Fourier Optics: Aberrations and Wavefront Sensors

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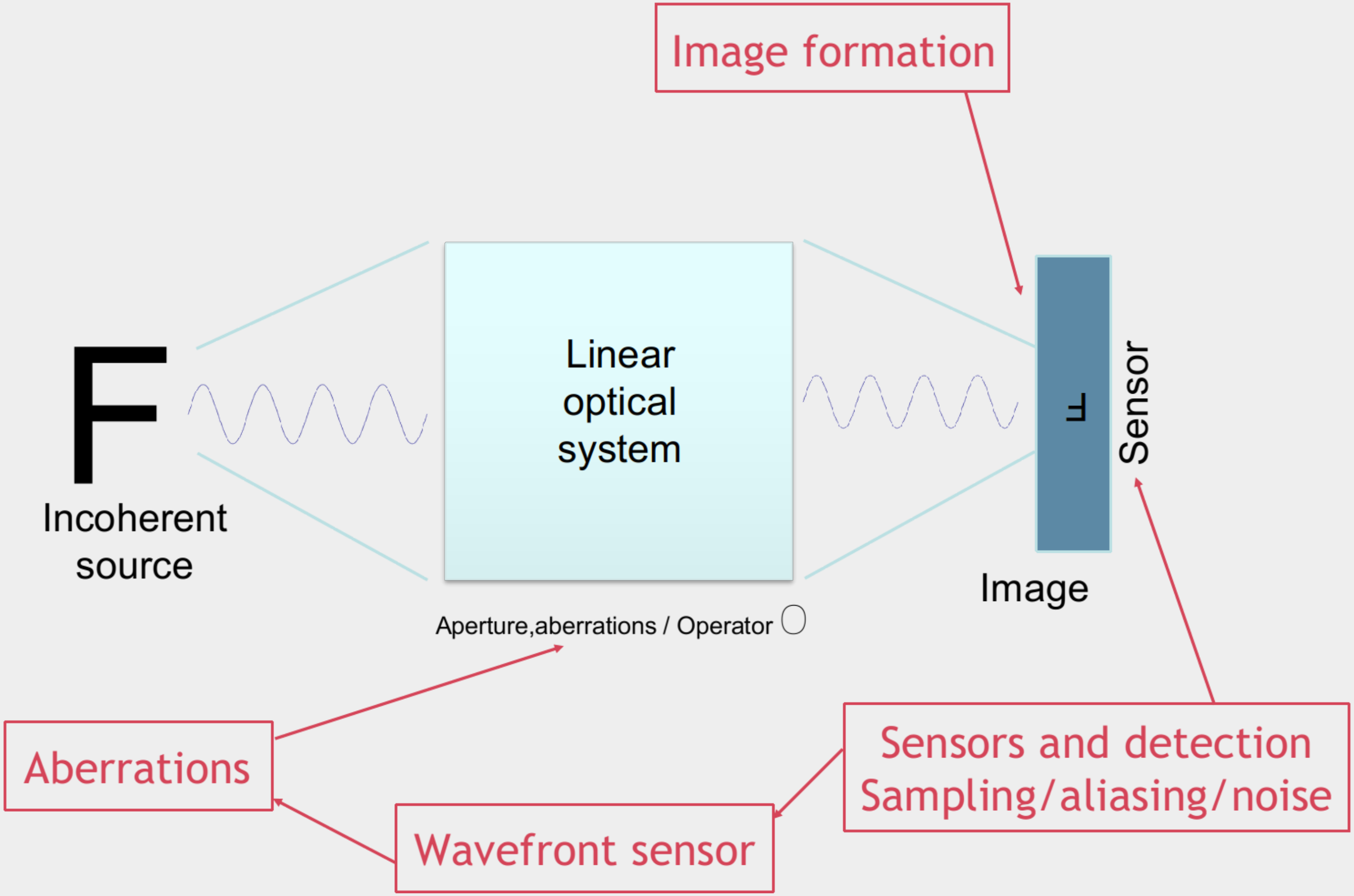
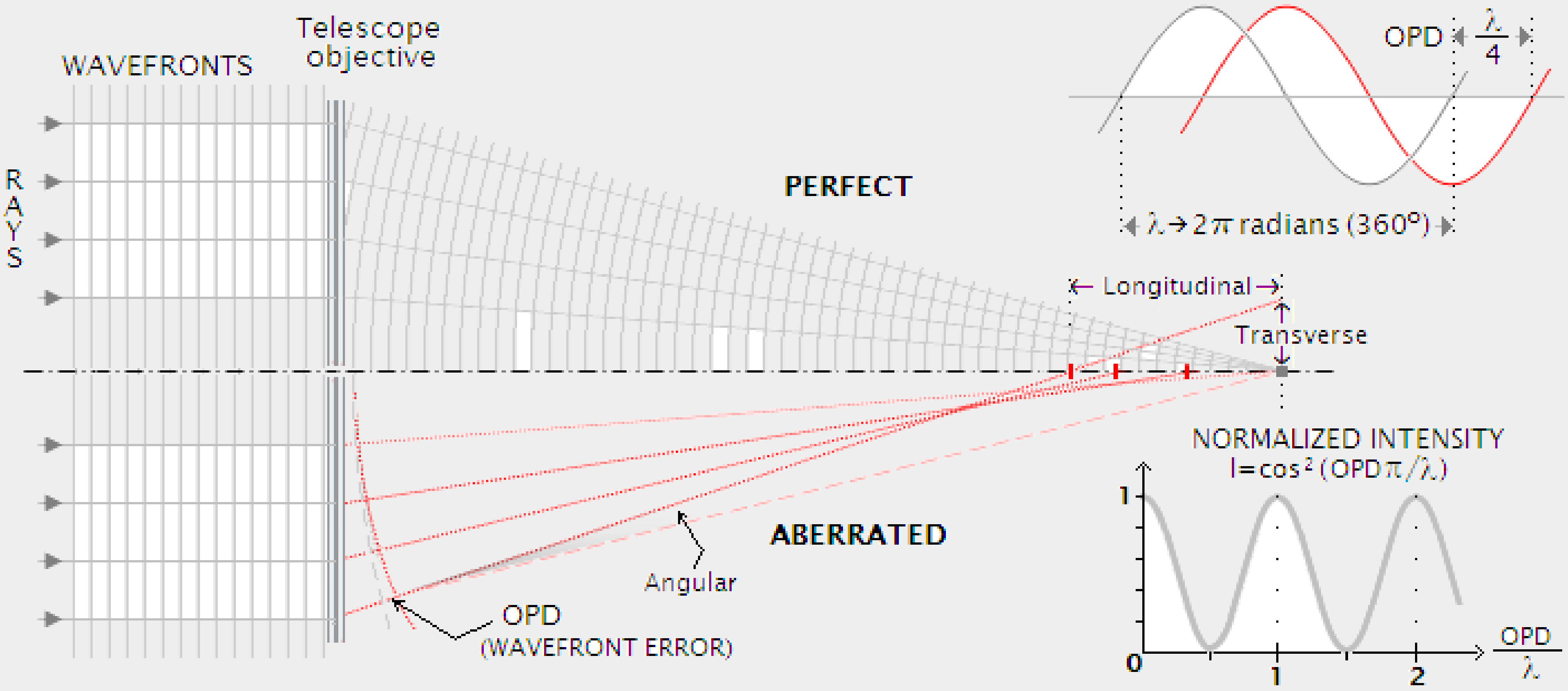
Linear Optical Systems



Phase Aberrations

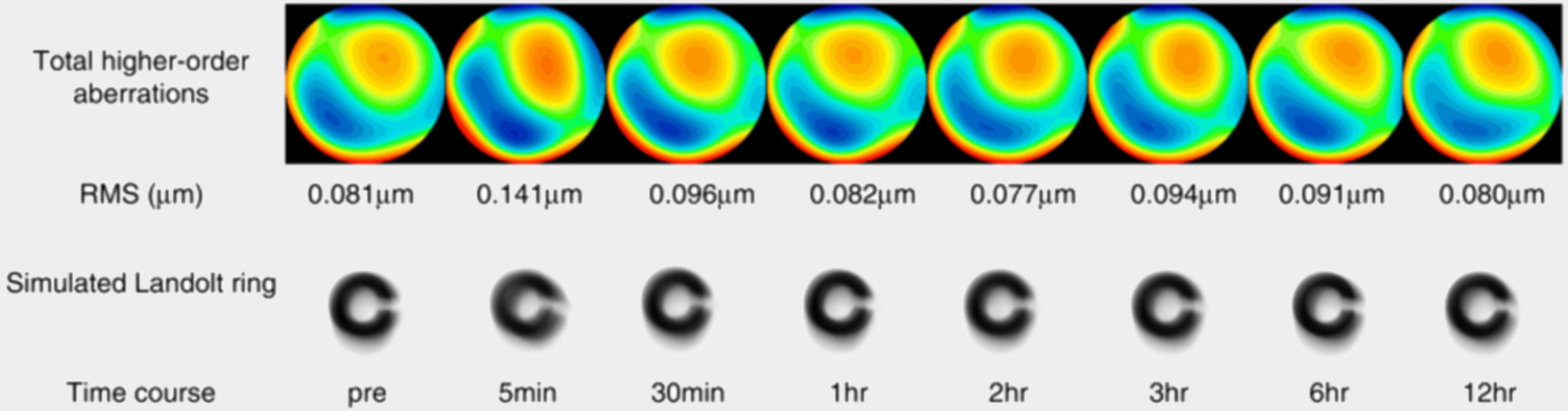
Phase Aberrations - Geometrical view

- Wavefront departs from flatness
- When focused, rays do not intersect at the same location



$$\Psi(x,y,t) = A(x,y,t)e^{i\varphi(x,x,t)}$$

- Now, $\varphi(x,y,t) \neq 0$, thus the PSF H departs from the simple square modulus of the aperture, as presented in previous lectures.
- This yields asymmetry and spread, and a loss of angular resolution as well as:
  - a loss of Strehl ratio
  - further attenuation of spatial frequencies in the OTF



Seidel Aberrations

SEIDEL ABERRATIONS

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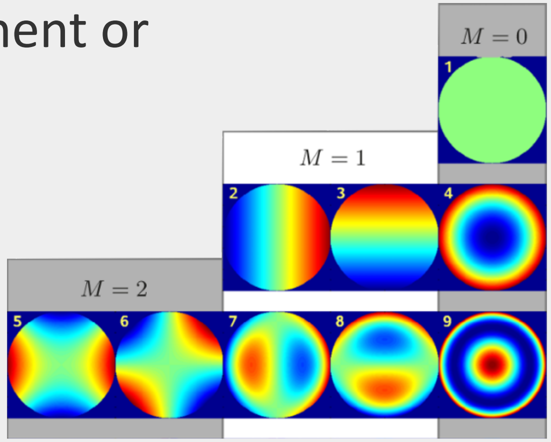
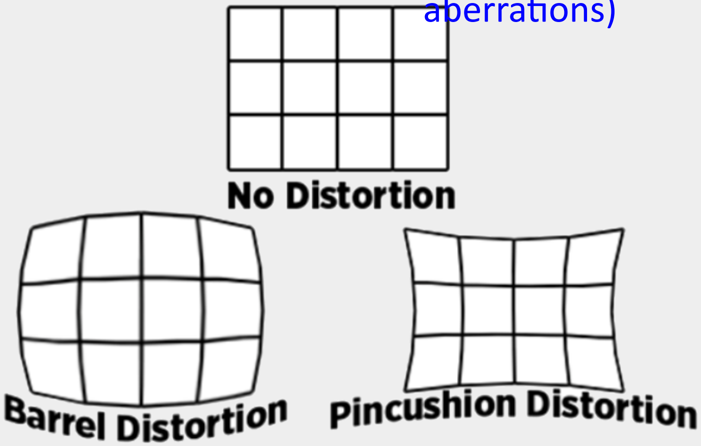
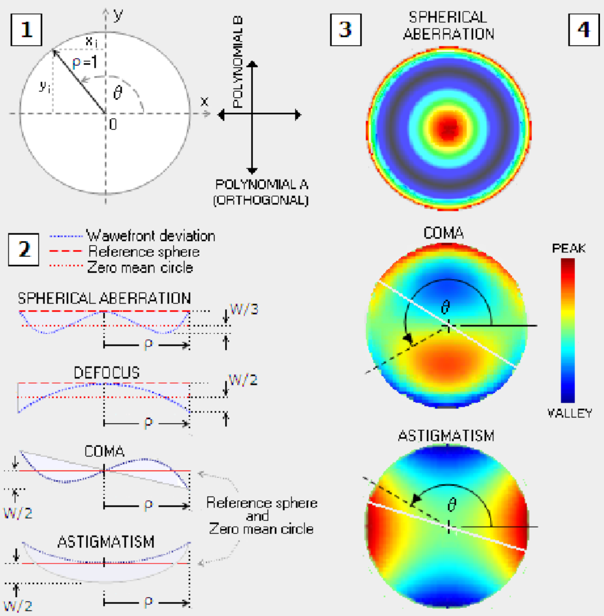
- Primary aberrations from optical system misalignment or manufacturing error
- Seidel aberrations for monochromatic light:
  1. Spherical aberration
  2. Coma
  3. Astigmatism
  4. Curvature of field
  5. Distortion
  6. Chromatic

(Not phase aberrations as described previously in this lecture, i.e. for a single point like object. Those are field dependent or wavelength dependant aberrations)

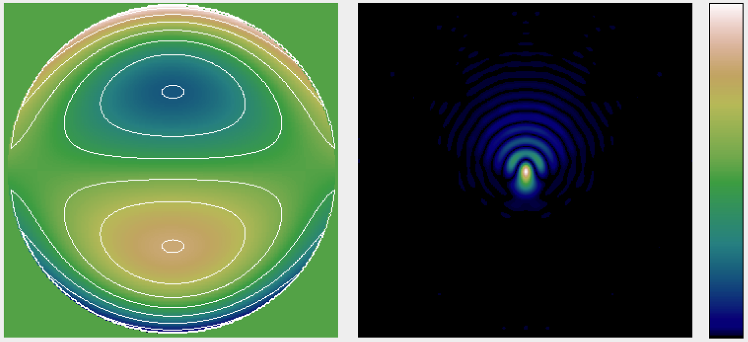


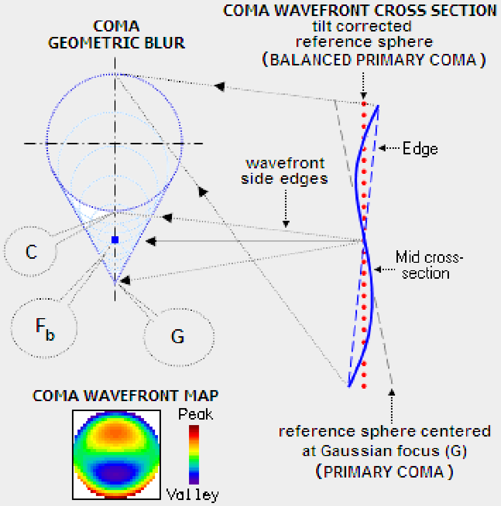


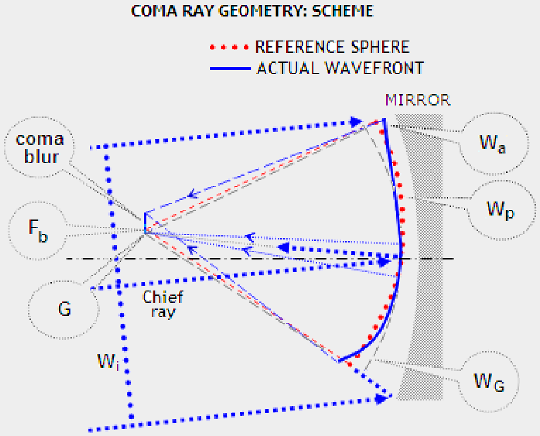


- Also a field aberration in many optical system (i.e. something you get when looking off-axis)
  - "field Aberration": an aberration that varies as a function of position in the output field (field = image plane)
- Characterised by core + tail

