# Neutrinos via Charm Decays in Astrophysical Sources

by

FRITZ ALI AGILDERE

fritz.agildere@udo.edu

# Abstract

# Contents

# Abbreviations

**AGN** Active Galactic Nucleus

**CKM** Cabibbo, Kobayashi & Maskawa

**CMB** Cosmic Microwave Background

**DSA** Diffusive Shock Acceleration

**EWT** Electroweak Theory

**FF** Fragmentation Function

**FFE** Force Free Electrodynamics

**GR** General Relativity

**GZK** Greisen, Sazepin & Kusmin

**LO** Leading Order

**MHD** Magnetohydrodynamics

**NLO** Next to Leading Order

**PDF** Parton Distribution Function

**PMNS** Pontecorvo, Maki, Nakagawa & Sakata

**QCD** Quantum Chromodynamics

**QED** Quantum Electrodynamics

**QFT** Quantum Field Theory

**QM** Quantum Mechanics

**SM** Standard Model

**SMBH** Supermassive Black Hole

**SR** Special Relativity

**UHECR** Ultrahigh Energy Cosmic Ray

# 1 Introduction

# 2 Background

While traversing space, messenger particles are subjected to many different influences, both in terms of type and magnitude, before being detected at earth. Understanding this propagation requires a thorough comprehension of physical processes at all scales of application, from production in astrophysical sources, through interactions with the vast radiation and matter fields that fill the cosmos, to entering the solar system and finally the terrestial atmosphere.

The following sections provide an incomplete overview of aspects relevant to the treatment of this complex topic, as well as references for further study on the various related subjects.

## 2.1 Particle Physics

In the modern view, interactions and categories of elementary particles are most accurately described by a construct called the *Standard Model* (SM) of particle physics. Underlying this formalism is the mathematical framework known as *Quantum Field Theory* (QFT) that combines and generalizes properties of *Quantum Mechanics* (QM) and *Special Relativity* (SR) to produce an extremely precise description of microscopic reality. Excitations in the associated fields are referred to as quanta and manifest as observable particles.

Since this work is mostly concerned with the statistical behaviour of large quantities instead of individual particle probabilistics, a more detailed explanation is omitted except for certain phenomena important to the justification of some later assumptions.

### Leptons

Neutrinos, Decays

### Hadrons

Confinement, Asymptotic Freedom, Residual Nuclear Force, Decays

## 2.2 Multimessenger Astronomy

### Gravitational Waves

### Cosmic Rays

### Photons

### Neutrinos

## 2.3 Astrophysical Sources

### Magnetars

### Active Galactic Nuclei

# 3 Methods

# 4 Results

# 5 Conclusion & Outlook

# Appendix

## A Reference Frames

Depending on the application, energies in particle physics are either given as viewed from a suitable rest frame or independent from the choice of coordinate system altogether. One widely adopted formulation uses the Mandelstam variables

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad u = (p_1 - p_4)^2$$

to assign different channels in scattering processes via the squared momentum carried by the exchanged mediating particle. Implied in this context is the Minkowski inner product, making the above quantities manifestly Lorentz invariant.

When working with parametrizations defined for use in different subdisciplines it often becomes necessary to convert from center of mass energies $\sqrt{s}$ to the energy $E$ of a projectile in the target rest frame. With $E^2 = \boldsymbol{P}^2c^2 + M^2c^4$ as well as momenta $P = (E, \boldsymbol{P}c)$ and $p = (mc^2, 0)$ one finds

$$s = (P + p)^2 = (E + mc^2)^2 - \boldsymbol{P}^2c^2 = 2Emc^2 + m^2c^4 + M^2c^4$$

for the invariant mass. This relation is typically approximated as $s = 2Emc^2$ at high energies.

## B Cross Sections

By defining an effective area perpendicular to the velocity vectors of projectiles and targets, cross sections measure the probability of collision processes in particle physics. Due to depending on the strength of an interaction, these quantities generally scale with energy. Distinct from the integrated case, differential cross sections are usually given with respect to some independent variable such as angle or momentum of the particle.

### Scattering

To model total cross sections in hadron proton scattering, this work uses the formula

$$\sigma_{hp} = H_h \ln^2(s/s_h) + P_h + R_h^1(s_h/s)^{\eta_1} + R_h^2(s_h/s)^{\eta_2} \tag{1}$$

as given in [1] for a universal analytic parametrization of the corresponding amplitudes.

All adjustable parameters are listed in table 1 together with relevant meson lifetimes for cooling. In this approach, the variable $M$ relates to $H = \pi(\hbar c/M)^2$ and $s_h = (m_h + m_p + M)^2$ as an effective mass. Coefficients in (1) denote Heisenberg, Pomeranchuk and Regge terms which have some qualitative motivation, though the formula itself is primarily a quantitative result.

