Marginal Unit Interpretation of Unconditional Quantile Regression  
and Recentered Influence Functions

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*Abstract -* When implementing Unconditional Quantile Regression as outlined in Firpo, Fortin and Lemieux (2009), one generates a Recentered Influence Function equalling the statistic in question (quantile value, distributional measure etc) or functional v(F) and an Influence Function (IF), which by definition has expected zero mean (E(IF)=0). Until now it has been difficult to isolate cleanly the functional v(F) from the IF. This paper combines RIF regressions with a linear combinations of dummy variable coefficients as outlined in Haisken-DeNew and Schmidt (1997) and centering of continuous variables to isolate exactly the v(F) of any RIF regression as the constant, and allowing interpretation of dummy variable coefficients as 1-percentage point increases in the respective dummy variable means from the weighted average (the functional v(F)) and continuous variable coefficients as a 1%-(native- unit increase. All dummy and continuous variables combined produce exactly the IF, cleanly separated from v(F), the constant. This methodology generalises to any RIF based regression using any functional v(F), keeping true to the nature of the IF, as being the infinitesimally small influence of explanatory variables on the IF. This is implemented straightforwardly in Stata, using this methodology.

JEL Codes: C21, C23, D63  
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1. Introduction
2. Methodology

The IF represents the re-scaled effect that a change in the distribution from has on statistic *v*, when the change is infinitesimally small, such as in for infinitesimally small increments of .

This leads to the following RIF formulation for Unconditional Quantile Regression

Firpo et al., (2009) write, “An important property of an IF is that its expectation is zero (e.g. an observation equal to the mean has no influence on the mean). The minor transformation of the IF into a RIF re-centers the IF so that its expectation is equal to the original distributional statistic v(F) ... This characteristic implies that the mean value of the RIF is equal to the statistic.”

Thus we need an effective method to introduce infinitesimally small changes into the IF. Surprisingly, we have a method to achieve that goal. Haisken-DeNew and Schmidt (1997) introduced Restricted Least Squares as a post-estimation linear combination of all dummy variable sets and the constant. The restriction implemented is weighted sum of the coefficients of each dummy variable set identically equal 0. This is implemented after running a standard regression of *K-1* dummies, dropping the *K*th dummy as the reference. Here for example, a linear regression, using the 4 regions on the map, is run using *North*, *South* and *East*, whilst *West* is dropped due to perfect collinearity. By augmenting the with an additional element, we regain the *West* coefficient after the transformation with the weighting matrix *W* indicating a matrix of the sample shares of the respective dummy variables.

The coefficient vector and variance-covariance matrix , now include the constant, adjusted for the deviations from the weighted average of the example dummy set (N, S, E, W). With any additional centered continuous variables, exposes the constant as being exactly the or . Collectively the linear combination of all centered continuous variables and all dummy variable sets comprise the , identically equal 0 in sum.

Fernando Rios-Avila (2020) writes something about 1 percentage point increases in the share of a dummy variable as opposed to changes from the weighted average to full 100% of the dummy variable.

We transform the coefficients one step further to allow for a 1 percentage point increase in the share of a dummy variable, as opposed to going from the mean to full 100% of the dummy variable, multiplying the coefficient vector and variance-covariance matrix , by 1/(1-), which is the respective dummy sample share, to arrive at the coefficient vector and variance-covariance matrix , both of which include the constant.

