1 Example 6

$$\begin{aligned}
min. \qquad & \frac{1}{4}\sum_{i=1}^{n}(x_i-1)^4 \\
s.t. \qquad & 4x_1 + 2x_2 = 10 \\
& 5 \leq 2x_1 + x_3 \\
& 1 \leq 2x_1 + 0.5x_i <= 2n, \qquad \forall i = 4, ..., n \\
& x_1 \qquad free \\
& 0.0 \leq x_2 \\
& 1.5 \leq x_3 \leq 10 \\
& x_i \geq 0.5, \qquad \forall i = 4, ..., n
\end{aligned}$$

The problem is convex.

$$[\nabla f(x)]_i = (x_i - 1)^3$$

$$[\nabla^2 f(x)]_{ij} \begin{cases} 3(x_i-1)^2 & i = j \\ 0 & i \neq j \end{cases}$$

2 Example 7

$$\begin{aligned}
min. \qquad & (2\alpha - 1)\frac{1}{4}\sum_{i=1}^{n}(x_i-1)^4 + \frac{1}{2}x^T x \\
s.t. \qquad & 4x_1 + 2x_2 = 10 \\
& 5 \leq 2x_1 + x_3 \\
& 1 \leq 2x_1 + 0.5x_i <= 2n, \qquad \forall i = 4, ..., n \\
& x_1 \qquad free \\
& 0.0 \leq x_2 \\
& 1.5 \leq x_3 \leq 10 \\
& x_i \geq 0.5, \qquad \forall i = 4, ..., n
\end{aligned}$$

For $\alpha < 0.5$, the problem is not convex, for $\alpha \geq 0.5$ it is convex. Optionally the constraints

$$-\infty <= 4x_1 + 2x_3 <= 19$$
$$4x_1 + 2x_2 = 10$$

can be added to make the problem rank deficient.

$$[\nabla f(x)]_i = (2\alpha - 1)(x_i - 1)^3 + x_i$$
$$[\nabla^2 f(x)]_{ij} \begin{cases} (2\alpha - 1)3(x_i - 1)^2 + 1 & i = j \\ 0 & i \neq j \end{cases}$$