

Properties of a P-Doped Germanium Semiconductor

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Abstract

In this experiment we explore the resistivity, charge densities, charge mobilities, and energy gap of a p-doped Germanium semiconductor as a function of temperature. We are able to obtain the following results by gathering resistance measurements while a Hall effect is induced on our semiconductor. We find the resistivity at room temperature to be $2.05(\text{ohm})\text{m} \pm 3.52 \cdot 10^{-4}(\text{ohm})\text{m}$ and the energy gap to be $1.22 \cdot 10^{-19}\text{J} \pm 2.16 \cdot 10^{-33}\text{J}$. Our energy gap value is consistent with the accepted value of $1.07 \cdot 10^{-19}\text{J}$.

Introduction

Semiconductors are the backbone to all high-technologies of the modern day. With their abilities to hold specific concentrations of charge carriers, and having an energy gap between their valence and conduction bands, we are able to exploit unique electrical properties when joining different types together. Transistors are the main result to these p and n type junctions; they act as little switches that can evidently store and manipulate information when billions are arranged properly. There are always advances and innovations being made to better the efficiency and sustainability of semiconductors, lots of condensed matter physicists are experimenting with carbon due to promising features that could one day maybe replace silicon and germanium.

Theory

Solid state physics is the field that builds the models of how semiconductors work. I am not going to get into too many formulas in this section since I will later show in detail the functions for the disable parameters we're looking for. Before we can dive into what happens in a semiconductor during the Hall effect we need to understand the energy structure of electrons in the solid. Unlike a typical conductor, electrons in semis are not easily unbound and able to absorb heat and light and move in an electric field, this is because the valence electrons of the atoms in the crystal create shared states which leads to an energy gap to the states where electrons can be unbound and conduct electricity. When an electron is excited to this conduction energy band it leaves behind a hole in its previous valence energy band, these holes also conduct electricity in the presence of an electric field. Although the holes are not a physical particle instead just an open state in the valence band for electrons to move through, physicists and engineers treat it as one so that the math and physical intuition is more digestible.

Electrons in the conduction band have a concentration, mobility, and effective mass, it is the same for holes in a valence band. The concentration is the amount of particles per unit volume in the semiconductor. The mobility is a factor that describes how fast the particle moves through the solid, on average hole mobility is lower than electron mobility. Effective mass is proportional to mobility since it describes how the mass changes with respect to the shape of the energy band. These parameters are tweaked when we introduce dopants. We can either add electron dopants in the conduction band (n-dopants) or add hole dopants in the valence band (p-dopants). p and n dopants are created by adding specific types of atoms (typically Phosphorus and Boron) that bind with silicon or germanium and either leave behind an extra electron in the conduction band or an extra hole in the valence band. Once we have a doped semiconductor we can measure its dopant concentration and mobility with help from the Hall effect.

The Hall effect is simply just measuring the voltage across a semiconductor while exposing it to a vertical magnetic field and applying a perpendicular current. The magnetic field causes the holes and electrons path to bend to one side of the semiconductor. This bending is due to the Lorentz force $(EQ.1) F = eBv_{(n,p)}$ which has an inherent Hall E field with a measurable voltage. From this Hall voltage we are to obtain something called the Hall coefficient, which is $(EQ.2) R_H = \frac{1}{e \cdot p}$ for the temperature region in which the conduction band is not heavily occupied for a p doped semiconductor. When the conduction band is occupied by a sufficient number of electrons the

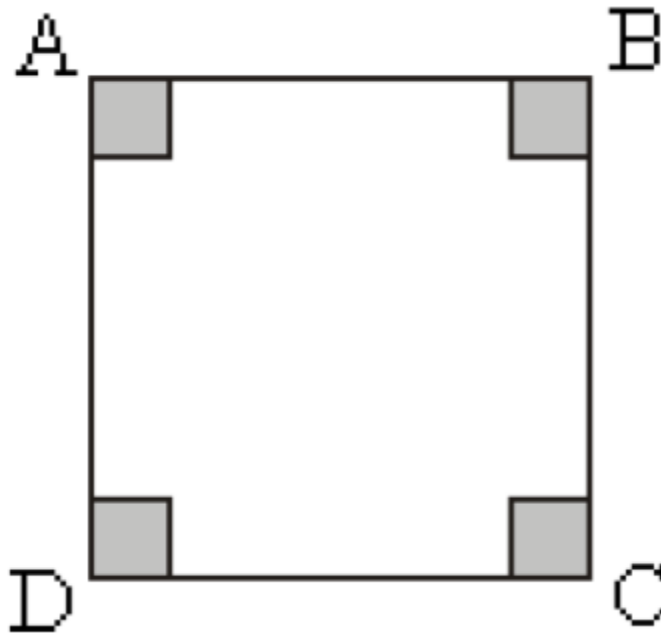
Hall coefficient takes a new form $(EQ.3) R_H = \frac{\left(p - n \left(\frac{\mu_n}{\mu_p}\right)^2\right)}{e \left(p + n \left(\frac{\mu_n}{\mu_p}\right)^2\right)}$ this is because electrons in conduction

