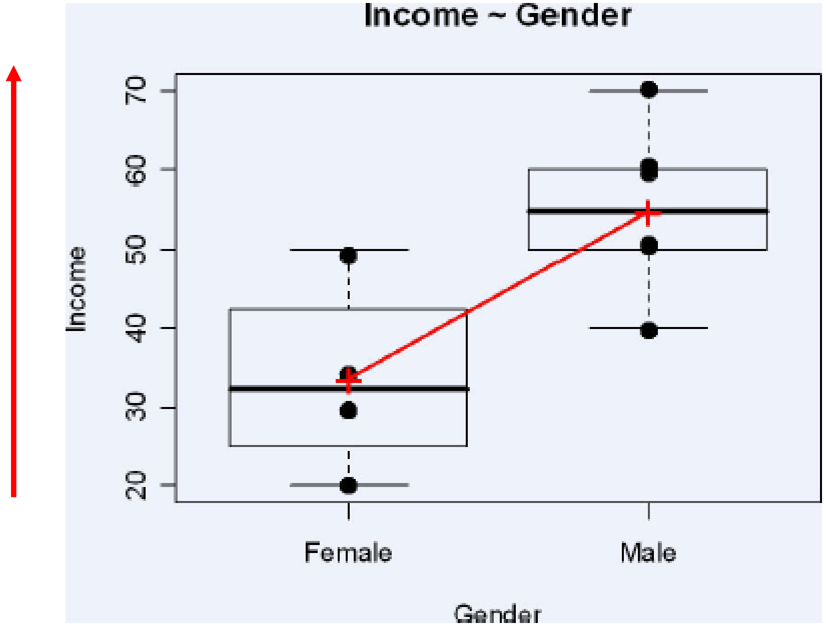
Think: What do I want to communicate? For what purpose?

Graphical methods: Quantitative data

Quantitative data (amounts) are naturally displayed in terms of
magnitude ∼ position along a scale



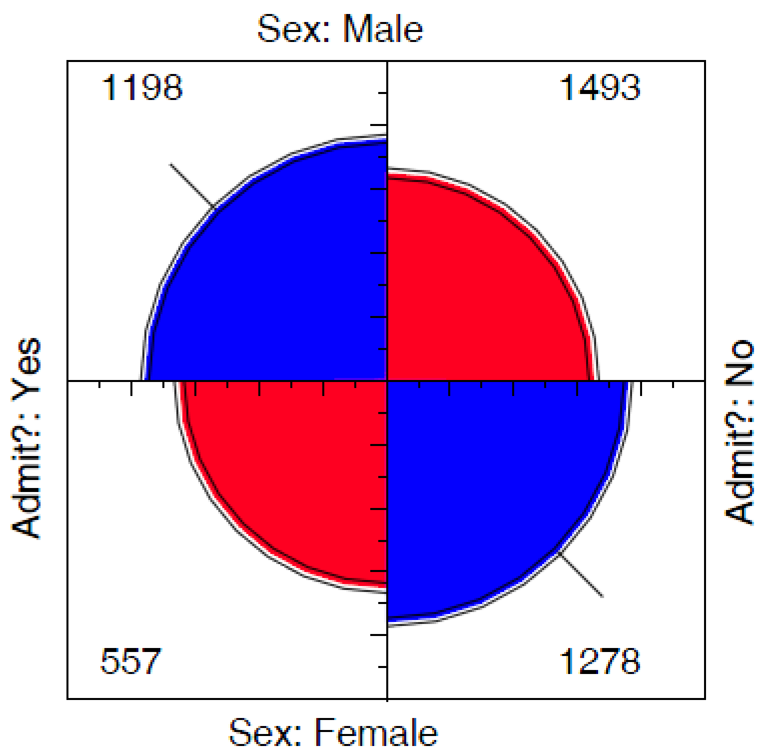
Scatterplot of Income vs. Experience



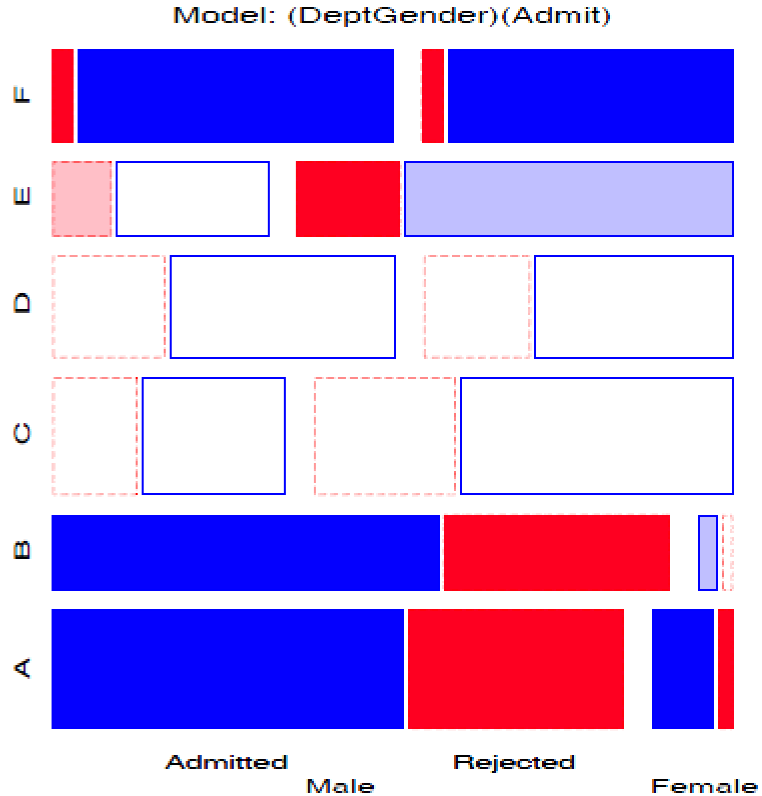
Boxplot of Income by Gender

Graphical methods: Categorical data

Frequency data (counts) are more naturally displayed in terms of **count** $\sim$ **area** (Friendly, 1995)