Coma (Z7)

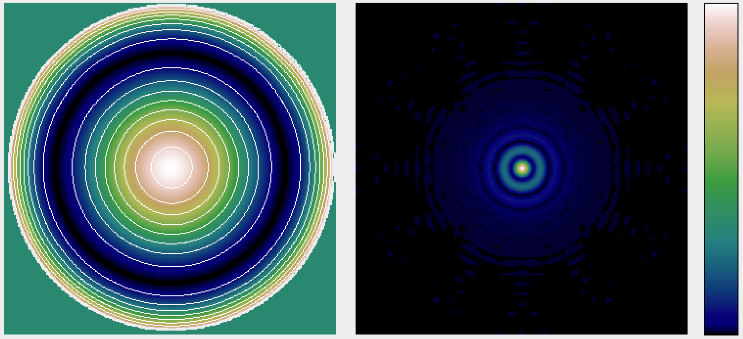


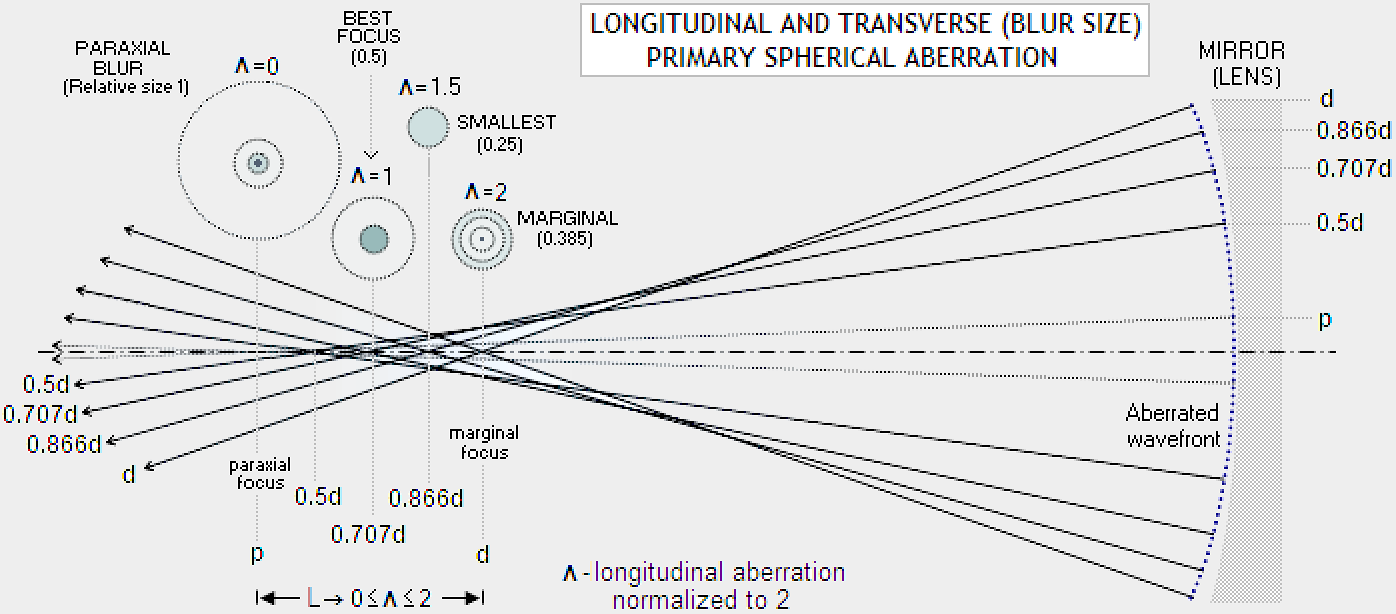




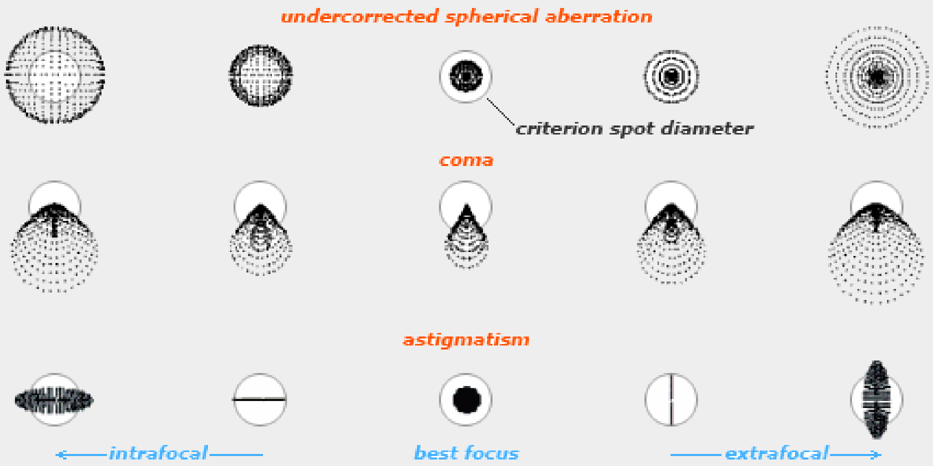
- Often due to polishing error
- The Hubble space telescope is an infamous example

Spherical (Z11)

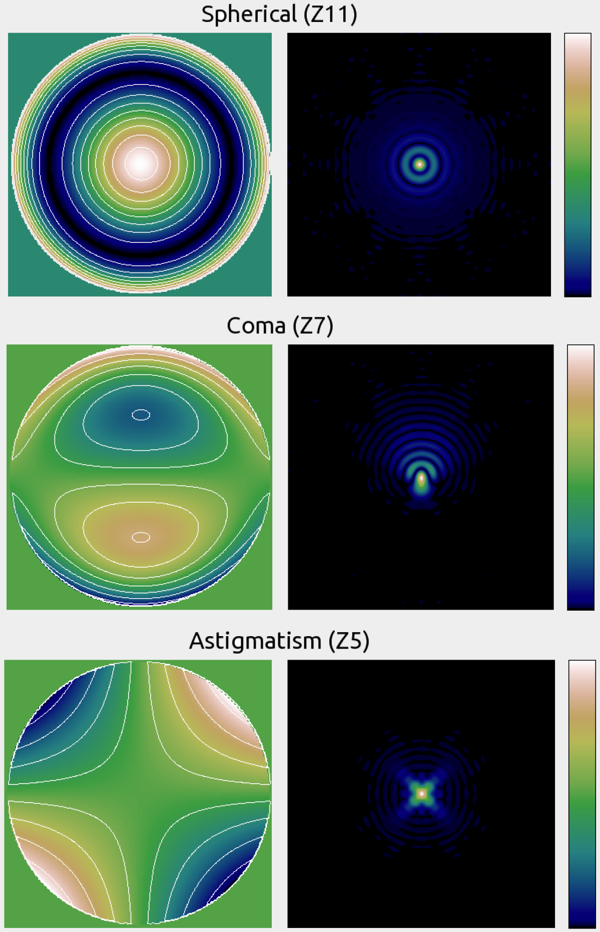




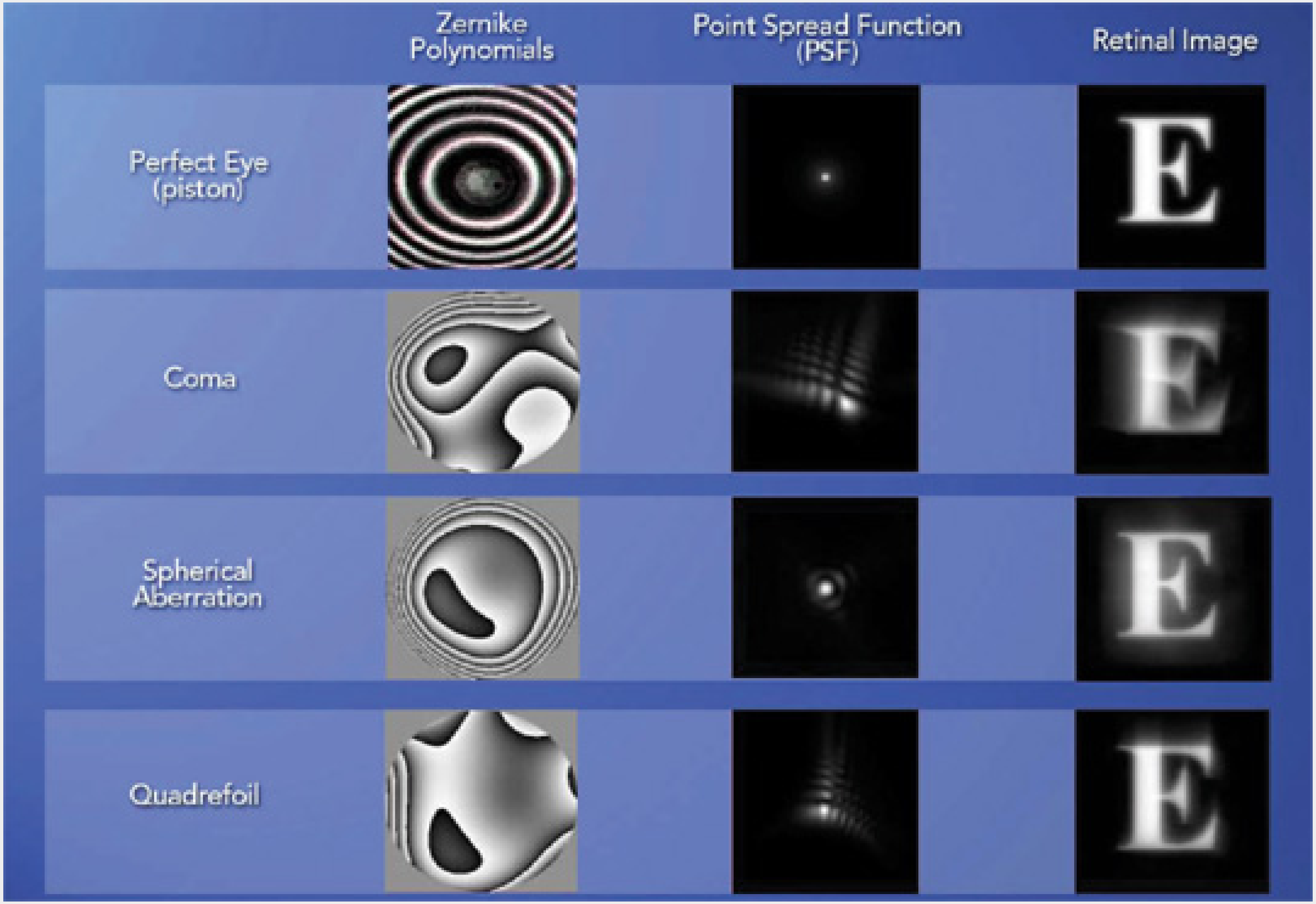
Geometrical (ray) view



Fourier optics view

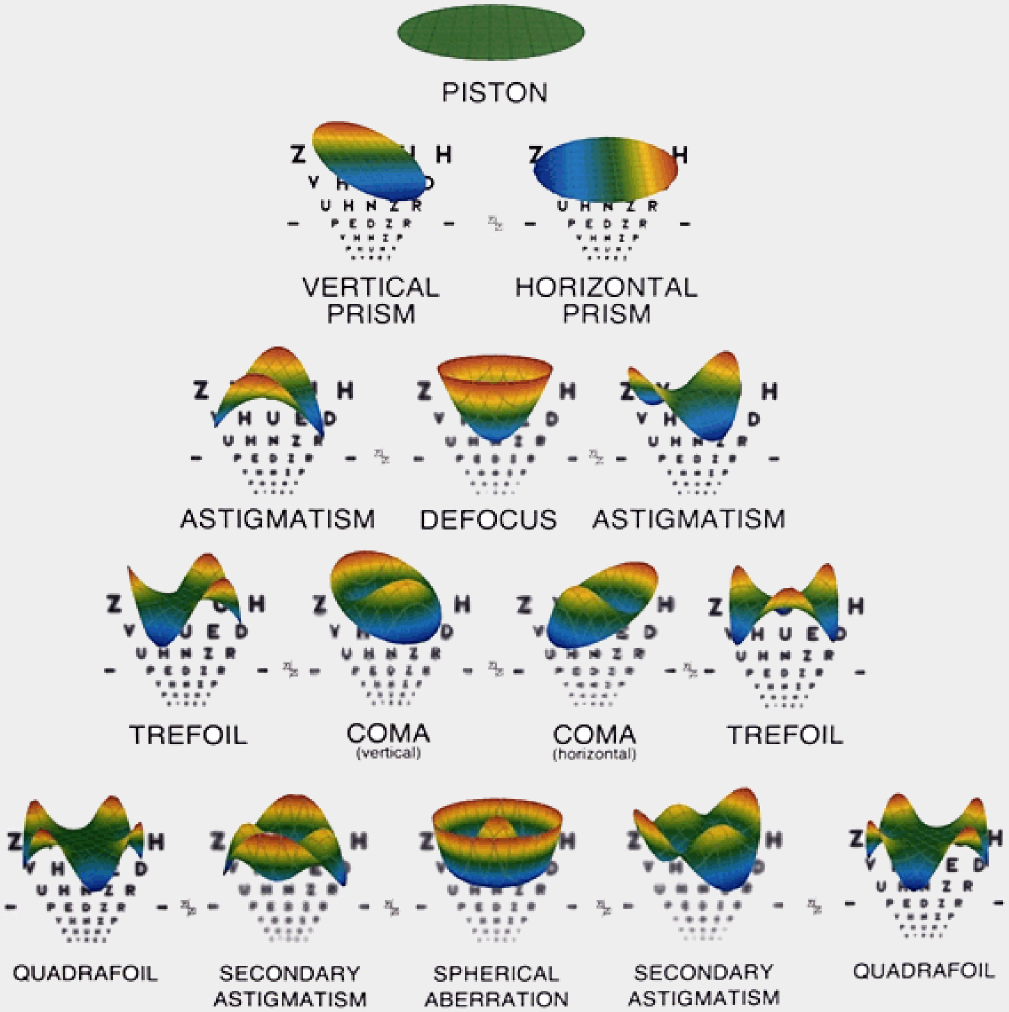


Beyond Seidel: real eye cases



Zernike polynomials to describe phase aberrations

Zernike polynomials/modes



Zernike polynomials: background

- The mathematical functions were originally described by Fritz Zernike in 1934.
- They were developed to describe the diffracted wavefront in phase contrast imaging.
- Zernike won the 1953 Nobel Prize in Physics for developing Phase Contrast Microscopy.