band also experience the Hall effect so we need now to account for the Lorentz forces on the electrons. More holes are also created because of the higher perturbation and they too experience the Hall effect and one would guess that the equal production of holes and electrons would cancel their hall effect out leaving the doped Hall voltage of the semiconductor unchanged, however this is not the case since electrons will have a far greater velocity than that of holes since $(EQ.4) v_{(p,n)} \propto \frac{1}{(p,n)}$. Since the concentration of holes is already so high from the dopants their velocity is much less than the electrons, so the Hall E field from the newly excited electrons is much greater than that of the Hall E field from the new holes.

As a result of higher temperature perturbation we see our Hall voltage drop as expected, this is because the electron Hall voltage starts to exponentially cancel out the hole hall voltage. Last thing worth mentioning is that electrons are able to overcome the relatively large energy gap and excite to the conduction band due to their temperature dependent probability of occupying different energy states. Later on I introduce the theoretical formula of electron and hole density as a function of temperature and all of its weird temperature dependency comes from the statistical nature of the system.

Experimental Design and Procedure

The experimental design is pretty simple; we have a big magnet which creates a B field perpendicular to the semiconductor, a current source that runs current along different axes of the material, a voltmeter that picks up voltages also along different axes, a cold deposit to cool down the material, a vacuum that eliminates any corrosive effects on the material, and a lab view program that takes all the current and voltage measurements at different temperatures. The diagram below shows the different axes of the semiconductor for which we will take measurements.



(Figure 1)

This figure is crucial to our experiment since we have tons of different voltages and currents to keep track of when calculating the Hall coefficient and resistivity. The notation throughout my analysis is pretty straightforward I_{ij} is the current across points i and j, I_{mij} is the switched current across points i and j, V_{ij} is the voltage across points i and j for non switched current, and V_{mij} is the voltage across points i and j for switched current.

The procedure for running the experiment goes as follows; first we turn everything on, next we pour liquid nitrogen into the cool depot, next we select a current, B field, and temperature range then start the lab view program, and lastly we export the data into our jupyter notebook for analysis. We run this procedure at 5 different currents values so that we have more range and can be more accurate when calculating our resistivity and Hall coefficient.

Analysis

Error Associated with Each Measurement:

$$V^{error} = \pm 1 \cdot 10^{-5} V$$

$$I^{error} = \pm 1 \cdot 10^{-7} A$$

$$d^{error} = \pm 5 \cdot 10^{-6} m$$

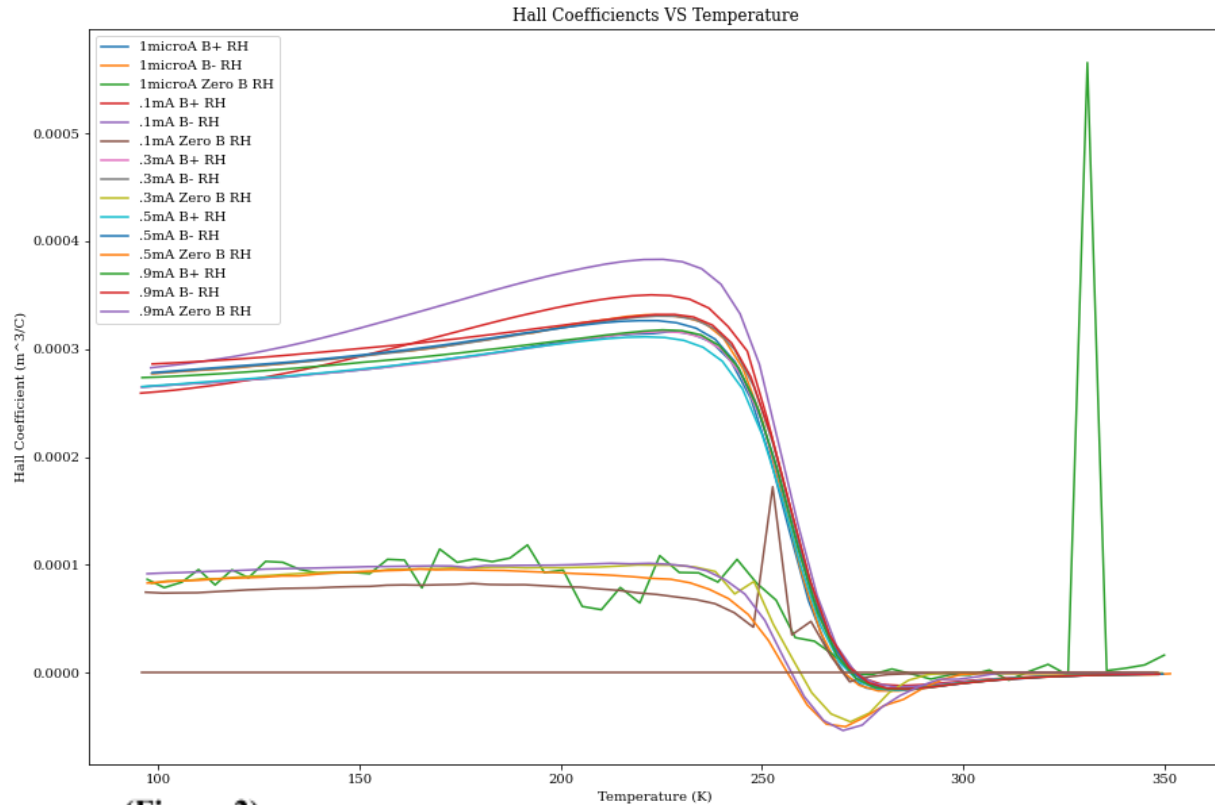
$$B^{error} = \pm 5 Gauss$$

Hall Coefficient: We first solve for R_H by relating it to the Hall voltage via the formula (EQ.5)

$R_H = \frac{V_H d}{I_x B_z}$ here V_H is either V_{AC} or V_{BD} , d is the thickness of the semiconductor

$.00125 \pm d^{error} m$, I_x is either I_{AC} for V_{BD} ; I_{mAC} for V_{mBD} ; I_{BD} for V_{AC} ; I_{mBD} for V_{mAC} , and B_z is the magnetic field that is perpendicular to the semiconductor and will be measured for both + and - orientations as well as zero. Figure 2 shows R_H 's for different currents from temperatures

95K – 350K. At room temperature $R_H = -1.02 \cdot 10^{-5} \pm 1.58 \cdot 10^{-9}$.



(Figure 2)

We from the data that on average it seems the R_H is pretty consistent no matter the the current or orientation of B field, what we do see is that when we have zero B field the R_H drops and has slightly different behavior. In the region from $95K - 240K$ we say that the material is extrinsic because R_H stays relatively constant as expected since we know the material is p-doped. We can also see that R_H is positive in this region and this tells us that our charge carriers are holes, the orientation of the B field does not matter here since the induced Hall voltage will be negative for a -B field and positive for a +B field. There is a slight increase in R_H in this region which means for some reason the concentration of holes decreases a little bit since $R_H \propto \frac{1}{p}$, we assume that there might be some electron-hole annihilation which is a reason for this decrease. The function fit to this slight increase is $R_H^{ex}(T) = (1.4 \cdot 10^{-9})T^2 + 2.5 \cdot 10^{-4}$.

In the region from $240K - 350K$ we say that the material is intrinsic because we can start to see the decrease in R_H as we discussed earlier since the electron Hall effect starts to cancel out the hole Hall effect. From around $275K - 325K$ R_H goes negative and this is because the electrons Hall E field completely cancels out the hole Hall E field and some which makes R_H negative. From $325K - 350K$ R_H goes to zero and this is because both electron and hole concentrations become so high that their relative velocities become tiny so their Hall E fields and Hall effects in general approach zero. I was not able to fit the extrinsic region of data nicely

however we know it will take on the form ${}_{(EQ.3)}R_H^{in}(T) = \frac{\left(p(T)-n(T)\left(\frac{\mu_n}{\mu_p}\right)^2\right)}{e\left(p(T)+n(T)\left(\frac{\mu_n}{\mu_p}\right)\right)^2}$. The reason I could not

