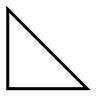
		ΨIJ		
Observation-based metrics				
o Originality	X	×		X
 Uniqueness 	X	X		
 Contribution 		X	×	×
Group-based metrics				
o Richness		X	×	X
 Divergence 	X	X		X
 Regularity 	×	×		X
Between groups metrics				
o Distance	×	×		×
o Overlap		X	X	×



	obs1	obs2	obs3	obs4	obs5	obs6	obs7
obs2	1						
obs3	2	1.5					
obs4	3	2.5	4				
obs5	0.5	2	5	6			
obs6	1	5	0.5	2	1		
obs7	2	2	6	1	1.5	2	
obs8	3	1	2	2	1	3.5	1

A. Observation-based metrics

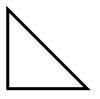
For observation 1

$$Originality_1 = \frac{(1+2+3+0.5+1+2+3)}{7} = 1.78$$

 $Uniqueness_1 = 0.5$

 $Contribution_1 = NA$

For originality, possibility to weight each distance with the relative abondance of the other species of the pair. No need to weight for uniqueness.



	obs1	obs2	obs3	obs4	obs5	obs6	obs7
obs2	1						
obs3	2	1.5					
obs4	3	2.5	4				
obs5	0.5	2	5	6			
obs6	1	5	0.5	2	1		
obs7	2	2	6	1	1.5	2	
obs8	3	1	2	2	1	3.5	1

B. Group-based metrics

Mean Pairwise distance (MPD)

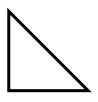
Richness =
$$NA$$

Divergence = $\frac{(1+2+1.5+3+2.5+4)}{6} = 2.33$

Regularity = FEve See details in Villeger et al. 2018

Possibility to weight each distance with the product of the relative abondance of the two species.

Indices RaoQ and FDis are also available for divergence but all are highly correlated between each others. See also Hill numbers for similar indices with different weights for abundance.



	obs1	obs2	obs3	obs4	obs5	obs6	obs7
obs2	1						
obs3	2	1.5					
obs4	3	2.5	4				
obs5	0.5	2	5	6			
obs6	1	5	0.5	2	1		
obs7	2	2	6	1	1.5	2	
obs8	3	1	2	2	1	3.5	1

C. Between groups metrics

The distance between the red and green is the mean of the pairwise distance calculated between the species of the two communities.

$$Distance_{G,R} = \frac{(0.5 + 2 + 5 + 6 + 1 + \dots)}{16} = 2.56$$

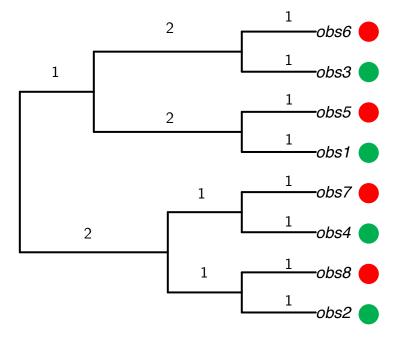
$$Beta Jaccard_{G,R} = NA$$

Possibility to weight each distance with the product of the relative abondance of the two species.

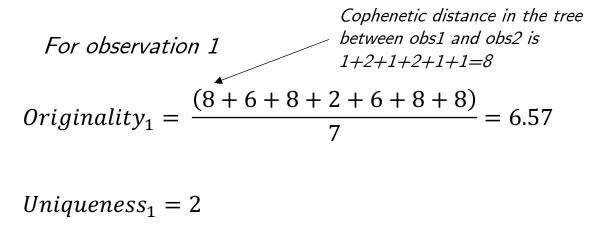
Index RaoQ for Beta diversity is also available for distance but it is highly correlated with the one above.



! Distances in a tree are called cophenetic distances



A. Observation-based metrics

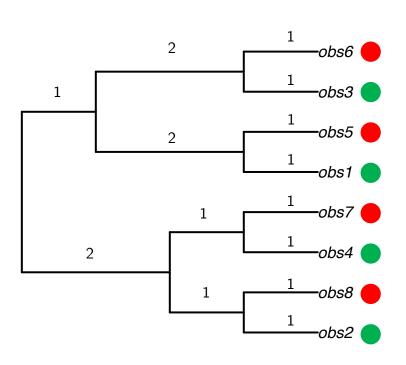


$$Contribution_1 = 1 + \frac{2}{2} + \frac{1}{4} = 2.25$$
 Branch length divided by the number of species sharing the branch

Possibility to weight each distance with the product of the relative abondance of the two species.