See Noll 1976 for additional information (Journal of the Optical Society of America, vol.66, 1976). There are two kind of Zernike, the even and odd:

$$\left.\begin{aligned} Z_{\text{even j}} &= Z_n^m(\rho,\theta) = \sqrt{2(n+1)}\; R_n^m(\rho)\;\cos(m\theta) \\ Z_{\text{odd j}} &= Z_n^m(\rho,\theta) = \sqrt{2(n+1)}\; R_n^m(\rho)\;\sin(m\theta) \end{aligned}\right\} \text{ for } m \neq 0$$

$$Z = Z_n^0(\rho,\theta) = \sqrt{n+1}\; R_n^0(\rho) \text{ for } m = 0$$

with

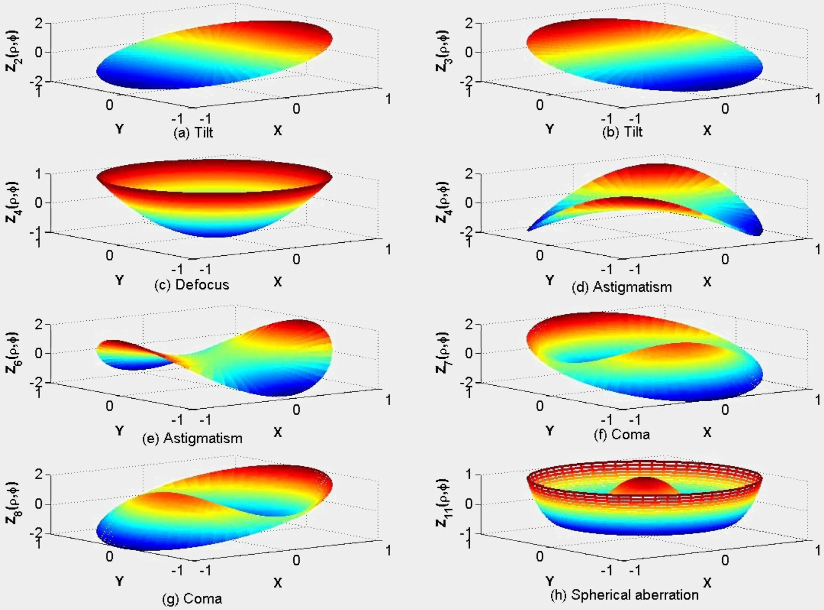
$$R_n^m(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!}\rho^{n-2s}$$

n and m are integers,with $m \leq n$ and $n-|m|$ even. These modes are defined on a disk of unity radius ($\rho \leq 1$). They are orthonormal:

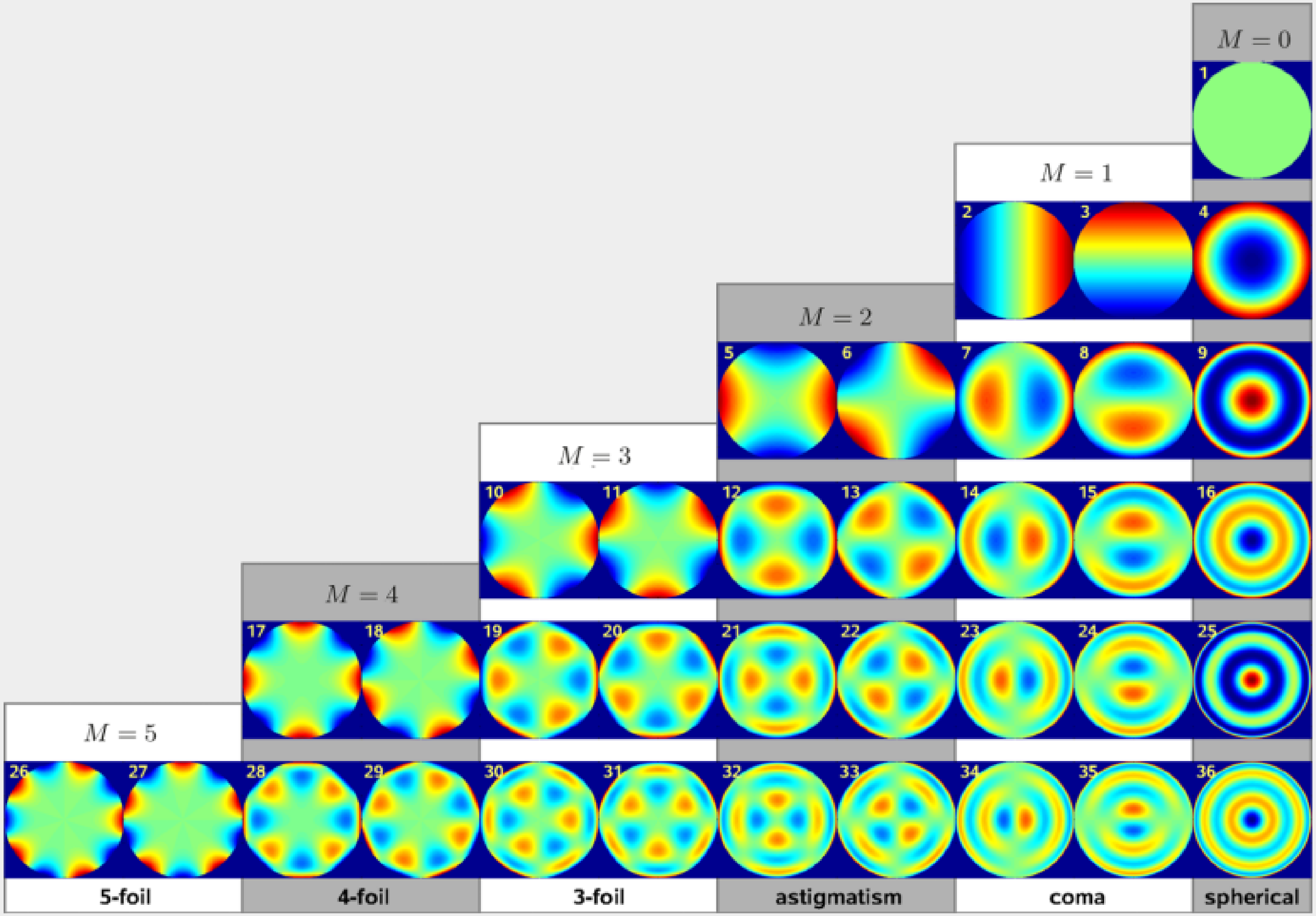
$$\iint_S Z_i Z_j \, W(r)\, dr\, d\theta = \delta_{ij} \quad \text{with} \quad W(r \leq 1) = 1/\pi \quad \text{and} \quad W(r > 1) = 0$$

Why Zernikes?

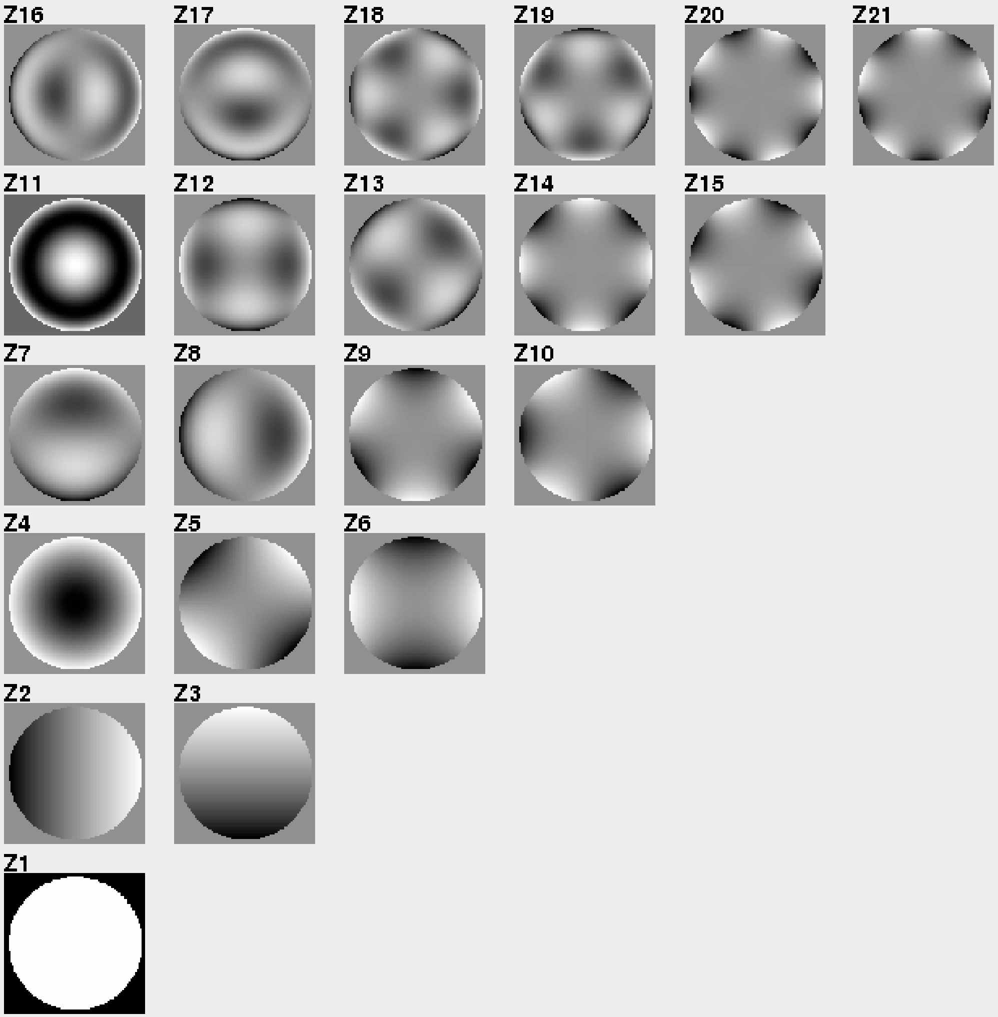
- Zernike polynomials have nice mathematical properties:
  - They are orthogonal over the continuous unit circle:
    $\iint_S Z_i(x, y,)Z_j(x, y)dS = \delta_{ij}$
  - All their derivatives are continuous.
- They efficiently represent common errors (e.g. coma, spherical aberration) seen in optics.
- They form a complete set, meaning that they can represent arbitrarily complex continuous surfaces given enough terms.