Fourfold display for 2×2 table

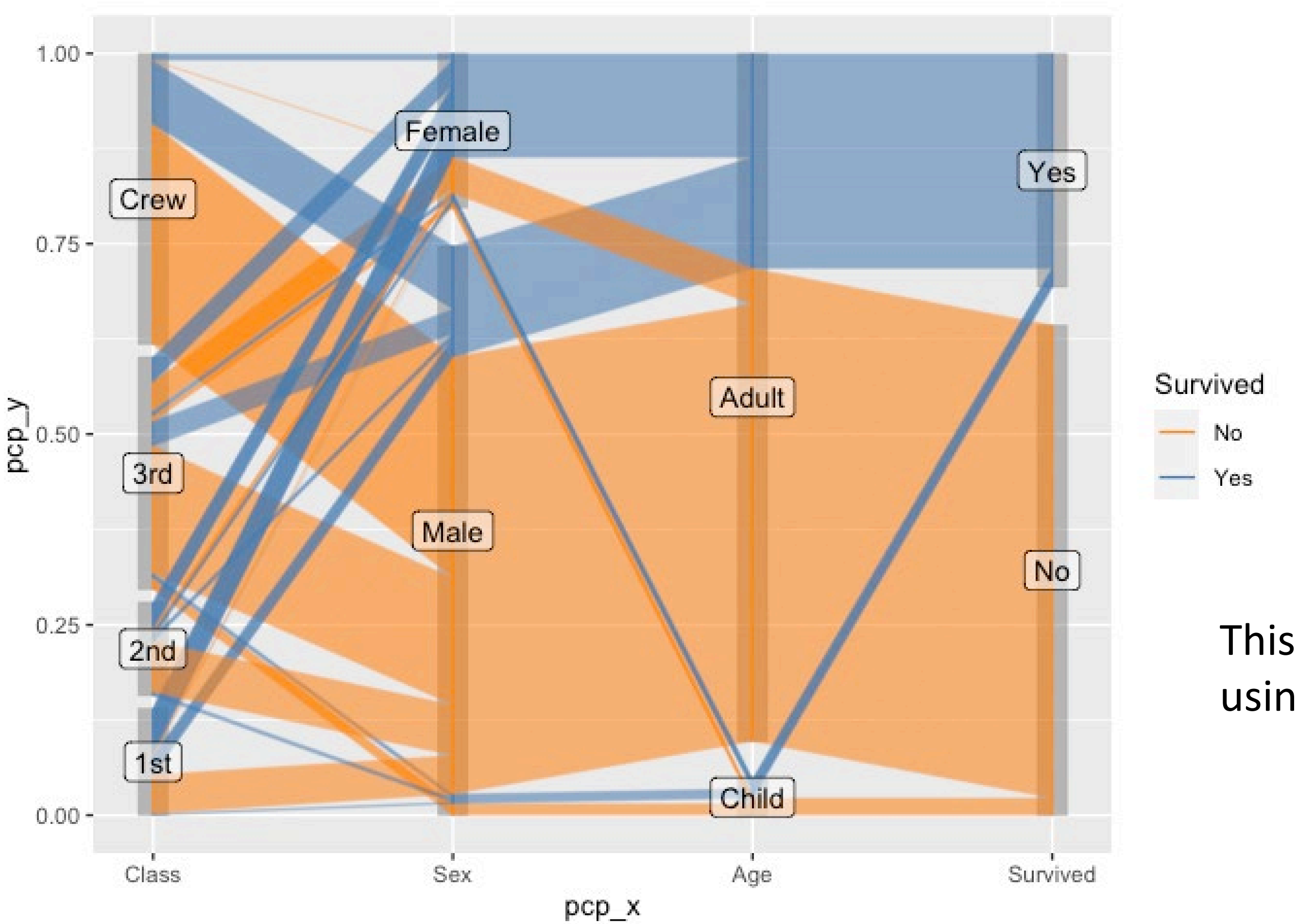


Mosaic plot for 3-way table

Friendly, M. (1995). Conceptual and visual models for categorical data. *American Statistician*, **49**: 153-160.

Categorical data: Parallel coordinates plot

Parallel coordinates plots show multiple variables, each along its’ own || axis
The categorical version uses the width of the band to show frequency



Survival on the Titanic, classified by:

- Class,
- Sex,
- Age,
- Survived

Band width ~ Freq

This plot was produced using the ggpcp package

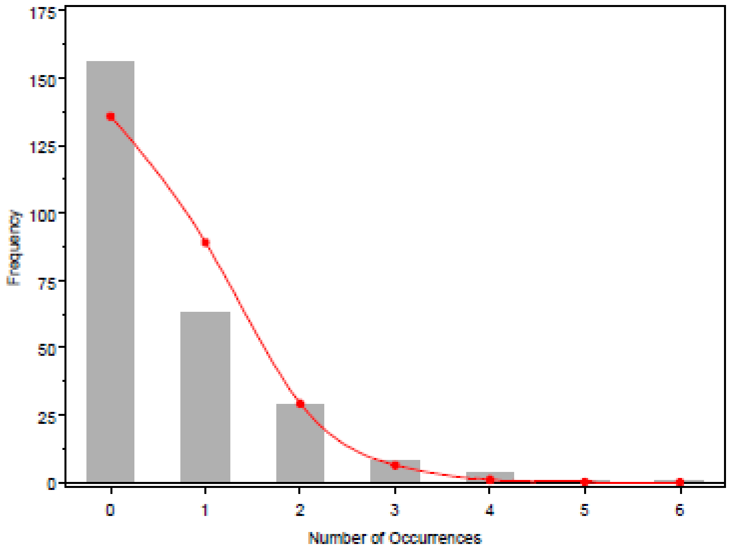
Effective data display

- Make the data stand out
  - Fill the data region (axes, ranges)
  - Use visually distinct symbols (shape, color) for different groups
  - Avoid chart junk, heavy grid lines that detract from the data
- Facilitate comparison
  - Emphasize the important comparisons visually
  - Side-by-side easier than in separate panels
  - “data” vs. a “standard” easier against a horizontal line
  - Show uncertainty where possible
- Effect ordering
  - For variables and unordered factors, arrange them according to the effects to be seen

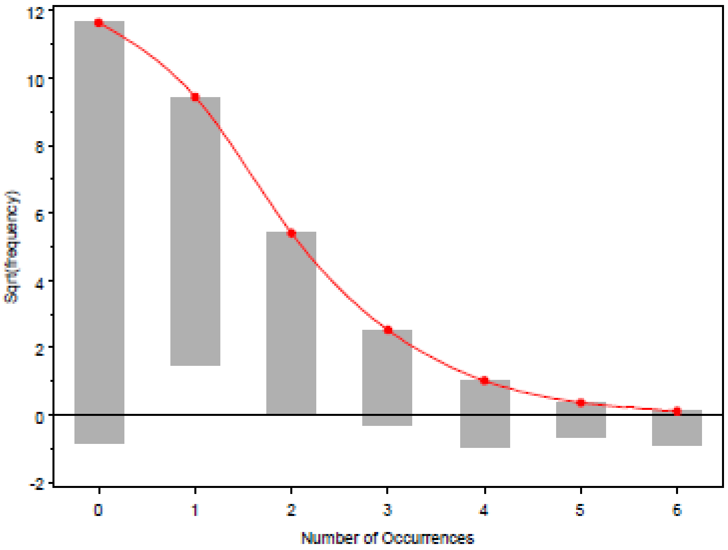
Facilitate comparison

Comparisons— Make visual comparisons easy

- Visual grouping— connect with lines, make key comparisons contiguous
- Baselines— compare *data* to *model* against a line, preferably horizontal
- Frequencies often better plotted on a square-root scale