Sum of the contribution is equal to the sum of all the branch lengths in the tree. Possibility to weight each contribution with the relative abondance of the species.





B. Group-based metrics

Sum of the branch lengths (Petchey & Gaston 2002)

$$Richness = 1 + 1 + 2 + 2 + 1 + 2 + 1 + 1 + 1 + 1 = 13$$

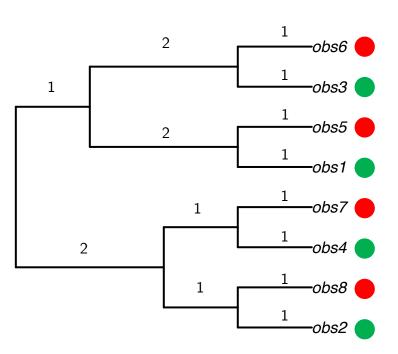
Mean pairwise cophenetic distance

$$Divergence = \frac{(8+6+8+8+4+8)}{6} = 7$$

Possibility to weight each distance with the product of the relative abondance of the two species.

Indices RaoQ with trees is also available for divergence but all are highly correlated between each others. See also Hill numbers for similar indices with different weights for abundance.





B. Group-based metrics

For *evenness*, we need the contribution of the four species in the green community: obs1=3.5; obs2=3; obs3=3.5; obs4=3

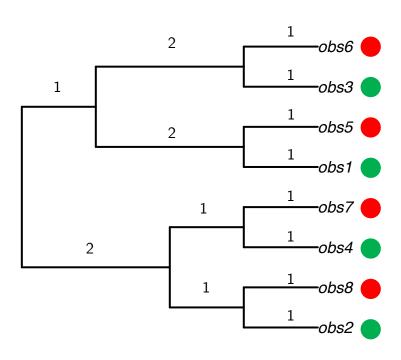
Camargo's formula
$$|2.25-2|+|2.25-2.25|+|2.25-2|+$$

$$Evenness = 1 - \frac{|2-2.25|+|2.25-2.25|+|2.25-2|}{6} = 0.84$$

1-the sum of the absolute difference between pairs of contributions divided by the number of pairs.

Possibility to weight each contribution with the relative abondance of the species.





C. Between groups metrics

As for the dissimilarity matrix, the distance between the red and green is the mean of the pairwise distance calculated between the species of the two communities.

$$Distance_{G,R} = \frac{(2+6+8+8+8+...)}{16} = 5.75$$

Possibility to weight each distance with the product of the relative abondance of the two species.



2 1 obs6 1 1 obs3 1 1 obs5 1 1 obs7 1 1 obs4 1 1 obs8

C. Between groups metrics

$$Beta Jaccard_{G,R} = \frac{b+c}{a+b+c}$$

a = Shared branch lengths between green and red

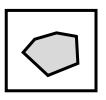
b = Branch lengths found only in green

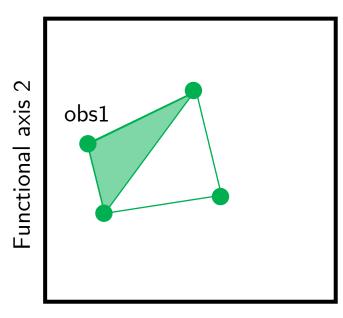
c = Branch lengths found only in red

$$Beta \ Jaccard_{G,R} = \frac{(4+4)}{4+4+9} = 0.47$$

Possibility to weight each branch length with the relative abondance of the species.

obs2





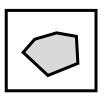
Functional axis 1

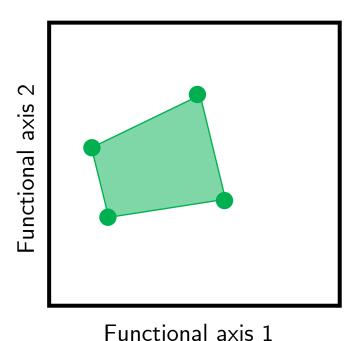
A. Observation-based metrics

$$Originality = NA$$

$$Uniqueness = NA$$

Contribution of each observation to the total surface/volume of a convex hull is calculated as the difference in surface/volume between the total convex hull and a second surface/volume lacking this specific observation.