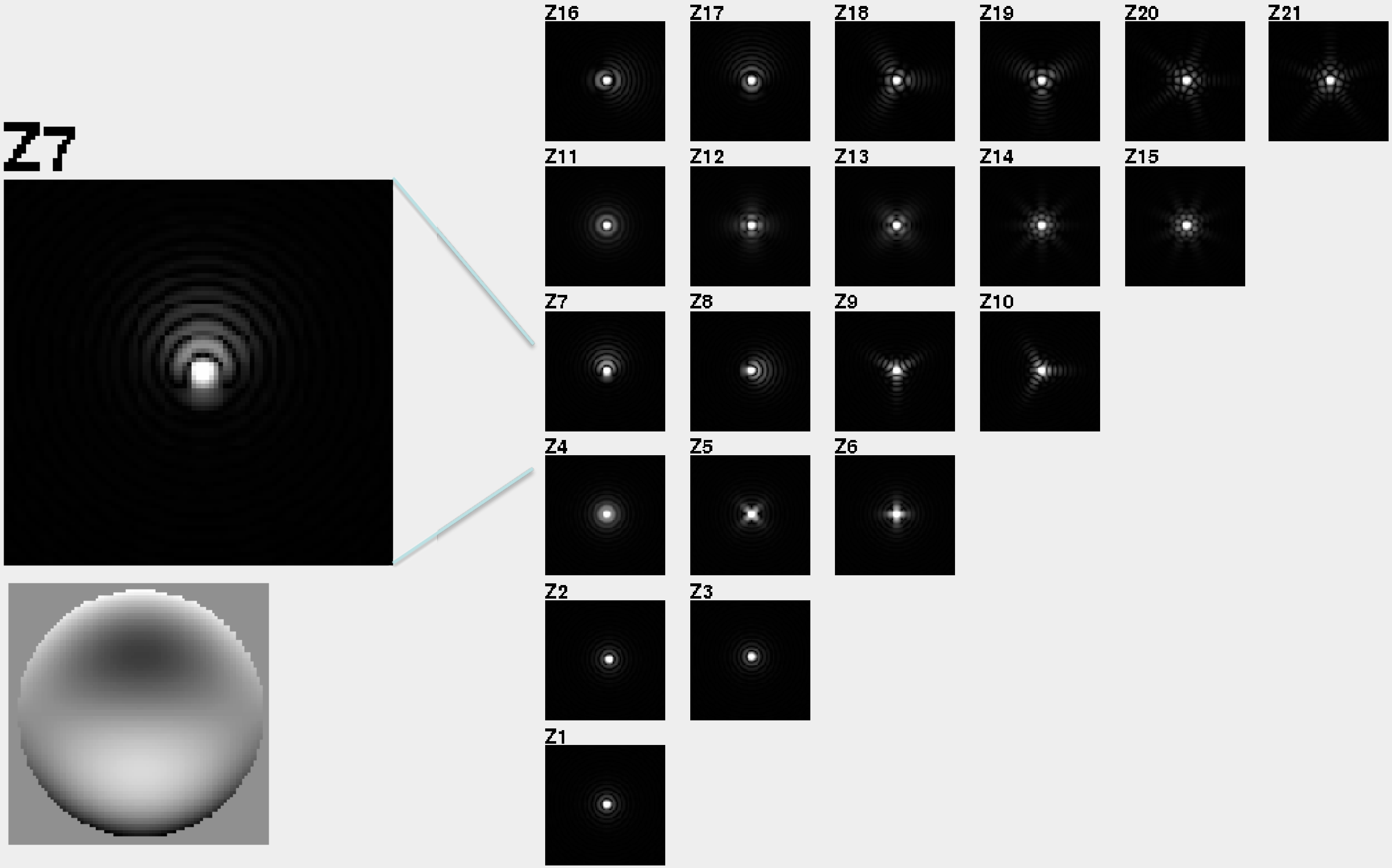
Zernike polynomials



ZERNIKE POLYNOMIALS…



… AND CORRESPONDING PSFs



Zernike polynomials to describe phase aberrations

Phase expansion and phase variance

The phase can be described as a **superposition** (sum) of Zernike polynomials

$$\varphi(x,y,t)\sum_{i=1}^{\infty}a_i(t)Z_i(x,y)$$

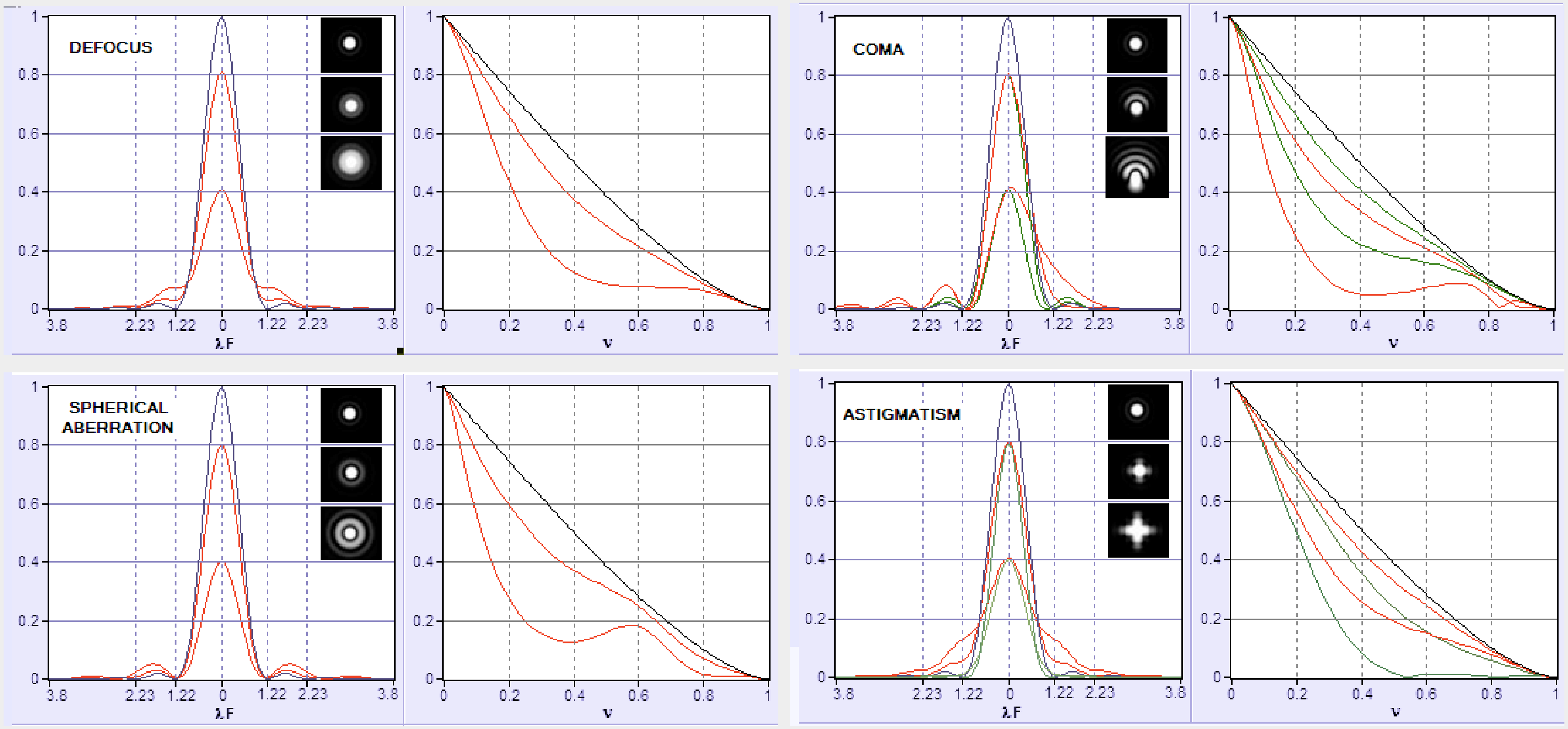
where the coefficients are calculated as follow:

$$a_i=\int_S W(r)\varphi(r,\theta)Z_i(r,\theta)\;r\;dr\;d\theta$$

The phase variance is then readily computed as:

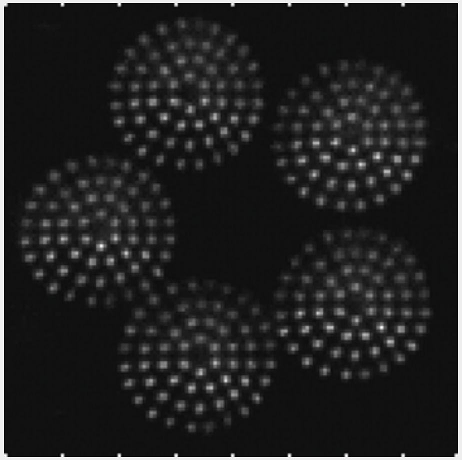
$$\sigma_\varphi^2=<\varphi^2(x,y,t)>_t=\sum_{i=1}^{\infty}a_i^2(t)\text{ given }\iint_S Z_i(x,y,)Z_j(x,y)dS=\delta_{ij}$$

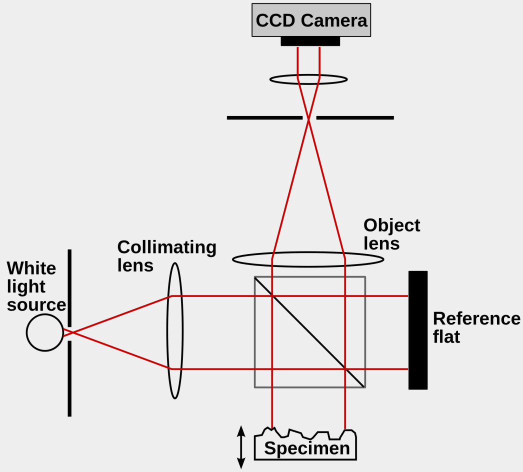
Impact on Optical Transfer Function

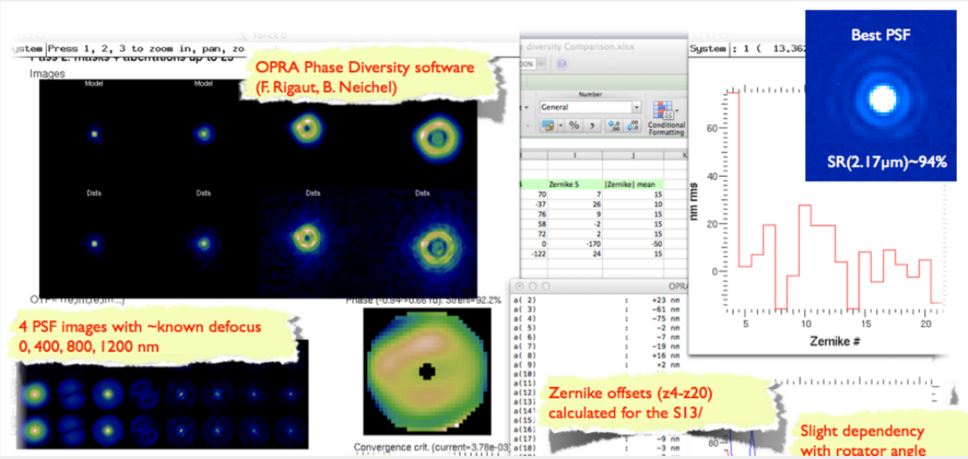


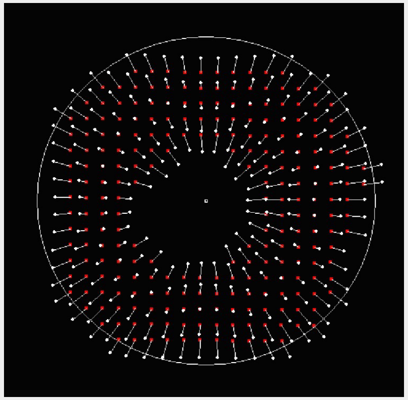
ABERRATION RETRIEVAL

- Using Wavefront sensors
  - Hartmann, Shack-Hartmann sensor
  - Foucault knife, pyramid sensor
  - Interferometer: Michelson, Mach-Zehnder, Fizeau,…
  - Self referenced interferometers: Shearing, point diffraction,…
- Using the image itself
  - Phase diversity



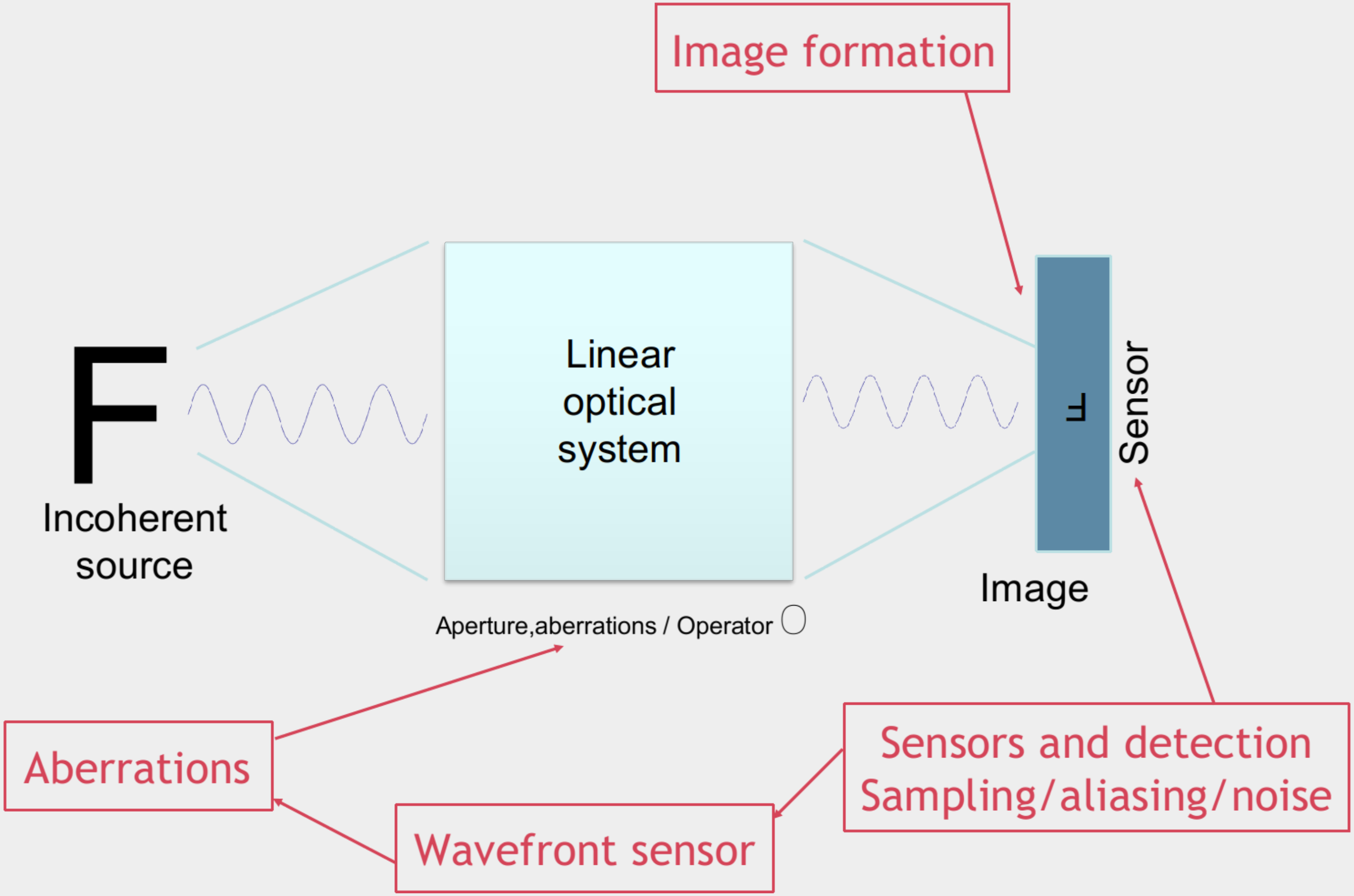






Wavefront Sensors

Linear Optical Systems

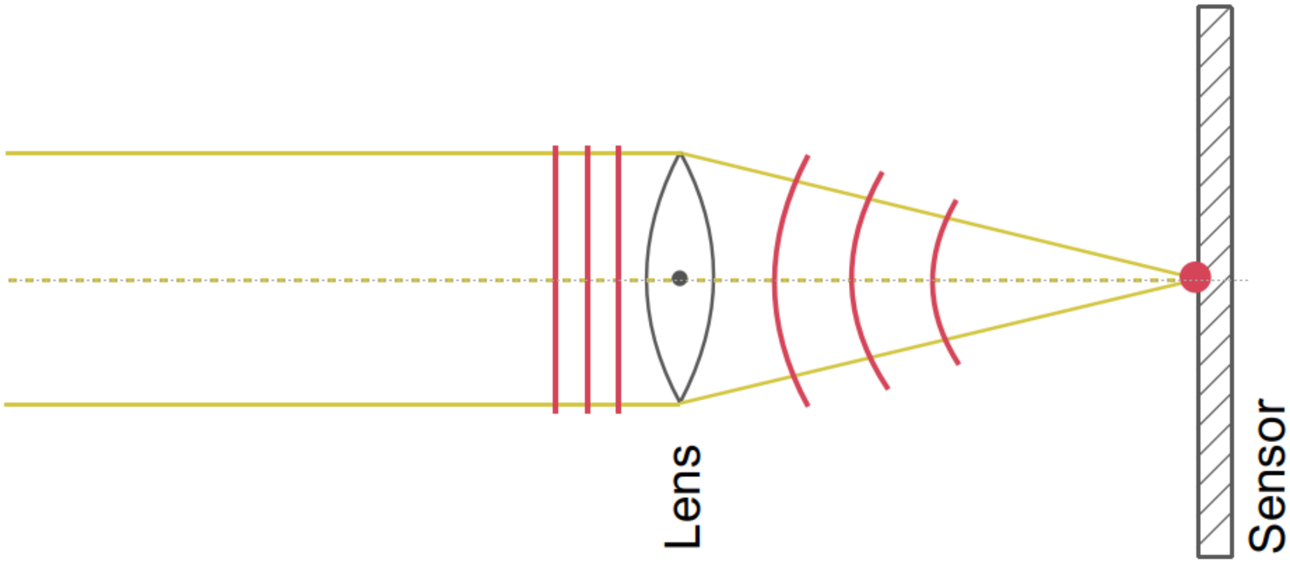


Wavefront Sensors (WFS)