Table 1: Fits to the total inclusive scattering cross sections in hadron proton collisions. Parameters are taken from [1] with $M = 2.121$ GeV for $H = 0.272$ mb as the rate of growth. Both $\eta_1 = 0.447$ and $\eta_2 = 0.5486$ are dimensionless exponents. Decay times $\tau_h$ and rest masses $m_h$ can be found in the particle listings [2] where the latter are given in natural units.

| $h$ | $P_h$ / mb | $R_h^1$ / mb | $R_h^2$ / mb | $\tau_h$ / ns | $m_h$ / GeV | $s_h$ / GeV$^2$ |
|---|---|---|---|---|---|---|
| $p$ | 34.41 | 13.07 | 7.39 | | 0.938 | 15.98 |
| $\pi$ | 18.75 | 9.56 | 1.767 | 26.03 | 0.140 | 10.23 |
| $K$ | 16.36 | 4.29 | 3.408 | 12.38 | 0.494 | 12.62 |

Assuming a quasi universal ratio $\mathcal{R}$ between elastic and total hadron cross sections, one obtains the inelastic cross section $\sigma_{\text{inel}} = (1 - \mathcal{R})\sigma_{\text{tot}}$ from $\sigma_{\text{el}} = \mathcal{R}\sigma_{\text{tot}}$ and $\sigma_{\text{el}} + \sigma_{\text{inel}} = \sigma_{\text{tot}}$ as a unitarity condition. Provided in [3] is the model independent parametrization

$$\mathcal{R}(s) = \frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)} = \mathcal{A} \tanh\big(\gamma_1 - \gamma_2 \ln(s) + \gamma_3 \ln^2(s)\big) \tag{2}$$

with a constant asymptote $\mathcal{A}$ at very high energies. Coefficients are given in table 2 for different physical settings. Both equations (1) and (2) use units of GeV$^2$ for the $s$ variables.

Table 2: Almost model independent ratio of hadronic elastic and total scattering cross sections. Factors $\gamma$ are taken from [3] for varying $\mathcal{A}$ asymptotes.

| $\mathcal{A}$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|---|---|---|---|
| 1/2 | 0.466 | 0.0259 | 0.00177 |
| 1 | 0.2204 | 0.0111 | 0.00076 |

Reference [4] tests the asymptotic rise $\sigma(s) \propto \ln^2(s)$ derived in [5] as a theoretical upper bound and concludes that it is somewhat exceeded. Additionally, a ratio $\mathcal{A} = 1/3$ due to diffraction as opposed to the black disc limit $\mathcal{A} = 1/2$ from optical theorem predictions is suggested. Because parameters are only available in the latter case, all calculations of $\mathcal{R}$ use function (2) as defined by an asymptote $\mathcal{A} = 1/2$ for this work. Data matching $\mathcal{A} = 1/3$ then implies underestimated values of $\sigma_{\text{inel}}(s)$ which should however not significantly influence the overall results.

## Production

For charm quark production in proton air collisions, reference [6] gives

$$x_F \frac{d\sigma}{dx_F}(x_F, E_p) = a x_F^b (1 - x_F^m)^n$$

as the parametrized differential cross section with components

$$a = a_1 \ln(E_p) - a_2 \qquad b = b_1 - b_2 \ln(E_p) \qquad n = n_1 - n_2 \ln(E_p)$$

for which table 3 lists all necessary constants. Here proton energies $E_p$ are defined as viewed by air nuclei at rest, while the Feynman scaling variable $x_F = p_c / p_s$ specifies magnitude ratios of produced charm quark longitudinal momentum to all available momentum in center mass coordinates of the colliding particles. Application of appendix A shows that this approximately fulfills $x_F = x_c$ where $x_c = E_c / E_p$ in the relevant energy ranges.

TABLE 3: Parametrization of the inclusive charm quark production differential cross section. Coefficients are calculated from [6] to write $E_p$ in units of GeV without needing redundant conversion steps. The number $m = 1.2$ is a constant at all energies. For the application at hand, energy ranges beyond the given validity intervals might be used when mentioned in the text.

| $E_p$ / GeV | $a_1$ / µb | $a_2$ / µb | $b_1$ | $b_2$ | $n_1$ | $n_2$ |
|---|---|---|---|---|---|---|
| $10^4 - 10^8$ | 826 | 8411 | 0.197 | 0.016 | 8.486 | 0.107 |
| $10^8 - 10^{11}$ | 403 | 2002 | 0.237 | 0.023 | 7.639 | 0.102 |

As in [7] it is assumed that the cross section scales linearly with nucleon number, yielding

$$\frac{d\sigma}{dx_c}(x_c, E_p) = A^{-1} \frac{d\sigma}{dx_F}(x_c, E_p)$$

for inclusive charm production in proton proton collisions. Approximating air as a gas mixture of roughly 75 % nitrogen and 25 % oxygen, one finds $A = 14.5$ for this scaling. Translation of charm quarks to charmed hadrons is achieved with a folding integral

$$\frac{d\sigma}{dx_h}(x_h, E_p) = \int_{x_h}^{1} dz\, z^{-1} \frac{d\sigma}{dx_c}(x_c, E_p)\, D_c^h(z) \tag{3}$$

where $z = E_h / E_c$ and $x_h = E_h / E_p$ as well as $x_c = x_h / z$ are fractional energies. Limits for the integration follow from a basic inequality $E_h \leq E_c \leq E_p$ to incorporate kinematic constraints. Furthermore, the probability of observing any final state $h$ originating from a $c$ quark is encoded in a *Fragmentation Function* (FF) $D_c^h(z)$ dependent on the fraction of hadron to charm energy. Reference [8] addresses the connection between this concept and that of a *Parton Distribution Function* (PDF) among other things.