Standard histogram with fit

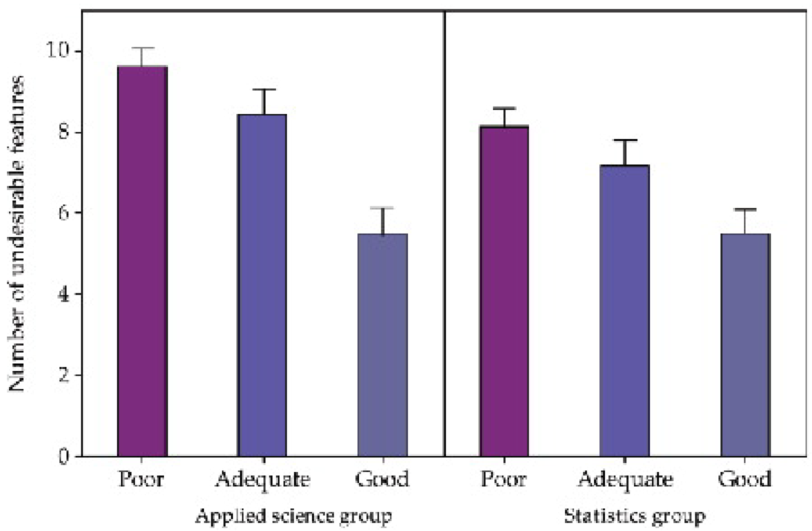


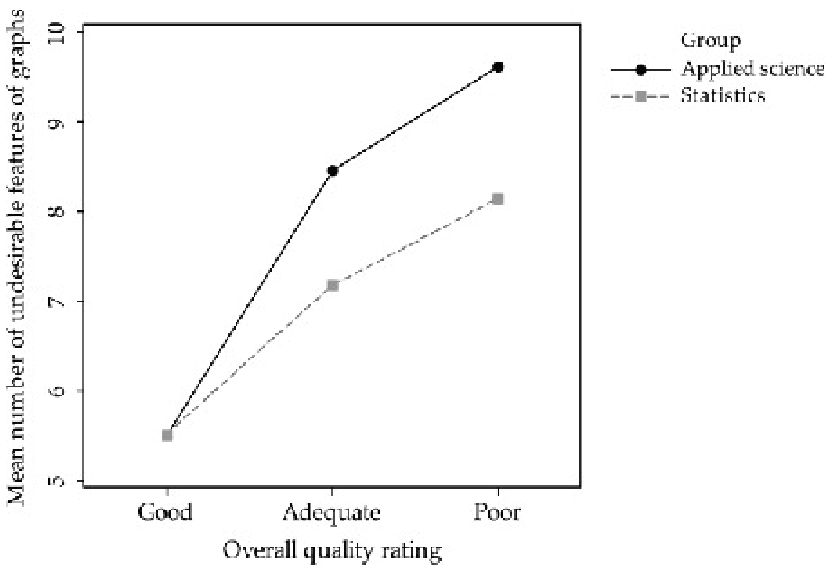
Suspended rootogram

Make comparisons *direct*

- Use points not bars (and don't dynamite them with ineffective error bars!)
- Connect similar circumstances to be compared by lines
- Same panel comparisons easier than different panels

Is there evidence of an interaction here?





???

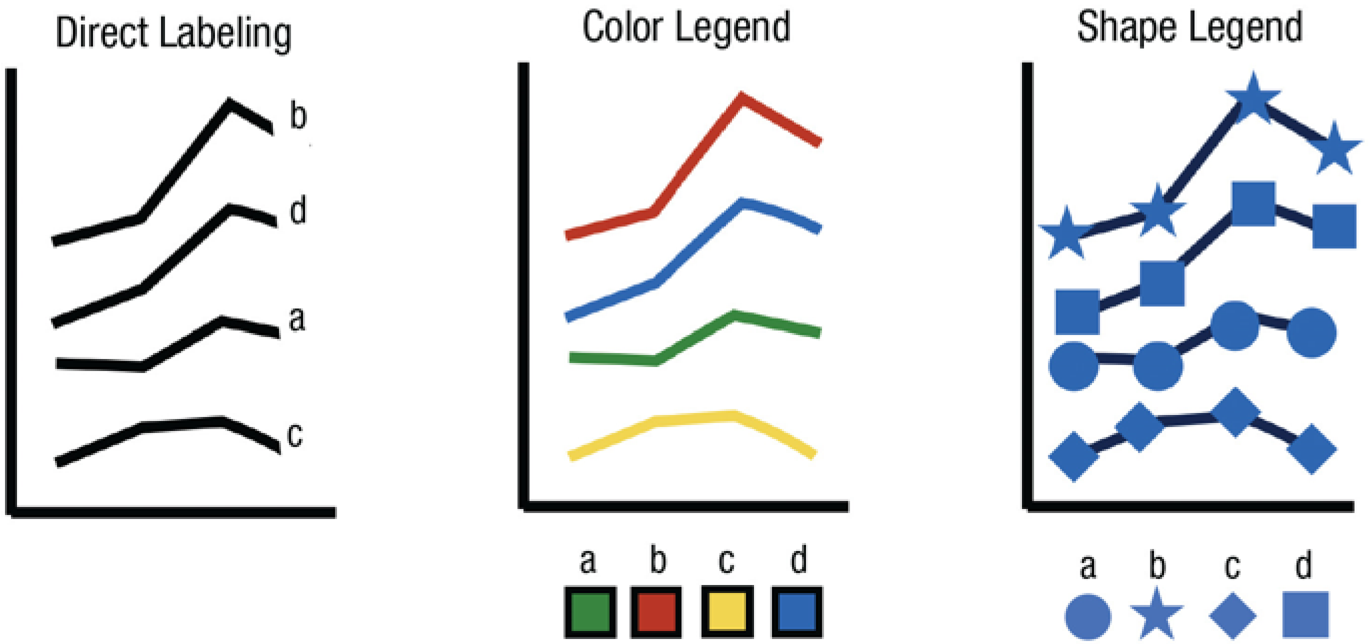
YES

Published in: Ian Gordon; Sue Finch; *Journal of Computational and Graphical Statistics* **2015,** 24, 1210-1229.
DOI: 10.1080/10618600.2014.989324

Direct labels vs. legends

Direct labels for points, lines and regions are usually easier and faster than legends

- Give the names of the four groups shown in the line graph at left in top-to-bottom order. (Answer: b, d, a, c.)
- Now do so for the graphs using color or shape legends
- You need to look back and forth between the graph and legend



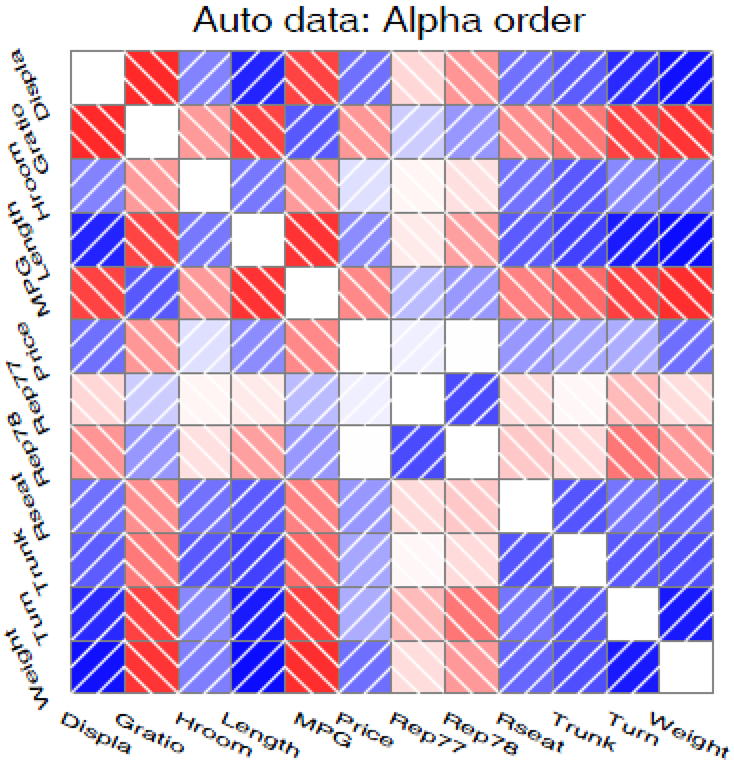
Source: Franconeri etal. DOI:10.1177/15291006211051956