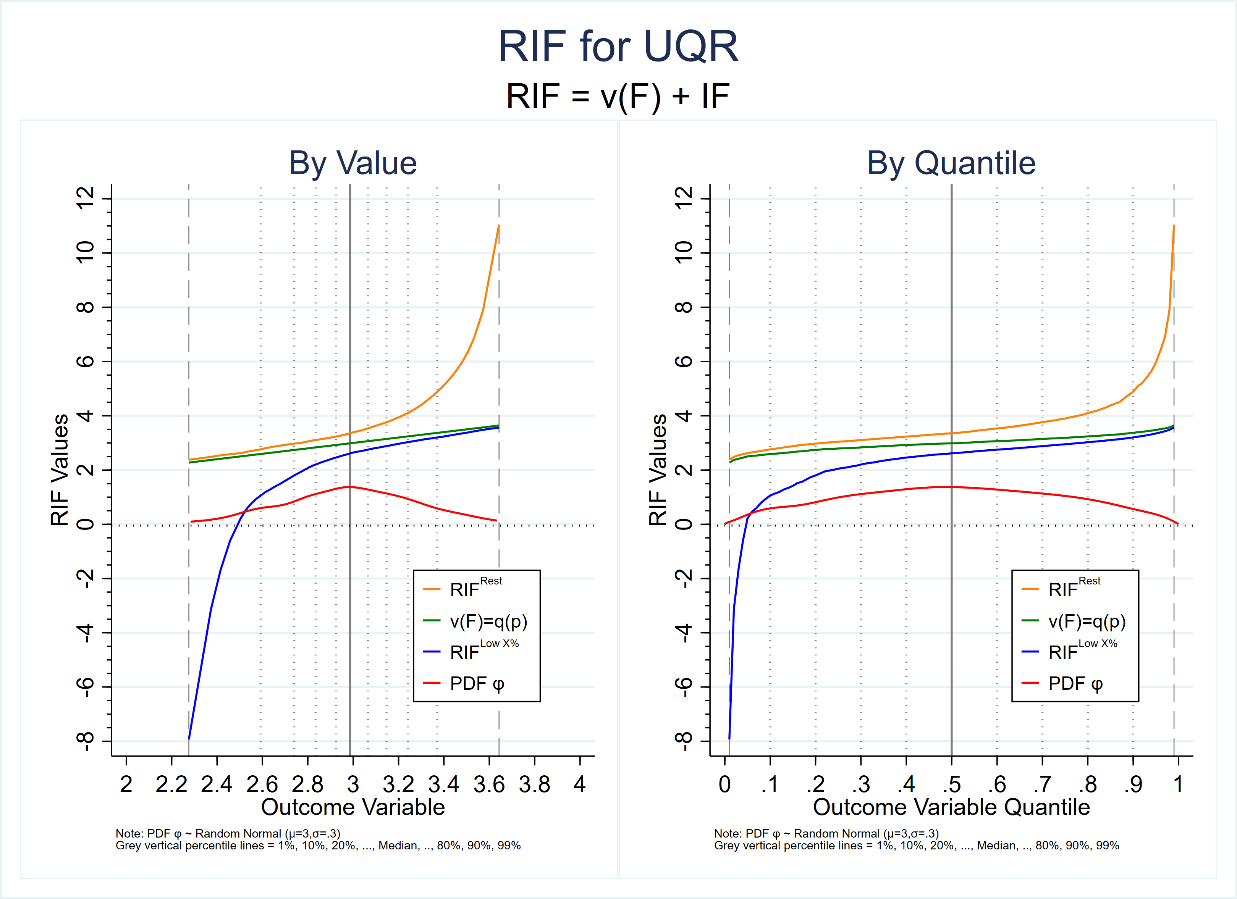
where the new weighting matrix TT for dummy variables is given by:

Finally, we are left with the issue of assessing the overall importance of note individual dummy variable coefficients, but summarizing entire groups of dummies. For a continuous variable, the simple magnitude of the single coefficient indicates the contribution that that variable has to the outcome variable, or in this case the Influence Function (IF). However, what happens with dummy variable sets? Does on examine the largest coefficient in the set, or the smallest? Haisken-DeNew and Schmidt (1997) answer this directly by focusing on the overall weighted standard deviation of all coefficients in a dummy variable set, assuming the coefficients have been transformed first in a manner suggested above. For regression equations containing more than one dummy variable set, is an optimal way of comparing.

The first component of before the minus sign contains the variation in the estimated coefficients and the second component contains the OLS sampling errors in variance-covariance matrix. Further, an F-test can be run for the collective significance of all coefficients in the dummy variable set (for all *k*; ) with degree of freedom (*K-1*, *N-K-1*) where N is the number of observations and K is the number of dummies in the dummy variable set.