Where a PDF represents the probability density of finding a parton with given momentum in a color neutral particle, probabilities for color neutral states existing inside individual partons are given by the appropriate FF instead. The partons described here are either quarks or gluons, which can be free only asymptotically at high energies due to carrying color charges. In this limit, the running coupling of QCD is small enough for a power series expansion to be a sensible approach, leading to the definition of terms such as LO and NLO in reference to exponent order. There exist different factorization methods to separate these parts from the nonperturbative contributions contained in any PDF and FF for the confined constituents of hadrons. By fitting to existing data or perturbative results, models can extrapolate to low momentum fractions that have not yet been probed experimentally. A similar procedure has lead [9] to obtain

$$D_c^h(z) = \frac{N_h z(1-z)^2}{\left((1-z)^2 + \epsilon_h z\right)^2} \tag{4}$$

with parameters from $e^+e^-$ data in table 4 as the charm hadron FF used throughout this work. It is important to note that such functions are invariant under charge conjugation, so that there is no differentiation between quark to particle or antiquark to antiparticle processes.

## C Spectral Distributions

Constructing spectra $dN_h/dE_h$ from proton injection requires folding of $F_{p\to h}$ as the hadronic distribution for a single $pp$ interaction with the number of protons per energy interval given by the function $dN_p/dE_p$ obtained from source specific modelling. An analogous approach can be applied to compute neutrino spectra $dN_\nu/dE_\nu$ from $dN_h/dE_h$ via distributions $F_{h\to\nu}$ formulated according to the involved decay modes.

### Charm

Spectral distributions for charmed hadron production are calculated according to [10] through

$$F_{p\to h}(E_h, E_p) = E_p^{-1}\sigma_{pp}^{-1}(E_p)\,\frac{d\sigma}{dx_h}(x_h, E_p)$$

with $E_h = x_h E_p$ translating between variables. This formula can be understood as a normalization of (3) against proton energies and the inelastic $pp$ cross section from appendix B to yield hadron numbers per unit energy.

To find neutrino spectra from charmed hadrons, the same approach as in [10] is used, which assumes an effective energy distribution approximated by three body decays for the semileptonic channel to a less massive pseudoscalar meson. By neglecting lepton masses, one obtains

$$\tilde{F}_{h\to\nu}(y) = D_h^{-1}\left(6b_h a_h^2 - 4a_h^3 - 12\lambda_h^3 a_h + 12\lambda_h^2 y - 6b_h y^2 + 4y^3 + 12\lambda_h^2 \ln\left((1-y)/\lambda_h\right)\right)$$

as a distribution with $y = E_\nu/E_h$ and $F_{h\to\nu}(E_\nu, E_h) = \mathcal{F}_h \tilde{F}_{h\to\nu}(y)/E_h$ for conversion.

Hadron specific coefficients for this equation are defined with the parameter $\lambda_h = \tilde{s}_h / m_h^2$ as

$$a_h = 1 - \lambda_h \qquad b_h = 1 - 2\lambda_h \qquad D_h = 1 - 8\lambda_h - 12\lambda_h^2 \ln(\lambda_h) + 8\lambda_h^3 - \lambda_h^4$$

where both $s_h$ and $m_h$ are listed in table 4 for all included charmed hadrons. The assumption of three body decays like $D^+ \to \overline{K}^0 e^+ \nu_e$ can be justified by consulting [2] for information on the relevant particles and comparing branching ratios, which indicate that purely leptonic modes are strongly suppressed. Hadronic channels such as $D^+ \to \pi^+ \pi^0$ or $D^+ \to K^- \pi^+ \pi^+$ are either very improbable as well or occur at significant rates but do not contribute many high energy neutrinos due to pions and kaons being subject to further cooling before decaying to leptons. By the same logic, secondary muon decay is neglected when determining the neutrino spectrum.

TABLE 4: Coefficients for charm hadron production, cooling and decay to neutrinos. All parameters $\epsilon_h$ are taken from fits to LO QCD results via the FF as defined and described in [9] with normalizations $N_h$ given by [10] to rescale the integration of (4) over [0,1] to approximately match the fractions $f_h$ provided in [11] from measurements. Effective masses $\sqrt{\tilde{s}_h}$ and branching fractions $\mathcal{F}_h$ are determined by [12] and [13] from fitting decay rates. Mean lifetimes $\tau_h$ and masses $m_h$ are adopted from [2] in the particle listings. Mass type quantities use natural units.

| $h$ | $N_h$ | $\epsilon_h$ | $\tau_h$ / fs | $\mathcal{F}_h$ | $\sqrt{\tilde{s}_h}$ / GeV | $m_h$ / GeV |
|---|---|---|---|---|---|---|
| $D^0$ | 0.577 | 0.101 | 410 | 0.067 | 0.67 | 1.86 |
| $D^+$ | 0.238 | 0.104 | 1033 | 0.176 | 0.63 | 1.87 |
| $D_s^+$ | 0.0327 | 0.0322 | 501 | 0.065 | 0.84 | 1.97 |
| $\Lambda_c^+$ | 0.0067 | 0.00418 | 203 | 0.045 | 1.27 | 2.29 |

## Pions & Kaons

By parametrizing event generator results, a neutral pion production spectrum of the form

$$\tilde{F}_\pi(x_\pi, E_p) = 4\alpha B x_\pi^{\alpha-1} \left( \frac{1 - x_\pi^\alpha}{1 + r x_\pi^\alpha (1 - x_\pi^\alpha)} \right)^4 \left( (1 - x_\pi^\alpha)^{-1} + \frac{r(1 - 2x_\pi^\alpha)}{1 + r x_\pi^\alpha (1 - x_\pi^\alpha)} \right) \left( 1 - \frac{m_\pi}{x_\pi E_p} \right)^{1/2}$$

is found in [14] with $m_\pi = 0.135\,\mathrm{GeV}$ [2] translated to natural units and parameters

$$B = \tilde{B} + C \qquad \alpha = \frac{\tilde{\alpha}}{\sqrt{C}} \qquad r = \frac{\tilde{r}}{\sqrt{C}}$$

where a low energy cutoff is enforced via $E_\pi = x_\pi E_p$ in the mass term. From

$$C = c_1 - c_2 \ln(E_p) + c_3 \ln^2(E_p)$$

results a dependence on projectile energy for the shape of this distribution.

