Model of Income and Measurement error: Simplification

Setup

Assume that E_r and U_r are the earnings and Ubenefits of individuals, based on registered data. And for simplicity, we assume they are measured without error.

In this setup, we would observe 4 cases for the earnings/ubenefits of individuals.

Table 1: Earnings Setup

Case	Description	Prob
I	$E_r = 0 \& U_r = 0$	NA
II	$E_r > 0 \& U_r = 0$	p1
III	$E_r = 0 \& U_r > 0$	p2
IV	$E_r>0\&U_r>0$	1-p1-p2

For now, lets assume CASE I is uninteresting.

From the Survey perspective we could have multiple cases:

- S1: Data is reported without error
- S2: Earnings are reported with error but Ubenefits are reported without error
- S3: Earnings are witout error but Ubenefits are reported with error
- S4: Both earnings and Ubenefits are reported with error

S1:
$$E_s = E_r$$
 and $U_s = U_r$

S2:
$$E_s = E_r + _1$$
 and $U_s = U_r$

S3:
$$E_s = E_r$$
 and $U_s = U_r + \epsilon_2$

S4:
$$E_s = E_r + \epsilon_3$$
 and $U_s = U_r + \epsilon_4$

Im using different ϵ_k for now.

S1: Combined with Cases of R

S1 & CII CIII CIV:
$$E_s = E_r$$
 and $U_s = U_r$ with p_s

S2: Combined with Cases of R

S2 & CII : - $ln(E_s)=ln(E_r)+\epsilon_2$ and and $U_s=0$ with probability p_ss2 - $E_s=0$ and and $U_s=0$ with probability $1-p_ss2$

S2 & CIII : -
$$ln(E_s) = \epsilon_1$$
 and and $ln(U_s) = ln(U_r)$

S2 & CIV : - $ln(E_s)=ln(E_r)+\epsilon_2$ and and $ln(U_s)=ln(U_r)$ with probability p_ss2 - $E_s=0$ and and $U_s=0$ with probability $1-p_ss2$

 ϵ_1 is similar to Mismatch, because people declare some earnings, even tho they dont have any $(E_r = 0)$.

 ϵ_2 is just measurement error.

 p_s2 is the probability of people declaring earnings, and 1-ps2 is the probability of people not declaring earnings.

S3: Combined with Cases of R

S3 & CII:

- $E_s = E_r$ and and $ln(U_s) = \lambda_1$

S3 & CIII : - $E_s=0$ and and $ln(U_s)=ln(U_r)+\lambda_2$ with probability p_ss3 - $E_s=0$ and and $U_s=0$ with probability $1-p_ss3$

S3 & CIV : - $E_s=E_r$ and and $ln(U_s)=ln(U_r)+\lambda_2$ with probability p_ss3 - $E_s=E_r$ and and $U_s=0$ with probability $1-p_ss3$

 λ_1 is similar to Mismatch, because people declare some Ubenefits even tho they dont have any $(U_r = 0)$. The caveat, we cannot identify cases here Registered UB data is positive, but Survey UB data is 0. But we can just add a Probability of this happening.

S4: Combined with Cases of R

This is the only one with very interesting cases.

S4 & CII:
$$-ln(E_s) = ln(E_r) + \epsilon_2$$
 and $ln(U_s) = \lambda_1 - E_s = 0$ and $ln(U_s) = \lambda_1$

S4 & CIII :
$$-ln(E_s) = \epsilon_1$$
 and $ln(U_s) = ln(U_r) + \lambda_2 - ln(E_s) = \epsilon_1$ and $U_s = 0$

S4 & CIV : -
$$ln(E_s) = ln(E_r) + \epsilon_2 + \gamma$$
 and $ln(U_s) = ln(U_r) + \lambda_2 - \gamma$ - $ln(E_s) = ln(E_r) + \epsilon_2 + \gamma$ and $U_s = 0$ - $E_s = 0$ and $ln(U_s) = ln(U_r) + \lambda_2 - \gamma$ - $E_s = 0$ and $U_s = 0$

$$\begin{split} &\text{II} \mid E_r > 0 \& U_r = 0 \mid p1 \mid \\ &\text{III} \mid E_r = 0 \& U_r > 0 \mid p2 \mid \\ &\text{IV} \mid E_r > 0 \& U_r > 0 \mid 1 - p1 - p2 \end{split}$$

S4:
$$E_s=E_r+\epsilon_3$$
 and $U_s=U_r+\epsilon_4$