Calculation of momentum velocity on collision of two masses

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In order to calculate the momentum between two colliding masses it is necessary to determine the partial velocity vector of a moving mass in direction to the second mass. Let's assume a mass m_1 moves with velocity $\vec{v}$ and would hit mass m_2. For simplicity m_2 remains stationary and the shapes of both masses are spherical. Even when the direction of $\vec{v}$ does not point directly to m_2, it behaves as if m_1 hits m_2 with a (partial) velocity $\vec{v_{m_2}}$ which points in the direction to m_2. We keep the calculation in two dimensional space. It should be easy to extend it to three dimensions.

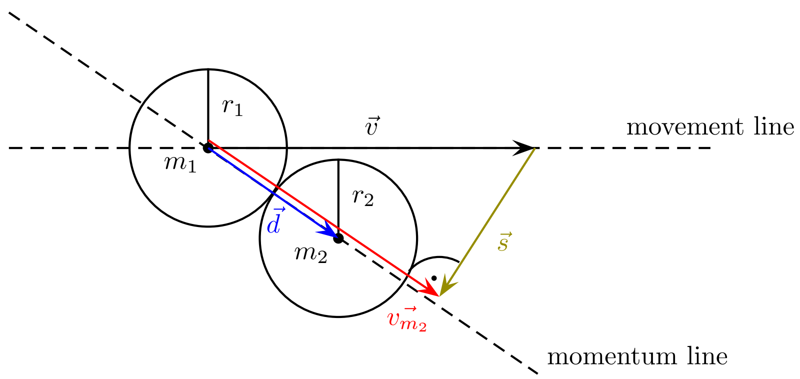

Figure 1: collision

We need to calculate $\vec{v_{m_2}}$. We define $\vec{d} = \vec{m_2} - \vec{m_1}$, where $\vec{m_1}$ and $\vec{m_2}$ are the positional vectors for m_1 resp. m_2. We require that the masses have a positive expansion ($r_1 > 0$, $r_2 > 0$), thus $\vec{d} \neq \vec{0}$ and $d_x \neq 0 \vee d_y \neq 0$ (collision takes place if $|\vec{d}| = r_1 + r_2$). Let $\vec{s}$ be a (the) vector with $\vec{v_{m_2}} = \vec{v} + \vec{s}$. Then $\vec{s}$ must be orthogonal to $\vec{v_{m_2}}$ and thus to $\vec{d}$. That is because $\vec{v_{m_2}}$ is a partial vector of $\vec{v}$ and points in the same direction as $\vec{d}$ which resides on the momentum line. See figure 1.

Be $\lambda \in \mathbb{R}$, then the following equations hold:

$$\vec{v_{m_2}} = \vec{v} + \vec{s} = \lambda \vec{d}$$
$$\vec{s} \cdot \vec{d} = 0$$

We end up with three equations and three unkown variables (s_x, s_y, λ), which can easily be solved:

$$v_x + s_x = \lambda d_x \tag{1}$$
$$v_y + s_y = \lambda d_y \tag{2}$$
$$s_x d_x + s_y d_y = 0 \tag{3}$$

case: $\boxed{d_x \neq 0}$

$$(3) <=> s_x = -\frac{s_y d_y}{d_x} \tag{4}$$
$$(2) <=> s_y = \lambda d_y - v_y \tag{5}$$
$$=> s_x = -\frac{(\lambda d_y - v_y) d_y}{d_x} \tag{6}$$
$$in\ \ (1) : v_x - \frac{(\lambda d_y - v_y) d_y}{d_x} = \lambda d_x \tag{7}$$
$$<=> v_x - \frac{\lambda d_y^2}{d_x} + \frac{v_y d_y}{d_x} = \lambda d_x \tag{8}$$
$$<=> v_x + \frac{v_y d_y}{d_x} = \lambda (d_x + \frac{d_y^2}{d_x}) \tag{9}$$
$$<=> \frac{v_x d_x}{d_x} + \frac{v_y d_y}{d_x} = \lambda (\frac{d_x^2 + d_y^2}{d_x}) \tag{10}$$
$$<=> \frac{(v_x d_x + v_y d_y) d_x}{d_x (d_x^2 + d_y^2)} = \lambda \tag{11}$$
$$<=> \lambda = \frac{v_x d_x + v_y d_y}{d_x^2 + d_y^2} = \frac{\vec{v}\vec{d}}{\vec{d}^2} \tag{12}$$

case: $\boxed{d_y \neq 0}$

Calculation is analogue. We end up with the same result for λ.

As a result we get:

$$v_{\vec{m_2}} = \frac{\vec{v}\vec{d}}{\vec{d}^2}\vec{d}$$

□