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# The two-way Mundlak estimator

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**ABSTRACT**
Mundlak shows that the fixed effects estimator is equivalent to the random effects estimator in the one-way error component model once the random individual effects are modeled as a linear function of *all* the averaged regressors over time. In the spirit of Mundlak, this paper shows that this result also holds for the two-way error component model once the individual and time effects are modeled as linear functions of *all* the averaged regressors across time and across individuals. Wooldridge also shows that the two-way fixed effects estimator can be obtained as a pooled OLS with the regressors augmented by the time and individual averages and calls it the two-way Mundlak estimator. While Mundlak used GLS rather than OLS on this augmented regression, we show that both estimators are equivalent for this augmented regression. This extends Baltagi's results from the one-way to the two-way error component model. The *F* test suggested by Mundlak to test for this correlation between the random effects and the regressors generate a Hausman type test that is easily generalizable to the two-way Mundlak regression. In fact, the resulting *F*-tests for the two-way error component regression are related to the Hausman type tests proposed by Kang for the two-way error component model.

**KEYWORDS**
Fixed and random effects; Hausman test; Mundlak regression; panel data; two-way error components model

**JEL CLASSIFICATION**
C33

## 1. Introduction

Many panel regressions have time-invariant variables as well as individual-invariant variables.[1] It is well known that the two-way fixed effects estimator wipes out these variables from the regression and does not provide their marginal effects on the dependent variable. In many empirical applications, these variables are important for policy decisions. For example, the effect of common language or distance on bilateral trade in a gravity trade model of countries over time, or the effect of gender or race on earnings in a Mincer wage equation. The advantages of the fixed effects estimator is that it wipes out any endogeneity between the regressors and the individual and time effects.

Mundlak (1978) presented this idea clearly using the one-way error component regression where the individual random effects are modeled as a linear function of *all* the regressors averaged over time. Mundlak explained that once this correlation of the random individual effects with the regressors is taken into account the random effects estimator obtained using GLS becomes the fixed effects estimator. The Within estimator can be obtained using least squares with individual dummy variables or the computationally simpler but equivalent Within transformation, see Baltagi (2021, Chapter 2).

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Many panel data regressions with large $N$ and large $T$ such as stock prices over time, or macro panel data across countries, or marketing data of household purchases over time use a two-way error component model. These can be fixed effects regressions with panel data of countries over time, or random effects regressions such as stock prices over time. As in the Mundlak (1978) one-way error component model, the two-way fixed effects regression allows the time and individual effects to be hopelessly correlated with *all* the regressors, whereas, the two-way random effects regression assumes that they are uncorrelated with *all* the regressors. The latter is a questionable assumption in economics and is usually tested using a Hausman (1978) test. Hausman's test was applied to the one-way error component model and was extended to the two-way error component model by Kang (1985).

The first contribution in this paper shows that Mundlak's (1978) result extends from the one-way to the two-way error component model. In fact, modeling the time effects as a linear function of *all* the averaged regressors over individuals, and the individual effects as a linear function of *all* the averaged regressors over time, this paper shows that the two-way random effects GLS estimator is equivalent to the two-way fixed effects estimator. The latter can be obtained by a least squares regression with time as well as individual dummies or a two-way Within estimator, especially when $N$ and $T$ are large, see Baltagi (2021, Chapter 3). This result has been alternatively derived by Wooldridge (2021) who showed that the two-way fixed effects estimator can also be obtained by running OLS on the original regression augmenting it with averaged regressors over time as well as averaged regressors across individuals. Wooldridge named this the two-way Mundlak (TWM) estimator. But Mundlak used GLS, Wooldridge uses OLS. Here we show these are the same.

For the one-way error component model, Mundlak (1978) ran GLS rather than OLS on this augmented regression as we review below, but Baltagi (2006) showed that OLS is equivalent to GLS for the one-way Mundlak augmented regression. This paper shows that Mundlak's result using GLS also holds for the two-way random error component model augmented with time and individual averages of *all* the regressors. In Mundlak's model, if the augmented regressors are insignificant, so that the individual and time effects are not correlated with the regressors, the resulting estimator will be a two-way random effects GLS estimator. The result is proven concisely using matrix algebra. This is in the spirit of Mundlak's (1978) original paper which shows that random effects estimator is no different from the fixed effects estimator if you allow correlation between the individual effects and all the regressors through their averages. This extends the Baltagi (2006) result from the one way to the two way error component model.