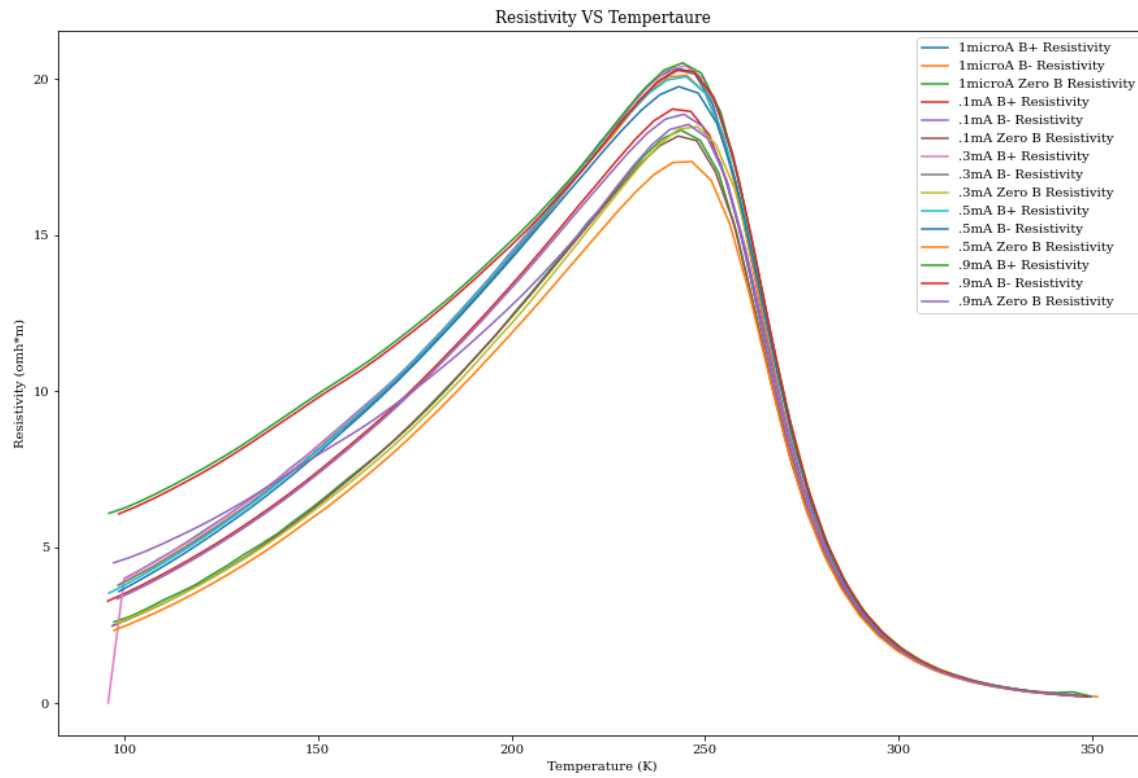
fit this function is because μ_n and μ_p are functions of temperature and my software did not have the computing power to determine these temperature dependencies accurately.

Resistivity: The next order of business is to solve for the ρ and we do this with the Van Der Pauw technique. In this technique we take the resistivities across different ends of the

