

Deep Learning Book

Chapter 7

Regularization for Deep Learning

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Generalization and Strategy

- How to make an algorithm that will perform well not just on the training data, but also on new inputs?
- Many strategies designed to reduce the test error, possibly at the expense of increased training error.
- These strategies are known collectively as **regularization**.
- Many regularization algorithm have been developed.
- Developing more effective regularization strategies is one of the major research efforts in the field.
- In this chapter, we describe regularization in more detail, focusing on regularization strategies for deep models or models that may be used as building blocks to form deep models.

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- There are many regularization strategies.
 1. Put extra constraints on a machine learning model. (Adding restrictions on the parameter values.)
 2. Add extra terms in the objective function that can be thought of as corresponding to a soft constraint on the parameter values.
- If chosen carefully, these extra constraints and penalties can lead to improved performance on the test set.
- Sometimes these constraints and penalties are designed to
 1. **encode** specific kinds of **prior knowledge**.
 2. Express a generic preference for a simpler model class in order to promote generalization.
 3. make an under-determined problem determined. (Provide more information)
- Other forms of regularization, known as ensemble methods, combine multiple hypotheses that explain the training data.

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- Principle: Treading increased bias for reduced variance.
- An effective regularizer is one that makes a profitable trade, reducing variance significantly while not overly increasing the bias.
- In practice, an overly complex model family does not necessarily include the target function or the true data generating process, or even a close approximation.
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Generalization and Strategy

- However, most applications of deep learning algorithms are to domains where the true data generating process is almost certainly outside the model family.
- Deep learning algorithms are typically applied to **extremely complicated domains** such as images, audio sequences and text, for which the true generation process essentially involves **simulating the entire universe**.
- To some extent, we are always trying to fit a square peg(the data generating process) into a round hole (our model family)『持方枘 (rui) 而欲内圆凿』.
- What this means is that controlling the complexity of the model is not a simple matter of finding the model of the **right size**, with the **right number of parameters**.
- Instead, we might find that the best fitting model is a large model that has been regularized appropriately.
- We now review several strategies for how to create such a large, deep, regularized model.

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Parameter Norm Penalties

- Regularization has been used for decades prior to the advent of deep learning.
- Linear models allow simple straightforward and effective regularization strategies.
- Most approaches are based on limiting the capacity of models by adding a **parameter norm penalty** $\Omega(\theta)$ to the objective function J :

$$\tilde{J}(\theta; X, y) = J(\theta; X, y) + \alpha \Omega(\theta)$$

where $\alpha \in [0, +\infty)$ weights the relative contribution of the norm penalty term.

- Setting α to 0 results in no regularization. Larger values of α correspond to more regularization.
- Optimize both J and norm
- Different Ω has different result.

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- We penalize **only the weights** of the affine transformation at each layer and leaves the biases unregularized.
- We do not induce too much variance by leaving the biases unregularized.
- Regularizing the bias parameters can introduce a significant amount of under-fitting.
- We therefore use the vector \mathbf{w} to indicate all of the weights that should be affected by a norm penalty, while the vector $\boldsymbol{\theta}$ denotes all of the parameters, including both \mathbf{w} and the unregularized parameters.
- Sometime we use a separate penalty with a different α coefficient for each layer.
- But it can be expensive to search for the correct value of multiple hyper-parameters, it is still reasonable to use the same weight decay at all layers just to reduce the size of search space.

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L^2 Parameter Regularization

- The L^2 norm penalty commonly known as *weight decay*.

$$\Omega(\theta) = \frac{1}{2} \|w\|_2^2$$

- This regularization strategy drives the weights closer to the origin. (as well as *ridge regression* or *Tikhonov regularization*)
- We can gain some insight into the behavior of weight decay regularization. (assume no bias for simplification)

$$\tilde{J}(w; X, y) = \frac{\alpha}{2} w^T w + J(w; X, y)$$

$$\nabla_w \tilde{J}(w; X, y) = \alpha w + \nabla_w J(w; X, y)$$

- The update

$$w \leftarrow w - \epsilon(\alpha w + \nabla_w J(w; X, y))$$

$$w \leftarrow (1 - \epsilon\alpha)w - \epsilon\nabla_w J(w; X, y)$$

- Shrink the weight vector by a constant factor on each step.
- What happens over the entire course of training?

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- First, let $\mathbf{w}^* = \arg \min_{\mathbf{w}} J(\mathbf{w})$

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