This augmented regression allows one to test the correlation between the regressors and the individual and time effects. In fact, Mundlak (1978) suggested a simple $F$-test for the significance of these augmented average regressors for the one-way model. It turns out that the Mundlak $F$-test is a Hausman (1978) type test. The second contribution of this paper shows that this $F$-test naturally extends to the two-way Mundlak augmented regression and one can test for correlation of the regressors with the individual and time effects. This allows us to relate several of these $F$-tests based on the augmented two-way Mundlak regression to the two-way error component Hausman type tests considered by Kang (1985).

Section 2 discusses the one-way and two-way Mundlak augmented regressions and derives the equivalence of GLS and OLS for the augmented two-way Mundlak regression. An $F$-test for the significance of the averaged regressors in this augmented regression, as in Mundlak (1978), allows one to compute a Hausman test for the correlation between these effects and the regressors. Section 3 concludes.

## 2. The Mundlak (1978) one-way and two-way regressions

Mundlak (1978) considered the one-way error component panel data regression

$$y = X\beta + Z_\mu \mu + \nu \tag{1}$$

where $y' = (y_{11}, \ldots, y_{1T}, y_{21}, \ldots, y_{2T}, \ldots, y_{N1}, \ldots, y_{NT})$ is $1 \times NT$, $X$ is $NT \times K$, $Z_\mu = I_N \otimes \iota_T$. $I_N$ is an identity matrix of dimension $N$, and $\iota_T$ is a vector of ones of dimension $T$. $\nu_{it} \sim$ IIN $(0, \sigma_\nu^2)$. Mundlak modeled the individual effects as follows:

$$\mu_i = \bar{X}_{i.}' \pi_\mu + \epsilon_i \tag{2}$$

where $\epsilon_i \sim$ IIN $(0, \sigma_\epsilon^2)$ and $\bar{X}_{i.}'$ is $1 \times K$ vector of observations on the explanatory variables averaged over time. In other words, Mundlak assumed that the individual effects are a linear function of the averages of *all* the explanatory variables across time. These individual effects are uncorrelated with the explanatory variables if and only if $\pi_\mu = 0$, otherwise this is a correlated random effects regression. Mundlak (1978) assumed, without loss of generality, that the $X$'s are deviations from their sample mean. In vector form (2) can be written as follows:

$$\mu = Z_\mu' X \pi_\mu / T + \epsilon \tag{3}$$

where $\mu' = (\mu_1, \ldots, \mu_N)$, and $\epsilon' = (\epsilon_1, \ldots, \epsilon_N)$. Substituting this auxiliary regressions for $\mu$, defined in (3), into the one-way error component regression defined in (1), one gets

$$y = X\beta + P_\mu X \pi_\mu + (Z_\mu \epsilon + \nu) \tag{4}$$

where $P_\mu = Z_\mu Z_\mu'/T = I_N \otimes \bar{J}_T$, and $\bar{J}_T = J_T/T$ is the averaging matrix over time, with $J_T = \iota_T \iota_T'$ denoting a matrix of ones of dimension $T$. Mundlak showed that GLS on this correlated random effects regression (4) yields the fixed effects estimator for $\beta$. Hence the random effects estimator for the one-way error component regression reduces to the fixed effects estimator once the correlated random individual effects are modeled as a linear function of *all* the regressors averaged over time.