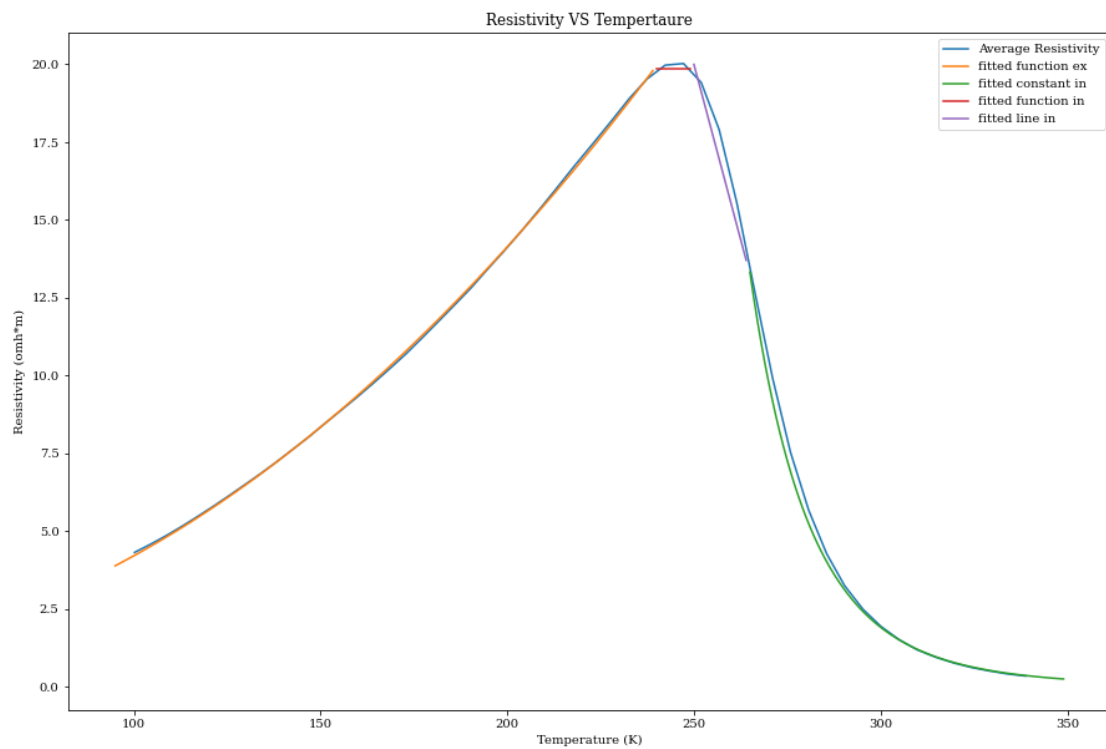
semiconductor and run them through ${}_{(EQ.4)}\rho = \frac{d\pi}{2\ln(2)} \left[\frac{V_{CD}}{I_{AB}} + \frac{V_{BC}}{I_{AD}} \right] \left[\cosh\left(\frac{\ln\left(\frac{V_{CD}I_{AD}}{V_{BC}I_{AB}}\right)}{2.403}\right) \right]^{-1}$.

We use this technique because it eliminates any extra resistivity within our measuring and current source devices. It does this by measuring voltage at the opposite side of an applied current. Figure 3 shows ρ 's for different currents for +, - and zero B field from temperatures

95K – 350K. At room temperature $\rho = 2.05 \pm .$



(Figure 3)



(Figure 4)

I am able to fit the the extrinsic region of the data with the function

$\rho^{ex}(T) = 3.31 \cdot 10^{-4} T^2 + .892$. From $240K - 250K$ I make the assumption that ρ is constant around 19.85, from $250K - 265K$ I fit the line $\rho^{in}(T) = -.45T + 132.5$, and from $265K - 350K$ I fit the function $\rho^{in}(T) = 7.67 \cdot 10^{-7} e^{\frac{4417.23}{T}}$. Figure 4 shows how my functions fit with the average resistivity.

Charge Carrier Concentration: For coming up with charge carrier concentrations as functions of temperature we fit to the statistical formula of the intrinsic concentration from Melissinos

which is $n^{in}(T) = 2 \left(\frac{K_B T}{2\pi \hbar_{bar}^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{\frac{-E_g}{2K_B T}}$. We make the assumptions that $m_e^* m_h^* \approx m_e^2$

then solve for E_g by finding the slope of $\ln\left(\frac{1}{\rho^{in}}\right)\left(\frac{1}{T}\right)$, from .003 -.004 in figure 6 you can see the line of interest and figure 5 you can see the code used to find E_g . After solving for the E_g and