Coefficients are specified in table 5 and recalculated for $E_p$ in GeV instead of TeV units. Under the assumption of a $\pi^0$ cross section approximately equal to the $\pi^\pm$ average and with identical spectra for charged pions, it follows that $F_\pi = \tilde{F}_\pi / E_p$ should describe pion production regardless of charge reasonably well for the purpose of this work.

TABLE 5: Parametrized spectral distribution for neutral pion production. Factors are taken from [14] and converted to write $E_p$ in units of GeV for $c_k$ coefficients.

| $\tilde{B}$ | $\tilde{\alpha}$ | $\tilde{r}$ | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|---|---|
| 0.25 | 0.98 | 2.6 | 1.515 | 0.206 | 0.075 |

For a convenient formulation of kaon production, references [15] and [16] indicate a constant ratio $\pi/K$ at moderately high energies. Similar fractions are retrieved from multiplicities given in [17] and lead to $F_K/F_\pi = 0.12$ as a simplifying assumption, the validity of which cannot be guaranteed for the application at hand. Calculations of kaon spectra still employ this approach but are subject to considerable reservations as a result.

Decays of pions and kaons to neutrinos are approximated via the $h \to \mu^+ \nu_\mu$ two body channel with branching fractions of $\mathcal{F}_\pi = 99.99\,\%$ and $\mathcal{F}_K = 63.56\,\%$ given in the [2] particle listings. By decaying, muons produced in these processes can significantly impact the neutrino spectrum. Results from [10] suggest that this is particularly relevant for pions. Muonic three body decays of type $\mu^- \to e^- \overline{\nu}_e \nu_\mu$ as well as cooling factors depend on the polarization of participating leptons due to the nature of weak force coupling. This complicates computations and is thus omitted in service of restricting the present work to a managable scope, though it should be remembered as an important caveat for the final results.

The remaining two body decays of ultrarelativistic hadrons $h$ to leptons $l$ obey a distribution

$$F_{h\to l}(E_l, E_h) = \mathcal{F}_h E_h^{-1} (1 - \lambda_h)^{-1}$$

with $m_\nu = 0$ and $m_\mu = 0.106\,\text{GeV}$ [2] as well as $\lambda_h = m_\mu^2 / m_h^2$ as a parameter. This formula is the same whether $l = \nu$ or $l = \mu$ because there is one muon for each neutrino. In addition, kinematic considerations lead to $E_\mu/E_h > \lambda_h$ and $E_\nu/E_h < 1 - \lambda_h$ for integral bounds $E_\mu < E_h < E_\mu/\lambda_h$ in the case of muons or $E_\nu/(1-\lambda_h) < E_h < E_p$ when considering neutrinos.

## D Cooling & Decay

Consider an infinitesimally thin slice of a target medium with particle number density $n$ and volume $V = S\,dx$ where $S$ and $dx$ measure surface area and thickness, respectively. Accordingly, there exist $\tilde{N} = nV$ targets that each have effective interaction cross sections $\tilde{\sigma}$ with a total coverage of $\tilde{S} = \tilde{\sigma}\tilde{N}$ as viewed by an incident projectile.

The probability for stopping such a beam constituent then corresponds to the ratio $\mathcal{P} = \tilde{S}/S$ of both areas or explicitly $\mathcal{P} = \tilde{\sigma} n\, dx$ in case of $dx$ as the covered distance. Expressing $\tilde{\sigma} = \kappa\sigma$ in terms of the inelastic scattering cross section $\sigma$ and a dimensionless factor $\kappa$ called inelasticity, one can identify a length scale $\lambda = (\kappa\sigma n)^{-1}$ as the mean free path between collisions. Multiplying with $\kappa$ includes the ratio of remaining to initital energy, which is taken to be constant. From this follows a reduction in beam particles $dN = -N\lambda^{-1}dx$ proportional to $N$ as the total projectile count and $\mathcal{P} = \lambda^{-1}dx$ for the reformulated probability which represents an ordinary differential equation of first order. The solution is found to follow an exponential law

$$N(x) = N_0 \exp\left(-\frac{x}{\lambda}\right)$$

where $N$ particles remain over some distance $x$ with $N_0$ as the initial amount. Furthermore,

$$P(x) = 1 - \exp\left(-\frac{x}{\lambda}\right)$$

gives the probability of a particle having been scattered after travelling $x$ length units. Similar steps for time instead of distance lead to the well known equation

$$N(t) = N_0 \exp\left(-\frac{t}{\tau}\right)$$

describing exponential decay. It commonly appears in the context of radioactive materials but also applies to hadrons and leptons or more generally any quantity which decreases at a rate proportional to itself. Analogous to the previous case, a particle decays with probability

