Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans: B. 0.2676
We want probability of manager not meeting his commitment after 60 minutes. i.e., $P(X > 60)$
But since he started doing work after 10 mins it becomes $P(X > 50)$.
Writing python code for it:

```
from scipy import stats
mean = 45
std = 8
1-stats.norm.cdf(50,loc=mean,scale=std)
```

//output

```
0.26598552904870054
```

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans A: False
Basically, we must check $P(X > 44) > P(38 < X < 44)$

```
mean = 38
std =6
P_44 = 1-stats.norm.cdf(44,mean,std)
P_38_44 = stats.norm.cdf(44,mean,std)-stats.norm.cdf(38,mean,std)
P_44>P_38_44
```

//output

```
False
```

Ans B: True
We have calculate P(X<30)*400

`stats.norm.cdf(30,mean,std)*400` //output `36.48`

This shows program will attract about 36 people if we round of the figure

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans:

For $2X_1$, we have:

Mean: $E(2X_1) = 2E(X_1) = 2\mu$

Variance: $Var(2X_1) = 4Var(X_1) = 4\sigma^2$

Distribution: $2X_1 \sim N(2\mu, 4\sigma^2)$

For $X_1 + X_2$, we have:

Mean: $E(X_1 + X_2) = E(X_1) + E(X_2) = 2\mu$

Variance: $Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 2\sigma^2$

Distribution: $X_1 + X_2 \sim N(2\mu, 2\sigma^2)$

Now we can see that both distributions have same mean 2μ. So, the probability distribution of both variables is centered around same point. Also, this value is twice the original mean of individual distributions.

The variance of $2X_1$ is twice than that of $X_1 + X_2$. So, the spread of $2X_1$ is more than $X_1 + X_2$. Both still are normal distribution but the shape of probability density function will differ cause of variance.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Ans: D. 48.5,151.5

We want to find the values of a and b such that the area under the normal curve between a and b is 0.99.

Since the normal distribution is symmetric about the mean, we can find a and b from inverse cdf of these values:

$1 - \frac{0.99}{2} = 0.05$ and $1 - 0.05 = 0.995$

```
stats.norm.ppf(0.005,100,20),stats.norm.ppf(0.995,100,20)
```

//output

```
(48.483413929021985, 151.516586070978)
```

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5th percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss each year?

Ans A. Rupee range that contains 95% probability for annual profit of company:

Approximately (9.9 crores, 98 crores)

```
# converting original mean and std in rupees
mean1 = 5*45
std1 = 3*45
mean2 = 7*45
std2 = 4*45
# calculating total mean and std
total_mean = mean1+mean2
total_std = np.sqrt(std1**2 + std2**2)
# rupee range
stats.norm.ppf(0.025,total_mean,total_std),stats.norm.ppf(0.975,tot
al_mean,total_std)
```

//output

```
(99.00810347848773, 980.9918965215122)
```

Ans B. 5 percentile of profit in Rupees for company

```
stats.norm.ppf(0.05,total_mean,total_std)
```

//output

```
169.9079339359186
```

Ans C. Code for loss

```
print('Division 1 loss probability:',stats.norm.cdf(0,mean1,std1))
print('Division 2 loss probability:',stats.norm.cdf(0,mean2,std2))
```

//output

```
Division 1 loss probability: 0.0477903522728147
Division 2 loss probability: 0.040059156863817086
```

Division 1 has larger probability of making loss each year