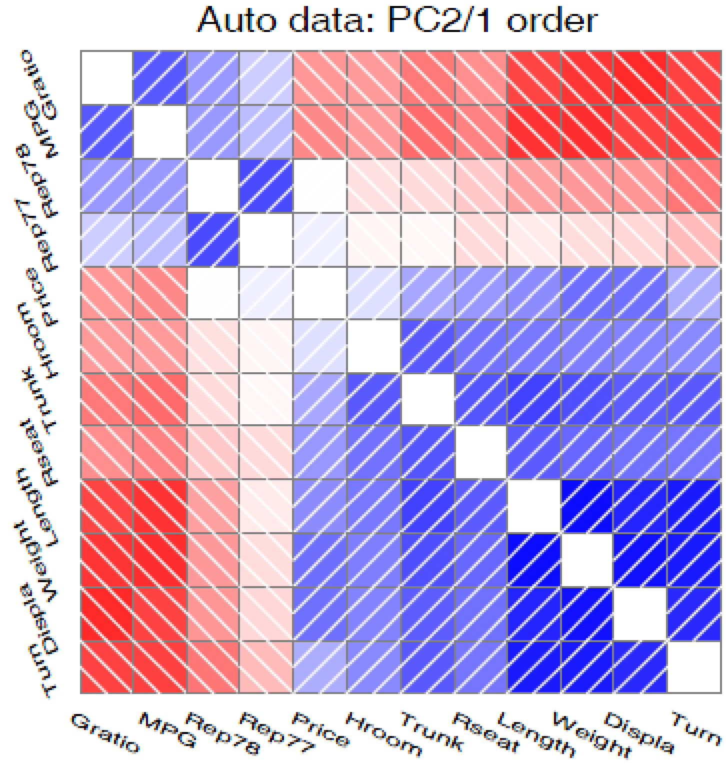
Effect ordering

- Information presentation is always ordered
  - in time or sequence (a talk or written paper)
  - in space (table or graph)
  - Constraints of time & space are dominant– can conceal or reveal the important message
- Effect ordering for data display
  - Sort the data by the effects to be seen
  - Order the data to facilitate the task at hand
    - lookup – find a value
    - comparison – which is greater?
    - detection – find patterns, trends, anomalies

Effect Ordering: Correlations

- **Effect ordering** (Friendly and Kwan, 2003)— In tables and graphs, sort unordered factors according to the effects you want to see/show.





Friendly & Kwan (2003). [Corrgrams: Exploratory displays for correlation matrices](). *American Statistician*, **54**(4): 316-324.

Tabular displays: Main effect ordering

- Tables are often presented with rows/cols ordered alphabetically
  - good for lookup
  - bad for seeing patterns, trends, anomalies

Table 1: Average Barley Yields (rounded), Means by Site and Variety

Variety	Site						*Mean*
	Crookston	Duluth	Grand Rapids	Morris	University Farm	Waseca	
Glabron	32	28	22	32	40	46	33.3
Manchuria	36	26	28	31	27	41	31.5
No. 457	40	28	26	36	35	50	35.8
No. 462	40	25	22	39	31	55	35.4
No. 475	38	30	17	33	27	44	31.8
Peatland	33	32	31	37	30	42	34.2
Svansota	31	24	23	30	31	43	30.4
Trebi	44	32	25	45	33	57	39.4
Velvet	37	24	28	32	33	44	33.1
Wisconsin No. 38	43	30	28	38	39	58	39.4
Mean	37.4	28.0	24.9	35.4	32.7	48.1	34.4

Tabular displays: Main effect ordering

- Better: sort rows/cols by means/medians
- Shade cells according to residual from additive model

Table 2: Average Barley Yields, sorted by Mean, shaded by residual from the model `Yield = Variety + Site`

Variety	Site: Grand Rapids	Duluth	University Farm	Morris	Crookston	Waseca	*Mean*
Svansota	23	24	31	30	31	43	30.4
Manchuria	28	26	27	31	36	41	31.5
No. 475	17	30	27	33	38	44	31.8
Velvet	28	24	33	32	37	44	33.1
Glabron	22	28	40	32	32	46	33.3
Peatland	31	32	30	37	33	42	34.2
No. 462	22	25	31	39	40	55	35.4
No. 457	26	28	35	36	40	50	35.8
Wisconsin No. 38	28	30	39	38	43	58	39.4
Trebi	25	32	33	45	44	57	39.4
Mean	24.9	28.0	32.7	35.4	37.4	48.1	34.4

Effect ordering: Frequency tables

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Alpha ordered

Eye color	Hair color: Blond	Black	Brown	Red
Blue	94	20	17	84
Brown	7	68	26	119
Green	10	15	14	54
Hazel	16	5	14	29

Model:	*Independence*: [Hair][Eye] χ^2 (9)= 138.29						
Color coding:	<-4	<-2	<-1	0	>1	>2	>4
n in each cell:	$n <$ expected				$n >$ expected		

There is an association, but it is hard to see the general pattern

Effect ordering: Frequency tables

- Effect ordering and high-lighting for tables

Table: Hair color - Eye color data: Effect ordered

Eye color	Hair color: Black	Brown	Red	Blond
Brown	68	119	26	7
Hazel	15	54	14	10
Green	5	29	14	16
Blue	20	84	17	94

Model:	*Independence*: [Hair][Eye] χ^2 (9)= 138.29						
Color coding:	$<$-4	$<$-2	$<$-1	0	$>$1	$>$2	$>$4
n in each cell:	$n <$ expected				$n >$ expected		

The pattern is clearer when the eye colors are permuted: light hair goes with light eyes & vice-versa

Sometimes, don’t need numbers at all

COVID transmission risk ~ Occupancy * Ventilation * Activity * Mask? * Contact.time

A complex 5-way table, whose message is clearly shown w/o numbers

A semi-graphic table shows the patterns in the data

There are 1+ unusual cells here. Can you see them?

From: N.R. Jones et-al (2020). Two metres or one: what is the evidence for physical distancing in covid-19? *BMJ* 2020;370:m3223, *doi: https://doi.org/10.1136/bmj.m3223*

Visual table ideas: Heatmap shading

Heatmap shading: Shade the background of each cell according to some criterion

The trends in the US and Canada are made obvious

NB: Table rows are sorted by Jan. value, lending coherence

Background shading ~ value:
US & Canada are made to stand out.

