

Lecture 7

Section 5 of Why Functional programming Matters

2nd glue: lazy evaluation

example from AI: computer playing games

we assume

data type Position

function moves :: Position -> [Positions]

reptree :: (a ->[a]) -> a -> Tree a

reptree f a = Node a (map (reptree f) (f a))

gametree :: (Position ->[Position]) -> Position -> Tree Position

gametree p = reptree moves p

Min-max and alpha-beta procedure

we consider the gametree

evaluate statically the positions in the tree, $\text{static} :: \text{position} \rightarrow \text{Int}$

value of a node=position measures how promising the position is for the computer

max when computer wins and min when it loses

The computer wants to always do the best move = take the move that leads to the position with maximum value

when the adversary moves, he/she should take the move that leads to the position with minimum value

$\text{maptree static} :: \text{Tree Position} \rightarrow \text{Tree Int}$

$\text{max,min} : [\text{Int}] \rightarrow \text{Int}$

$\text{maximize,minimize} :: \text{Tree Int} \rightarrow \text{Int}$

$\text{maximize (Node n sub)} = \text{max (map minimize sub)}$

$\text{minimize (Node n sub)} = \text{min (map maximize sub)}$

for leaves

$\text{maximize (Node n [])} = n$ static evaluation of the position

$\text{minimize (Node n [])} = n$

`evaluate :: (Position -> [Position]) -> Position -> Int`
`evaluate = maximize . maptree static . gametree`

it doesn't work for infinite trees

even finite game trees may be too big

we prune the tree to a fixed depth:

`prune :: Int -> Tree a -> Tree a`

`prune 0 (Node a x) = Node a []`

`prune (n+1) (Node a x) = Node a (map (prune n) x)`

`evaluate = maximize . (maptree static) . (prune 5) . gametree`

`evaluate = maximize . (maptree static) . (prune 5) . gametree`

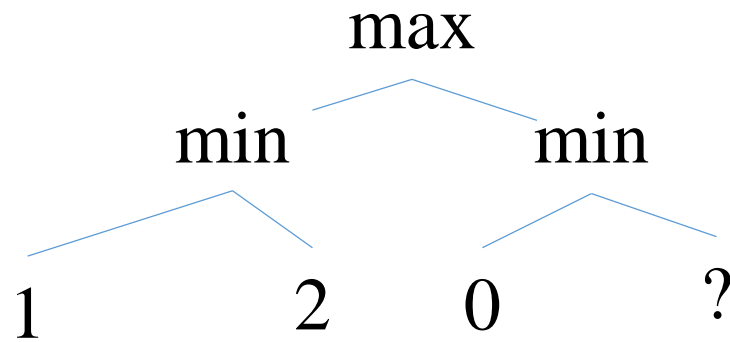
higher-order functions `reptree` and `maptree` allow us to construct and manipulate possibly infinite trees

lazy evaluation allow us to separate the construction of the game tree and the computations on it

without lazy evaluation, in place of `(prune 5) . gametree` we should have a unique function that computes the tree counting 5 levels

`(maptree static) . (prune 5) . gametree` constructs the parts of the tree that `maximize` needs ! Again, thanks to lazy evaluation

we improve maximize so that it may not need parts of the pruned tree



1 is the min of the first subtree and 0 or less will be the result of the 2nd subtree => drop the ? it won't matter

incorporate this idea in maximize and minimize

$\text{maximize} = \text{max} . \text{maximize}'$ and similarly for minimize

$\text{maximize}', \text{minimize}' :: \text{Tree Int} \rightarrow [\text{Int}]$

$\text{maximize}' (\text{Node } n []) = n : []$

$\begin{aligned} \text{maximize}' (\text{Node } n l) &= \text{map minimize } l \\ &= \text{map } (\text{min} . \text{minimize}') l \\ &= \text{map min } (\text{map minimize}' l) \\ &= \text{mapmin } (\text{map minimize}' l) \\ &\quad \text{where mapmin} = \text{map min} \end{aligned}$

$\text{map minimize}' l :: [[\text{Int}]]$ and $\text{map min } (\text{map minimize}' l) :: [\text{Int}]$ that is the list of the minima of those lists, but some minima may be useless

smarter mapmin:

```
mapmin :: [[Int]] -> [Int]
```

```
mapmin (nums : rest) = (min nums) : (omit (min nums) rest)
```

```
omit pot [] = []
```

```
omit pot (nums : rest) | minleq nums pot = omit pot rest --nums dropped  
                        | otherwise       = (min nums) : (omit (min nums) rest)
```

```
minleq [] pot = False
```

```
minleq (n:rest) | n <= pot = True  
                | otherwise = minleq rest pot
```

```
evaluate = max . maximize'. (maptree static) . (prune 8) . gametree
```