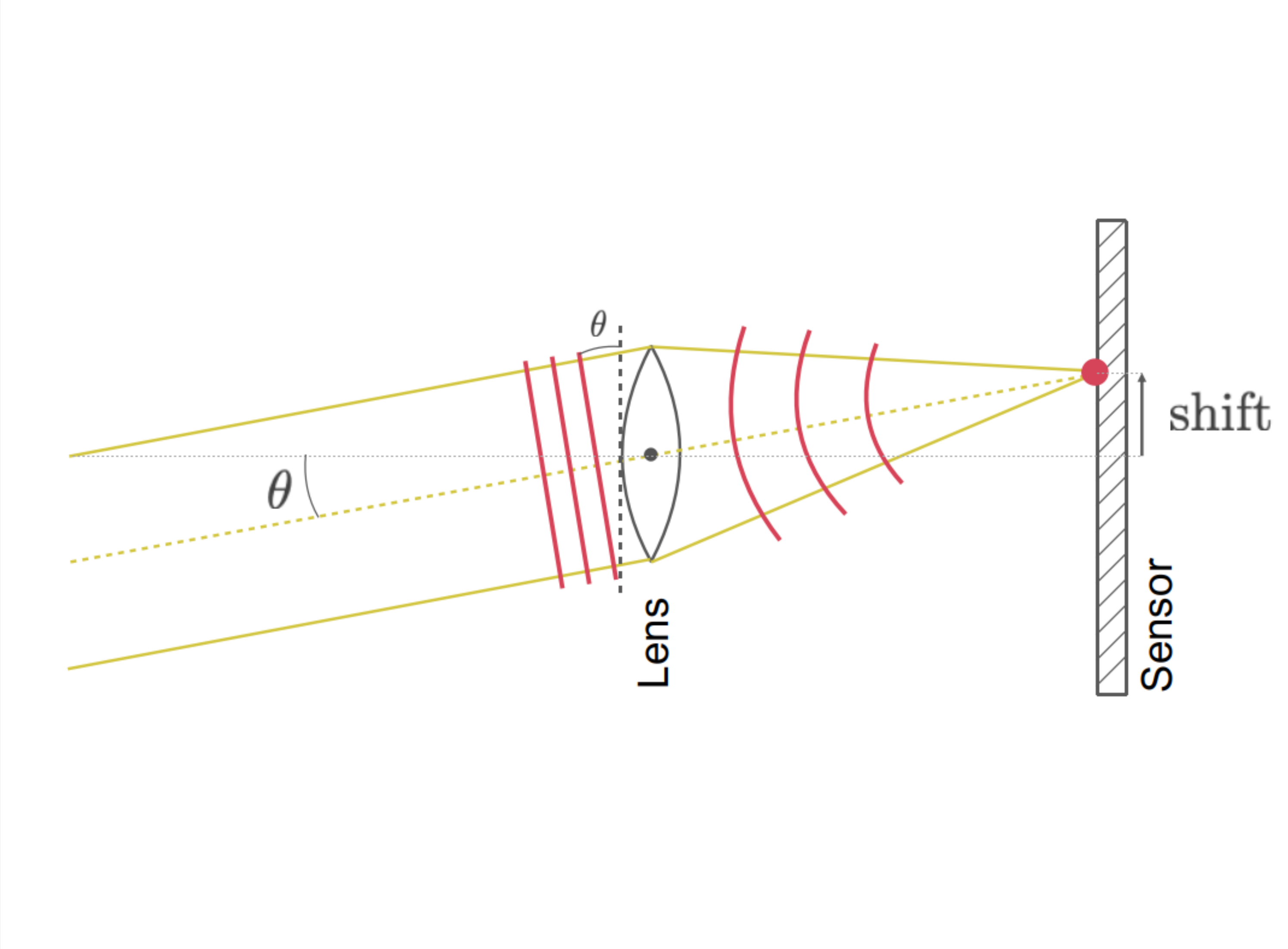
- A device that is measuring the wavefront phase (and potentially amplitude)
- Many wavefront sensors measure the phase first derivative (local slope) or second derivative (local curvature), some use a mix of both
  - First derivative wavefront sensors:
    - Shack-Hartmann
    - Shearing interferometers (lateral, radial, rotation)
    - Pyramid, Foucault knife
- Second derivative wavefront sensors:
  - Curvature
- And then some device measure the phase difference with some reference wave:
  - Point diffraction interferometer, Michelson, …
- But all do that through an intensity measurement of some sort

Shack-Hartmann Wavefront Sensor: how does it work?

