$$
\begin{aligned}
LNN =& \int \left[1 - \sum_{i=1}^{c} P^2(\omega_i|x) \right] p(x)dx \\
=& \int \left[1 - \sum_{i=1}^{c} \left(\frac{P(x|\omega_i)P(\omega_i)}{p(x)} \right)^2 \right] p(x)dx \\
=& \int p(x) - \sum_{i=1}^{c} \frac{P(x|\omega_i)^2 P(\omega_i)^2}{p(x)} dx \\
=& \int p(x)dx - \int \sum_{i=1}^{c} \frac{P(x|\omega_i)^2 P(\omega_i)^2}{p(x)} dx \\
=& 1 - \int \sum_{i=1}^{c} \frac{P(x|\omega_i)^2 P(\omega_i)^2}{p(x)} dx \\
=& 1 - \int_0^{\frac{cr}{c-1}} \frac{1}{p(x)c^2} dx - \sum_{i=1}^{c} \int_i^{i+1-\frac{cr}{c-1}} \frac{1}{p(x)c^2} dx \\
=& 1 - \frac{cr}{p(x)c^2(c-1)} - \frac{c(c-1) - c^2 r}{p(x)c^2(c-1)} \\
=& 1 - \frac{r - c + 1 + cr}{p(x)c(c-1)} \\
=& 1 - \frac{(1-r)(c-1)}{p(x)c(c-1)} \\
=& 1 - \frac{(1-r)}{p(x)c}
\end{aligned}
$$

Where if we think that:

$$\int p(x)dx = 1$$

And more precisely, given $P(x|\omega_i)$ as stated in the question, we constrain $\int p(x)$ to the support of the density:

$$\int_0^{\frac{cr}{c-1}} p(x)dx + \sum_{i=1}^{c} \int_i^{i+1-\frac{cr}{c-1}} p(x)dx = 1$$

It would be tempting to say that if p(x) is constant (yet I'm not sure of this assumption):

$$\begin{aligned}
p(x)\Big|_0^{\frac{cr}{c-1}} + \sum_{i=1}^{c} p(x)\Big|_i^{i+1-\frac{cr}{c-1}} &= 1 \\
p(x)\left(\frac{cr}{c-1} + \sum_{i=1}^{c}\left[1 - \frac{cr}{c-1}\right]\right) &= 1 \\
p(x)\left(\frac{cr + c(c-1) - c^2 r}{c-1}\right) &= 1 \\
p(x)c(1-r) &= 1
\end{aligned}$$

And therefore:

$$p(x) = \frac{1}{c(1-r)}$$

But if we plug this above, we get:

$$\begin{aligned}
LNN =& 1 - \frac{(1-r)}{\frac{c}{c-cr}} \\
LNN =& 1 - \frac{(1-r)}{\frac{1}{1-r}} \\
LNN =& 1 - (1-r)^2 \\
LNN =& 1 - (1 - 2r + r^2) \\
LNN =& 2r - r^2
\end{aligned}$$

However, if for some reason integrating the $\frac{1}{p(x)}$ part were to yield $\frac{1}{c}$, then we would have:

$$LNN = 1 - (1-r) = r$$

Could you please give me any pointers or hints as to what I'm doing wrong?