Mundlak suggested an $F$-test for $\pi_\mu = 0$ to determine whether the random effects are correlated with the regressors in (3). Mundlak (1978) applied GLS to this regression because the disturbances have a random one-way error component variance-covariance structure even after including the time averaged regressors $P_\mu X$ with typical element $\bar{X}_{i.}$. Baltagi (2006) showed that GLS on this regression (4) is equivalent to OLS. The OLS estimator of $\beta$, the coefficients of $X$ yield the one-way fixed effects estimator or Within estimator denoted by $\tilde{\beta}_{fe}$. This is also known as the Within estimator because it can be obtained from a regression of $(y_{it} - \bar{y}_{i.})$ on $(x_{it} - \bar{x}_{i.})$ for every regressor in $X$, see Baltagi (2021). A test for the significance of the coefficients of $\bar{X}_{i.}$ is a test for $\pi_\mu = 0$, and not rejecting the null means that the random effects are not correlated with the regressors. In this case, GLS for the random effects is the efficient estimator. The OLS estimator of $\pi_\mu$ turns out to be $\hat{\pi}_\mu = (\hat{\beta}_{between} - \tilde{\beta}_{fe})$ which is a Hausman (1978) test for testing whether $E(\mu_i | X_{it}) = 0$. $\hat{\beta}_{between}$ is the between estimator based on the time averaged regression of $\bar{y}_{i.}$ on $\bar{X}_{i.}$. Not rejecting the null means that the random effects estimator is efficient. Rejecting the null does not mean that the fixed effects estimator is efficient. However, empirical researchers settle on reporting it as their preferred estimator when the null is rejected. This is not at odds with the Mundlak (1978) idea that once the correlation of the random effects with *all* the regressors through their time mean is accounted for, the random effects estimator of $\beta$ becomes the fixed effects estimator.

Hausman and Taylor (1981) showed that this Hausman test can also be performed using other contrasts like $(\hat{\beta}_{re} - \tilde{\beta}_{fe})$ or $(\hat{\beta}_{re} - \hat{\beta}_{between})$. Here, $\hat{\beta}_{re}$ is the random effects estimator based on GLS on (4) assuming $\pi_\mu = 0$. Hausman and Taylor (1981) also argued that not *all* the right hand side regressors have to be correlated with the random individual effects as in the Mundlak regression and proposed an alternative instrumental variable GLS estimator based on which regressors are not correlated with the individual effects. They also provided Hausman (1978) type tests for the choice of these non-correlated regressors. Hausman and Taylor (1981) allow one to estimate time-invariant variables wiped out by the fixed effects estimator. Empirical applications using the Hausman and Taylor (1981) estimator include Cornwell and Rupert (1988) who estimated a wage equation especially the effects of time-invariant variables like education, race and gender. Egger and Pfaffermayr (2004) who estimated the effects of distance on trade and foreign direct investment. Serlenga and Shin (2007) who estimated a gravity equation of bilateral trade flows finding that country-specific variables, like common language have a large effect on bilateral trade.

The two-way Mundlak regression was recently proposed by Wooldridge (2021). Using simple derivations, Wooldridge showed that once *all* the time averaged regressors and *all* the individual averaged regressors are included in the regression along with the original regressors, the resulting OLS estimator of $\beta$ is the two-way fixed effects or Within estimator. Here, in the spirit of Mundlak (1978) we show that random effects and fixed effects for the two-way error component model are the same once we model the individual effects as linear function of the time averaged regressors and the time effects as a linear function of the individual averaged regressors. To show this result, consider the two-way error components model given in Baltagi (2021, Chapter 3) as:

$$y = X\beta + Z_\mu \mu + Z_\lambda \lambda + \nu \tag{5}$$

with $\lambda' = (\lambda_1, \ldots, \lambda_T)$ and $Z_\lambda = \iota_N \otimes I_T$. In addition to modeling the individual effects as in Mundlak (1978), see equation (3) above, we model the time effects as follows:

$$\lambda_t = \bar{X}'_{.t}\pi_\lambda + \varepsilon_t \tag{6}$$

where $\varepsilon_t \sim$ IIN $(0, \sigma_\varepsilon^2)$ and $\bar{X}'_{.t}$ is $1 \times K$ vector of observations on the explanatory variables averaged over individuals. In other words, this assumes that the time effects are a linear function of *all* the explanatory variables averaged across individuals. These time effects are uncorrelated with the explanatory variables if and only if $\pi_\lambda = 0$. In vector form (6) can be written as follows:

