3. Assignment – Robotics

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1. Drive Control and Parking Maneuver

https://github.com/frimuell/catkin_ws_user/tree/master/tasks_wise1819/ub03

(See video in the repo)

2. Homogenous Transformation Matrix

$$ {}^{B}_{A}T = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

3. Rotating a Vector using Quaternion Rotation and Axis-Angle Representation as Quaternion

Assignment 3

Exercise 3 - rotating a vector using quaternion rotation

$(0,0,1)$ = z-axis $\quad x = (2,0,0)$

$-\frac{3\pi}{2} = -270°$

rotating a vector (x,y,z) about the z-axis through an angle α

$$\underbrace{\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{z-axis rotation matrix}} \cdot \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\text{vector to rotate}} = \begin{pmatrix} x\cos\alpha \cdot y\sin\alpha \\ x\sin\alpha + y\cos\alpha \\ z \end{pmatrix}$$

here: rotated vector x': $\begin{pmatrix} 2\cos(270°) \\ 2\sin(-270°) \\ 0 \end{pmatrix} = \begin{pmatrix} 2\cos(90°) \\ 2\sin(90°) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

Exercise 3 - axis and angle correlation to a quaternion (w,x,y,z)

correlation: $\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) \\ x' \sin\left(\frac{\alpha}{2}\right) \\ y' \sin\left(\frac{\alpha}{2}\right) \\ z' \sin\left(\frac{\alpha}{2}\right) \end{pmatrix}$ where (x', y', z') = axis

here: $\alpha = 2\cos^{-1}(0{,}5) = 60°$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2\sin^{-1}(-0{,}5) \cdot \frac{1}{2} \\ 2\sin^{-1}(-0{,}5) \cdot \frac{1}{2} \\ 2\sin^{-1}(0{,}5) \cdot \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

4. normal vector calculation

Assignment 3

Exercise 4b - vector for the z-axis of ~~a~~ frame A

A is spun by $\vec{x} = \begin{pmatrix} -\sqrt{5} \\ \sqrt{5} \\ 0 \end{pmatrix}$ and $\vec{y} = \begin{pmatrix} \sqrt{5} \\ \sqrt{5} \\ 0 \end{pmatrix}$

z-axis vector = normal vector to the frame spun by $\vec{x}$ and $\vec{y}$ and is equal to their crossproduct

$$\vec{x} \times \vec{y} = \begin{pmatrix} 0 \cdot \sqrt{5} - 0 \cdot \sqrt{5} \\ 0 \cdot \sqrt{5} - -\sqrt{5} \cdot 0 \\ -\sqrt{5} \cdot \sqrt{5} - \sqrt{5} \cdot \sqrt{5} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$$

5. Driving two Circles

The radius of the circle on the track was 45 cm.

The radius of the circle on the carpet was about 10cm greater and rather difficult to measure due to the cars lack of grip on the carpet and thereby not being able to drive a proper circle.

(See videos in the repo)