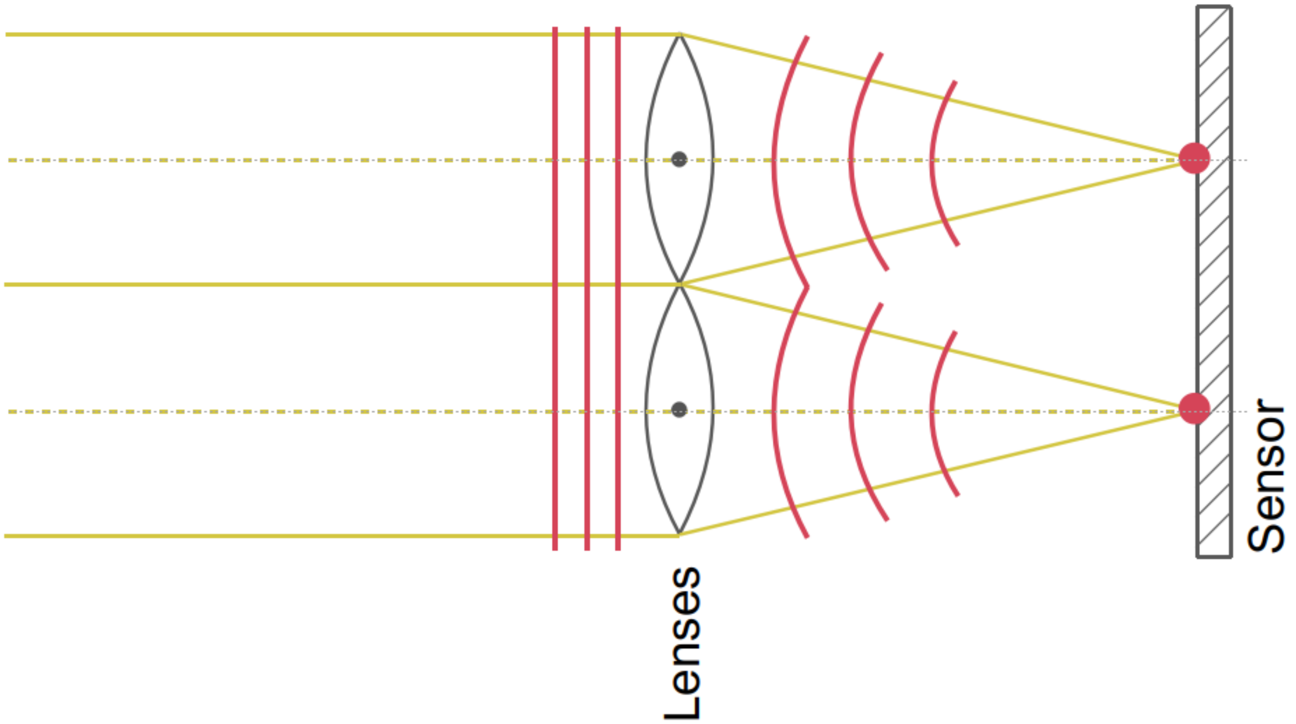
Shack-Hartmann WFS



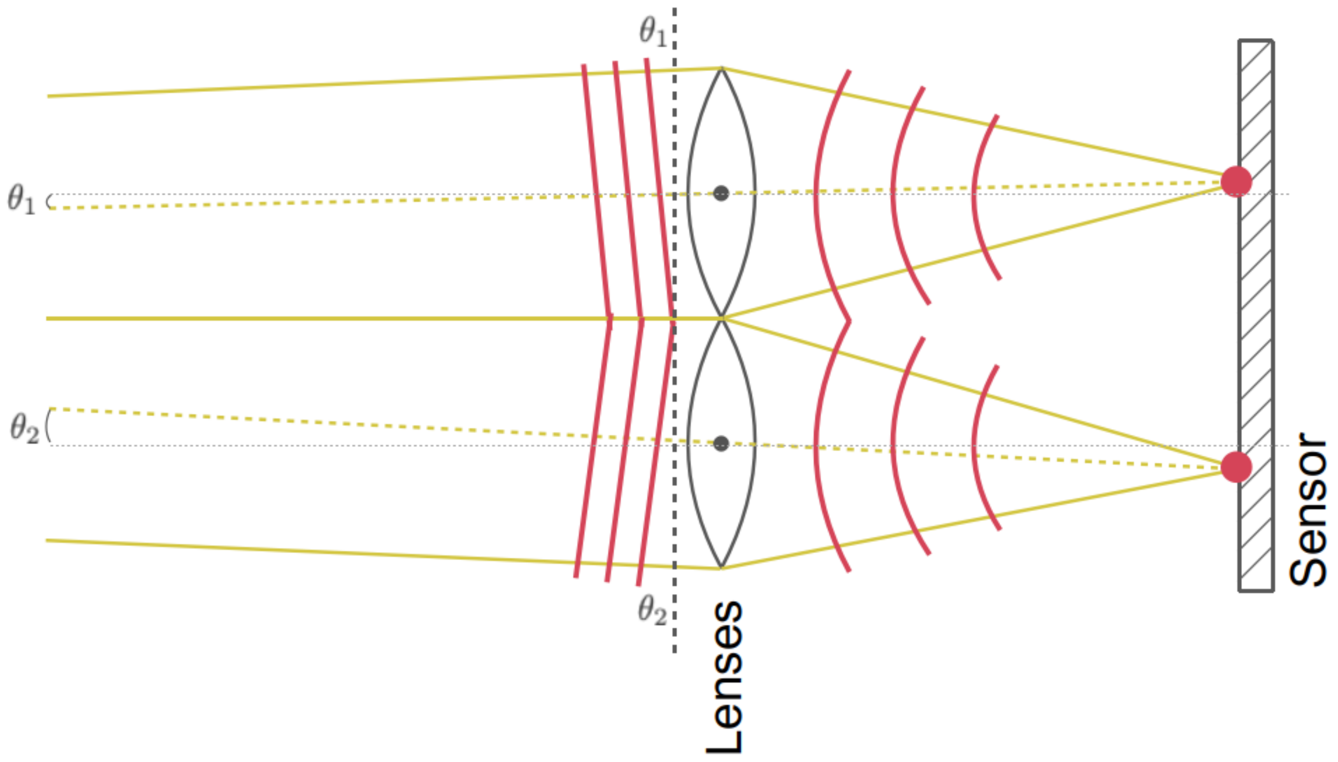
Shack-Hartmann WFS



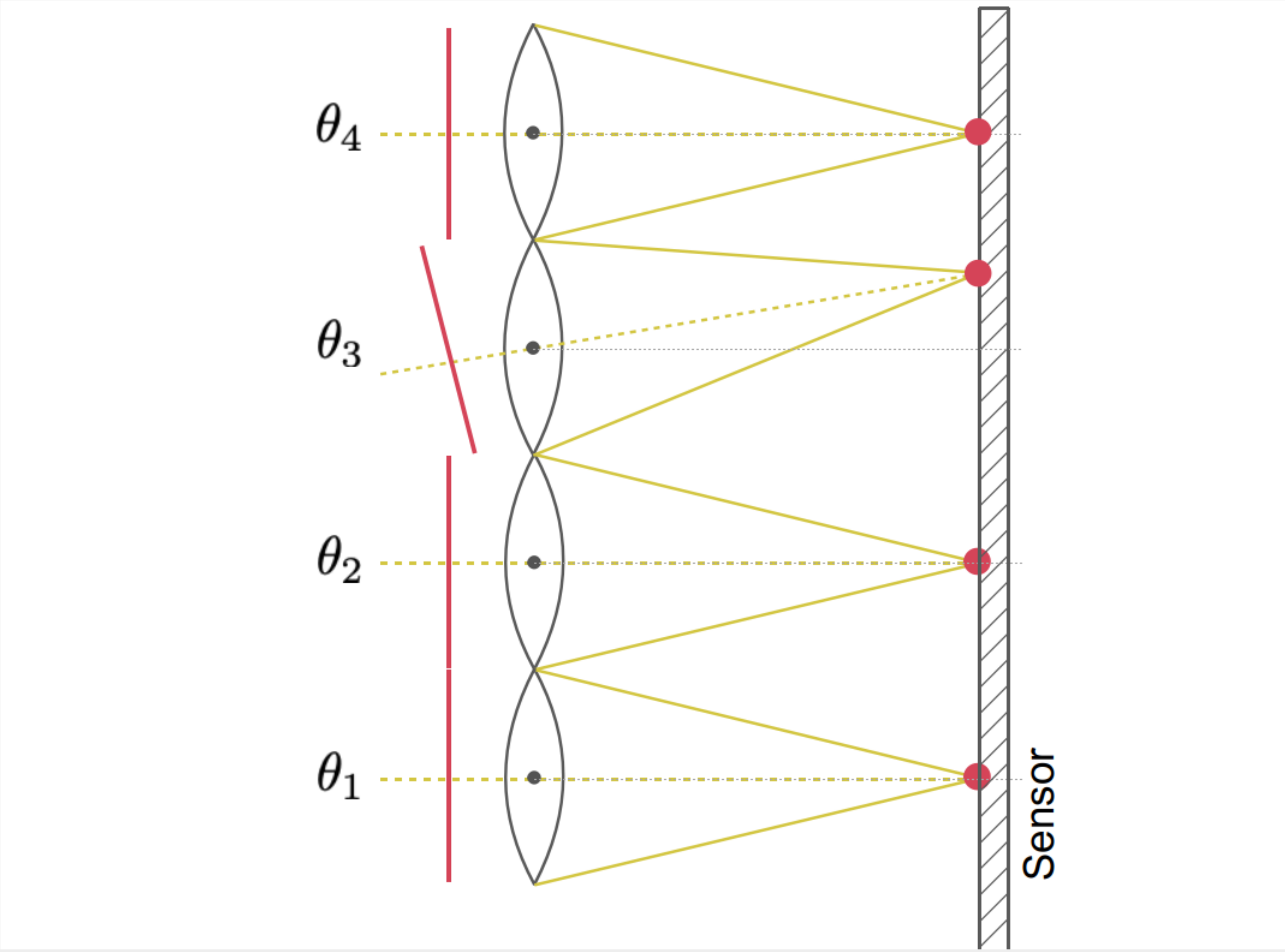
Shack-Hartmann WFS



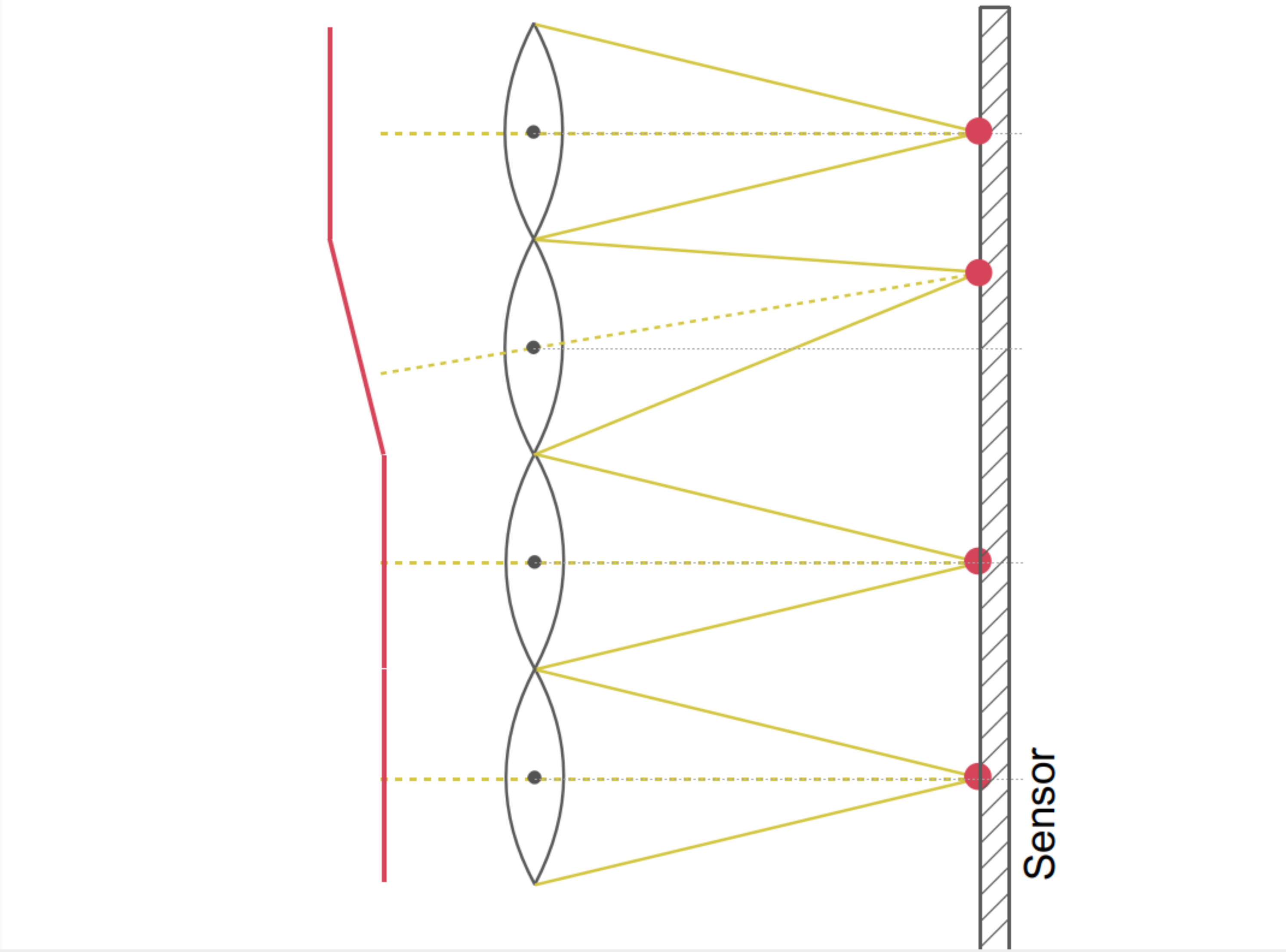
Shack-Hartmann WFS



Shack-Hartmann WFS

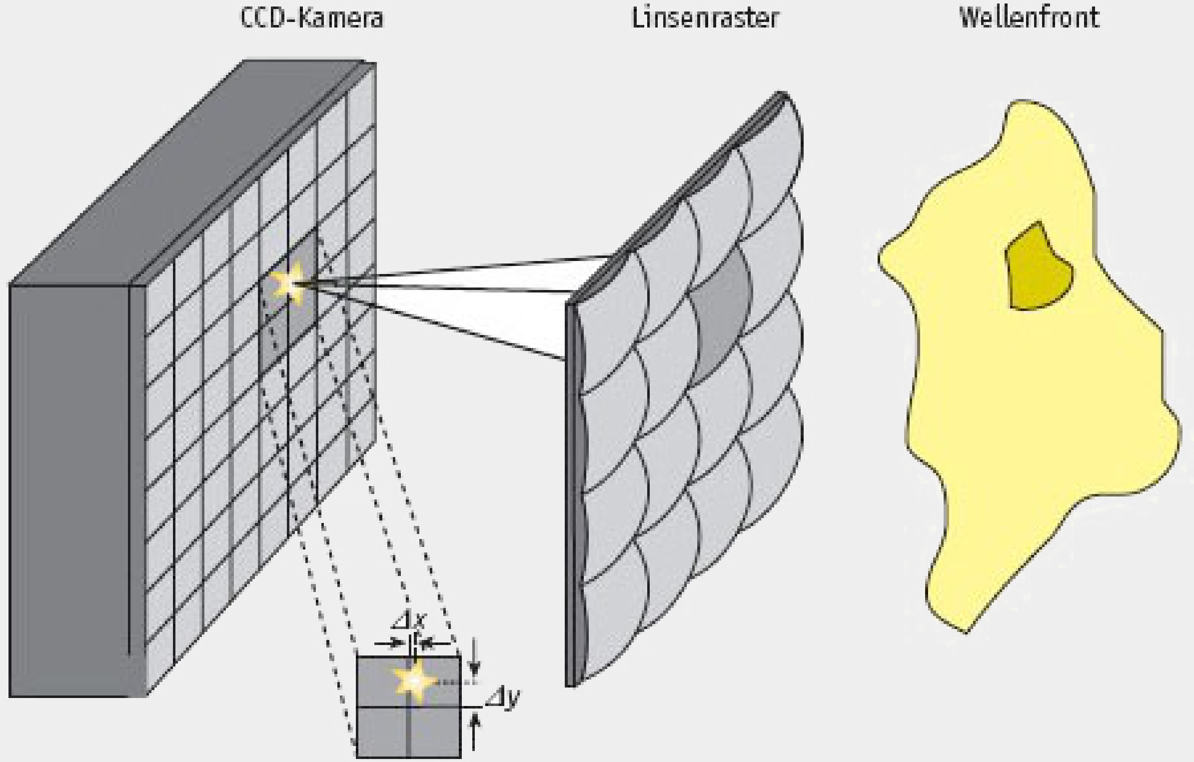


Shack-Hartmann WFS



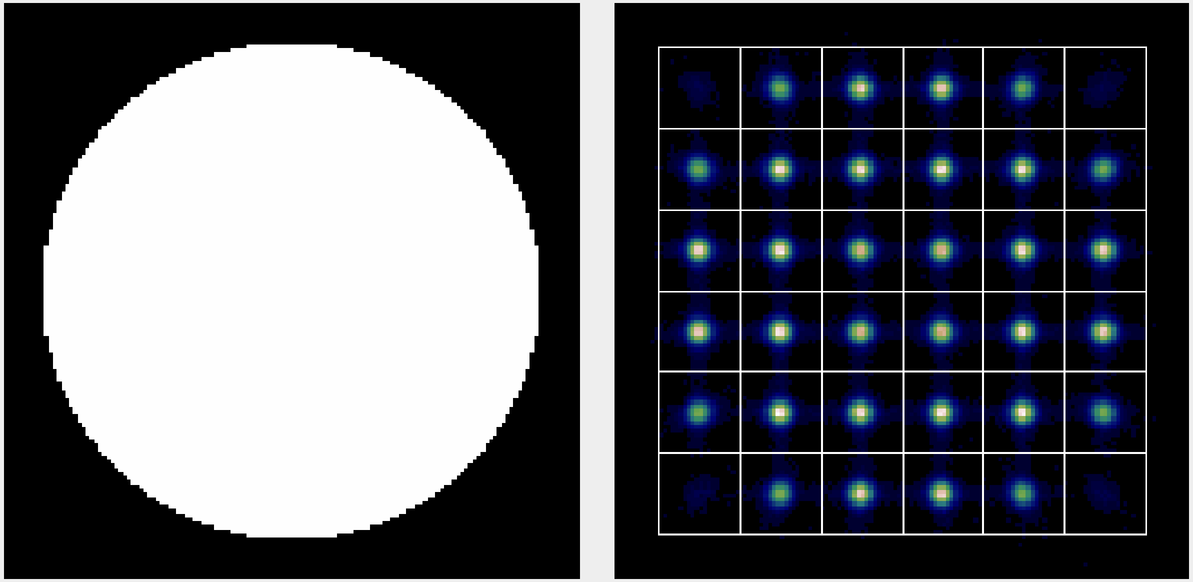
SHWFS Extension in two dimensions

Shack-Hartmann in 2D

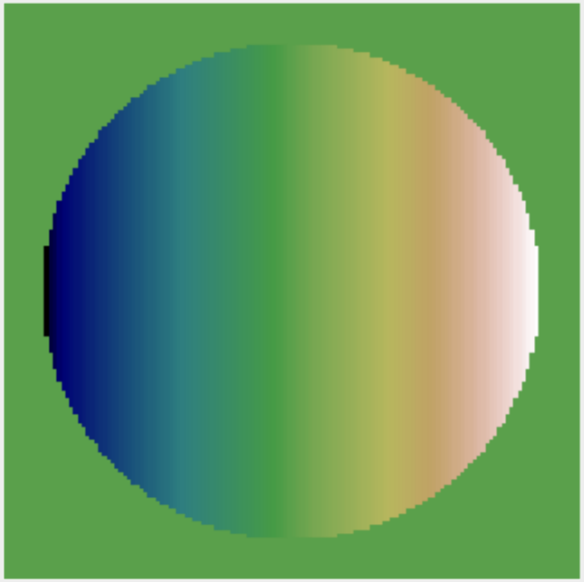


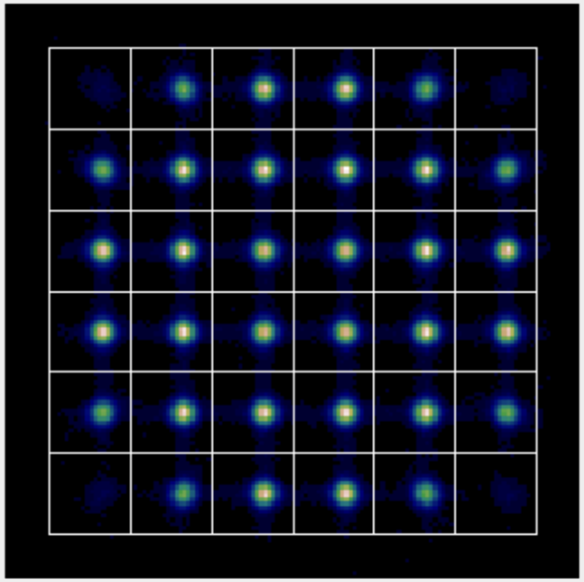
- A Shack-Hartmann sensor measure the average X and Y gradient over the subaperture

No Aberration

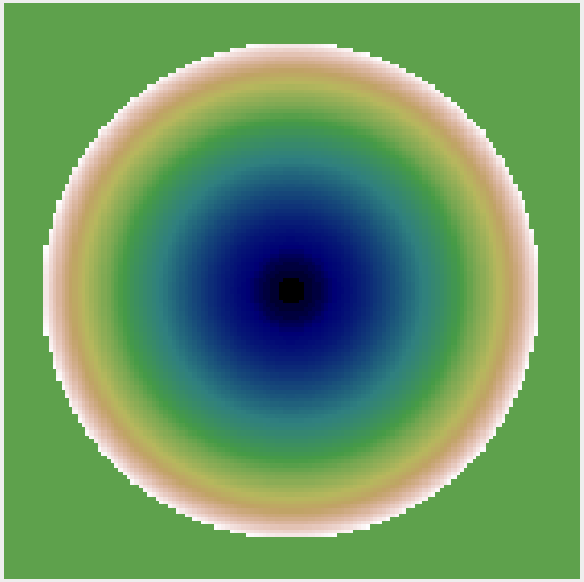


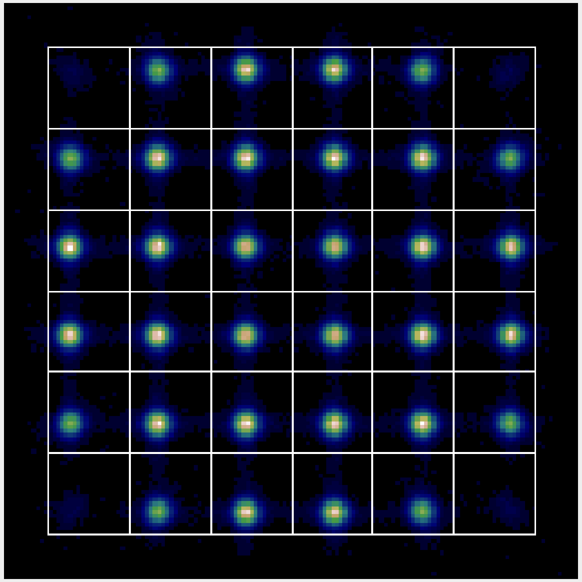
Tilt (Z2)



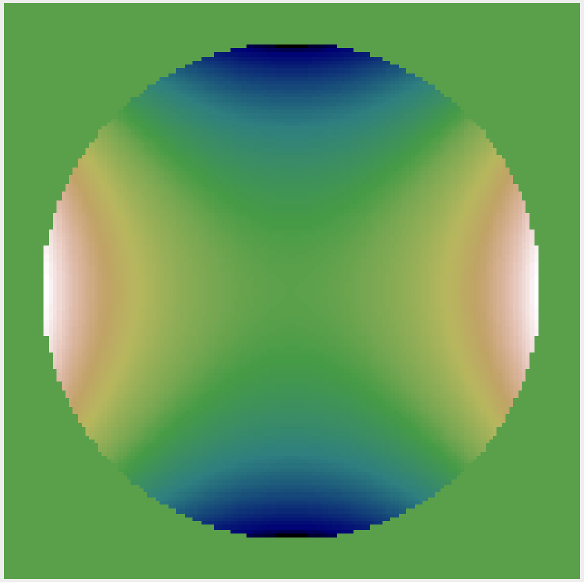


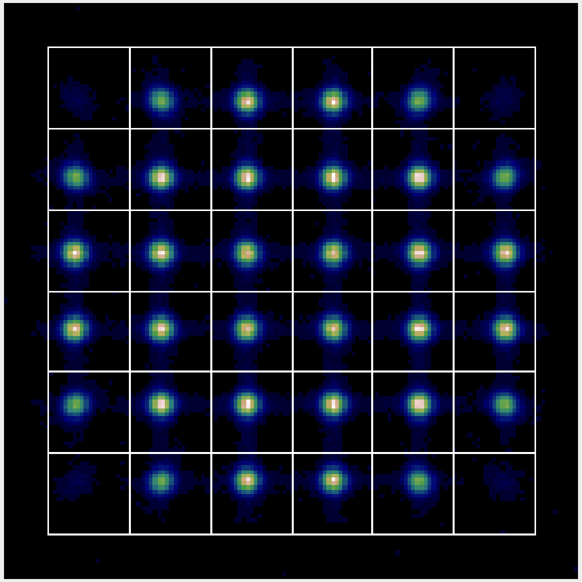
Defocus (Z4)



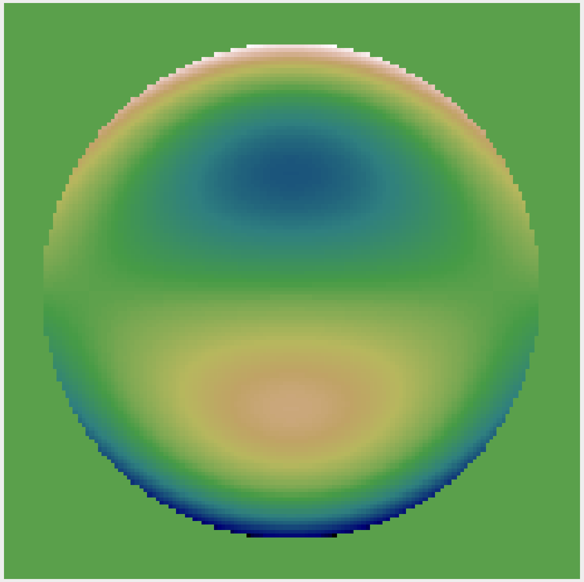


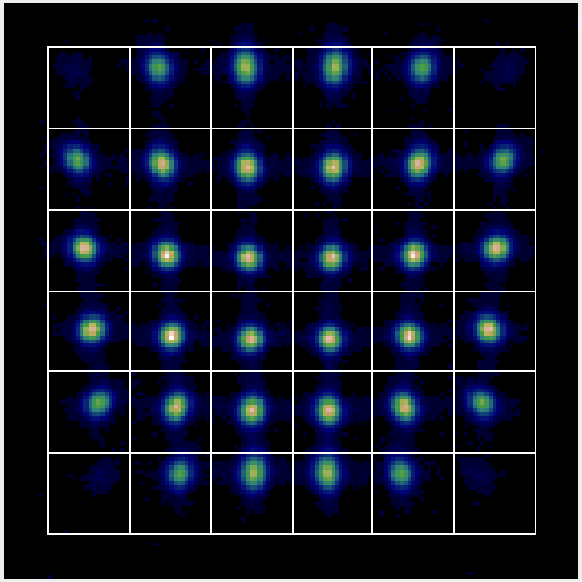
Astigmatism (Z6)



