Neutrinos via Charm Decays in Astrophysical Sources

by

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Abstract

Acknowledgements

Abbreviations

AGN Active Galactic Nucleus

CKM Cabibbo, Kobayashi & Maskawa

DSA Diffusive Shock Acceleration

EWT Electroweak Theory

FF Fragmentation Function

GR General Relativity

GZK Greisen, Sazepin & Kusmin

LO Leading Order

NLO Next to Leading Order

PDF Parton Distribution Function

PMNS Pontecorvo, Maki, Nakagawa & Sakata

QCD Quantum Chromodynamics

QED Quantum Electrodynamics

QFT Quantum Field Theory

QM Quantum Mechanics

SM Standard Model

SR Special Relativity

UHECR Ultrahigh Energy Cosmic Ray

Contents

1 Introduction

2 Background

While traversing space, messenger particles are subjected to many different influences, both in terms of type and magnitude, before being detected at earth. Understanding this propagation requires a thorough comprehension of physical processes at all scales of application, from production in astrophysical sources, through interactions with the vast radiation and matter fields that fill the cosmos, to entering the solar system and finally the terrestial atmosphere.

The following sections provide an incomplete overview of aspects relevant to the treatment of this complex topic, as well as references for further study on the various related subjects.

2.1 Particle Physics

In the modern view, interactions and categories of elementary particles are most accurately described by a construct called the *Standard Model* (SM) of particle physics. Underlying this formalism is the mathematical framework known as *Quantum Field Theory* (QFT) that combines and generalizes properties of *Quantum Mechanics* (QM) and *Special Relativity* (SR) to produce an extremely precise description of microscopic reality. Excitations in the associated fields are referred to as quanta and manifest as observable particles.

Since this work is mostly concerned with the statistical behaviour of large quantities instead of individual particle probabilistics, a more detailed explanation is omitted except for certain phenomena important to the justification of some later assumptions.

Fundamental Interactions

Electromagnetism is described by *Quantum Electrodynamics* (QED) and mediated by photons γ with Abelian U(1) symmetry, whereas the weak force is carried by Z and $W^{\pm}$ bosons related to the SU(2) group. In terms of *Electroweak Theory* (EWT) all these particles actually result from symmetry breaking of virtual isospin and hypercharge fields, unifying U(1) × SU(2) under a single coherent structure. The previously massless gauge bosons then gain their masses via the Higgs mechanism, where only the γ is excluded and remains without mass.

Notably, the weak interaction exclusively couples to left handed fermions, violating parity.

Chirality, Helicity, Decay, Mixing, Quarks, Color Charge

Carriers of the strong force are named gluons and arise from the SU(3) symmetry group.

Weak, Electromagnetic, Electroweak, Higgs

Parity Violation, Mixing

Limits, Dark Matter, Dark Energy, Neutrino Oscillation, Quantum Gravity

Vector Bosons, Scalar Boson, Boson Spins, Fermion Spins

General Relativity

Leptons

Neutrinos, Decays

Hadrons

Confinement, Asymptotic Freedom, Residual Nuclear Force, Decays

2.2 Multimessenger Astronomy

Gravitational Waves

Cosmic Rays

Photons

Neutrinos

2.3 Astrophysical Sources

Magnetars

Active Galactic Nuclei

3 Methods

4 Results

5 Discussion

6 Conclusion & Outlook

Appendix

A Reference Frames

Depending on the application, energies in particle physics are either given as viewed from a suitable rest frame or independent from the choice of coordinate system altogether. One widely adopted formulation uses the Mandelstam variables

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

to assign different channels in scattering processes via the squared momentum carried by the exchanged mediating particle. Implied in this context is the Minkowski inner product, making the above quantities manifestly Lorentz invariant.

When working with parametrizations defined for use in different subdisciplines it often becomes necessary to convert from center of mass energies $\sqrt{s}$ to the energy E of a projectile in the target rest frame. With $E^2 = \boldsymbol{P}^2c^2 + M^2c^4$ as well as momenta $P = (E, \boldsymbol{P}c)$ and $p = (mc^2, 0)$ one finds

$$s = (P + p)^2 = (E + mc^2)^2 - \boldsymbol{P}^2c^2 = 2Emc^2 + m^2c^4 + M^2c^4$$

for the invariant mass. This relation is typically approximated as $s = 2Emc^2$ at high energies.

B Cross Sections

By defining an effective area perpendicular to the velocity vectors of projectiles and targets, cross sections measure the probability of collision processes in particle physics. Due to depending on the strength of an interaction, these quantities generally scale with energy. Distinct from the integrated case, differential cross sections are usually given with respect to some independent variable such as angle or momentum of the particle.

