Calculation of momentum velocity on collision of two masses

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April 9, 2017

In order to calculate the momentum between two colliding masses it is necessary to determine the partial velocity vector of a moving mass in direction to the second mass. Let's assume a mass m_1 moves with velocity $\vec{v}$ and would hit mass m_2. For simplicity m_2 remains stationary and the shapes of both masses are spherical. Even when the direction of $\vec{v}$ does not point directly to m_2, it behaves as if m_1 hits m_2 with a (partial) velocity $\vec{v_{m_2}}$ which points to the direction of m_2. The calculation holds for any dimension.

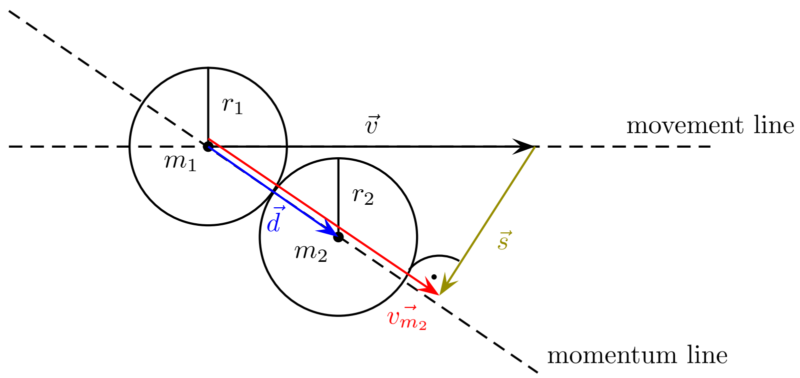

Figure 1: collision

We need to calculate $\vec{v_{m_2}}$. We define $\vec{d} = \vec{m_2} - \vec{m_1}$, where $\vec{m_1}$ and $\vec{m_2}$ are the positional vectors for m_1 resp. m_2. We require that the masses have a positive expansion ($r_1 > 0$, $r_2 > 0$), thus $\vec{d} \neq \vec{0}$ (collision takes place if $|\vec{d}| = r_1 + r_2$). Let $\vec{s}$ be a (the) vector with $\vec{v_{m_2}} = \vec{v} + \vec{s}$. Then $\vec{s}$ must be orthogonal to $\vec{v_{m_2}}$ and thus to $\vec{d}$. That is because $\vec{v_{m_2}}$ is a partial vector of $\vec{v}$ and points in the same direction as $\vec{d}$ which resides on the momentum line. See figure 1.

Be $\lambda \in \mathbb{R}$, then the following equations hold:

$$\vec{v_{m_2}} = \vec{v} + \vec{s} = \lambda\vec{d} \tag{1}$$

$$\vec{s} \cdot \vec{d} = 0 \tag{2}$$

This can easily be solved:

$$(1), (2) => (\lambda\vec{d} - \vec{v})\vec{d} = 0 \tag{3}$$

$$<=> \lambda\vec{d}^2 - \vec{v}\vec{d} = 0 \tag{4}$$

$$<=> \lambda = \frac{\vec{v}\vec{d}}{\vec{d}^2} \tag{5}$$

As a result we get:

$$\vec{v_{m_2}} = \frac{\vec{v}\vec{d}}{\vec{d}^2}\vec{d}$$

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