Tech note: use light text on a darker background

Unemployment rate in selected countries

January-August 2020, sorted by the unemployment rate in January.

country	Jan ▲	Feb	Mar	Apr	May	Jun	Jul	Aug
Japan	2.4%	2.4%	2.5%	2.6%	2.9%	2.8%	2.9%	3.0%
Netherlands	3.0%	2.9%	2.9%	3.4%	3.6%	4.3%	4.5%	4.6%
Germany	3.4%	3.6%	3.8%	4.0%	4.2%	4.3%	4.4%	4.4%
Mexico	3.6%	3.6%	3.2%	4.8%	4.3%	5.4%	5.2%	5.0%
US	**3.6%**	**3.5%**	**4.4%**	**14.7%**	**13.3%**	**11.1%**	**10.2%**	**8.4%**
South Korea	4.0%	3.3%	3.8%	3.8%	4.5%	4.3%	4.2%	3.2%
Denmark	4.9%	4.9%	4.8%	4.9%	5.5%	6.0%	6.3%	6.1%
Belgium	5.1%	5.0%	5.0%	5.1%	5.0%	5.0%	5.0%	5.1%
Australia	5.3%	5.1%	5.2%	6.4%	7.1%	7.4%	7.5%	6.8%
Canada	**5.5%**	**5.6%**	**7.8%**	**13.0%**	**13.7%**	**12.3%**	**10.9%**	**10.2%**
Finland	6.8%	6.9%	7.0%	7.3%	7.5%	7.8%	8.0%	8.1%

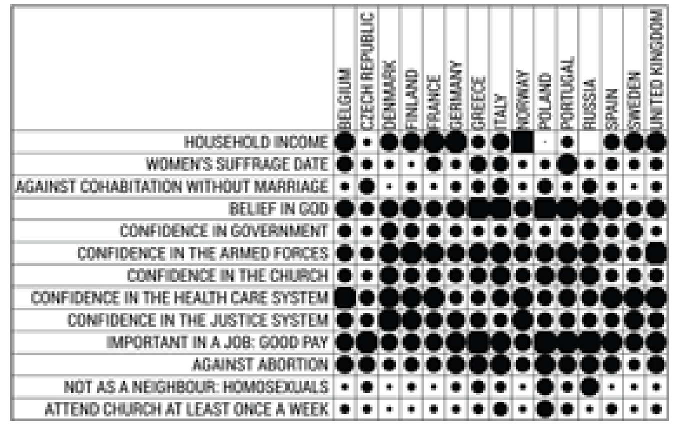
Source: OECD • Get the data • Created with Datawrapper

Bertifier: Turning tables into graphs

a attitudes & attributes

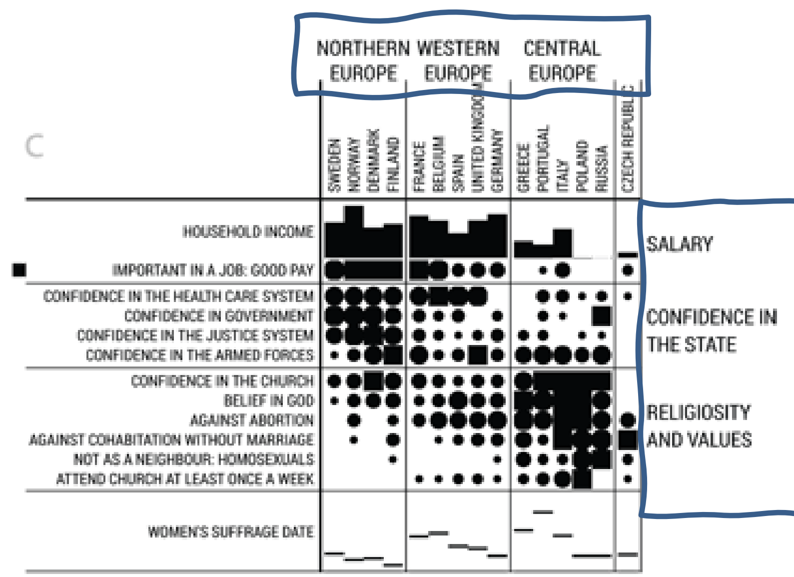
	Belgi	Czech	Denr	Finla	Fran	Gerr	Gree	Ita	Norv	Pola	Port	Russ	Sp	Swe	United
Household income	2687	16957	2468	2573	2831	2879	2044	24	3145	1537	1936	1528	22	2624	26904
Women's suffrage dat	1948	1920	1915	1906	1944	1918	1952	19	1913	1918	1976	1918	19	1921	1928
Against cohabitation w	12	42	4	18	8	20	30	46	12	39	17	39	16	6	19
Belief in God	61	36	63	69	52	63	93	91	56	96	86	77	76	46	65
Confidence in Goverm	32	21	55	42	34	29	22	28	51	23	30	60	35	54	19
Confidence in the arm	50	34	72	83	73	58	70	75	57	63	75	73	57	41	89
Confidence in the chur	36	20	63	47	41	40	52	67	44	65	67	67	31	39	36
Confidence in the heal	91	42	75	73	78	34	39	54	74	44	58	51	79	75	80
Confidence in the justi	50	35	87	73	56	58	50	36	78	44	48	41	42	69	51
Important in a job: goo	60	85	54	58	58	73	94	76	56	93	88	93	77	62	75
Against abortion	56	51	28	40	44	60	65	72	42	75	61	63	57	25	57
Not as a neighbour: ho	7	22	5	12	5	16	30	21	6	52	21	61	5	7	10
Attend church at least	15	13	5	7	11	12	19	35	9	54	25	8	21	9	17

b encode values by size & shape



(a) Table: attitudes and attributes by country
(b) Visual: encode values by size, shape
(c) Sort & group by themes, country regions

c



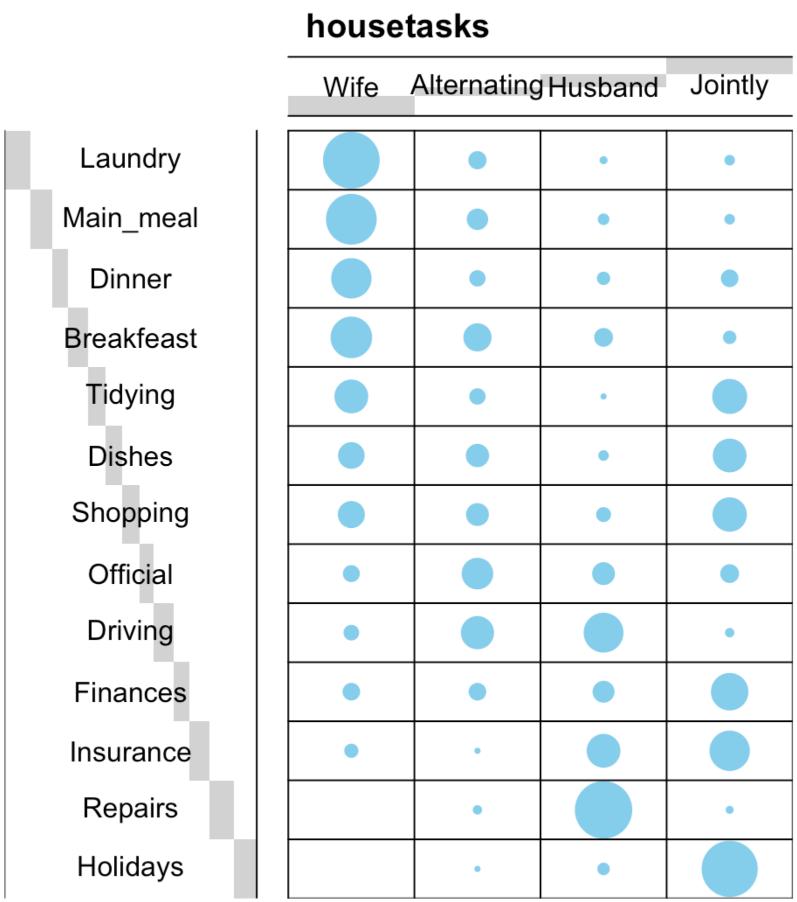
Bertifier: Bertin's reorderable matrix
See: http://www.aviz.fr/bertifier

Example: Household tasks

Who does what in households?