Scattering

To model total cross sections in hadron proton scattering, this work uses the formula

$$\sigma_{hp} = H_h \ln^2(s/s_h) + P_h + R_h^1(s_h/s)^{\eta_1} + R_h^2(s_h/s)^{\eta_2} \tag{1}$$

as given in [1] for a universal analytic parametrization of the corresponding amplitudes.

All adjustable parameters are listed in table 1 together with relevant meson lifetimes for cooling. In this approach, the variable M relates to $H = \pi(\hbar c/M)^2$ and $s_h = (m_h + m_p + M)^2$ as an effective mass. Coefficients in (1) denote Heisenberg, Pomeranchuk and Regge terms which have some qualitative motivation, though the formula itself is primarily a quantitative result.

TABLE 1: Fits to the total inclusive scattering cross sections in hadron proton collisions. Parameters are taken from [1] with $M = 2.121$ GeV for $H = 0.272$ mb as the rate of growth. Both $\eta_1 = 0.447$ and $\eta_2 = 0.5486$ are dimensionless exponents. Decay times τ_h and rest masses m_h can be found in the particle listings [2] where the latter are given in natural units.

h	P_h / mb	R_h^1 / mb	R_h^2 / mb	τ_h / ns	m_h / GeV	s_h / GeV^2
p	34.41	13.07	7.39		0.938	15.98
π	18.75	9.56	1.767	26.03	0.140	10.23
K	16.36	4.29	3.408	12.38	0.494	12.62

Assuming a quasi universal ratio $\mathcal{R}$ between elastic and total hadron cross sections, one obtains the inelastic cross section $\sigma_{\text{inel}} = (1-\mathcal{R})\sigma_{\text{tot}}$ from $\sigma_{\text{el}} = \mathcal{R}\sigma_{\text{tot}}$ and $\sigma_{\text{el}} + \sigma_{\text{inel}} = \sigma_{\text{tot}}$ as a unitarity condition. Provided in [3] is the model independent parametrization

$$\mathcal{R}(s) = \frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)} = \mathcal{A}\tanh\big(\gamma_1 - \gamma_2 \ln(s) + \gamma_3 \ln^2(s)\big) \tag{2}$$

with a constant asymptote $\mathcal{A}$ at very high energies. Coefficients are given in table 2 for different physical settings. Both equations (1) and (2) use units of GeV^2 for the s variables.

TABLE 2: Almost model independent ratio of hadronic elastic and total scattering cross sections. Factors γ are taken from [3] for varying $\mathcal{A}$ asymptotes.

$\mathcal{A}$	γ_1	γ_2	γ_3
1/2	0.466	0.0259	0.00177
1	0.2204	0.0111	0.00076

Reference [4] tests the asymptotic rise $\sigma(s) \propto \ln^2(s)$ derived in [5] as a theoretical upper bound and concludes that it is somewhat exceeded. Additionally, a ratio $\mathcal{A} = 1/3$ due to diffraction as opposed to the black disc limit $\mathcal{A} = 1/2$ from optical theorem predictions is suggested. Because parameters are only available in the latter case, all calculations of $\mathcal{R}$ use function (2) as defined by an asymptote $\mathcal{A} = 1/2$ for this work. Data matching $\mathcal{A} = 1/3$ then implies underestimated values of $\sigma_{\text{inel}}(s)$ which should however not significantly influence the overall results.

Production

For charm quark production in proton air collisions, reference [6] gives

$$x_F \frac{d\sigma}{dx_F}(x_F, E_p) = a x_F^b (1 - x_F^m)^n$$

as the parametrized differential cross section with components

$$a = a_1 - a_2 \ln(E_p) \qquad b = b_1 - b_2 \ln(E_p) \qquad n = n_1 - n_2 \ln(E_p)$$

for which table 3 lists all necessary constants. Here proton energies E_p are defined as viewed by air nuclei at rest, while the Feynman scaling variable $x_F = p_c / p_s$ specifies magnitude ratios of produced charm quark longitudinal momentum to all available momentum in center mass coordinates of the colliding particles. Application of appendix A shows that this approximately fulfills $x_F = x_c$ where $x_c = E_c / E_p$ in the relevant energy ranges.

TABLE 3: Parametrization of the inclusive charm quark production differential cross section. Coefficients are calculated from [6] to write E_p in units of GeV without needing redundant conversion steps. The number $m = 1.2$ is a constant at all energies. For the application at hand, energy ranges beyond the given validity intervals might be used when mentioned in the text.