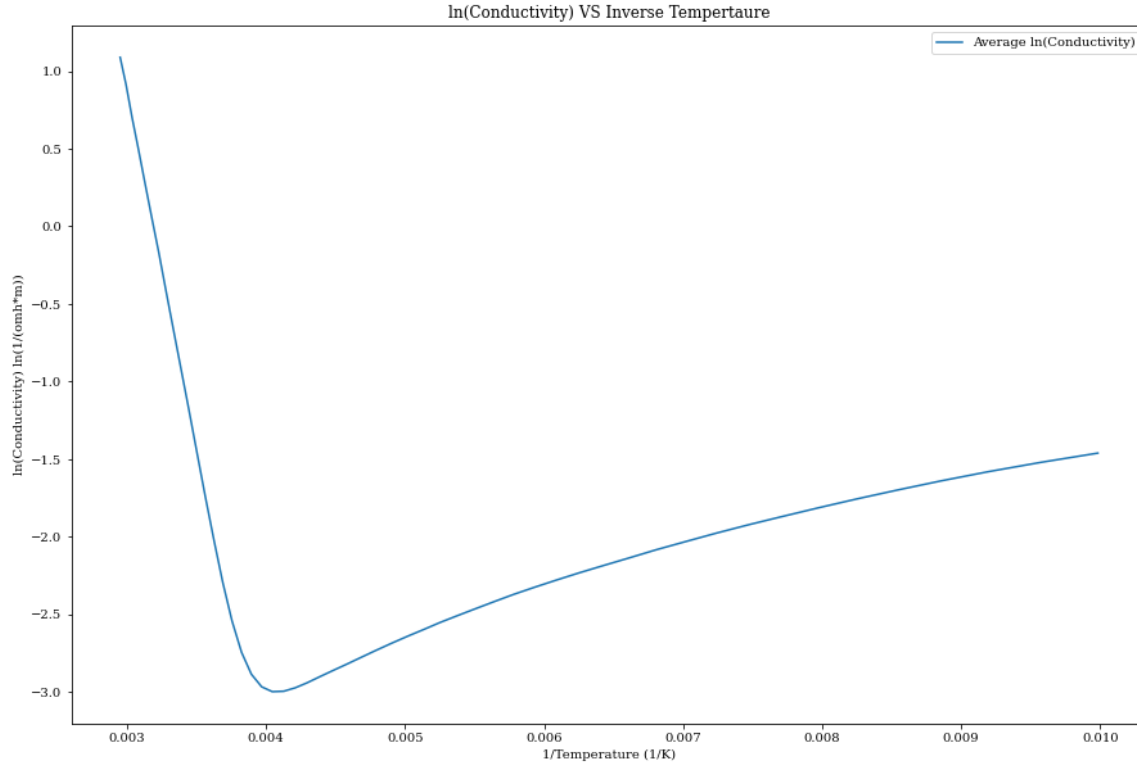
plugging in m_e^2 we get $n^{in}(T) = 4.83 \cdot 10^{15} T^{3/2} e^{\frac{-4417.23}{T}}$, this is equal to the number of electron in the conduction band as function of temperature however we make the assumption that from $95K - 240K$ the concentration is zero since the concentration of holes is much higher in this region. For holes we say $p^{ex}(T) = \frac{1}{eR_H^{ex}(T)}$ in the extrinsic region and

$p^{in}(T) = p_{max}^{ex}(240K) + n^{in}(T)$ in the intrinsic region.

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#Fritz Ahern SHE Lab spring 2022 Phsyics 111B
#n(int) curve fitting, we use a formula and find the Eg for the material via slope of
#ln(cond) vs 1/T
T=1/Run1["Temperature (K)"][:,3][36:54]
def e_func(T,a,b):
    return a*T+b
popt,pcov=sp.curve_fit(e_func,T,np.log(1/A_rho_ps[36:54]), sigma=None, absolute_sigma=False)
a_err_rho_ex, b_rho_RH_ex= np.sqrt(np.diag(pcov))
print('a and b values for intrinsic n function',popt)
print('statistical uncertainty of a and b values', a_err_rho_ex, b_err_rho_ex)
print('a = -Eg/2K => Experimental Eg for Germanium = 2aK =',-2*popt[0]*1.380649e-23)
print('Actual Eg for Germanium = 1.0735e-19')

a and b values for intrinsic n function [-4417.23097111    14.0847294 ]
statistical uncertainty of a and b values 2.161393763286172e-10 7.050931224918922e-06
a = -Eg/2K => Experimental Eg for Germanium = 2aK = 1.2197291046072298e-19
Actual Eg for Germanium = 1.0735e-19
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(Figure 5)



(Figure 6)

Mobilities: We want the Hall mobility, extrinsic hole mobility and extrinsic electron mobility.

Hall mobility is given by (EQ.6) $\mu_{Hall} = \frac{R_H}{\rho}$, we can see its behavior in figure 7. I was only able to

fit the extrinsic region with the function $\mu_{Hall}^{ex}(T) = \frac{R_H^{ex}(T)}{\rho^{ex}(T)}$ since I do not have a functions for

$R_H^{in}(T)$ nor $\rho^{in}(T)$. For hole and electron mobility we use the formula (EQ.6) $\frac{1}{\rho} = en\mu_n + ep\mu_p$ for