$$P(t) = 1 - \exp\left(-\frac{t}{\tau}\right)$$

before a time $t$ has passed and for $\tau$ as the mean lifetime. Translating this from rest frame to laboratory coordinates defines the decay timescale $t_{\mathrm{dec}} = \tau\Gamma$ with a Lorentz factor $\Gamma = E/m$ via projectile energy and invariant mass. This is equivalent to a characteristic decay length given by $\lambda_{\mathrm{dec}} = vt_{\mathrm{dec}}$ where the velocity $v = c$ can be set for highly relativistic particles. Additionally, mean free path and cooling distance $\lambda_{\mathrm{cool}} = (\kappa\sigma n)^{-1}$ refer to exchangeable concepts. Particles lose energy in every collision, which is the same as reducing temperature from a thermodynamics perspective. Dividing by the speed of light $c$ translates this expression to $t_{\mathrm{cool}} = (\kappa\sigma nc)^{-1}$ as a cooling timescale. Analogously, the distance $\lambda_{\mathrm{dec}} = c\tau E/m$ can be rewritten as $t_{\mathrm{dec}} = \tau E/m$ in units of time. Substituting into the decay formula yields a cooling factor

$$C = 1 - \exp\left(-\frac{t_{\mathrm{cool}}}{t_{\mathrm{dec}}}\right) \tag{5}$$

which scales spectra from direct production to account for decay processes taking place after collisional energy losses have occured. This is in some sense a core mechanism for the hypothesis that neutrinos from charm dominate pion and kaon contributions at high energy regimes. While longer lived particles experience significant cooling due to time dilation, charmed hadron decay is prompt in comparison. One requirement for the validity of (5) is that $\lambda_{\mathrm{dec}} \ll d$ holds with a target field size $d$ to ensure decays occur exclusive inside this region.

## E High Energy Cutoff

At very high energies, protons and nuclei interact with cosmic photons, which can be blueshifted up to extreme gamma regimes due to the Doppler effect. In these processes, the photon spin is absorbed to produce a delta resonance $\Delta^+$ representing an excited proton state. This decays almost immediately to pairs of nucleons and pions, leading to $p\gamma \to n\pi^+$ or $p\gamma \to p\pi^0$ as probable reaction channels and resulting in a change of momentum for the produced particles.

For CMB radiation, a close to perfect black body spectrum has been measured with

$$\frac{dn}{d\epsilon}(\epsilon) = \frac{\epsilon^2}{\pi^2 \hbar^3 c^3 \left(e^{\epsilon/k_B T} - 1\right)}$$

given by [18] as the photon density. This predicts that at a temperature $T = 2.725\,\mathrm{K}$ significant numbers of photons exist with energies around $\epsilon = 10^{-12}$ GeV in the CMB rest frame. To find a threshold in center of mass coordinates, the scalar product is used for

$$s = \left(p_p + p_\gamma\right)^2 = p_p^2 + p_\gamma^2 + 2p_p p_\gamma = m_p^2 c^4 + 2E\epsilon = M^2 c^4$$

where $M = m_p + m_\pi$ and $m_\gamma = 0$ as well as head on collisions have been assumed. The solution

$$E = \left(\left(m_p + m_\pi\right)^2 c^4 - m_p^2 c^4\right)/(2\epsilon)$$

or roughly $E = 10^{11}$ GeV for the so called GZK cutoff. Energies that exceed this value when viewed from CMB coordinates lead to significant losses, making the universe opaque to such protons. If pair production $\gamma \to e^+ e^-$ via interaction with a proton is considered, one finds

$$E = \left(\left(m_p + 2m_e\right)^2 c^4 - m_p^2 c^4\right)/(2\epsilon)$$

approximated to $E = 10^9$ GeV by setting $M = m_p + 2m_e$ instead. For alpha particles and heavier nuclei, similar steps apply, while electrons are limited mainly through Compton downscattering. Additionally, there exist infrared and radio backgrounds as potentially relevant radiation fields. From the multiple competing mechanisms for energy losses and momentum isotropization, it is as a consequence extremely challenging to reliably interpret cosmic ray signals and practically impossible to reconstruct any information about specific sources.

## F Magnetic Field Scales

Particles carrying electric charge $Q = Ze$ and moving at velocity $v$ orthogonal to a homogenous magnetic field $B$ are acted on by the Lorentz force $F = QvB$ resulting in a gyrating motion. This must then be equal to the relativistic centripetal force $F = m\Gamma v^2/R$ on a circular path with $R$ as the Larmor radius. Solving by rearranging and identifying $p = m\Gamma v$ leads to

$$R = \frac{p}{QB}$$

on which a condition $R < D$ with the magnetic field extent $D$ can be placed.

In case of highly relativistic energies one can write $E = pc$ to replace momentum and obtain an inequality $E < QcBD$ from the above considerations. Realistic astrophyiscal magnetic fields are not ordered, following turbulences travelling through the plasma instead. To incorporate effects of moving scattering centers, a factor $\beta$ proportional to the Alfvén velocity is included, giving

$$E < Ze\beta cBD \tag{6}$$

as the Hillas criterion, named after its description in [19] to constrain source region sizes based on prevailing magnetic fields for UHECR acceleration.