In the following Figure 1, we plot the RIF function for all percentiles of a variable randomly distributed with a corresponding probability density function shown in red. The vertical grey lines bound the density at the 1 percentile, the 10th percentile, the 20th percentile, … the median, the 60th percentile, … 90th percentile and finally the 99th percentile. At each percentile, there are 2 distinct values for the RIF, which on average produce the functional or as . The lower of the two values is given by the blue line, which for any percentile, takes on the value and the lowest are assigned this value. The remaining (100- are assigned the higher of the two values, , given by the orange line. Thus, the weighted sum or the two values is identically equal to the or . From our derivation above, the green line is the constant found in and the remaining coefficients from the centered continuous variables and transformed dummy variable sets, comprise the , or simultaneously the orange and blue lines. We show the graphic of the outcome variable with itself (“By Value”) on the X-axis as well as the percentile (“By Quantile”) in a corresponding graphic adjacent.

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Figure 1 : Unconditional Quantile Regression and RIF

1. Empirical Application

We use the publicly available example Stata dataset oaxaca.dta[[2]](#footnote-2) from the Oaxaca (1973) to illustrate the methodology. We use the publicly available RIF estimators[[3]](#footnote-3) combined with a post-estimation version of Restricted Least Squares[[4]](#footnote-4). We combine these using a wrapper for both estimation procedures[[5]](#footnote-5) into one command with many options.

in which is the dependent variable, the constant, the explanatory variables which include continuous and dummy variables, and the normally distributed error term. This can be estimated using OLS in which case the RIF of is simply the variable itself. However we can estimate this for any unconditional quantile using the RIFs outline above.

The the explanatory variables comprise dummy variable sets: *female*, *married*, *kids6*, *kids714*, and *isco*. Continuous variables include: *educ*, *exper*, *tenure*. The continuous variables and dummy variables are transformed separately as outlined above.

We include 3 regressions: (a) the standard regression without any transformations of the explanatory variables producing and ; (b) the transformed continuous variable and dummy variable set coefficient vector and variance-covariance matrix and (c) the final coefficient vector (in which contained in is exactly the or ) and variance-covariance matrix most appropriate to RIF regression.

Talk about coefficients

Talk about weighted standard deviation of dummy differentials and F-tests

1. Conclusions

Using a novel combination of the RIF regression methodology outlined in Firpo, Fortin, and Lemieux (2009) and a system of linear combinations outlined in Haisken-DeNew and Schmidt (1997), we identify an optimal interpretation of continuous variable and dummy variable set coefficients, focusing on infinitesimally small changes, keeping true to the spirit of the Influence Function or IF. As such, our continuous variables are centered on zero and the coefficients represent a 1% increase from the mean of those variables. This allows us to expose the functional or as this then becomes the constant which is automatically estimated in the model. This is by far the most intuitive manner with which to estimate distributional RIF based regressions, such as the standard FGT measures for poverty outlined by Foster, Greer, Thorbecke (1984, 2010), with the coefficient of the constant (with standard error) identifying the distributional measure of interest. This is the first paper to transform the RIF regressions to allow this direct interpretation and is valid for any RIF based linear models. By transforming the coefficients and variable in the manner we outline, the researcher has the unique advantage of not having to worry about the arbitrary use of one dummy reference over another, for each and every dummy variable set; has transformed the coefficients appropriate to the interpretation of the Influence Function (IF), and finally that constant identifies the functional or and provides a point estimate of the fuctional with standard error etc. This is applicable to all UQR estimators and any other RIF based estimators supported by RIF variable generation.

1. References

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Table1: Descriptive Statistics

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | Label | Count | W Sum | Mean | Var | SD | Min | Max | Sum |
| lnwage |  | 1434 | 1434 | 4814.80 | 0.28 | 0.53 | 0.51 | 5.26 | 4814.80 |
| educ |  | 1647 | 1647 | 18778.00 | 5.64 | 2.37 | 5.00 | 17.50 | 18778.00 |
| c\_\_educ |  | 1647 | 1647 | -92.23 | 0.96 | 0.98 | -2.70 | 2.46 | -92.23 |
| exper |  | 1434 | 1434 | 18861.75 | 99.36 | 9.97 | 0.00 | 49.17 | 18861.75 |
| c\_\_exper |  | 1434 | 1434 | 0.00 | 1.00 | 1.00 | -1.32 | 3.61 | 0.00 |
| tenure |  | 1434 | 1434 | 11272.58 | 65.94 | 8.12 | 0.00 | 44.83 | 11272.58 |
| c\_\_tenure |  | 1434 | 1434 | 0.00 | 1.00 | 1.00 | -0.97 | 4.55 | 0.00 |
| female |  | 1647 | 1647 | 888.00 | 0.25 | 0.50 | 0.00 | 1.00 | 888.00 |
| married |  | 1647 | 1647 | 862.00 | 0.25 | 0.50 | 0.00 | 1.00 | 862.00 |
| kids6 |  | 1647 | 1647 | 469.00 | 0.44 | 0.66 | 0.00 | 4.00 | 469.00 |
| kids714 |  | 1647 | 1647 | 542.00 | 0.50 | 0.71 | 0.00 | 4.00 | 542.00 |
| isco |  | 1434 | 1434 | 5757.00 | 4.24 | 2.06 | 1.00 | 9.00 | 5757.00 |

