Discussion Worksheet 1
Review of Calculus I Integration Techniques[1]

MATH 52 Calculus II
with Professor Stankova

1 Substitution Rule

1. Evaluate the indefinite integrals:

   (a) $\displaystyle\int \frac{x}{1+x^4}\,dx$; (b) $\displaystyle\int \frac{\cos\frac{\pi}{x}}{x^2}\,dx$.

2. Evaluate the definite integrals:

   (a) $\displaystyle\int_0^{\pi} \sin x\cos(\cos x)\,dx$; (b) $\displaystyle\int_{-2}^{-1} x^2\sqrt{2+x}\,dx$; (c)* $\displaystyle\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx$.

2 Definite Integrals. Shortcuts

1. Calculate the integrals using FTC II (Fundamental Theorem of Calculus, Part II):

   (a) $\displaystyle\int_1^3 (3^x + x + e^x)\,dx$; (b) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 4x\,dx$; (c) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin 2x}{\cos x}\,dx$; (d) $\displaystyle\int_2^3 \frac{(x-2)^2}{x^2}\,dx$.

2. Calculate $\displaystyle\int_{-2}^{2} x\sqrt{x^2+\cos x}\,dx$. (*Hint:* No antiderivative is needed here!)

3. Let $f(x)=\sqrt{\ln x}$. Evaluate $\displaystyle\int_1^4 f(x)\,dx + \int_3^6 f(x)\,dx + \int_6^1 f(x)dx - \int_3^4 f(x)\,dx$.

   (*Hint:* No antiderivative is needed here!)

3 Challenges

1. Compute the following derivative: $\displaystyle\frac{d}{dx}\int_{-1}^{x^3} (t+2)\,dt$.

2. Find the value of the real number c such that $\displaystyle\int_0^4 \left(c+\sqrt{4x-x^2}\right)\,dx = 0$.

3. Let $g(x)$ be a function such that $g''(x)$ exists and is continuous at all real numbers. If $g(1)=2$, $g(3)=-9$, $g'(1)=0$, and $g'(3)=4$, compute $\displaystyle\int_1^3 \left(g''(x)+g'(x)\right)\,dx$.

4. For any integrable function $f(x)$ on $[a,b]$, prove that $\displaystyle\left|\int_a^b f(x)\,dx\right| \leqslant \int_a^b |f(x)|\,dx$. (*Hint:* $-|c| \leqslant c \leqslant |c|$ for any $c\in\mathbb{R}$.)

4 True/False Jeopardy

For each question, briefly justify your answer in one or two sentences.
If True, briefly explain why. If False, provide a counterexample or explain.

(1) Ⓣ Ⓕ $\displaystyle\int_{-2}^{2} x^2 \cos x \sin x + e^{2x^2}\, dx = 2\int_{0}^{2} e^{2x^2}\, dx.$

(2) Ⓣ Ⓕ Since the antiderivative of $\dfrac{1}{x^2+1}$ is $(\arctan x + C)$, then

$$\int_{-2}^{e} \frac{1}{x^2+1} dx = \arctan e - \arctan(-2).$$

(3) Ⓣ Ⓕ If $f(x) = (\ln x)^2$, then $\displaystyle\int_{1}^{2} f(x)\, dx - \int_{4}^{1} f(x)\, dx - \int_{2}^{5} f(x)\, dx + \int_{4}^{5} f(x)\, dx = 0.$

(4) Ⓣ Ⓕ If $F'(x) = f(x)$, and $F(x)$ is continuous on $[a, b]$ and $c, d \in [a, b]$, then

$$\int_{c}^{d} f(x)\, dx = F(c) - F(d).$$

(5) Ⓣ Ⓕ For a differentiable function f on $\mathbb{R}$, FTC says that the area under f' from $x = 1$ to $x = 2$ is always equal to $f(2) - f(1)$.

(6) Ⓣ Ⓕ There are two different ways to calculate definite integrals by SR (substitution rule): forget temporarily about the bounds of integration, find an antiderivative, and use FTC II; or go directly forward with SR while not forgetting to change the bounds of integration.

5 Deadline for HW Submission

Many students procrastinate submitting their HWs until 10-15 minutes before the deadline and then get stuck "in traffic" on Gradescope, along with hundreds more such students submitting at the last minute. Keep in mind that we will **not** reopen Gradescope for any reason to allow for late submissions, regardless of the reasons given: personal emergencies, technical difficulties, the Gradescope taking too long to access a file submission, and so on.

The top 35 out 38 (or 39) HW assignments will count towards the final grade. Thus, use the 3 HWs that will be dropped for personal emergencies and do not wait until the last minute to submit your HW. If you have technical difficulties submitting your HW, you need to talk to a GSI in OH to help you understand how to submit on Gradescope. Nevertheless, HWs that are not submitted by the deadline, for whatever reasons, will be counted as 0.