## G Stochastic Acceleration

Due to its wide applicability in different astrophysical scenarios, probabilistic collisions are often viewed as one of the more plausible mechanisms responsible for accelerating cosmic rays to high energies. The general case is described in [20] and supposes that for each collision, particles gain energy proportional to a constant factor $\eta$ and remain in the region of acceleration with fixed probability $\varsigma$ on average. With initial conditions $N_0$ for the particle number and $E_0$ as the mean energy, this results in $N = N_0\varsigma^k$ and $E = E_0\eta^k$ after $k$ collisions. Using $\ln x^k = k \ln x$ in

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln(\varsigma)}{\ln(\eta)}$$

eliminates the exponent and by rearranging gives the relation

$$N = N_0 \left(\frac{E}{E_0}\right)^{\ln(\varsigma)/\ln(\eta)}$$

connecting energy and number of particles. This integrated spectrum incidentally follows a power law, which is an almost ubiquitous feature observed in cosmic ray physics. One obtains

$$\frac{dN}{dE} = \frac{N_0}{E_0}\left(\frac{E}{E_0}\right)^{\alpha}$$

for the differential spectrum where the spectral index

$$\alpha = \frac{\ln(\varsigma)}{\ln(\eta)} - 1$$

is constrained by $\ln(\varsigma)/\ln(\eta) < 0$ due to $\varsigma < 1$ and $\eta > 1$ as implied per the definitions.

The basic case of DSA considers strong shock fronts moving with velocity $\beta = v/c$ in a fully ionized gas. Requiring momentum isotropization without significant energy losses on both sides of the discontinuity results in $\ln(\varsigma)/\ln(\eta) = -1$ for a $dN/dE \propto E^{-2}$ spectral dependence that is discussed by [20] as well. A slightly steeper index around $\alpha = -2.5$ can be derived when nonlinear effects are accounted for to better reproduce measurements.

Energy gain increasing linearly with $\beta$ leads this mechanism to be categorized as Fermi type acceleration of first order, whereas the originally proposed formulation scales like $\beta^2$ or as second order. Though shocks exceed the local speed of sound in the astrophysical medium, relativistic velocities are typically not achieved. Consequently, ratios $\beta \ll 1$ mean that lower order processes are much more efficient in reaching high particle energies.

## H Pulsar Spindown

To explain observations of rapidly spinning neutron stars or pulsars, there has to exist some mechanism by which rotational energy is lost. Reference [21] gives a brief overview of possible radiation candidates such as gravitational quadrupolar or higher order electromagnetic moments. Because this work is limited in its scope and concerns itself with the acceleration of electric charges, a pure magnetic dipole approach will be adopted. For more compact and convenient notation, Gaussian units are used.

In an idealized view like [22] of stars as sharply bound and uniformly magnetized spheres, one finds $\mu = R^3 B/2$ for the external magnetic moment. The parameters $R$ and $B$ measure stellar radius and polar magnetic flux density, respectively. In case of a rotating dipole in vacuo,

$$L = \frac{2\mu^2 \omega^4 \sin^2 \chi}{3c^3} \tag{7}$$

is the exact expression for radiant power derived in [23] as a standard reference, where a static angle $\chi$ between the dipole field and rotational axis with angular frequency $\omega$ is assumed. If instead a FFE limit is applied, variational calculations [24] and MHD simulations [25] indicate

$$L = \frac{\mu^2 \omega^4 \left(1 + \sin^2 \chi\right)}{c^3} \tag{8}$$

as an appropriate expression of the luminosity. This is likely somewhat more accurate than the previous result, as it has been shown by [26] that the surroundings of a neutron star cannot support a vacuum but must instead be filled with a plasma of charge carriers originating from instabilities at the surface. However, one immediately encounters the problem that $\boldsymbol{E} \cdot \boldsymbol{B} = 0$ as a condition of force free magnetospheres prevents particle acceleration. As [27] and [28] discuss, this can be overcome by introducing deviations from such a global solution on local scales.

Spindown is described by an exponential energy decay $E = E_0 \exp\left(-t/t_{\text{sd}}\right)$ with a characteristic timescale $t_{\text{sd}}$ and $\dot{E} = -E/t_{\text{sd}}$ for the time derivative. Assuming all energy is stored in rotational form, the expressions $E = I\omega^2/2$ and $\dot{E} = I\omega\dot{\omega}$ are also valid with $I$ as the neutron star moment of inertia. Equations (7) and (8) can be generalized as $L = K\omega^4$ by using a coefficient $K$ containing all other information except the dipolar $\omega^4$ dependence. Identifying this with the energy loss via $\dot{E} = -L$ and evaluating at $t = 0$ yields $L_0 = K\omega_0^4$ as well as $\dot{E}_0 = -I\omega_0^2/2t_{\text{sd}}$ for an expression of $t_{\text{sd}} = I/2K\omega_0^2$ as the spindown time. It further follows for any $t$ that $I\dot{\omega} = -K\omega^3$ must hold. This is a special case of the power law differential equation $\dot{\omega} \propto \omega^n$ with $n = 3$ being the braking index characteristic of a rotating magnetic dipole. Separating variables and integrating leads to finding $\omega = \omega_0\left(1 + t/t_{\text{sd}}\right)^{-1/2}$ as the time dependent frequency solution.

## I Accretion Disks

Diffusely distributed material in orbit around a central massive object naturally produces disk like structures. This is a consequence of gravitational forces being compensated in the radial plane by rotational effects while matter is relatively free to collapse in the axial direction.

Compression and friction during inward spiraling heat the disk, thereby emitting intense thermal radiation. Settings where these phenomena likely occur are protoplanetary disks surrounding newly formed stars or accretion flows on a SMBH inside the galactic core region, among others. The following describes some related effects that are for example discussed by [20] as well.