$$\lambda = Z'_\lambda X \pi_\lambda / N + \varepsilon \tag{7}$$

where $\varepsilon' = (\varepsilon_1, \ldots, \varepsilon_T)$. Substituting the auxiliary regressions for $\mu$ and $\lambda$ (defined in (3) and (7)), into the regression defined in (5), one gets

$$y = X\beta + P_\mu X \pi_\mu + P_\lambda X \pi_\lambda + (Z_\mu \epsilon + Z_\lambda \varepsilon + \nu) \tag{8}$$

where $P_\lambda = \bar{J}_N \otimes I_T$, and $\bar{J}_N = J_N/N$ is the averaging matrix over individuals, with $J_N = \iota_N \iota'_N$ denoting a matrix of ones of dimension $N$. Using the fact that the $\epsilon$, $\varepsilon$ and $\nu$ are uncorrelated, the two-way error component term in parentheses has zero mean and variance-covariance matrix

$$\Omega = E(Z_\mu \epsilon + Z_\lambda \varepsilon + \nu)(Z_\mu \epsilon + Z_\lambda \varepsilon + \nu)' = \sigma_\epsilon^2 (I_N \otimes J_T) + \sigma_\varepsilon^2 (J_N \otimes I_T) + \sigma_\nu^2 I_{NT} \tag{9}$$

We apply the Wansbeek and Kapteyn (1982) trick by replacing $J_N$ by $N\bar{J}_N$, $I_N$ by $E_N + \bar{J}_N$, $J_T$ by $T\bar{J}_T$ and $I_T$ by $E_T + \bar{J}_T$ and collect terms with the same matrices. In Wansbeek and Kapteyn's notation, $E_N = I_N - \bar{J}_N$ is by definition the deviations from mean matrix, with $E_N$ and $\bar{J}_N$ being symmetric and idempotent and summing to the identity matrix $I_N$. This gives

$$\Omega = \sum_{i=1}^{4} w_i Q_i \tag{10}$$

where $w_1 = \sigma_\nu^2, w_2 = T\sigma_\epsilon^2 + \sigma_\nu^2, w_3 = N\sigma_\varepsilon^2 + \sigma_\nu^2$ and $w_4 = T\sigma_\epsilon^2 + N\sigma_\varepsilon^2 + \sigma_\nu^2$. Correspondingly, $Q_1 = E_N \otimes E_T, Q_2 = E_N \otimes \bar{J}_T = P_\mu - (\bar{J}_N \otimes \bar{J}_T), Q_3 = \bar{J}_N \otimes E_T = P_\lambda - (\bar{J}_N \otimes \bar{J}_T)$, and $Q_4 = \bar{J}_N \otimes \bar{J}_T$, respectively. The $w_i$ are the distinct characteristic roots of $\Omega$ and the $Q_i$ are the corresponding matrices of eigenprojectors. $w_1$ is of multiplicity $(N-1)(T-1)$, $w_2$ is of multiplicity $(N-1)$, $w_3$ is of multiplicity $(T-1)$ and $w_4$ is of multiplicity 1. Each $Q_i$ is symmetric and idempotent with its rank equal to its trace. Moreover, the $Q_i$ are pairwise orthogonal and sum to the identity matrix. The advantages of this spectral decomposition are that

$$\Omega^r = \sum_{i=1}^{4} w_i^r Q_i \tag{11}$$

where $r$ is an arbitrary scalar. This obtains the inverse $\Omega^{-1}$ for $r = -1$, and $\Omega^{-1/2}$ for $r = -1/2$.