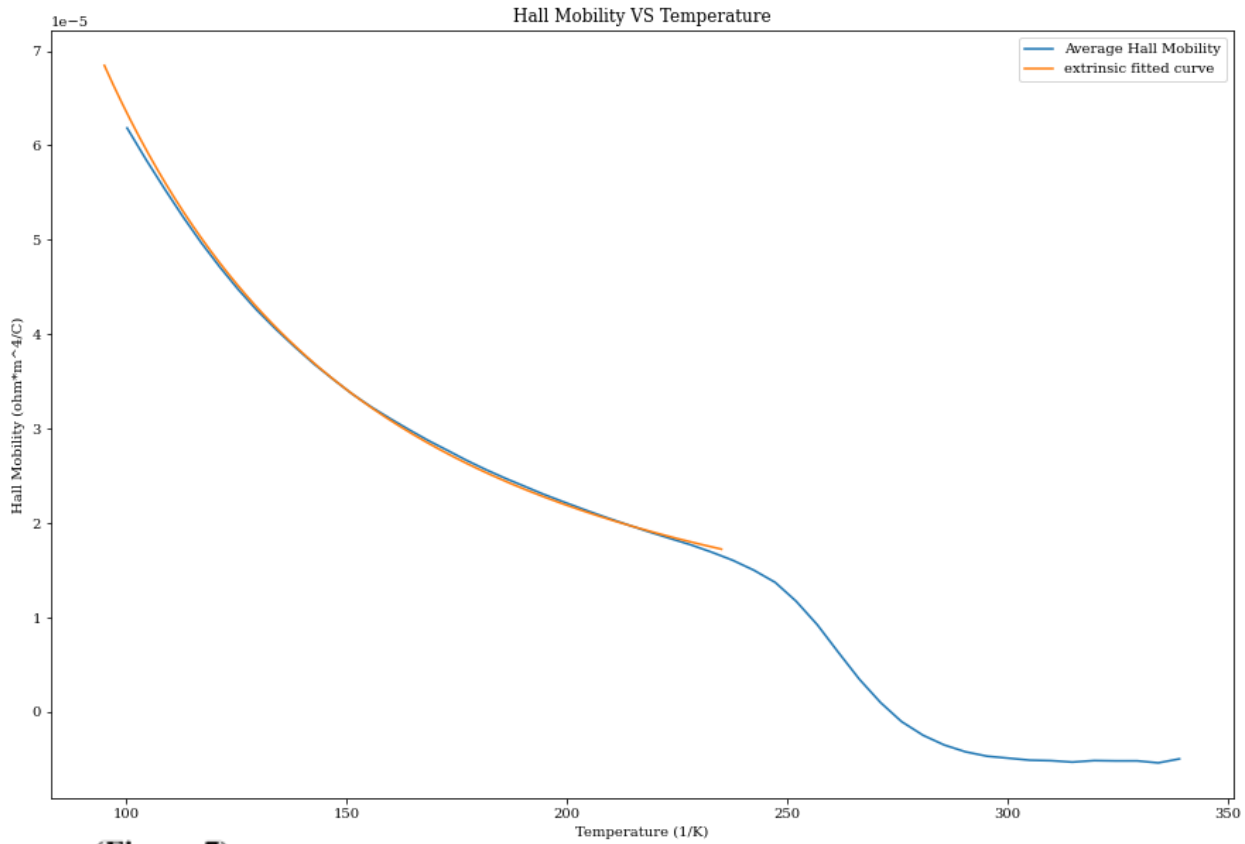
the extrinsic region since we assume $n^{ex}(T) \approx 0$ so we can then $\mu_p^{ex}(T) = \mu_{Hall}^{ex}(T)$. Since we

know $\mu_n \propto \frac{1}{n}$ and we know $n^{ex}(T)$ is tiny we can assume that $\mu_n^{ex} \gg 1$. In general mobility is a

tricky thing to measure and even trickier to calculate theoretically. Another form for mobility is

(EQ.7) $\mu = \frac{e\tau}{m^*}$ where (EQ.6) $m^* = \hbar_{bar}^2 \left(\frac{d^2 E_{band}}{dk^2} \right)^{-1}$ and τ is the scattering time of the particle, this is

tricky to calculate because the τ and m^* are dependent on lots of factors. We try our best to find $\mu_{p,n}(T)$ when fitting EQ.3 with our data, but as stated earlier I had no luck finding a proper fit.

