# Neutrinos via Charm Decays in Astrophysical Sources

by

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# Abstract

# Acknowledgements

# Abbreviations

**AGN** Active Galactic Nucleus

**CKM** Cabibbo, Kobayashi & Maskawa

**EWT** Electroweak Theory

**GR** General Relativity

**GZK** Greisen, Sazepin & Kusmin

**PMNS** Pontecorvo, Maki, Nakagawa & Sakata

**QCD** Quantum Chromodynamics

**QED** Quantum Electrodynamics

**QFT** Quantum Field Theory

**QM** Quantum Mechanics

**SM** Standard Model

**SR** Special Relativity

# Contents

# 1 Introduction

# 2 Background

While traversing space, messenger particles are subjected to many different influences, both in terms of type and magnitude, before being detected at earth. Understanding this propagation requires a thorough comprehension of physical processes at all scales of application, from production in astrophysical sources, through interactions with the vast radiation and matter fields that fill the cosmos, to entering the solar system and finally the terrestial atmosphere.

The following sections provide an incomplete overview of aspects relevant to the treatment of this complex topic, as well as references for further study on the various related subjects.

## 2.1 Particle Physics

In the modern view, interactions and categories of elementary particles are most accurately described by a construct called the *Standard Model* (SM) of particle physics. Underlying this formalism is the mathematical framework known as *Quantum Field Theory* (QFT) that combines and generalizes properties of *Quantum Mechanics* (QM) and *Special Relativity* (SR) to produce an extremely precise description of microscopic reality. Excitations in the associated fields are referred to as quanta and manifest as observable particles.

Since this work is mostly concerned with the statistical behaviour of large quantities instead of individual particle probabilistics, a more detailed explanation is omitted except for certain phenomena important to the justification of some later assumptions.

### Fundamental Interactions

Electromagnetism is described by *Quantum Electrodynamics* (QED) and mediated by photons $\gamma$ with Abelian U(1) symmetry, whereas the weak force is carried by $Z$ and $W^{\pm}$ bosons related to the SU(2) group. In terms of *Electroweak Theory* (EWT) all these particles actually result from symmetry breaking of virtual isospin and hypercharge fields, unifying U(1) × SU(2) under a single coherent structure. The previously massless gauge bosons then gain their masses via the Higgs mechanism, where only the $\gamma$ is excluded and remains without mass.

Notably, the weak interaction exclusively couples to left handed fermions, violating parity.

Chirality, Helicity, Decay, Mixing, Quarks, Color Charge

Carriers of the strong force are named gluons and arise from the SU(3) symmetry group.

Weak, Electromagnetic, Electroweak, Higgs

Parity Violation, Mixing

Limits, Dark Matter, Dark Energy, Neutrino Oscillation, Quantum Gravity

Vector Bosons, Scalar Boson, Boson Spins, Fermion Spins

General Relativity

### Leptons

Neutrinos, Decays

### Hadrons

Confinement, Asymptotic Freedom, Residual Nuclear Force, Decays

## 2.2 Multimessenger Astronomy

### Gravitational Waves

### Cosmic Rays

### Photons

### Neutrinos

## 2.3 Astrophysical Sources

### Magnetars

### Active Galactic Nuclei

# 3 Methods

# 4 Results

# 5 Discussion

# 6 Conclusion & Outlook

# Appendix

## A Reference Frames

Depending on the application, energies in particle physics are either given as viewed from a suitable rest frame or independent from the choice of coordinate system altogether. One widely adopted formulation uses the Mandelstam variables

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

to assign different channels in scattering processes via the squared momentum carried by the exchanged mediating particle. Implied in this context is the Minkowski inner product, making the above quantities manifestly Lorentz invariant.

When working with parametrizations defined for use in different subdisciplines it often becomes necessary to convert from center of mass energies $\sqrt{s}$ to the energy $E$ of a projectile in the target rest frame. With $E^2 = \boldsymbol{P}^2c^2 + M^2c^4$ as well as momenta $P = (E, \boldsymbol{P}c)$ and $p = (mc^2, 0)$ one finds

$$s = (P + p)^2 = (E + mc^2)^2 - \boldsymbol{P}^2c^2 = 2Emc^2 + m^2c^4 + M^2c^4$$

for the invariant mass. This relation is typically approximated as $s = 2Emc^2$ at high energies.

## B Cross Sections

### Scattering

### Production

$$x_F \frac{d\sigma}{dx_F}(x_F, E_p) = a x_F^b (1 - x_F^m)^n$$

[1]

$$a = a_1 - a_2 \ln\left(E_p\right) \qquad b = b_1 - b_2 \ln\left(E_p\right) \qquad n = n_1 - n_2 \ln\left(E_p\right)$$

$m = 1.2$

TABLE 1: Parameters for the charm quark differential cross section. Coefficients in the given energy ranges are calculated from [1] to represent $E_p$ in units of GeV without requiring unnecessary conversions.

| $E_p$ / GeV | $a_1$ / μb | $a_2$ / μb | $b_1$ | $b_2$ | $n_1$ | $n_2$ |
|---|---|---|---|---|---|---|
| $10^4 - 10^8$ | 826 | 8411 | 0.197 | 0.016 | 1.061 | 0.107 |
| $10^8 - 10^{11}$ | 403 | 2002 | 0.237 | 0.023 | 7.639 | 0.102 |

As in [2] it is assumed that the cross section scales linearly with the nucleon number, yielding

$$\frac{d\sigma}{dx_c}(x_c, E_p) = A^{-1}\frac{d\sigma}{dx_F}(x_c, E_p)$$

for the inclusive $pp \to cX$ production.