Size of symbols in a balloon plot shows the frequencies

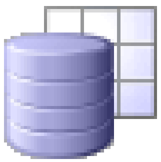
	Who_does_it?			
	Alternating	Husband	Jointly	Wife
Breakfeast	36	15	7	82
Dinner	11	7	13	77
Dishes	24	4	53	32
Driving	51	75	3	10
Finances	13	21	66	13
Holidays	1	6	153	0
Insurance	1	53	77	8
Laundry	14	2	4	156
Main_meal	20	5	4	124
Official	46	23	15	12
Repairs	3	160	2	0
Shopping	23	9	55	33
Tidying	11	1	57	53



Rows and columns were permuted to show the relationship more clearly

Data, pictures, models & stories

Goal: Tell a credible story about some real data problem



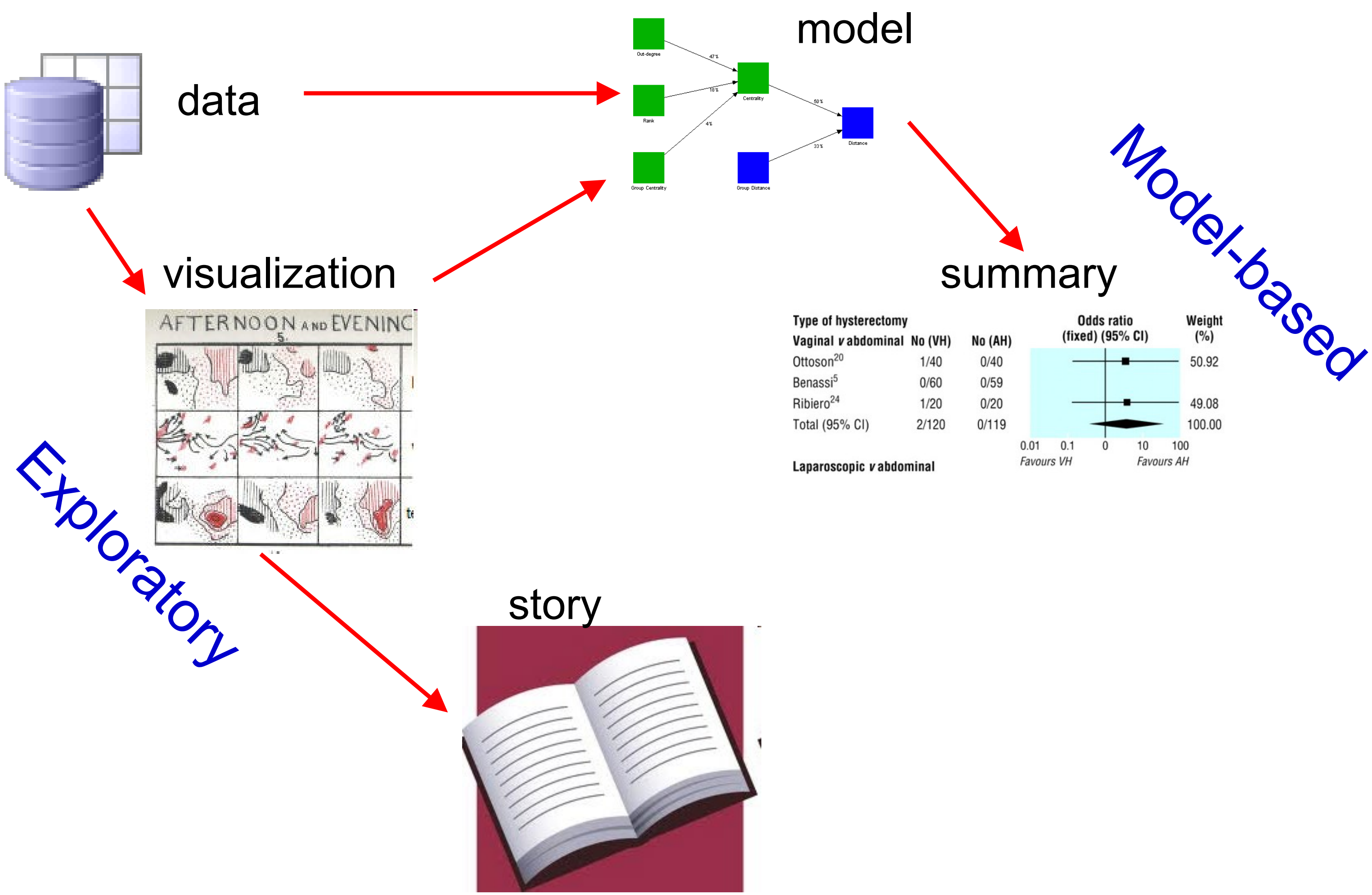
data

Gender bias
Measles vaccination
Global warming
...