Rewrite $X$ as $(\sum_{i=1}^{4} Q_i)X$, since $\sum_{i=1}^{4} Q_i = I_{NT}$ and collect like terms, one gets

$$y = Q_1 X\beta + P_\mu X(\pi_\mu + \beta) + P_\lambda X(\pi_\lambda + \beta) + (Z_\mu \epsilon + Z_\lambda \varepsilon + \nu) \tag{12}$$

where we used the fact that the $X's$ are deviations from their sample mean, so that $(\bar{J}_N \otimes \bar{J}_T)X = 0$. Note that $Q_1 X, P_\mu X$ and $P_\lambda X$ are orthogonal to each other, since $E_T \bar{J}_T = E_N \bar{J}_N = 0$ and $P_\mu P_\lambda X = (\bar{J}_N \otimes \bar{J}_T)X = 0$. Shortly, we will prove that OLS on this regression (12) is equivalent to GLS. In fact, OLS on (12) yields

$$\hat{\beta}_{OLS} = (X'Q_1X)^{-1}X'Q_1 y = \tilde{\beta}_w \tag{13}$$

$$(\widehat{\pi_\mu + \beta})_{OLS} = (X'P_\mu X)^{-1}X'P_\mu y = \hat{\beta}_{between1} \tag{14}$$

$$(\widehat{\pi_\lambda + \beta})_{OLS} = (X'P_\lambda X)^{-1}X'P_\lambda y = \hat{\beta}_{between2} \tag{15}$$

where $\tilde{\beta}_w$ is the two-way Within estimator which runs $\tilde{y} = Q_1 y$ with typical element $\tilde{y}_{it} = (y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{y}_{..})$ on $\tilde{X} = Q_1 X$ with typical element $(x_{it} - \bar{x}_{i.} - \bar{x}_{.t} + \bar{x}_{..})$ for every regressor in $X$, see Baltagi (2021, Chapter 3). $\hat{\beta}_{between1}$ is the between estimator running OLS on the averaged equation over time, while $\hat{\beta}_{between2}$ is the between estimator running OLS on the averaged equation over individuals. This means that $\hat{\pi}_\mu = \hat{\beta}_{between1} - \tilde{\beta}_w$. Also, $\hat{\pi}_\lambda = \hat{\beta}_{between2} - \tilde{\beta}_w$. A joint test for $H_0^a; \pi_\mu = \pi_\lambda = 0$ would test jointly whether there is correlation between the averaged regressors (over time and over individuals) with the time and individual effects. This can be done with an $F$ test from the OLS regression of $y$ on $\tilde{X}$, $\bar{X}_{i.}, \bar{X}_{.t}$, see Wooldridge (2021). For the two-way Mundlak extension, the test for $H_0^a$; $\pi_\mu = \pi_\lambda = 0$ is a joint test of two between estimators (one averaging across time and one averaging across individuals), being equal to a two-way Within estimator. Note that one can also test $H_0^b; \pi_\mu = 0$; as well as $H_0^c; \pi_\lambda = 0$; using the $F$ tests. These are conditional $F$-tests based on testing $\pi_\mu = 0$ assuming $\pi_\lambda \neq 0$. Also, testing $\pi_\lambda = 0$ assuming $\pi_\mu \neq 0$. Having rejected $H_0^a$, this may shed some light on whether this is due to correlated individual effects or correlated time effects, or both. This adds insight to the Hausman tests for the two-way error component model studied by Kang (1985). In fact, Kang (1985) suggested a Hausman test assuming the $\mu_i's$ fixed and testing $E(\lambda_t|X_{it}) = 0$ based on $\hat{\beta}_{between2} - \tilde{\beta}_w$. Similarly, Kang suggested a Hausman test assuming the $\lambda_t's$ fixed and testing $E(\mu_i|X_{it}) = 0$ based on $\hat{\beta}_{between1} - \tilde{\beta}_w$. In the context of the two-way Mundlak regression, this corresponds to testing $H_0^b; \pi_\mu = 0$ and $H_0^c; \pi_\lambda = 0$, respectively. Note that the equivalence of the Hausman test based on $\hat{\beta}_{between} - \tilde{\beta}_{fe}$ for the one-way model with that based on $\hat{\beta}_{re} - \tilde{\beta}_{fe}$ and $\hat{\beta}_{re} - \hat{\beta}_{between}$ derived by Hausman and Taylor (1981) no longer hold for the two-way model as clear from the existence of two between estimators and made clear by the Mundlak two-way regression. In fact, Kang (1985) suggested a Hausman test based on $\hat{\beta}_{re} - \tilde{\beta}_w$ for the two-way error component model where $\tilde{\beta}_w$ assumes both the $\mu_i's$ and $\lambda_t's$ are fixed, whereas $\hat{\beta}_{re}$ assumes that $E(\mu_i|X_{it}) = E(\lambda_t|X_{it}) = 0$ and is based on GLS on (8) assuming $\pi_\mu = \pi_\lambda = 0$.