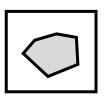


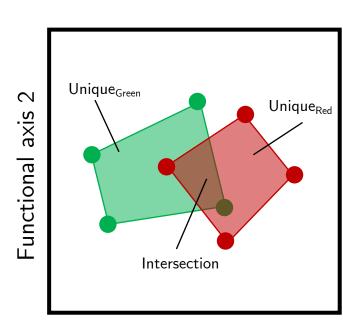
B. Group-based metrics

Richness is the minimum convex hull which includes all the species of the group; Richness is then the surface / volume inside this hull.

Divergence = NA

Evenness = NA





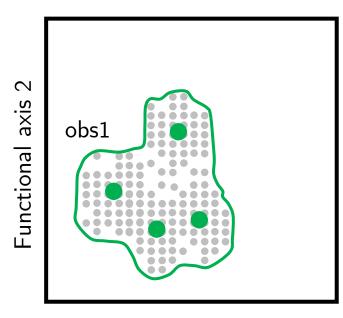
Functional axis 1

C. Between groups metrics

$$Distance_{G,R} = NA$$

$$Beta Jaccard_{G,R} \\ = \frac{Unique_{Green} + Unique_{Red}}{Unique_{Green} + Unique_{Red} + Intersection}$$





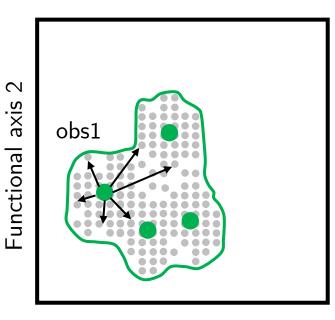
Functional axis 1

Incorporating abundance data:

Each observation can be weighted by replicating it based on its abundance in the estimation of the hypervolume. If Obs1 is replicated 10 times (e.g. 10 individuals for a species), it will appears 10 times during the construction of hypervolume.

All following metrics can be calculated with or without abundance data.





Functional axis 1

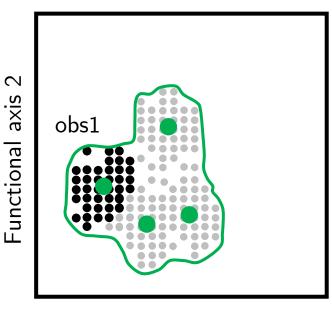
A. Observation-based metrics

For observation 1

Originality₁ is average distance between each observation to a sample of stochastic points within the boundaries of the hypervolume

 $Uniqueness_1 = NA$





Functional axis 1

A. Observation-based metrics

For observation 1

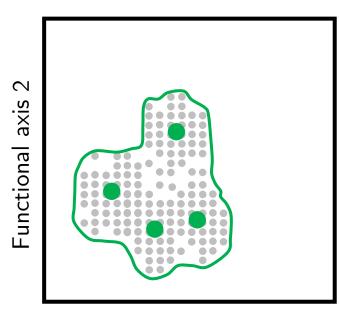
 $Originality_1$ is average distance between each observation to a sample of stochastic points within the boundaries of the hypervolume

 $Uniqueness_1 = NA$

Contribution₁ is measured as the proportion of random points that is closer to obs1 multiplied by the total hypervolume of the group.

Contribution can also be measured using the « leave one out » apporach as for convex-hull.





Functional axis 1

B. Group-based metrics

Richness is the total surface/volume of the functional hyperspace.



Functional axis

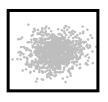
Centroid

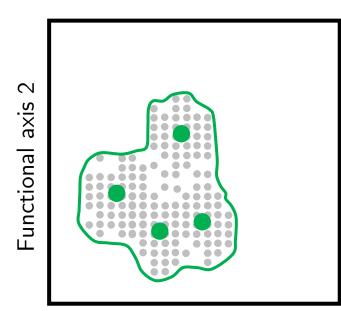
Functional axis 1

B. Group-based metrics

Richness is the total surface/volume of the functional hyperspace.