story

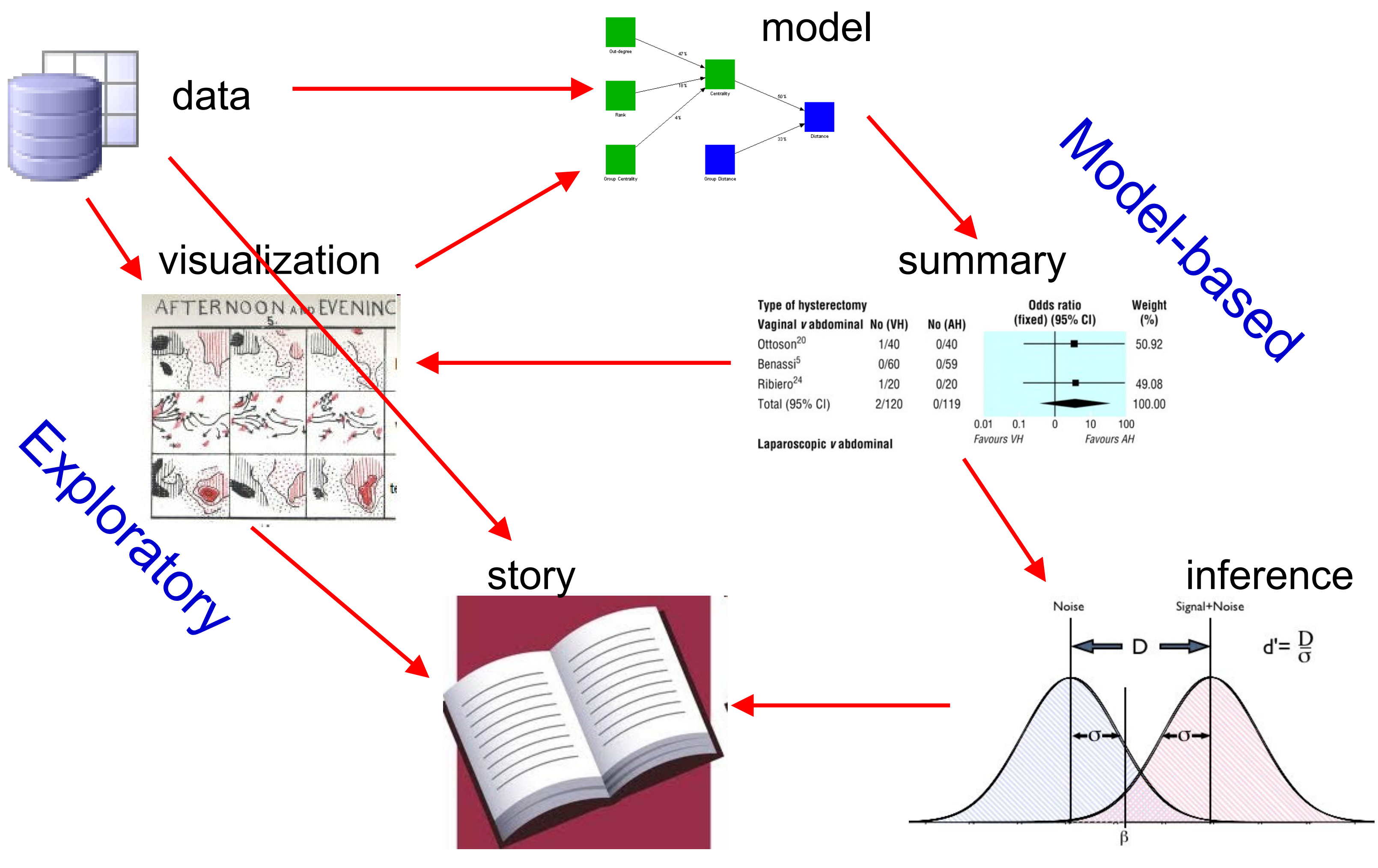
Data, pictures, models & stories

Two paths to enlightenment



Data, pictures, models & stories

Now, tell the story!



Gender Bias at UC Berkeley?

Science, 1975, **187:** 398--403

Sex Bias in Graduate Admissions: Data from Berkeley

Measuring bias is harder than is usually assumed, and the evidence is sometimes contrary to expectation.

P. J. Bickel, E. A. Hammel, J. W. O'Connell

Determining whether discrimination because of sex or ethnic identity is being practiced against persons seeking passage from one social status or locus to another is an important problem in our society today. It is legally impor-

decision to admit or to deny admission. The question we wish to pursue is whether the decision to admit or to deny was influenced by the sex of the applicant. We cannot know with any certainty the influences on the evaluators in the

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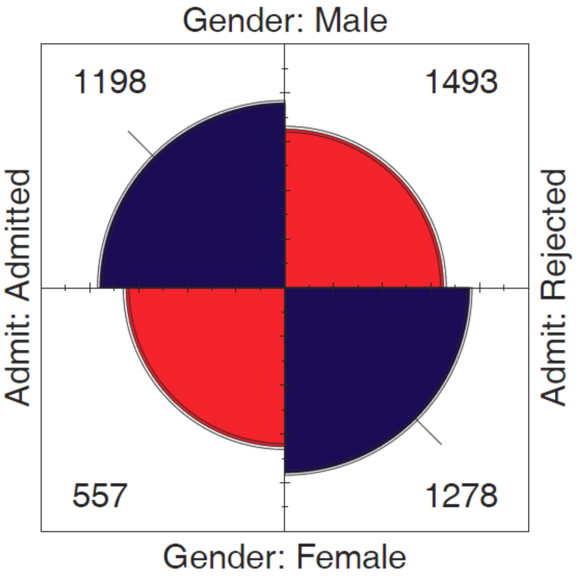
2 × 2 Frequency Tables: Fourfold displays

Table: Admissions to Berkeley graduate programs

	Admitted	Rejected	Total	% Admit	Odds(Admit)
Males	1198	1493	2691	44.52	0.802
Females	557	1278	1835	30.35	0.437
Total	1755	2771	4526	38.78	0.633

odds ratio (θ) = 1.84



Males nearly twice as likely to be admitted

- Is this a "significant" association?
- Is it evidence for gender bias?
- How to measure strength of association?
- How to visualize?

Fourfold display:

- quarter circles, area ~ frequency
- ratio of areas: odds ratio (θ)
- confidence bands: overlap iff $\theta \approx 1$
- visualize significance!