**Theorem.** *The OLS estimator is equivalent to the GLS estimator for the two-way Mundlak regression in (12) and there is no need to run GLS to get the TWM estimator despite the two-way error component variance covariance matrix.*

*Proof.* Using Zyskind's (1967) necessary and sufficient condition for the equivalence of OLS and GLS, one has to show that $P_Z\Omega = \Omega P_Z$ where $Z = [Q_1X, P_\mu X, P_\lambda X]$ is the matrix of regressors in (12) and $P_Z = Z(Z'Z)^{-1}Z'$ with $\Omega = \sum_{i=1}^{4} w_iQ_i$ from (10). The regressors are orthogonal, so

$$P_Z = P_{Q_1X} + P_{P_\mu X} + P_{P_\lambda X}$$

and

$$P_Z\Omega = w_1P_{Q_1X} + w_2P_{P_\mu X} + w_3P_{P_\lambda X} = \Omega P_Z$$

This uses the properties of the $Q_i's$. For example, $P_{Q_1X}\Omega = w_1P_{Q_1X}$ since $Q_1\Omega = w_1Q_1$; also $X'P_\mu\Omega = w_2X'P_\mu$ and $X'P_\lambda\Omega = w_3X'P_\lambda$.

Zyskind's (1967) necessary and sufficient condition is satisfied and we do not need to run GLS on (12). In fact, OLS suffices and yields the estimators in (13) to (15). This extends Baltagi's (2006) results from the one-way to the two-way Mundlak augmented regression. This also shows that the TWM estimator derived by Wooldridge (2021) can be computed using OLS rather than GLS as Mundlak proposed.

The asymptotics for the two-way error component model were carried out by Wallace and Hussain (1969) who found that when the regressors are exogenous, both the random effects GLS and fixed effects Within estimators for the two-way error component model are (i) asymptotically normal, (ii) unbiased and consistent and that (iii) the random effects GLS estimator has a smaller generalized variance (i.e., more efficient) in finite samples. However, both estimators have equivalent asymptotic variance-covariance matrices, as both $N$ and $T \to \infty$. Hansen (2007) also showed that the two-way fixed effects estimator is consistent and asymptotically normal without strict exogeneity, as both $N$ and $T \to \infty$, with independent sampling in the cross-sectional dimension and weak dependence in the time-series dimension.

Wooldridge (2021) showed the usefulness of this two-way Mundlak regression for event studies allowing for considerable heterogeneity as well as relaxing the common trends assumption. Yang (2022) derived similar results for the correlated random effects model with multiple fixed effects. For unbalanced panels, excellent readings on the one-way Mundlak (1978) and Chamberlain (1982) reduced form regressions are given by Wooldridge (2019) and Abrevaya (2013). For an extension of the one-way Mundlak augmented regression to spatial panel data, see Debarsy (2012).

## 3. Conclusion

This paper shows that Mundlak's (1978) results for the one-way error component model with individual effects carry over to the two-way error component model with time and individual effects. In other words, the random and fixed effects estimators for the two-way model are the same once the random effects are modeled as a linear function of all the regressors averaged over time and over individuals. Wooldridge (2021) showed that the two-way fixed effects estimator is the same as pooled OLS on this augmented regression with time and individual averages of the regressors. He named the latter estimator TWM estimator. Mundlak applied GLS rather than OLS to this augmented regression, but we show in this paper that OLS is equivalent to GLS. This paper also shows that the $F$-tests for testing the significance of the augmented averages generate Hausman type tests as in the one-way Mundlak paper. We relate these to the Hausman tests for the two-way error component model proposed by Kang (1985).

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