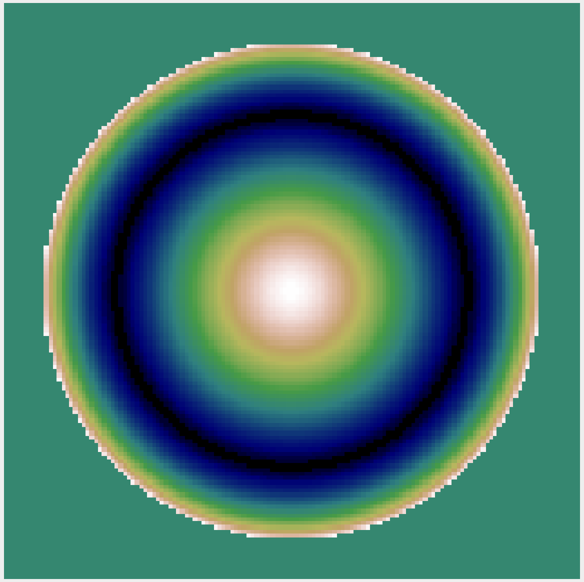


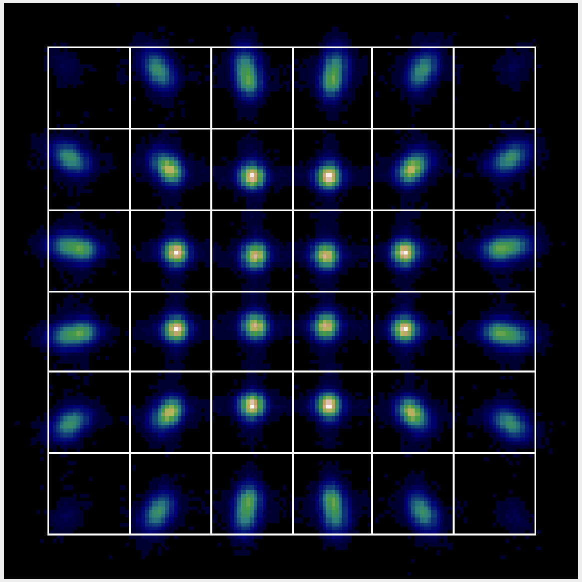
Coma (Z8)



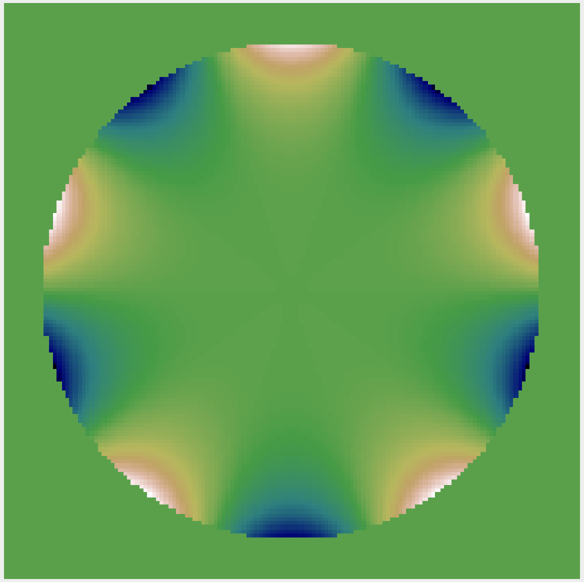


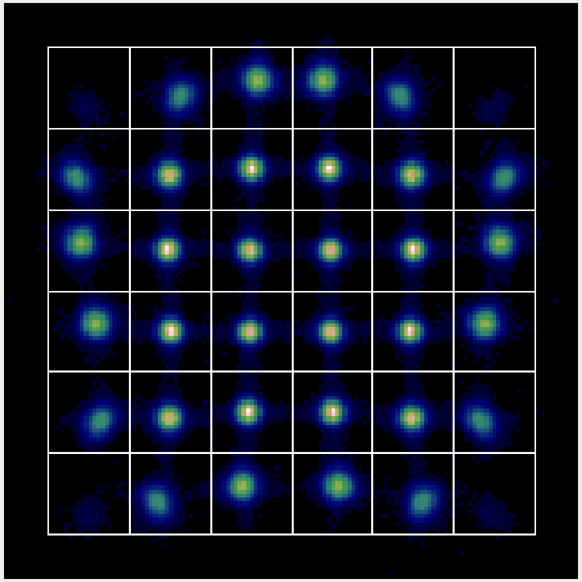
Spherical (Z11)





High order (e.g. Z21)



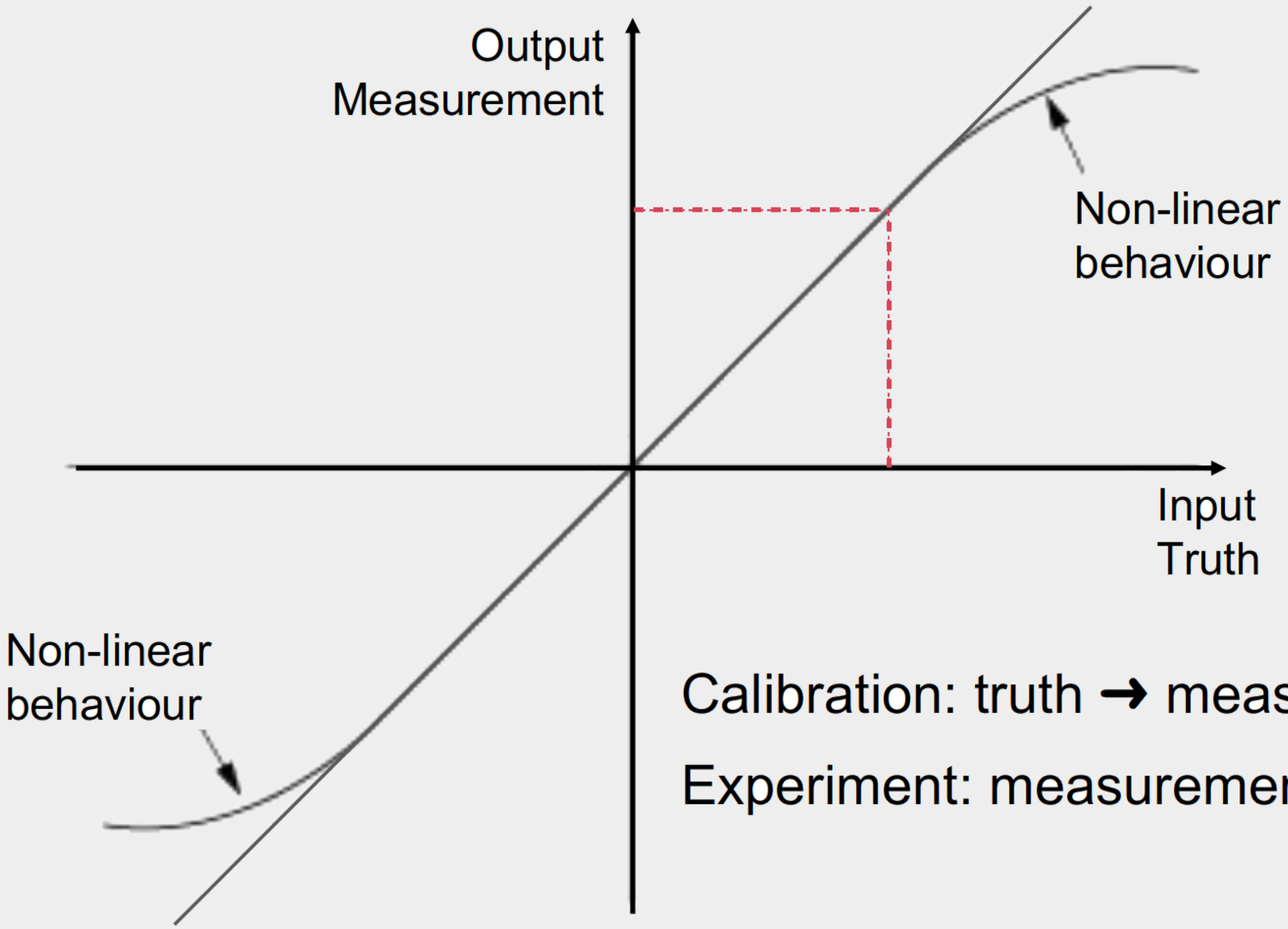


Real-World Considerations

What WFS to choose?

- Noise, usage of light
- Dynamical range
- Linearity, hysteresis
- Cost
- Polychromaticity
- Spatial aliasing
- Speed, computational requirements
- Solution uniqueness
- Extended sources
- Self referenced
- Ease of implementation