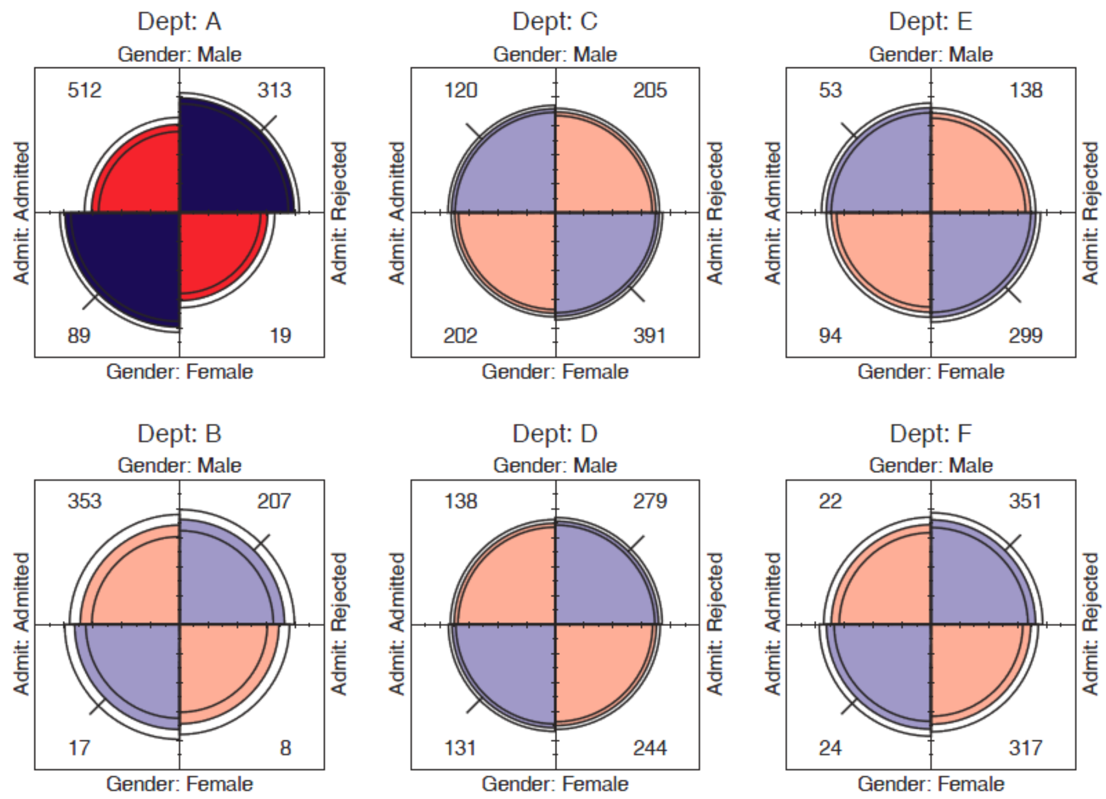
2 × 2 × k Stratified tables

The data arose from 6 graduate departments

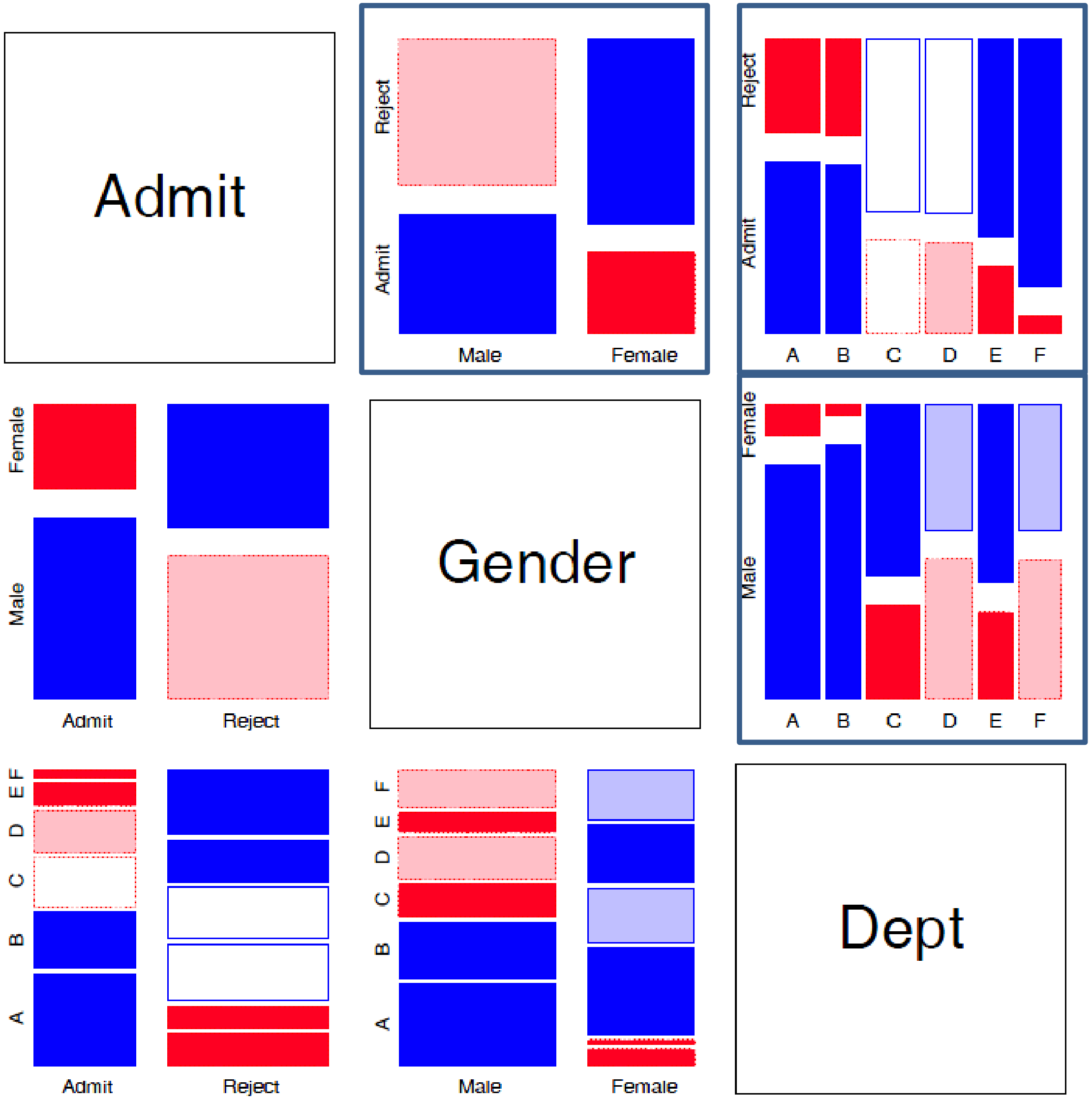
No difference between males & females, except in Dept A where women more likely to be admitted!

Design:

- small multiples
- encode direction by color
- encode signif. by shading



Mosaic matrices



Scatterplot matrix analog for categorical data

All pairwise views
Small multiples → comparison

The answer: Simpson's Paradox

- Depts A, B were easiest
- Applicants to A, B mostly male
- ∴ Males more likely to be admitted overall

Measures & models

If the focus is on the association between gender and admission for each department the odds ratio: odds(Admit|Male) / odds(Admit|Female) is a good summary

odds = Pr(Admit) / Pr (Reject)
OR = odds(Admit|M) / odds(Admit|F)

OR = 1 → M/F equally likely admitted

LOR = **log(OR): 0** → M/F equally likely admitted

Std errors & CIs provide individual signif tests

Models provide a comprehensive summary

Now, we can tell the story !

Graphical methods for categorical data

These share similar ideas & scope with methods for quantitative data

Exploratory methods

- Minimal assumptions (like non-parametric methods)
- Show the *data*, not just *summaries*
- But can add summaries: smoothed curve(s), trend lines, ...
- Help detect *patterns, trends, anomalies*, suggest hypotheses

Plots for model-based methods

- Residual plots - departures from model, omitted terms, ...
- Effect plots - estimated probabilities of response or log odds
- Diagnostic plots - influence, violation of assumptions

Plots: Data, Model, Data+Model

- **Data plots**: well-known. Help to answer:
  - What do the data look like?
  - Are there unusual features? (outliers, non-linear relations)
  - What kinds of summaries would be useful?
- **Model plots**
  - What does the model look like? (plot predicted values)
  - How does the model change when parameters change? (plot competing models)
  - How does the model change when the data is changed? (influence plots)
- **Data+Model plots**
  - How well does model fit the data? (focus on residuals)
  - Does model fit uniformly good/bad, or just in some regions?
  - Model uncertainty: show confidence regions
  - Data support: where is data too thin to make a difference?

Summary

- Categorical data involves some new ideas
  - Discrete variables: unordered or ordered
  - Counts, frequencies as outcomes
- New / different data structures & functions
  - tables – 1-way, 2-way, 3-way, … `table()`, `xtabs()`
  - similar in matrices or arrays `matrix()`, `array()`
  - datasets:
    - frequency form
    - case form
- Graphical methods: often use area ~ Freq
  - Consider: graphical comparisons, effect order
- Models: Most are ≅ natural extensions of lm()