## Hydrostatic Equilibrium

In Newtonian physics, the gravitational force at distance $R$ from a mass $M$ is given as

$$F = \frac{GMm}{R^2} \tag{9}$$

for particles with masses $m$ orbiting the central object. The resulting acceleration is therefore of magnitude $g = GM/R^2$ with vertical component $\ddot{z} = g\sin(\vartheta)$ with $\vartheta$ as the elevation angle. In case of $\vartheta \ll \pi/2$ one can approximate $\sin(\vartheta) = z/R$ to obtain

$$\ddot{z} = \frac{GMz}{R^3}$$

which is related to pressure $p$ and density $\rho$ via $dp/dz = -\rho\ddot{z}$ for a stable equilibrium to exist. Using the speed of sound $u^2 = dp/d\rho$ allows reformulation to

$$\frac{d\rho}{dz} = -\rho\frac{GMz}{u^2R^3}$$

and $\rho = \rho_0 \exp\left(-z^2/2h^2\right)$ after integration. According to this result, the disk density follows a centered Gaussian distribution with height parameter $h$ defined via

$$h^2 = \frac{u^2R^3}{GM}$$

as the standard deviation. Assuming $M \gg m$ and circular Keplerian orbits, expression (9) equals the acting centripetal force, leading to $v^2 = GM/R$ for the orbital velocity. One is then able to write $h^2 = u^2R^2/v^2$ and from condition $h \ll R$ necessary for the small angle approximation to be valid follows $u \ll v$ or that orbital velocities must greatly exceed the disk medium specific speed of sound. This special case is called a thin disk and can further be extended to slim or even thick types of accretion structures.

## Luminosity Limit

Any luminous object radiating with spherical symmetry excerts a pressure

$$P = \frac{L}{4\pi R^2 c}$$

as a function of luminosity $L$ at distance $R$ from the source. Suppose a gaseous cloud containing particles with masses $m$ falling towards the same bright central mass $M$ due to gravitational attraction. During this process, each particle experiences $F = \varkappa mP$ as an opposing radiative force where the opacity $\varkappa$ measures cross section per unit mass.

Balancing of (9) and rearranging leads to the Eddington limit

$$L = \frac{4\pi GMc}{\varkappa} \tag{10}$$

on the luminosity beyond which additional matter is immediately blown away from the central object. By assuming that infalling material consists exclusively of ionized hydrogen, one can approximate $\varkappa = m/\sigma$ with proton mass $m$ and Thomson cross section $\sigma$ from electron scattering. Though originally applied in the context of stellar structures, this approach can also be used to describe accretion disks. If a compact object increases its mass with rate $\dot{M}$ due to accreting matter, some of the corresponding gravitational potential may be converted to radiation. In terms of rest energy, a luminosity $L = \eta \dot{M} c^2$ is obtained where $\eta$ denotes the efficiency of this mechanism. With (10) one finds an analogous steady state limit

$$\dot{M} = \frac{4\pi GM}{\eta c \varkappa}$$

for the accretion rate, enforced by continuous balancing of radiation pressure and gravitational forces as a natural feedback process.

# J Implementation

In order to calculate neutrino spectra from hadronic distributions, several integrals have to be computed. Discretizing this task allows the general case

$$F(x, y) = \int_{z_-}^{z_+} dz\, G(x, z)\, H(z, y)$$

to be rewritten as a Riemann sum. Assuming $G$ and $H$ are integrable over a given interval,

$$F_{ij} = \sum\nolimits_k D_{kk}\, G_{ik}\, H_{kj}$$

converges to the exact solution for sufficiently small steps. Transforming variables

$$x \to x_i \qquad\qquad y \to y_j \qquad\qquad z \to z_k$$

and defining $D_{kk} = z_{k+1} - z_k$ leads to the above notation. It is easily shown how this expression in terms of indices translates to the product of corresponding matrices

$$\boldsymbol{F} = \boldsymbol{G}\boldsymbol{D}\boldsymbol{H}$$

as an equivalent formulation. Here the output $\boldsymbol{F} \in \mathbb{R}^{m\times n}$ is obtained from the inputs $\boldsymbol{G} \in \mathbb{R}^{m\times l}$ and $\boldsymbol{H} \in \mathbb{R}^{l\times n}$ as well as the square matrix $\boldsymbol{D} \in \mathbb{R}^{l\times l}$ that encodes all step sizes on its diagonal. These results enable a quick and efficient implementation of the required calculations as program code, where array arithmetic operations can greatly increase execution speed.[1]

[1] In service of reproducability, all implementations can be viewed in this repository.

# Bibliography


[1] V. Belousov, V. Ezhela, Y. Kuyanov, N. Tkachenko, *Physics of Atomic Nuclei* **79**, 113–117, DOI: 10.1134/S1063778816010075 (2016).

[2] P. D. Group *et al.*, *Progress of Theoretical and Experimental Physics* **2022**, 083C01, ISSN: 2050-3911, DOI: 10.1093/ptep/ptac097 (2022).

[3] D. Fagundes, M. Menon, *Nuclear Physics A* **880**, 1–11, ISSN: 0375-9474, DOI: 10.1016/j.nuclphysa.2012.01.017, arXiv: 1112.5115 [hep-ph] (2012).

[4] D. A. Fagundes, M. J. Menon, P. V. R. G. Silva, *Journal of Physics G: Nuclear and Particle Physics* **40**, 065005, ISSN: 1361-6471, DOI: 10.1088/0954-3899/40/6/065005, arXiv: 1208.3456 [hep-ph] (2013).