Divergence is calculated as the average distance between a sample of stochastic points and the hypervolume centroid.





Functional axis 1

B. Group-based metrics

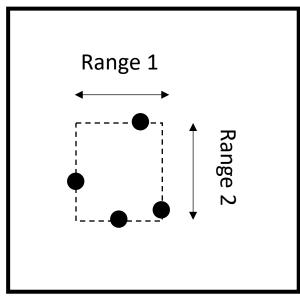
Richness is the total surface/volume of the functional hyperspace.

Divergence is calculated as the average distance between a sample of stochastic points and the hypervolume centroid.

Evenness is calculated as the overlap between the calculated hypervolume and a second, imaginary hypervolume where traits are evenly distributed within their possible range (and abundance evenly distributed between the observations!)



Functional axis 2



Functional axis 1

B. Group-based metrics

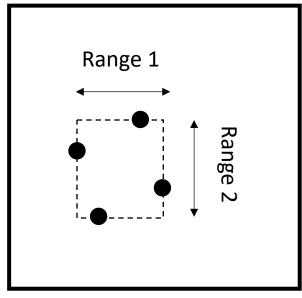
Richness is the total surface/volume of the functional hyperspace.

Divergence is calculated as the average distance between a sample of stochastic points and the hypervolume centroid.

Evenness is calculated as the overlap between the calculated hypervolume and a second, imaginary hypervolume where traits are evenly distributed within their possible range



Functional axis 2



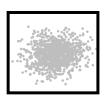
Functional axis 1

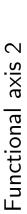
B. Group-based metrics

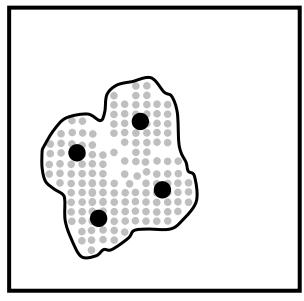
Richness is the total surface/volume of the functional hyperspace.

Divergence is calculated as the average distance between a sample of stochastic points and the hypervolume centroid.

Evenness is calculated as the overlap between the calculated hypervolume and a second, imaginary hypervolume where traits are evenly distributed within their possible range







Functional axis 1

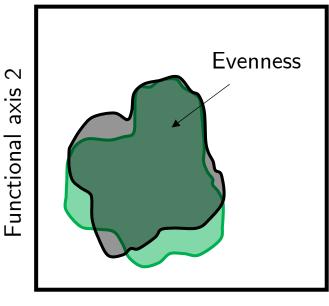
B. Group-based metrics

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Functional axis 1

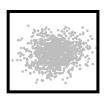
B. Group-based metrics

Richness is the total surface/volume of the functional hyperspace.

Divergence is calculated as the average distance between a sample of stochastic points and the hypervolume centroid.

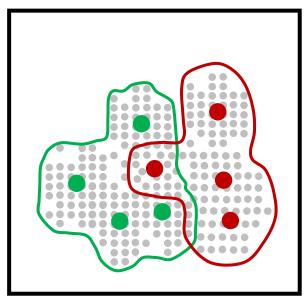
Evenness is calculated as the overlap between the calculated hypervolume and a second, imaginary hypervolume where traits are evenly distributed within their possible range.

If the 2 hypervolumes are the same = overlap is max and evenness is 1.



Functional axis 2

C. Between groups metrics



Functional axis 1



Functional axis 2

Functional axis 1

C. Between groups metrics

Distance = Still the possibility to calculate the distance between the hypervolume centroids as for the dissimilarity-based framework.



Unique axis Company Unique Red

Functional axis 1

C. Between groups metrics

Distance = Still the possibility to calculate the distance between the hypervolume centroids as for the dissimilarity-based framework.

$$Beta Jaccard_{G,R} \\ = \frac{Unique_{Green} + Unique_{Red}}{Unique_{Green} + Unique_{Red} + Intersection}$$