Table 2: Comparing Standard RIF to Recentering and Linear Combinations: RIF Median Regression q(50)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | | (1) | | (2) | | (3) | |
|  | | Standard Median Reg | | 1% Continuous/ Full Dummy | | 1% Continuous/ 1% point Dummy | |
| [C:mean] years of education | | 0.0399\*\*\* | | 0.00460\*\*\* | | 0.00460\*\*\* | |
|  | | (7.65) | | (7.65) | | (7.65) | |
|  | |  | |  | |  | |
| [C:mean] years of work experience | | 0.00856\*\*\* | | 0.00113\*\*\* | | 0.00113\*\*\* | |
|  | | (6.12) | | (6.12) | | (6.12) | |
|  | |  | |  | |  | |
| [C:mean] years of job tenure | | 0.00799\*\*\* | | 0.000628\*\*\* | | 0.000628\*\*\* | |
|  | | (4.64) | | (4.64) | | (4.64) | |
|  | |  | |  | |  | |
| sex of respondent (1=female)=0 | | 0 | | 0.0405\*\*\* | | 0.000849\*\*\* | |
|  | | (.) | | (3.48) | | (3.48) | |
|  | |  | |  | |  | |
| sex of respondent (1=female)=1 | | -0.0849\*\*\* | | -0.0445\*\*\* | | -0.000849\*\*\* | |
|  | | (-3.48) | | (-3.48) | | (-3.48) | |
|  | |  | |  | |  | |
| married=0 | | 0 | | -0.00563 | | -0.000120 | |
|  | | (.) | | (-0.44) | | (-0.44) | |
|  | |  | |  | |  | |
| married=1 | | 0.0120 | | 0.00635 | | 0.000120 | |
|  | | (0.44) | | (0.44) | | (0.44) | |
|  | |  | |  | |  | |
| number of childern ages 6 and younger=0 | | 0 | | -0.0229\*\*\* | | -0.00151\*\*\* | |
|  | | (.) | | (-4.41) | | (-4.41) | |
|  | |  | |  | |  | |
| number of childern ages 6 and younger=1 | | 0.108\*\* | | 0.0850\* | | 0.000940\* | |
|  | | (2.59) | | (2.28) | | (2.28) | |
|  | |  | |  | |  | |
| number of childern ages 6 and younger=2 | | 0.242\*\*\* | | 0.220\*\*\* | | 0.00230\*\*\* | |
|  | | (4.45) | | (4.26) | | (4.26) | |
|  | |  | |  | |  | |
| number of childern ages 6 and younger=3 | | 0.128 | | 0.105 | | 0.00106 | |
|  | | (1.32) | | (1.10) | | (1.10) | |
|  | |  | |  | |  | |
| number of childern ages 6 and younger=4 | | 0.505\*\*\* | | 0.482\*\*\* | | 0.00482\*\*\* | |
|  | | (12.48) | | (12.27) | | (12.27) | |
|  | |  | |  | |  | |
| number of children ages 7 to 14=0 | | 0 | | -0.0235\*\*\* | | -0.00128\*\*\* | |
|  | | (.) | | (-4.05) | | (-4.05) | |
|  | |  | |  | |  | |
| number of children ages 7 to 14=1 | | 0.113\*\* | | 0.0898\*\* | | 0.00100\*\* | |
|  | | (2.91) | | (2.61) | | (2.61) | |
|  | |  | |  | |  | |
| number of children ages 7 to 14=2 | | 0.148\*\* | | 0.125\*\* | | 0.00134\*\* | |
|  | | (3.25) | | (2.96) | | (2.96) | |
|  | |  | |  | |  | |
| number of children ages 7 to 14=3 | | 0.198 | | 0.175 | | 0.00177 | |
