2. The role of Controls on 2x2 Canonical DID

The standard 2x2 DID estimator described above is able to identify the ATT under the unconditional parallel trends assumption (UPTA) if the control and treated groups have similar characteristics, or if the change in outcome does not change systematically across characteristics. If this is not the case, the canonical DID cannot identify the ATT. To address this case, Sant’Anna and Zhao (2020) impose what is known as the conditional parallel trend assumption. This assumption states that PTA holds but only when considering groups with the same characteristics:

Where is the expected outcome conditional on observed characteristics .

In other words, as described in Sant’Anna and Zhao (2020), the role of characteristics is to balance differences between treatment and control groups, such that we can predict the outcome change for the treatment group, using data from the control group.

Under this conditional PTA, the DID estimator becomes:

Where is the expected change in the outcome for treated units, absent of treatment, and after accounting for the distribution of characteristics in the treated group.

The Sant’Anna and Zhao (2020) emphasize that should be a vector of pre-treatment characteristics. In principle, these allows for time varying characteristics, as long as the variation is strictly exogenous. Otherwise, time varying variables are bad controls, and introduce a bias in the estimator, also known as a collider variable bias (Cunninham, 2020).

To better understand the role of controls in the DID framework stated above, we can modify our data generating process as follows. Assume that the outcome for control units at time is a linear function of observed characteristics which can be time constant or time varying, and some idiosyncratic error :

In the panel data case, the observed outcome change for unit can be written as follows:

This shows that the changes in the outcome , absent of treatment, is explained by changes in returns to characteristics , or by changes in characteristics . From the identification point of view, this equation also suggests that the parameters and can only be identified if , or more specifically , is exogenous to and . This assumption is reasonable in the canonical 2x2 DID design, in absence of any direct or indirect spillover effects in the control group.

The main problem occurs if the strict exogeneity assumption of does not hold in the treatment group. This could happen if the treatment effect has a direct impact on characteristics X (treatment effects), or if there are indirect effects, such as units changing characteristics X so they qualify to receive treatment. If this happens, controlling for time varying characteristics will bias the estimation of , because part of the treatment effect will be incorrectly accounted by the changes in observed characteristics, when is estimated in the treated sample.

To abstract from this problem, the panel data estimators proposed in Sant’Anna and Zhao (2020) treat all covariates as time constant, by only using information from the pre-treatment period in the estimators. For the case of repeated cross-section, they follow Abadie (2005) and assume all covariates are stationary. This implies that either all controls should be consider as time fixed within unit, or that the expected change in characteristics across groups is zero.

3. DID Estimators:

Based on the framework of Sant’Anna and Zhao (2020), three types of DID estimators are discussed when using panel data and repeated crossection. These are the outcome regression approach (OR), Reweighted-Inverse Probability approach (IPW), and doubly robust estimator (DR). All these strategies aim to provide different estimators for counterfactual outcome for treated units at time , as if they never received treatment, .

Under the conditional PTA, an approximation for this is given by:

Thus, different estimators proposed and discussed in Sant’Anna and Zhao (2020) come from different alternatives of how can be approximated. For simplicity, we refer to the panel-data estimators here.

**Outcome Regression Approach**

This approach proposes using a linear regression model to obtain an estimate for , using data for the control units only. This is done by estimating the following regression:

Afterwards, is estimated by substituting with the predicted outcome . Thus the OR estimator for the ATT becomes:

The properties of this estimator depend on the correct model specification of equation XX

**Inverse Probability Weighting**

This approach proposes to use a reweighted strategy to obtain an estimatefor , and estimate the ATT. This is done by reshaping the distribution of characteristics of the control units, so they more closely resemble that of the treated units, and use a weighted average of among the control units as the approximation for

The first step is to estimate a propensity score using a binomial model, for example a logit, where the dependent variable is an indicator that a unit belongs to the treated group, as a function of pretreatment characteristics:

Using the predicted scores, , define the Inverse probability weights as:

Then, the approximation for the change in the outcome in absence of treatment is defined as:

This the estimator becomes

Double Robust Estimator