75 % nitrogen and 25 % oxygen $A = 14.5$

$$\frac{d\sigma}{dx_E}(x_E, E_p) = \int_{x_E}^{1} dz\, z^{-1}\frac{d\sigma}{dx_c}(x_c, E_p)\, D_c^h(z)$$

$$D_c^h(z) = \frac{N_h z(1-z)^2}{\big((1-z)^2 + \epsilon_h z\big)^2} \tag{1}$$

Decay

$$\tilde{F}_{h\to\nu} = D_h^{-1}\Big(6b_h a_h^2 - 4a_h^3 - 12\lambda_h^3 a_h + 12\lambda_h^2 y - 6b_h y^2 + 4y^3 + 12\lambda_h^2 \ln\big((1-y)/\lambda_h\big)\Big)$$

$$a_h = 1 - \lambda_h \qquad b_h = 1 - 2\lambda_h \qquad D_h = 1 - 8\lambda_h - 12\lambda_h^2\ln\big(\lambda_h\big) + 8\lambda_h^3 - \lambda_h^4$$

TABLE 2: Parameters for charm hadron production, cooling and decay to neutrinos. Coefficients $\epsilon_h$ are taken from fits to LO QCD results via the fragmentation function in [3] with normalizations $N_h$ given by [4] to scale the integration of (1) over [0,1] to approximately match the fractions $f_h$ provided in [5] from measurements. Effective masses $\sqrt{\tilde{s}_h}$ have been determined by [6] from fitting decay rates, except for $D_s^+$ which is copied from [4] because no original value could be found. Mean lifetimes $\tau_h$ and rest masses $m_h$ are adopted from [7] in the particle listings. For mass type quantities, natural units $c = 1$ are used.

| $h$ | $N_h$ | $\epsilon_h$ | $\tau_h$ / fs | $\sqrt{\tilde{s}_h}$ / GeV | $m_h$ / GeV |
|---|---|---|---|---|---|
| $D^0$ | 0.577 | 0.101 | 410 | 0.67 | 1.86 |
| $D^+$ | 0.238 | 0.104 | 1033 | 0.63 | 1.87 |
| $D_s^+$ | 0.0327 | 0.0322 | 501 | 0.84 | 1.97 |
| $\Lambda_c^+$ | 0.0067 | 0.00418 | 203 | 1.27 | 2.29 |

# C Spectral Distributions

## Pions & Kaons

## Charm

# D Pulsar Spindown

# E Stochastic Acceleration

Due to its wide applicability in different astrophysical scenarios, probabilistic collisions are often viewed as one of the more plausible mechanisms responsible for accelerating cosmic rays to high energies. The general case is described in [8] and supposes that for each collision, particles gain energy proportional to a constant factor $\eta$ and remain in the region of acceleration with fixed probability $\varsigma$ on average. With initial conditions $N_0$ for the particle number and $E_0$ as the mean energy, this results in $N = N_0\varsigma^k$ and $E = E_0\eta^k$ after $k$ collisions. Using $\ln x^k = k \ln x$ in

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln(\varsigma)}{\ln(\eta)}$$

eliminates the exponent and by rearranging gives the relation

$$N = N_0 \left(\frac{E}{E_0}\right)^{\ln(\varsigma)/\ln(\eta)}$$

as the integrated spectrum. Accordingly, one finds

$$\frac{dN}{dE} = \frac{N_0}{E_0}\left(\frac{E}{E_0}\right)^{\alpha}$$

for the differential spectrum where

$$\alpha = \frac{\ln(\varsigma)}{\ln(\eta)} - 1$$

defines the spectral index. From $\varsigma < 1$ and $\eta > 1$ follows $\alpha + 1 < 0$ as a general constraint.

$\alpha = -2$

$dN/dE \propto E^{-2}$

# F Implementation

In order to calculate neutrino spectra from hadronic distributions, several integrals have to be computed. Discretizing this task allows the general case

$$F(x, y) = \int dz\, G(x, z)\, H(z, y)$$

to be rewritten as a Riemann sum. Assuming $G$ and $H$ are integrable over a given interval,

$$F_{ij} = \sum\nolimits_k D_{kk}\, G_{ik}\, H_{kj}$$

converges to the exact solution for sufficiently small steps. Transforming variables

$$x \rightarrow x_i \qquad y \rightarrow y_j \qquad z \rightarrow z_k$$

and defining $D_{kk} = z_{k+1} - z_k$ leads to the above notation. It is easily shown how this expression in terms of indices translates to the product of corresponding matrices

$$\boldsymbol{F} = \boldsymbol{G}\boldsymbol{D}\boldsymbol{H}$$

as an equivalent formulation. Here the output $\boldsymbol{F} \in \mathbb{R}^{m\times n}$ is obtained from the inputs $\boldsymbol{G} \in \mathbb{R}^{m\times l}$ and $\boldsymbol{H} \in \mathbb{R}^{l\times n}$ as well as the square matrix $\boldsymbol{D} \in \mathbb{R}^{l\times l}$ that encodes all step sizes on its diagonal. These results enable a quick and efficient implementation of the required calculations as program code, where array arithmetic operations can greatly increase execution speed.[1]

[1] In service of reproducability, all implementations can be viewed in this repository.

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# Figures

# Tables