|  | | (1.61) | | (1.43) | | (1.43) | |
|  | |  | |  | |  | |
| number of children ages 7 to 14=4 | | -0.172 | | -0.195 | | -0.00196 | |
|  | | (-0.59) | | (-0.67) | | (-0.67) | |
|  | |  | |  | |  | |
| occupation (ISCO)=1 | | 0 | | 0.193\*\*\* | | 0.00206\*\*\* | |
|  | | (.) | | (4.81) | | (4.81) | |
|  | |  | |  | |  | |
| occupation (ISCO)=2 | | -0.0599 | | 0.133\*\*\* | | 0.00159\*\*\* | |
|  | | (-1.23) | | (4.77) | | (4.77) | |
|  | |  | |  | |  | |
| occupation (ISCO)=3 | | -0.133\*\* | | 0.0601\*\*\* | | 0.000863\*\*\* | |
|  | | (-2.88) | | (3.53) | | (3.53) | |
|  | |  | |  | |  | |
| occupation (ISCO)=4 | | -0.165\*\* | | 0.0284 | | 0.000335 | |
|  | | (-3.10) | | (0.98) | | (0.98) | |
|  | |  | |  | |  | |
| occupation (ISCO)=5 | | -0.338\*\*\* | | -0.145\*\*\* | | -0.00164\*\*\* | |
|  | | (-6.30) | | (-4.75) | | (-4.75) | |
|  | |  | |  | |  | |
| occupation (ISCO)=6 | | -0.492\*\*\* | | -0.299\*\*\* | | -0.00303\*\*\* | |
|  | | (-5.21) | | (-3.54) | | (-3.54) | |
|  | |  | |  | |  | |
| occupation (ISCO)=7 | | -0.379\*\*\* | | -0.185\*\*\* | | -0.00211\*\*\* | |
|  | | (-7.36) | | (-6.35) | | (-6.35) | |
|  | |  | |  | |  | |
| occupation (ISCO)=8 | | -0.237\*\* | | -0.0442 | | -0.000458 | |
|  | | (-3.04) | | (-0.68) | | (-0.68) | |
|  | |  | |  | |  | |
| occupation (ISCO)=9 | | -0.517\*\*\* | | -0.323\*\*\* | | -0.00335\*\*\* | |
|  | | (-7.59) | | (-6.10) | | (-6.10) | |
|  | |  | |  | |  | |
| Constant | | 2.953\*\*\* | | 3.406\*\*\* | | 3.406\*\*\* | |
|  | | (37.28) | | (311.36) | | (311.36) | |
| Observations | | 1434 | | 1434 | | 1434 | |
| female\_sd | |  | | 0.040626 | | 0.000813 | |
| female\_p | |  | | 0.000522 | | 0.000522 | |
| isco\_sd | |  | | 0.130262 | | 0.001472 | |
| isco\_p | |  | | 0.000000 | | 0.000000 | |
| kids6\_sd | |  | | 0.056780 | | 0.001468 | |
| kids6\_p | |  | | 0.000000 | | 0.000000 | |
| kids714\_sd | |  | | 0.046174 | | 0.001205 | |
| kids714\_p | |  | | 0.001052 | | 0.001052 | |
| married\_sd | |  | | 0.000000 | | 0.000000 | |
| married\_p | |  | | 0.658850 | | 0.658850 | |

*t* statistics in parentheses

\* *p* < 0.05, \*\* *p* < 0.01, \*\*\* *p* < 0.001

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2. See http://fmwww.bc.edu/repec/bocode/o/oaxaca.dta [↑](#footnote-ref-2)
3. See Rios-Avila (2021) for his estimator rifhdreg.ado on RIF regressions using Stata. [↑](#footnote-ref-3)
4. See Haisken-DeNew and Schmidt (1997) and the Stata procedure fvhds97.ado. [↑](#footnote-ref-4)
5. See Rios-Avila and de New (2021) for rifhdregc.ado. [↑](#footnote-ref-5)