E_p / GeV	a_1 / µb	a_2 / µb	b_1	b_2	n_1	n_2
$10^4 - 10^8$	826	8411	0.197	0.016	1.061	0.107
$10^8 - 10^{11}$	403	2002	0.237	0.023	7.639	0.102

As in [7] it is assumed that the cross section scales linearly with nucleon number, yielding

$$\frac{d\sigma}{dx_c}(x_c, E_p) = A^{-1} \frac{d\sigma}{dx_F}(x_c, E_p)$$

for inclusive charm production in proton proton collisions. Approximating air as a gas mixture of roughly 75 % nitrogen and 25 % oxygen, one finds $A = 14.5$ for this scaling. Translation of charm quarks to charmed hadrons is achieved with a folding integral

$$\frac{d\sigma}{dx_h}(x_h, E_p) = \int_{x_h}^{1} dz\, z^{-1} \frac{d\sigma}{dx_c}(x_c, E_p)\, D_c^h(z)$$

where $z = E_h / E_c$ and $x_h = E_h / E_p$ as well as $x_c = x_h / z$ are fractional energies. Limits for the integration follow from a basic inequality $E_h \leq E_c \leq E_p$ to incorporate kinematic constraints. Furthermore, the probability of observing any final state h originating from a c quark is encoded in a *Fragmentation Function* (FF) $D_c^h(z)$ dependent on the fraction of hadron to charm energy. Reference [8] addresses the connection between this concept and that of a *Parton Distribution Function* (PDF) among other things.

Where a PDF represents the probability density of finding a parton with given momentum in a color neutral particle, probabilities for color neutral states existing inside individual partons are given by the appropriate FF instead. The partons described here are either quarks or gluons, which can be free only asymptotically at high energies due to carrying color charges. In this limit, the running coupling of QCD is small enough for a power series expansion to be a sensible approach, leading to the definition of terms such as LO and NLO in reference to exponent order. There exist different factorization methods to separate these parts from the nonperturbative contributions contained in any PDF and FF for the confined constituents of hadrons. By fitting to existing data or perturbative results, models can extrapolate to low momentum fractions that have not yet been probed experimentally. A similar procedure has lead [9] to obtain

$$D_c^h(z) = \frac{N_h z(1-z)^2}{\big((1-z)^2 + \epsilon_h z\big)^2} \tag{3}$$

with parameters from e^+e^- data in table 4 as the charm hadron FF used throughout this work.

C Spectral Distributions

Pions & Kaons

$$F(x, E_p) = 4\alpha B x^{\alpha-1}\left(\frac{1-x^\alpha}{1+rx^\alpha(1-x^\alpha)}\right)^4\left(\frac{1}{1-x^\alpha}+\frac{r(1-2x^\alpha)}{1+rx^\alpha(1-x^\alpha)}\right)\left(1-\frac{m_0}{xE_p}\right)^{1/2}$$

[10]

Charm

$$\tilde{F}_{h\to\nu}(y) = D_h^{-1}\Big(6b_h a_h^2 - 4a_h^3 - 12\lambda_h^3 a_h + 12\lambda_h^2 y - 6b_h y^2 + 4y^3 + 12\lambda_h^2 \ln\big((1-y)/\lambda_h\big)\Big)$$

$$a_h = 1-\lambda_h \qquad b_h = 1-2\lambda_h \qquad D_h = 1 - 8\lambda_h - 12\lambda_h^2\ln\big(\lambda_h\big) + 8\lambda_h^3 - \lambda_h^4$$

Table 4: Coefficients for charm hadron production, cooling and decay to neutrinos. All parameters ϵ_h are taken from fits to LO QCD results via the FF as defined and described in [9] with normalizations N_h given by [11] to rescale the integration of (3) over [0,1] to approximately match the fractions f_h provided in [12] from measurements. Effective masses $\sqrt{\tilde{s}_h}$ have been determined by [13] from fitting decay rates, except for D_s^+ which is copied from [11] because no original value could be found. Mean lifetimes τ_h and rest masses m_h are adopted from [2] in the particle listings. For mass type quantities, natural units are used.

h	N_h	ϵ_h	τ_h / fs	$\sqrt{\tilde{s}_h}$ / GeV	m_h / GeV
D^0	0.577	0.101	410	0.67	1.86
D^+	0.238	0.104	1033	0.63	1.87
D_s^+	0.0327	0.0322	501	0.84	1.97
Λ_c^+	0.0067	0.00418	203	1.27	2.29