[5] M. Froissart, *Phys. Rev.* **123**, 1053–1057, DOI: 10.1103/PhysRev.123.1053 (3 1961).

[6] V. P. Gonçalves, M. V. Machado, *Journal of High Energy Physics* **2007**, 028–028, ISSN: 1029-8479, DOI: 10.1088/1126-6708/2007/04/028, arXiv: hep-ph/0607125 (2007).

[7] A. Bhattacharya, R. Enberg, M. H. Reno, I. Sarcevic, A. Stasto, *Journal of High Energy Physics* **2015**, ISSN: 1029-8479, DOI: 10.1007/jhep06(2015)110, arXiv: 1502.01076 [hep-ph] (2015).

[8] A. Metz, A. Vossen, *Progress in Particle and Nuclear Physics* **91**, 136–202, ISSN: 0146-6410, DOI: 10.1016/j.ppnp.2016.08.003, arXiv: 1607.02521 [hep-ex] (2016).

[9] B. A. Kniehl, G. Kramer, *Physical Review D* **74**, ISSN: 1550-2368, DOI: 10.1103/physrevd.74.037502, arXiv: hep-ph/0607306 (2006).

[10] J. A. Carpio, K. Murase, M. H. Reno, I. Sarcevic, A. Stasto, *Physical Review D* **102**, ISSN: 2470-0029, DOI: 10.1103/PhysRevD.102.103001, arXiv: 2007.07945 [astro-ph.HE] (2020).

[11] M. Lisovyi, A. Verbytskyi, O. Zenaiev, *The European Physical Journal C* **76**, ISSN: 1434-6052, DOI: 10.1140/epjc/s10052-016-4246-y, arXiv: 1509.01061 [hep-ex] (2016).

[12] E. V. Bugaev, A. Misaki, V. A. Naumov, T. S. Sinegovskaya, S. I. Sinegovsky, N. Takahashi, *Physical Review D* **58**, ISSN: 1089-4918, DOI: 10.1103/physrevd.58.054001, arXiv: hep-ph/9803488 (1998).

[13] A. Bhattacharya, R. Enberg, Y. S. Jeong, C. Kim, M. H. Reno, I. Sarcevic, A. Stasto, *Journal of High Energy Physics* **2016**, ISSN: 1029-8479, DOI: 10.1007/jhep11(2016)167, arXiv: 1607.00193 [hep-ph] (2016).

[14] S. R. Kelner, F. A. Aharonian, V. V. Bugayov, *Phys. Rev. D* **74**, 034018, DOI: 10.1103/PhysRevD.74.034018, arXiv: astro-ph/0606058 (3 2006).

[15] G. I. Lykasov, A. I. Malakhov, A. A. Zaitsev, *The European Physical Journal A* **57**, ISSN: 1434-601X, DOI: 10.1140/epja/s10050-021-00408-9, arXiv: 2012.02451 [hep-ph] (2021).

[16] G. I. Lykasov, A. I. Malakhov, A. A. Zaitsev, *The European Physical Journal A* **58**, ISSN: 1434-601X, DOI: 10.1140/epja/s10050-022-00773-z, arXiv: 2201.10301 [hep-ph] (2022).

[17] H. B. J. Koers, A. Pe'er, R. A. M. J. Wijers, arXiv: hep-ph/0611219 (2006).

[18] T. K. Gaisser, R. Engel, E. Resconi, *Cosmic Rays and Particle Physics* (Cambridge University Press, Cambridge, Second Edition, 2016), ISBN: 9781139192194, DOI: 10.1017/CBO9781139192194.

[19] A. M. Hillas, *Annual Review of Astronomy and Astrophysics* **22**, 425–444, DOI: 10.1146/annurev.aa.22.090184.002233 (1984).

[20] M. S. Longair, *High Energy Astrophysics* (Cambridge University Press, Cambridge, Third Edition, 2011), ISBN: 9780511778346, DOI: 10.1017/CBO9780511778346.

[21] C. Alvarez, A. Carramiñana, *Astronomy & Astrophysics* **414**, 651–658, ISSN: 1432-0746, DOI: 10.1051/0004-6361:20031627, arXiv: astro-ph/0311267 (2004).

[22] A. J. Deutsch, *Annales d'Astrophysique* **18**, 1 (1955).

[23] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, NY, Third Edition, 1999), ISBN: 9780471309321.

[24] A. Gruzinov, *The Astrophysical Journal* **647**, L119–L122, ISSN: 1538-4357, DOI: 10.1086/506590, arXiv: astro-ph/0510751 (2006).

[25] A. Spitkovsky, *The Astrophysical Journal* **648**, L51–L54, ISSN: 1538-4357, DOI: 10.1086/507518, arXiv: astro-ph/0603147 (2006).

[26] P. Goldreich, W. H. Julian, *Astrophysical Journal* **157**, 869, DOI: 10.1086/150119 (1969).

[27] J. Li, A. Spitkovsky, A. Tchekhovskoy, *The Astrophysical Journal* **746**, 60, DOI: 10.1088/0004-637X/746/1/60, arXiv: 1107.0979 [astro-ph.HE] (2012).

[28] S. E. Gralla, N. Iqbal, *Physical Review D* **99**, ISSN: 2470-0029, DOI: 10.1103/physrevd.99.105004, arXiv: 1811.07438 [hep-th] (2019).

# Figures

# Tables

# Acknowledgements