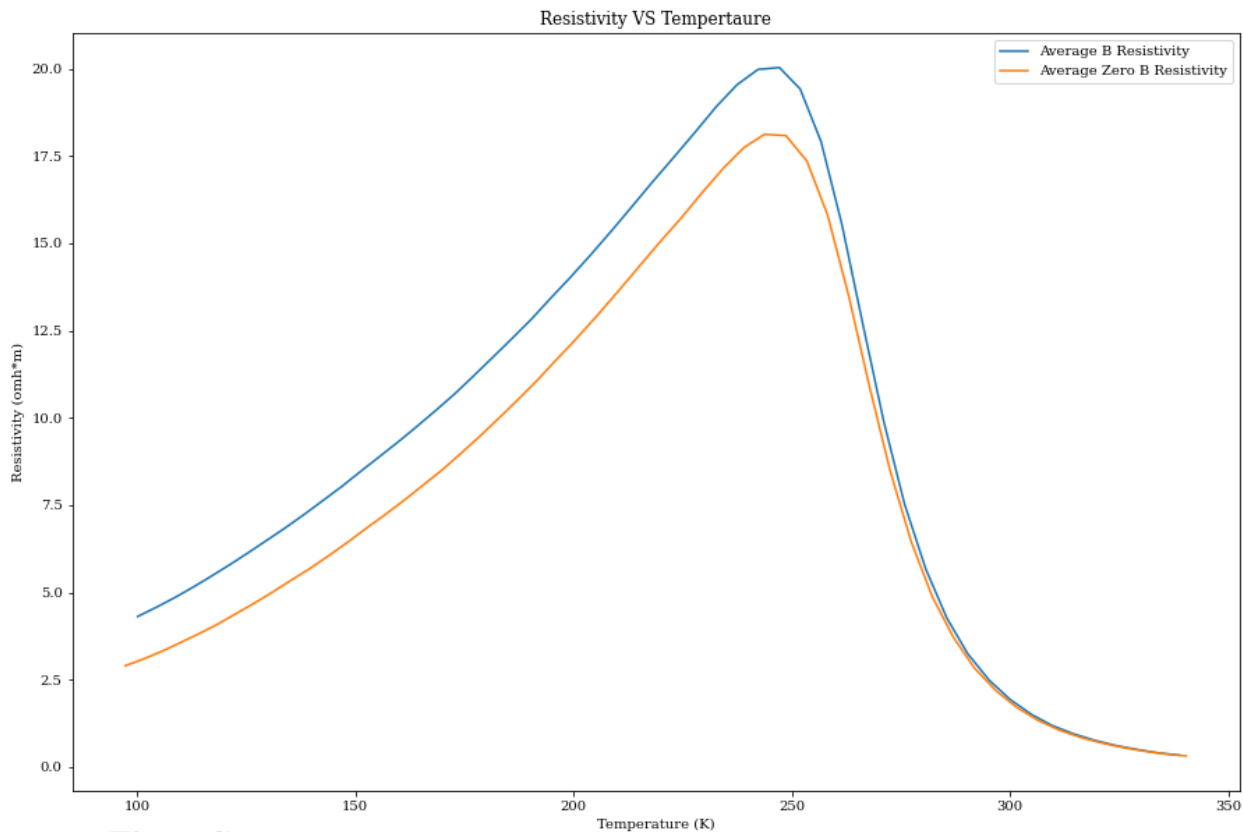


(Figure 7)

B-Field at $E_H=E_x$: In this calculation we are assuming that the Hall E field is equal to the applied E field and we are looking to see what the magnetic field will be. We know $R_H = \frac{V_H d}{I_x B_z}$;

$V_H = E_H s$; and $I_x = \frac{sdE_x}{\rho}$. Plugging these in we see that $B_z = \frac{\rho}{R_H}$.

Zero Field Resistivity vs Magnetoresistivity: In figure 8 we see that the resistivity is slightly higher when we have the magnetic field on, this happens because the magnetic field increases the path that the charge carriers have because of the bending caused by the Hall effect(the Lorentz force).



(Figure 8)

Conclusion

This experiment was successful in that we were able to replicate the energy gap of Germanium. Lots was learned about band structure, charger density, mobility, and semiconductor in general. The data analysis was quite intensive compared to the other labs so I was able to sharpen my python skills as well. If I were to redo my data collection I would aim for higher currents since we saw the best results when applying them. I would also seek more help with curve fitting when trying to find the $R_H^{in}(T)$. I think this lab should be required since semiconductors are so crucial for all technologies in this day and age.

References

Melissinos, Adrian. "[The Hall Effect and Properties of Semiconductors](#)." Experiments in Modern Physics, pp. 80-98. Academic Press (1966). Eqs. 1,3, 5,7

Lab Manual Eqs. 2,4,6

Kittel, Charles "Introduction to solid state physics" for Eq. 8