D Cooling & Decay

$$\frac{dN}{dx} = -\frac{N}{\lambda}$$

$$N(x) = N_0 \exp\left(-\frac{x}{\lambda}\right)$$

$$P(x) = 1 - \exp\left(-\frac{x}{\lambda}\right)$$

$x = vt\, v = c\, t = \Gamma\tau\,\Gamma = E/m$

$\lambda = (\kappa\sigma n)^{-1}$

$t_{\mathrm{dec}} = \Gamma\tau$

$t_{\mathrm{cl}} = \lambda/c$

$$\frac{dN}{dt} = -\frac{N}{\tau}$$

$$N(t) = N_0 \exp\left(-\frac{t}{\tau}\right)$$

$$P(t) = 1 - \exp\left(-\frac{t}{\tau}\right)$$

E High Energy Cutoff

F Expansive Magnetic Fields

G Stochastic Acceleration

Due to its wide applicability in different astrophysical scenarios, probabilistic collisions are often viewed as one of the more plausible mechanisms responsible for accelerating cosmic rays to high energies. The general case is described in [14] and supposes that for each collision, particles gain energy proportional to a constant factor η and remain in the region of acceleration with fixed probability ς on average. With initial conditions N_0 for the particle number and E_0 as the mean energy, this results in $N = N_0\varsigma^k$ and $E = E_0\eta^k$ after k collisions. Using $\ln x^k = k \ln x$ in

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln(\varsigma)}{\ln(\eta)}$$

eliminates the exponent and by rearranging gives the relation

$$N = N_0 \left(\frac{E}{E_0}\right)^{\ln(\varsigma)/\ln(\eta)}$$

connecting energy and number of particles. This integrated spectrum incidentally follows a power law, which is an almost ubiquitous feature observed in cosmic ray physics. One obtains

$$\frac{dN}{dE} = \frac{N_0}{E_0}\left(\frac{E}{E_0}\right)^{\alpha}$$

for the differential spectrum where the spectral index

$$\alpha = \frac{\ln(\varsigma)}{\ln(\eta)} - 1$$

is constrained by $\ln(\varsigma)/\ln(\eta) < 0$ due to $\varsigma < 1$ and $\eta > 1$ as implied per the definitions.

The basic case of DSA considers strong shock fronts moving with velocity $\beta = v/c$ in a fully ionized gas. Requiring momentum isotropization without significant energy losses on both sides of the discontinuity results in $\ln(\varsigma)/\ln(\eta) = -1$ for a $dN/dE \propto E^{-2}$ spectral dependence that is discussed by [14] as well. A slightly steeper index $\alpha \approx -2.5$ can be produced when nonlinear effects are accounted for to more closely match observations.

Energy gain increasing linearly with β leads this mechanism to be categorized as Fermi type acceleration of first order, whereas the originally proposed formulation scales like β^2 or as second order. Though shocks exceed the local speed of sound in the astrophysical medium, relativistic velocities are typically not achieved. Consequently, ratios $\beta \ll 1$ mean that lower order processes are much more efficient in reaching high particle energies.

H Pulsar Spindown

I Luminosity Limit

J Implementation

In order to calculate neutrino spectra from hadronic distributions, several integrals have to be computed. Discretizing this task allows the general case

$$F(x,y) = \int_{z_-}^{z_+} dz\, G(x,z)\, H(z,y)$$

to be rewritten as a Riemann sum. Assuming G and H are integrable over a given interval,

$$F_{ij} = \sum_k D_{kk}\, G_{ik}\, H_{kj}$$

converges to the exact solution for sufficiently small steps. Transforming variables

$$x \to x_i \qquad y \to y_j \qquad z \to z_k$$

and defining $D_{kk} = z_{k+1} - z_k$ leads to the above notation. It is easily shown how this expression in terms of indices translates to the product of corresponding matrices

$$\boldsymbol{F} = \boldsymbol{G}\boldsymbol{D}\boldsymbol{H}$$

as an equivalent formulation. Here the output $\boldsymbol{F} \in \mathbb{R}^{m\times n}$ is obtained from the inputs $\boldsymbol{G} \in \mathbb{R}^{m\times l}$ and $\boldsymbol{H} \in \mathbb{R}^{l\times n}$ as well as the square matrix $\boldsymbol{D} \in \mathbb{R}^{l\times l}$ that encodes all step sizes on its diagonal. These results enable a quick and efficient implementation of the required calculations as program code, where array arithmetic operations can greatly increase execution speed.[1]

[1] In service of reproducability, all implementations can be viewed in this repository.

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Figures

Tables