Assignment 0

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1 Summarize selected parts of chapters 0 and 1

THEOREM 0.20

For any two sets A and B, $\overline{A \cup B} = \overline{A} \cap \overline{B}$

THEOREM 0.21

For every graph G, the sum of the degrees of all the nodes in G is an even number.

THEOREM 0.22

For each even number n greater than 2, there exists a 3-regular graph with n nodes.

THEOREM 0.24

$\sqrt{2}$ is rational.

THEOREM 0.25

For each $t \geq 0$,

$$P_t = PM^t - Y\left(\frac{M^t - 1}{M - 1}\right)$$

DEFINITION 1.5

A **finite automaton** is a 5-tuple (Q, $\sum$, δ, q_0, F)

1. Q is a finite set called the states,
2. $\sum$ is a finite set called the alphabet,
3. $\delta : Q \times \sum \rightarrow Q$ is the transition function,

4. $q_0 \in Q$ is the start state, and
5. F $\subseteq$ Q is the set of accept states.

DEFINITION 1.16

A language is called a regular language if some finite automaton recognizes it.

DEFINITION 1.23

Let A and B be languages. We define the regular operations union, concatenation, and star as follows:

- **Union:** $A \cup B = \{x|x \in A \text{ or } x \in B\}$
- **Concatenation:** $A \circ B = \{x|x \in A \text{ and } x \in B\}$
- **Star:** $A^* = \ \{x_1x_2...x_k|k \geq 0 \text{ and each } x_i \in A\}$

THEOREM 1.25

The class of regular languages is closed under the union operation.

THEOREM 1.26

The class of regular languages is closed under the concatenation operation.

DEFINITION 1.37

A **nondeterministic finite automaton** is a 5-tuple(Q, $\sum$, δ, q_0, F), where

1. Q is a finite set of states,
2. $\sum$ is a finite alphabet,
3. $\delta : Q \times \sum_\epsilon \rightarrow \rho(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. F $\subseteq$ Q is the set of accept states.

THEOREM 1.39

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

COROLLARY 1.40

A language is regular if and only if some nondeterministic finite automaton recognizes it.

THEOREM 1.45

The class of regular languages is closed under the union operation.

THEOREM 1.47

The class of regular languages is closed under the concatenation operation

THEOREM 1.49

The class of regular languages is closed under the star operation

DEFINITION 1.52

Say that R is a **regular expression if** R is

1. a for some a in the alphabet $\sum$,
2. ε,
3. $\emptyset$,
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions or
6. (R_1^*), where R_1 is a regular expression.

In items 1 and 2, the regular expressions a and ε represent the languages $\{a\}$ and $\{\varepsilon\}$, respectively.

In item 3, the regular expression $\emptyset$ represents the empty language.

In items 4, 5, and 6, the expressions represent the languages obtained by taking the union or concatenation of the languages R1 and R2, or the star of the language R1, respectively.

THEOREM 1.54

A language is regular if and only if some regular expression describes it.

LEMMA 1.55

If a language is described by a regular expression, then it is regular

LEMMA 1.60

If a language is regular, then it is described by a regular expression.

DEFINITION 1.64

A **generalized nondeterministic finite automaton** is a 5-tuple (Q, $\sum$, δ, q_{start}, q_{accept}), where

1. Q is the finite set of states,
2. $\sum$ is the input alphabet,
3. $\delta : (Q - q_{accept}) \times (Q - q_{start}) \rightarrow R$ is the transition function,
4. q_{start} is the start state, and
5. q_{accept} is the accept state.

CLAIM 1.65

For any GNFA G, CONVERT(G) is equivalent to G.

THEOREM 1.70

Pumping lemma

If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0, xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

Proof of THEOREM 0.21

Every edge in G is connected to two nodes. Each edge contributes 1 to the degree of each node to which it is connected. Therefore, each edge contributes 2 to the sum of the degrees of all the nodes.

Hence, if G contains e edges, then the sum of the degrees of all the nodes of G is 2e, which is an even number

2 Sipser exercises

0.1

(a) Set of all odd natural numbers

(b) Set of all even integers

(c) Set of all even natural numbers

(d) Natural numbers divisible by two and three

(e) Set of all integers such that each number plus one equals the same number. This is an empty set.

0.2

(a) $\{n|n = 10^i \text{ and } i \in \{0, 1, 2\}\}$

(b) $\{n \mid n \in \mathbb{Z} \text{ and } n > 5\ \}$

(c) $\{n \mid n \in \mathbb{N} \text{ and } n < 5\ \}$

(d) $\{aba\}$

(e) $\{\varepsilon\}$

(f) $\{\}$

0.5

There are 2^c elements in the power set of C.

Since cardinality of power set is $2^{|c|}$ where c is the cardinality of set C.

0.6

$X \rightarrow \{1, 2, 3, 4, 5\}$

$Y \rightarrow \{6, 7, 8, 9, 10\}$

$f : X \rightarrow Y$

$G : X \times Y \rightarrow Y$

(a) $f(2) = 7$

(b) range(co domain) of f= $\{6, 7\}$ and domain of f= $\{1, 2, 3, 4, 5\}$

(c) $g(2, 10) = 6$

(d) range(co domain) of g= $\{6, 7, 8, 9, 10\}$ and domain of g= $\{(1, 6), (1, 7), (1, 8), (1, 9), (1, 10), (2, 6), (2, 7), (2, 8), (2, 9), (2, 10)$ $(3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10),\ (5, 6), (5, 7), (5, 8), (5, 9), (5, 10)\}$

(e) $g(4, f(4)) = g(4, 7) = 8$