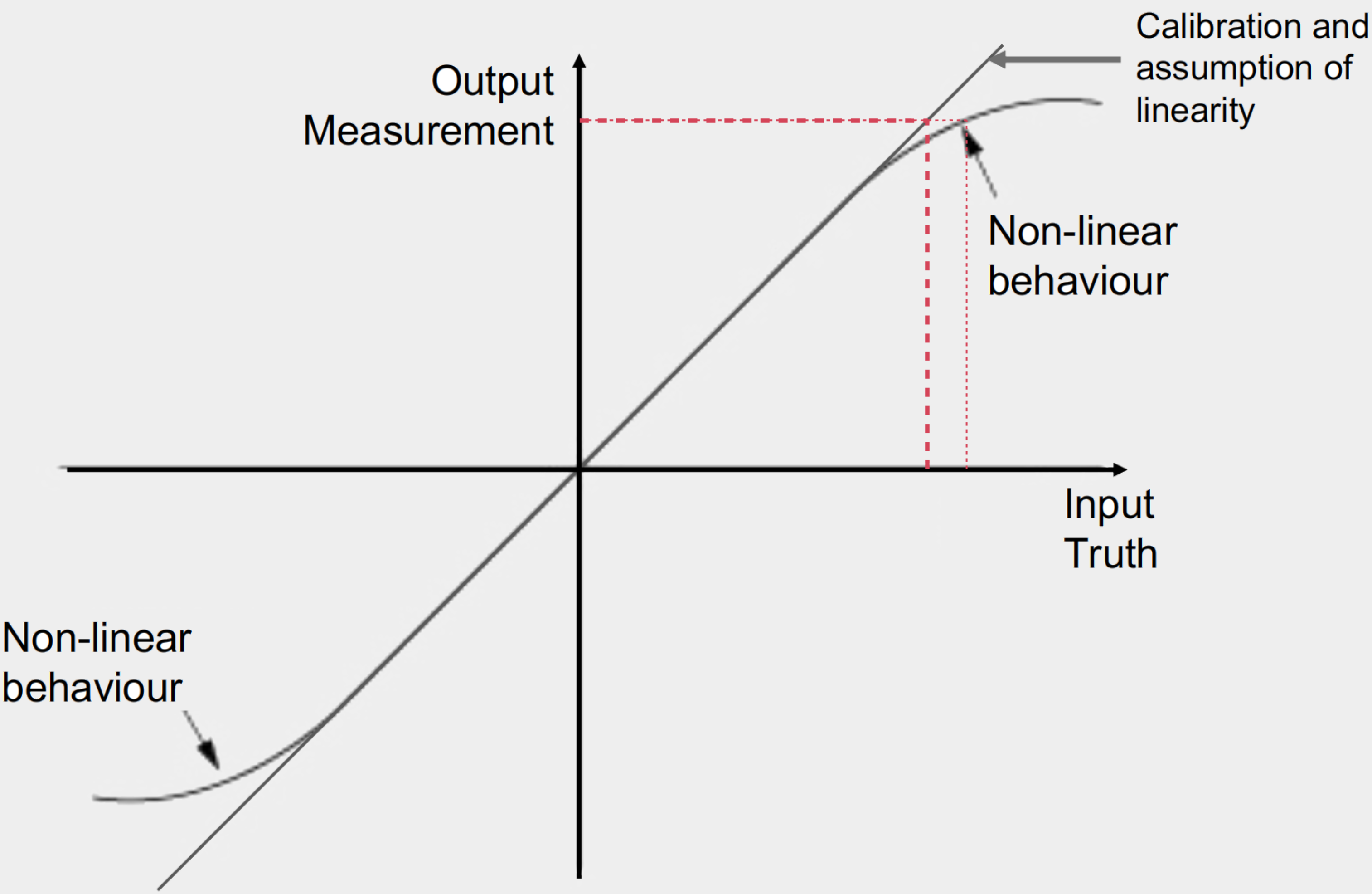
Sensor Transfer function



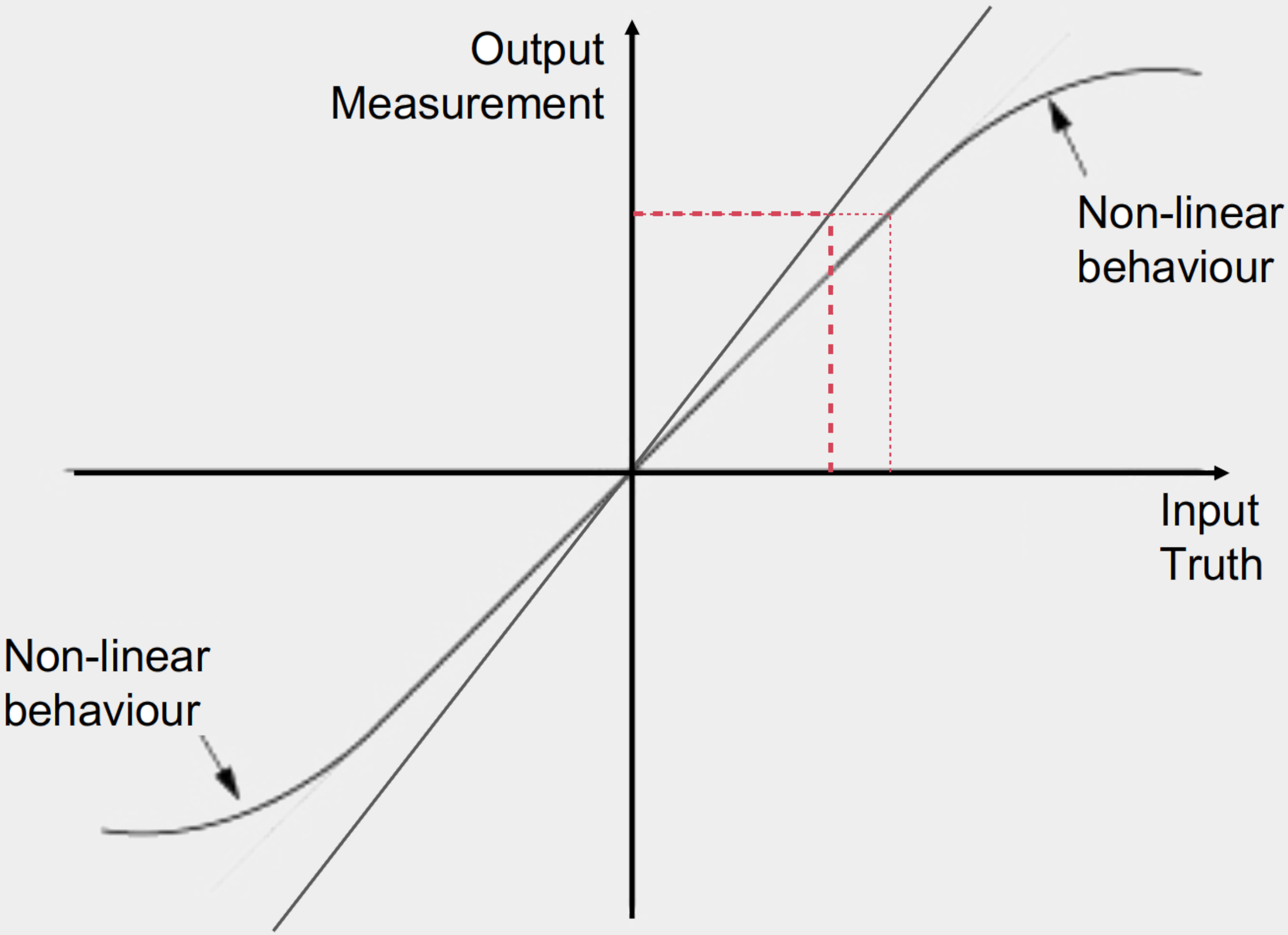
Calibration: truth ➜ measurement

Experiment: measurement ➜ truth

Sensing issues: non linearity



Sensing issues: Calibration error



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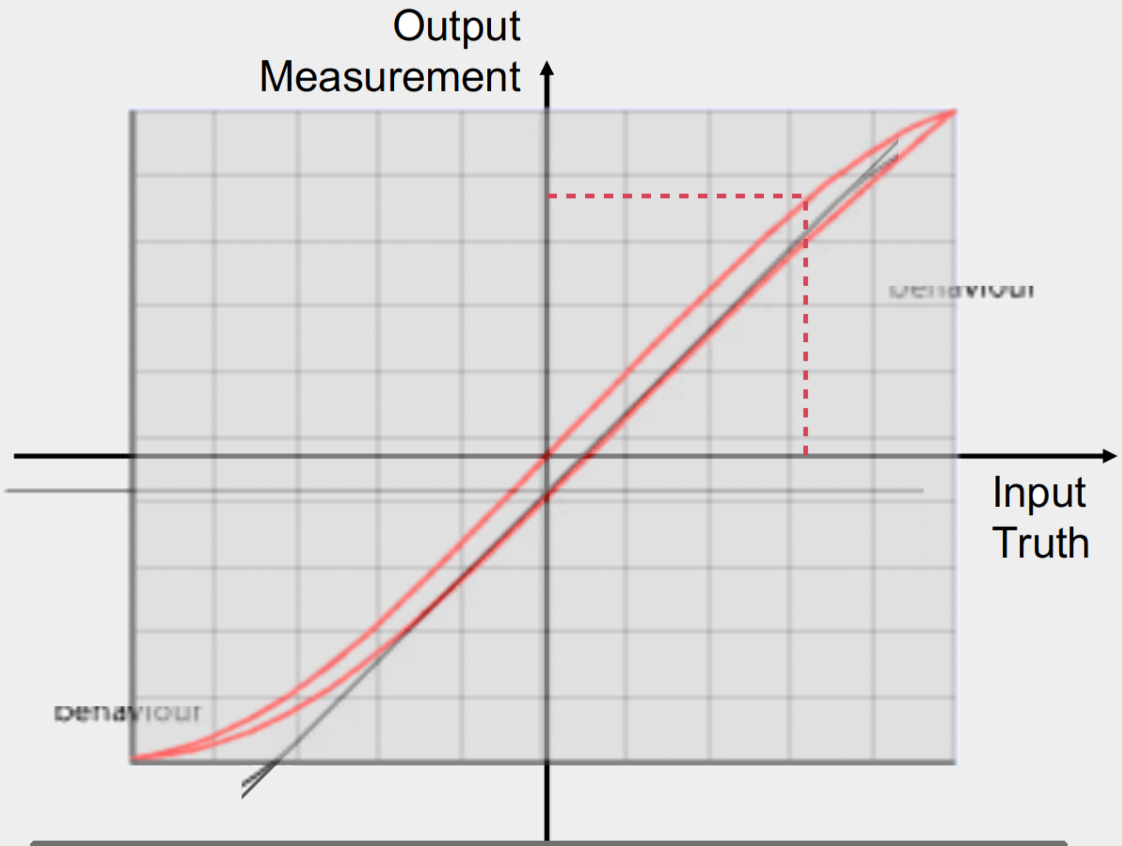
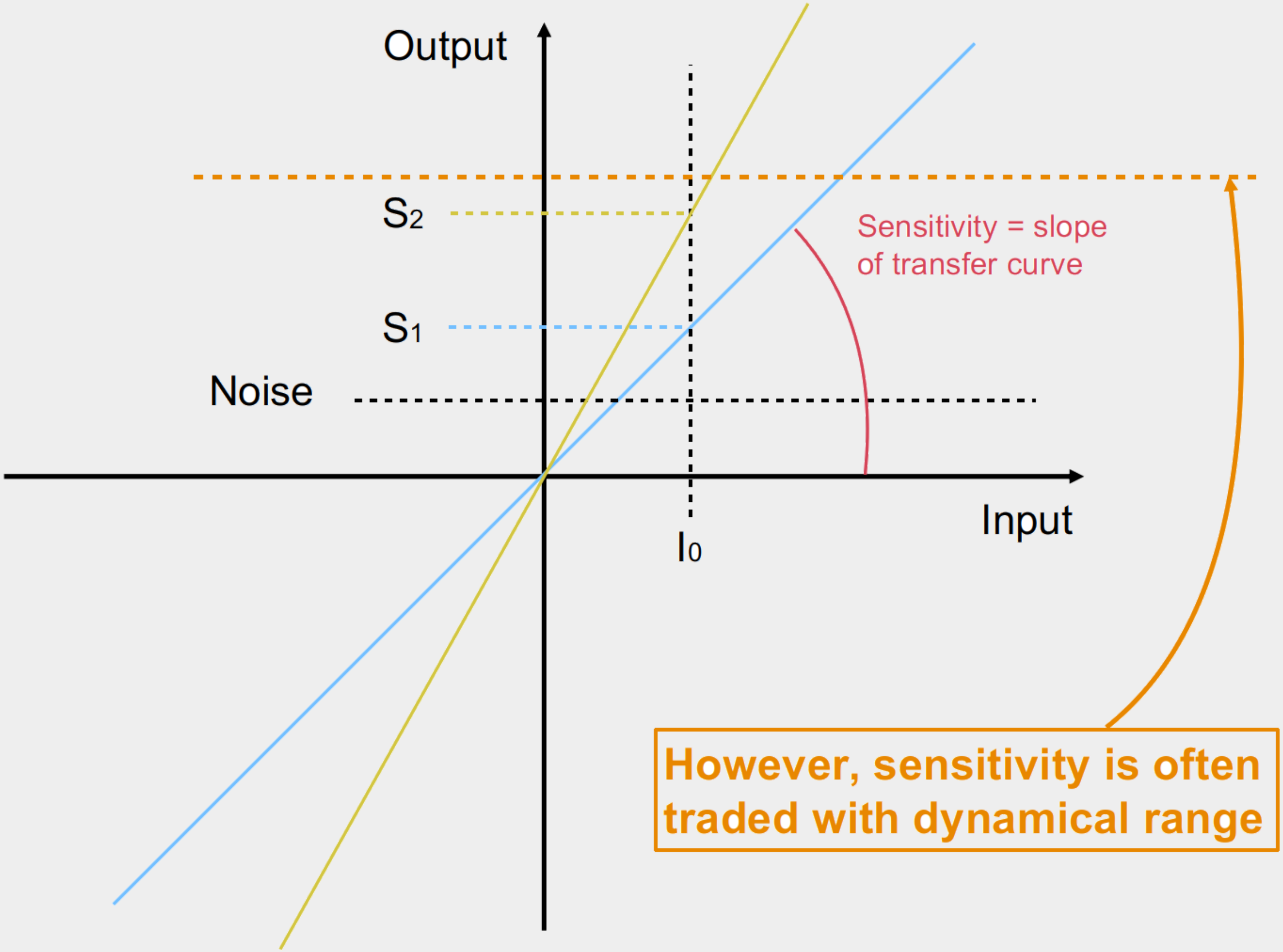


Proper calibration & Full Modelling:
Challenging for Hysteresis

Noise and SNR



Trade-offs and choice drivers

- Noise, usage of light
- Dynamical range
- Linearity, hysteresis
- Cost
- Polychromaticity
- Spatial aliasing
- Speed, computational requirements
- Solution uniqueness
- Extended sources
- Self referenced
- Ease of implementation

Does it need noiseless detector?
Does it need expensive optics, shutters, detectors?

Typically standard interferometric methods are out

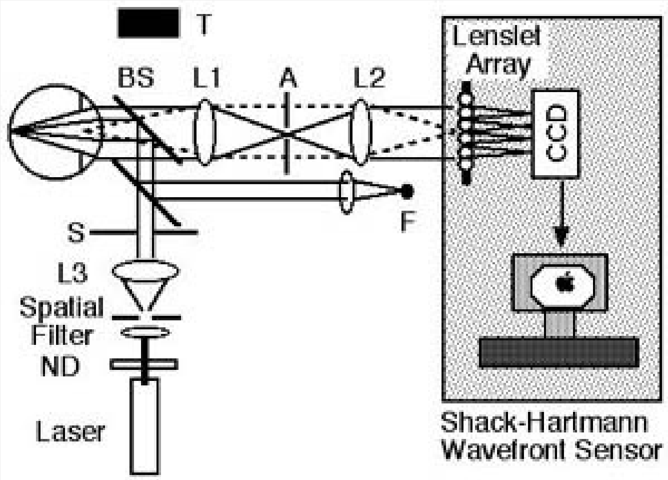
Iterative methods start with handicap
Order of the method $O(n, n^2, \text{etc})$

Focal plane methods (Gerchberg-Saxton or Phase Diversity) start with a disadvantage

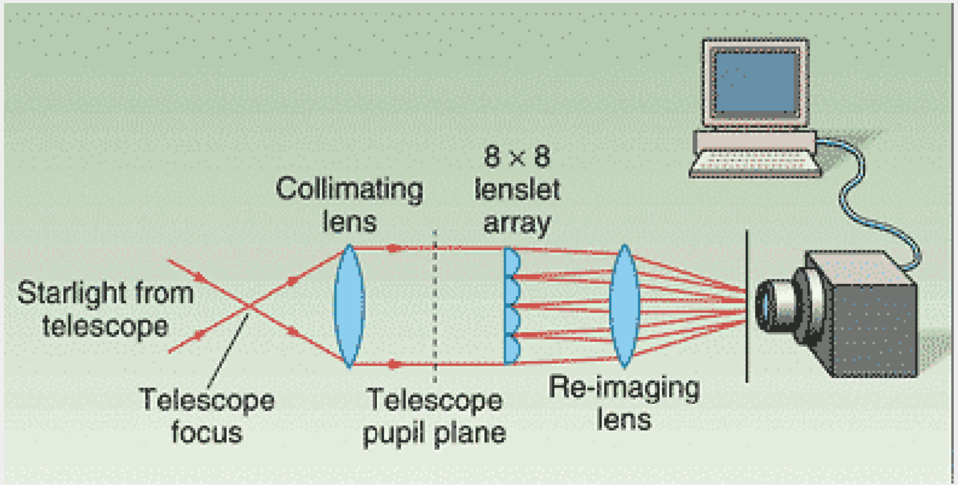
Non-reference methods a no-go when medium to analyse is remote

Other Real-World Considerations

In Astronomy



In Ophthalmology



- Field stop
- Collimating optics (usually lenslet 1F behind collimator)
- Lenslet array (most commercial arrays have pitch of 100-1000 µm)
- 2D Sensor: Most CCDs/CMOS have pixels of 2 (CMOS for phones) to 20 microns (CCDs for science applications). Typical format 128^2 to 2048^2.

Applications & Needs

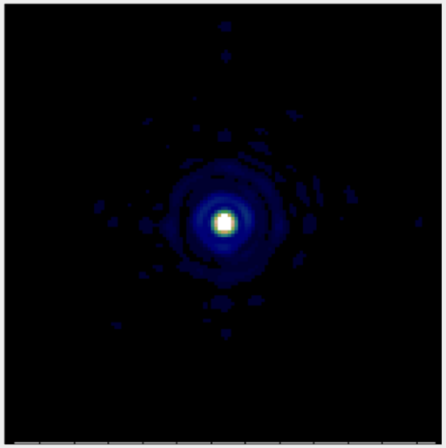
	Astronomy	Medicine	Lasers
Noise, usage of light	+++	++	+
Dynamical range	++	+++	++
Linearity, hysteresis	++	++	++
Cost	+	++	++
Polychromaticity	+++	+	-
Spatial aliasing	+	++	++
Speed, computational req.	+++	+	++
Solution uniqueness	+	+	++
Extended sources	++	+++	-
Self referenced	N/A	+	+
Ease of implementation	+	+	+

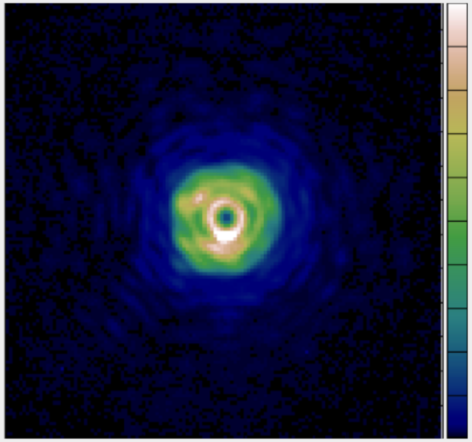
Alternatives WFS techniques

Alternative: Phase diversity

- Focal plane method
- Acquire an "in focus" image
- but (wavefront) phase is lost during the image formation $\mathcal{I} = |\mathcal{F}(Ae^{i\varphi})|^2$
- ...and an image with some added phase "diversity", e.g. focus $\mathcal{I}' = |\mathcal{F}(Ae^{i(\varphi+\varphi_0)})|^2$
- The second image lift the sign uncertainty
- Then use a minimisation package (Steepest descent, Conjugate Gradient, Levenberg–Marquardt, etc) to find the phase that reproduce best the images, or AI.

“In focus” image





“Out of focus” image

Actual images from NACO, VLT, ESO. 2.2 microns.

- The problem: Given data points y_i at x_i, find model parameters β so that the least square distance model-data S is minimum:
  $S(\beta_1, \beta_2) = \sum_{i=1}^{m} (y_i - f(x_i, \beta_1, \beta_2))^2$
- Iterative methods
- Steepest descent, Conjugate Gradient, Levenberg–Marquardt
- Issue with local minima: