EE 672 System Identification: Assignment 2, Part 1

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1 Data Splitting and Preprocessing

Details of Dryer data (like visual inspection and statistical properties) was provided in a great detail in the first assignment. So, it will not be covered in this report. In Dryer data, there are 1000 samples/measurement points. Estimation data set and validation data set for system identification of hair dryer plant will be generated by splitting whole Dryer data into two pieces of 75% and 25% respectively. It is similar to data splitting process in machine learning problems but without shuffling since it is a dynamic system unlike most of the machine learning problems. Estimation data set is identified as the first 75% of the original Dryer data by setting Samples as from 1 to 750 in the Select Range window. Then, this data set is called Dryer_Estimation (can be seen in green color on data window). Validation data set is chosen as the last 25% of the original Dryer data by setting Samples from 751 to 1000 in the Select Range window. Then, this data set is called Dryer_Validation (can be seen in red color on data window). Both estimation and validation data sets can be seen together in Figure 1.

In Preprocess section (in the middle of SID toolbox), there is a "Remove means" option in the middle of the pop-up menu. The corresponding mean of estimation and validation will be removed from estimation and validation data set separately. Then, newly updated data sets for estimation and validation are called Dryer_Estimation_Preprocessed (can be seen in blue color on data window) and Dryer_Validation_Preprocessed (can be seen in pink color on data window). The preprocessed estimation and validation data sets can be seen in Figure 2. The preprocessed estimation data set is used to train the model (to build the mathematical model by finding its parameters). The preprocessed validation data set is used to test the performance of the model for unseen data.

2 Estimating Four Polynomial Models: ARX, ARMAX, Output-Error (OE), and Box-Jenkins (BJ)

In this report, four different types of polynomial models will be trained with preprocessed estimation dryer data set and then the accuracies on preprocessed validation data set will be

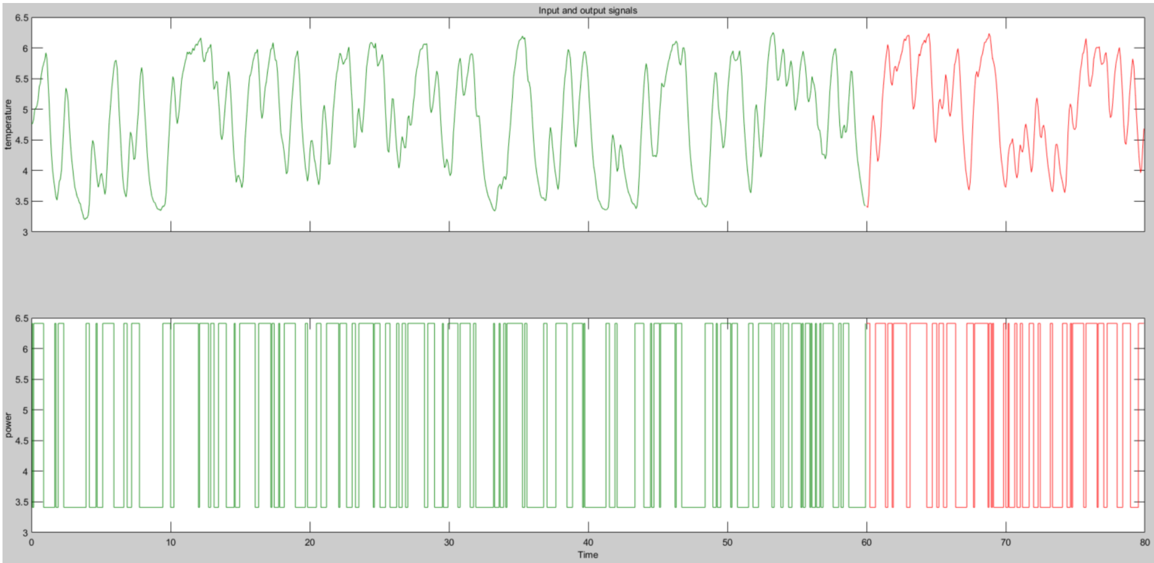

Figure 1: Estimation (in green color) and Validation (in red color) Data Set for Hair Dryer Plant

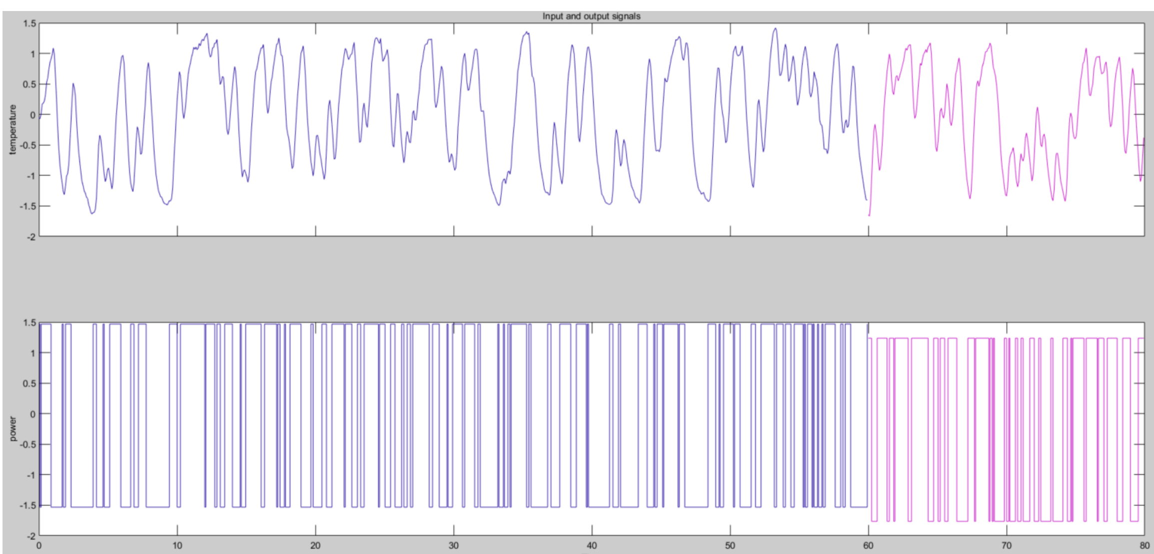

Figure 2: Estimation and Validation Data Set for Hair Dryer Plant After Remove Means Operation

discussed. These models are ARX, ARMAX, OE, and BJ. The ultimate aim is to compare the performance of different types of models in different focus settings like simulation and k-step ahead predictions. That's why the parameters in each model will not be tuned to observe the model performance in an unbiased environment.

3 Details of Model Settings

ARX model can be mathematically represented as $Ay = Bu + e$ where e is white-noise, u represents the input signal, and y stands for the output of the system. In our hair dryer plant, y is the temperature of dryer's outlet and u is the electrical power supplied to the hair drier. In ARX model, there exist three parameters: $n_k = 1$ for the number of pure delay in the system, $n_a = 4$ for the degree of polynomial A, and $n_b = 4$ for the degree of polynomial B. There exist four options for the focus of ARX model, but simulation and prediction will be used. Since how parameter (degrees of polynomials) tuning will affect the accuracy in ARX is another critical discussion, the order selection property for ARX on Polynomial Models window will be applied. By default, the order for each three parameters on ARX model will be optimized by adjusting each of three parameters from 1 to 10. Among two optimized suggestions (MDL choice and Best Fit) for three parameters, Best Fit optimization where $n_a = 6$, $n_b = 9$, and $n_k = 2$ will be selected. Two different polynomial models are created according to these two model structures with two different focuses. So, there are four ARX models named as arx692_pred, arx692_sim, arx441_pred, and arx441_sim. "_sim" name extension for models demonstrates that corresponding model is created with the focus setting of simulation option. "_pred" name extension for models demonstrates that corresponding model is generated with the focus setting of prediction option. These naming convention will be utilized for upcoming models too.

In ARMAX model, there exist four parameters: $n_k = 1$ for the number of pure delay in the system, $n_a = 2$ for the degree of polynomial A, $n_b = 2$ for the degree of polynomial B, and $n_c = 2$ for the degree of polynomial C. There exist four options for the focus of ARMAX model, but simulation and prediction will be used. Two different ARMAX models are generated with respect to two different focuses. So, there exist two ARMAX models named as amx2221_pred and amx2221_sim.

In Output-Error model, there exist three parameters: $n_k = 1$ for the number of pure delay in the system, $n_b = 2$ for the degree of polynomial B, and $n_f = 2$ for the degree of polynomial F. There exist two options for the focus of ARX model which are simulation and filter. Only simulation option will be applied. So, one OE model is built and named as oe221_sim.

In Box-Jenkins model, there exist five parameters: $n_k = 1$ for the number of pure delay in the system, $n_b = 2$ for the degree of polynomial B, $n_f = 2$ for the degree of polynomial F, $n_c = 2$ for the degree of polynomial C, and $n_d = 2$ for the degree of polynomial D. There exist four options for the focus of BJ model, but simulation and prediction will be used. Two different BJ models are generated with respect to two different focuses. So, there exist two BJ models named as bj22221_pred and bj22221_sim.

4 Estimation Performance Comparison

In order to assess the training performance of these nine models, training fit accuracy in percentage and two error metric are calculated: final prediction error (FPE), and mean squared error (MSE). These three criteria for all nine models can be examined in great detail in Figure 3. It is observed that as the error (FPE or MSE) decreases in the estimation process, the accuracy increases as expected. The same type of models with the same parameters (degree of polynomials) is examined to see the effect of the focus setting. For instance, consider two BJ models: the BJ model with prediction focus (bj22221_pred) has almost 6% better estimation accuracy performance than the same BJ model with simulation focus (bj2222_sim). This deduction is the same for other types of models too. The reason is that not only previous input signals (up until t-1 times for k-step ahead prediction) but also previous output signals (up until t-k times for k-step ahead prediction) are used in prediction. However, in simulation, only the input signal is used. That difference between prediction and simulation results in higher estimation performance for the models created with respect to the prediction focus.

It is not known how Dryer data is generated. So, the data generation mechanism for our data is missing. However, by just inspecting training accuracy, we can establish an idea. Although the flexibility of ARX model is a bit limited compared to ARMAX, BJ, and OE type of models, ARX types of models have the biggest training fit accuracy among others as is seen from Figure 3. So, ARX model has the least bias among others. Thus, it can be inferred that it is more likely that Dryer data is generated with an ARX model. By applying polynomial order optimization in ARX model, very small training accuracy around 0.2% and 0.7% is gained for prediction and simulation focus respectively.

Model Type	Focus	Model Name	Model Parameters						Training Fit Accuracy (%)	FPE	MSE
			n_a	n_b	n_c	n_d	n_f	n_k			
ARX	Prediction	arx692_pred	6	9	NA	NA	NA	2	95.7	0.001402	0.001315
ARX	Prediction	arx441_pred	4	4	NA	NA	NA	1	95.46	0.001515	0.001467
ARX	Simulation	arx692_sim	6	9	NA	NA	NA	2	89.19	0.00269	0.008315
ARX	Simulation	arx441_sim	4	4	NA	NA	NA	1	88.89	0.005667	0.00878
ARMAX	Prediction	amx2221_pred	2	2	2	NA	NA	1	90.16	0.007005	0.006894
ARMAX	Simulation	amx2221_sim	2	2	2	NA	NA	1	84.84	0.009928	0.01635
Box Jenkins	Prediction	bj22221_pred	NA	2	2	2	2	1	90.58	0.006442	0.006306
Box Jenkins	Simulation	bj22221_sim	NA	2	2	2	2	1	84.84	0.00893	0.01635
Output-Error	Simulation	oe221_sim	NA	2	NA	NA	2	1	84.84	0.01652	0.01635

Figure 3: Training Performance Comparison for All Models

5 Validation Performance Comparison

Models can have two different kinds of outputs: simulated model output and the predicted model output with different horizons (number of steps ahead). Generated model's output for the validation data set is different for the simulated model option and the predicted model option. Thus, two cases must be investigated separately.

5.1 Simulated Model Output for Validation Data

In the simulated output case, only inputs are used for validating the performance of the models. In Figure 4, the validation performance of five models is examined in terms of Best Fits (in percentage). These five models are arx692_pred, arx441_pred, amx2221_pred, bj22221_sim, and oe221_sim. Best validation accuracy belongs to ARX types of models again as it is previously the case for training accuracy. But, it must be noted that although BJ and OE models were not used previous outputs as feedback in the estimation, they have around 85% best fits accuracy in the validation. Both BJ and OE models outperformed ARMAX model which used previous outputs in the estimation.

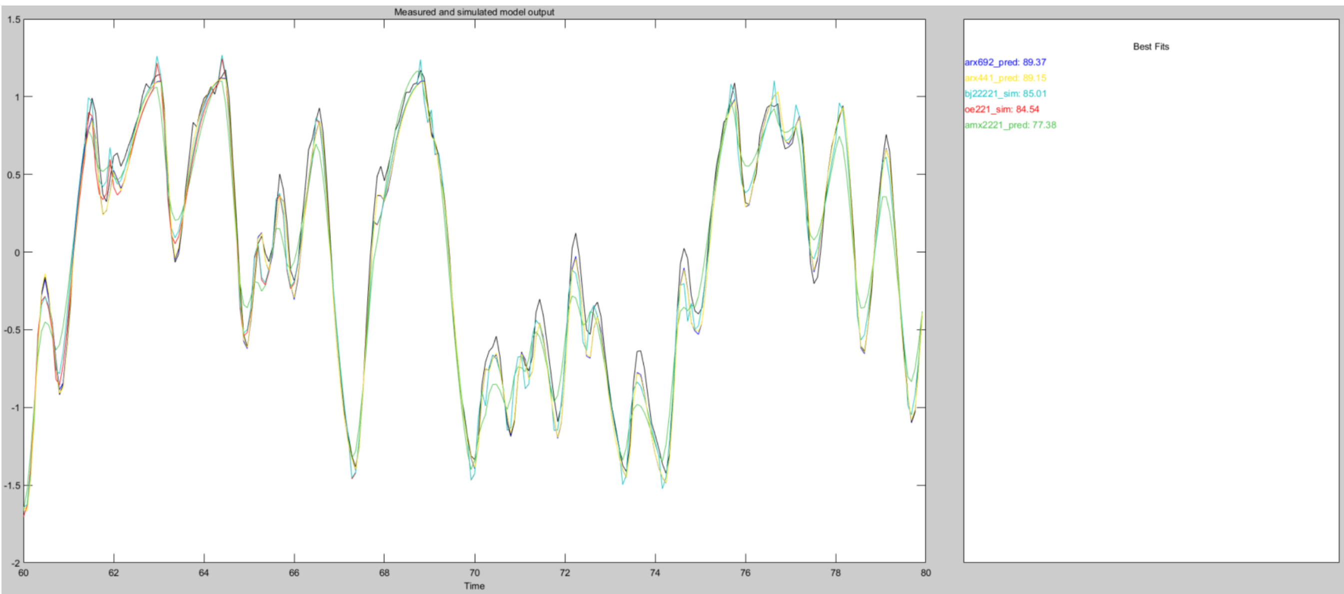

Figure 4: Simulated Outputs of 5 Models

5.2 Predicted Model Outputs for Validation Data

In the predicted output case, not only previous inputs (up until t-1 times for k-step ahead prediction) but also previous outputs (up until t-k times for k-step ahead prediction) are used for validating the performance of the models. 1-step, 5-step, 10-step, 20-step, 100-step ,and 1000-step predictions for all previously mentioned five models can be observed in Figure 5, 6, 7, 8, 9, and 10.

5.3 ARX and ARMAX Models with Simulation Focus

As it can be seen in Figure 11, there is not much difference between models with prediction and simulation focus for ARX type of model in terms of validation accuracy. However, when the training accuracy for ARX models is considered as in Figure 3, there is a big difference of around 6%. It means that ARX model is not learning its parameters good enough in the

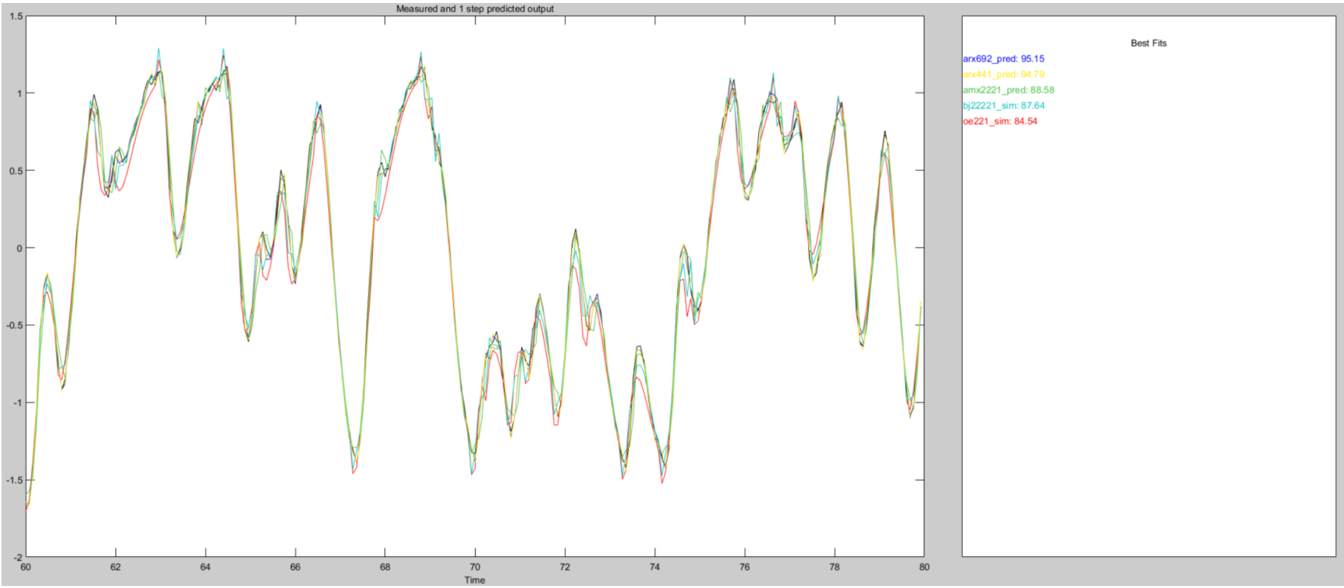

Figure 5: 1 Step Ahead Predicted Outputs of 5 Models

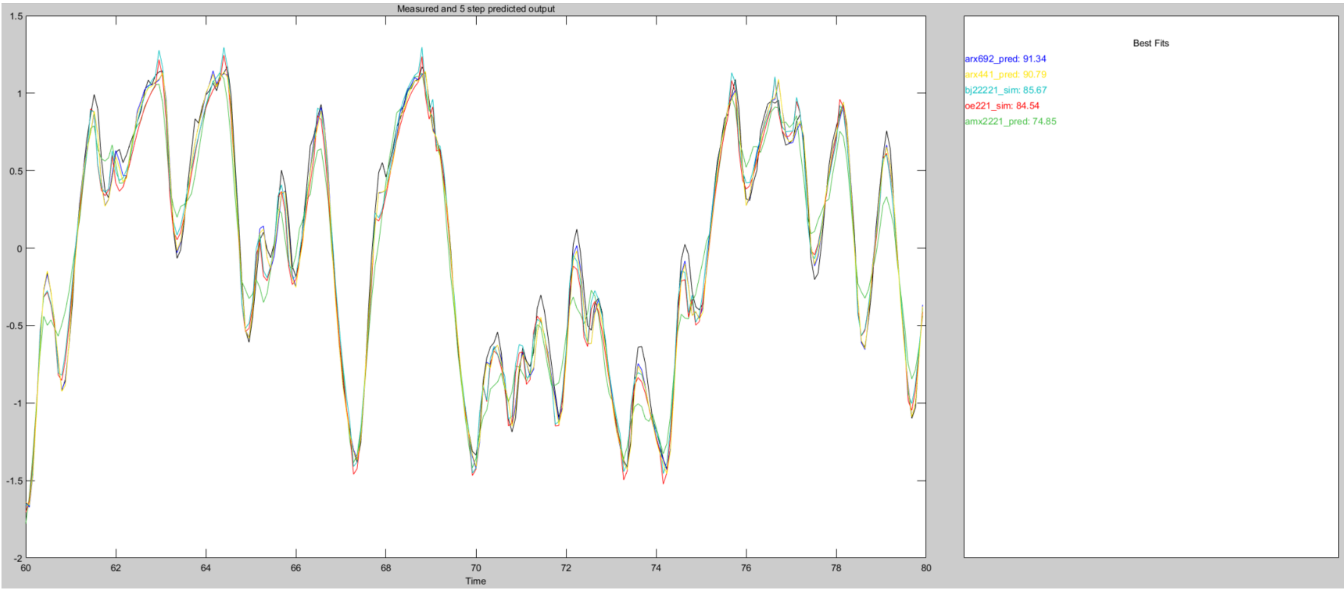

Figure 6: 5 Step Ahead Predicted Outputs of 5 Models

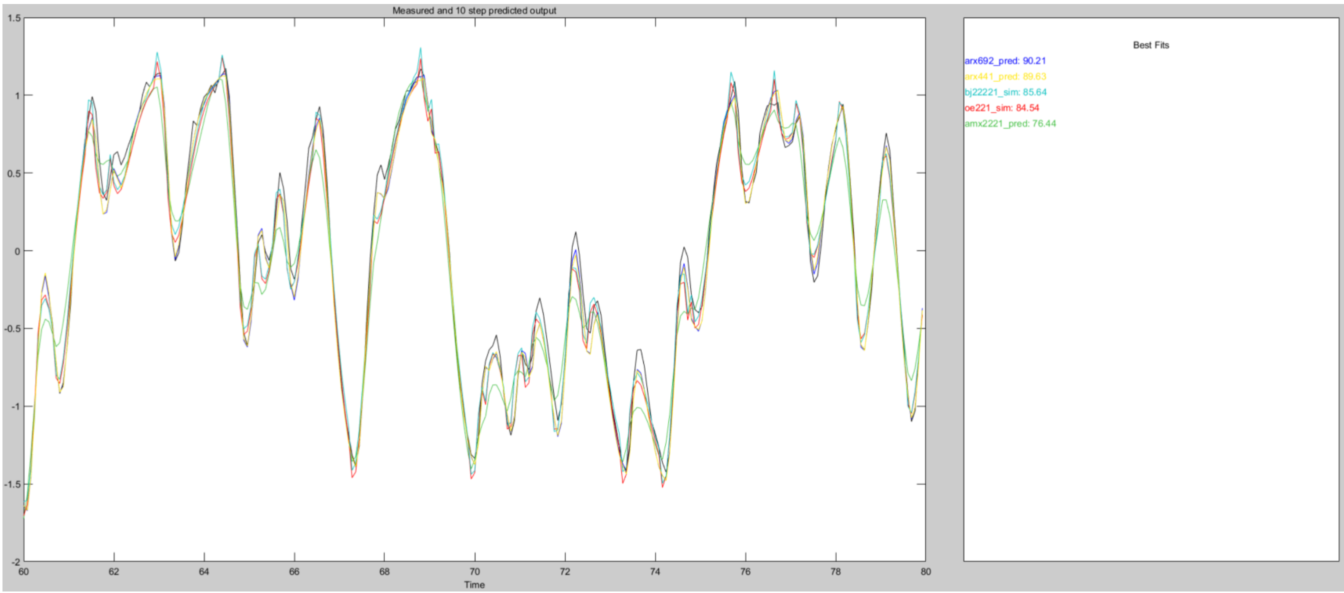

Figure 7: 10 Step Ahead Predicted Outputs of 5 Models

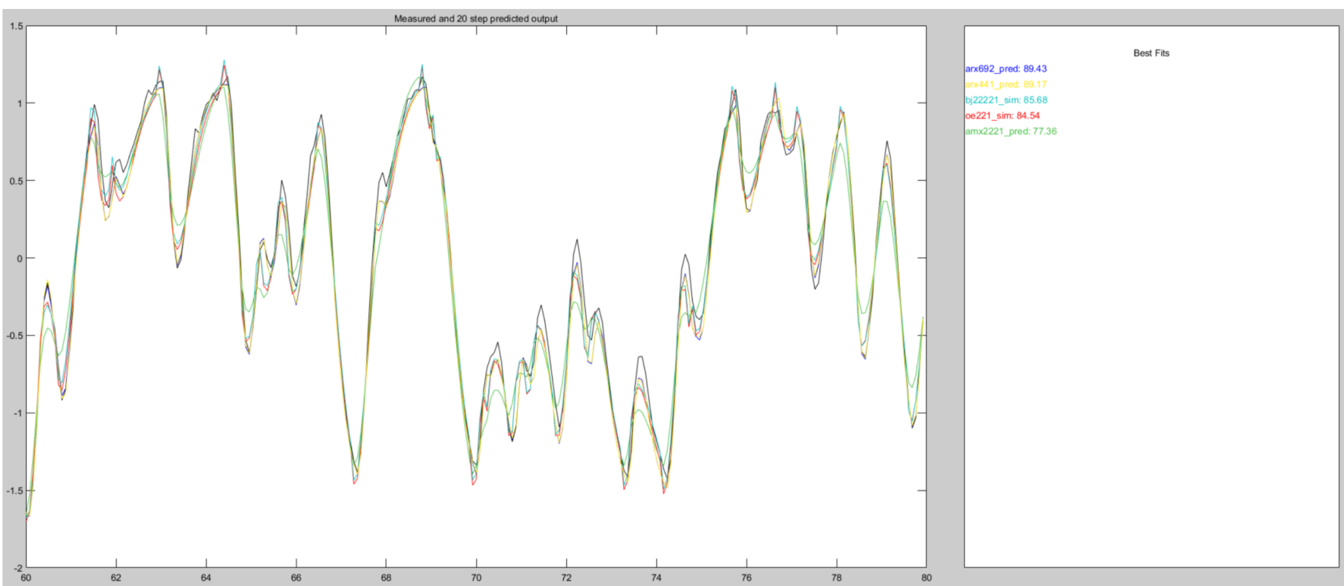

Figure 8: 20 Step Ahead Predicted Outputs of 5 Models

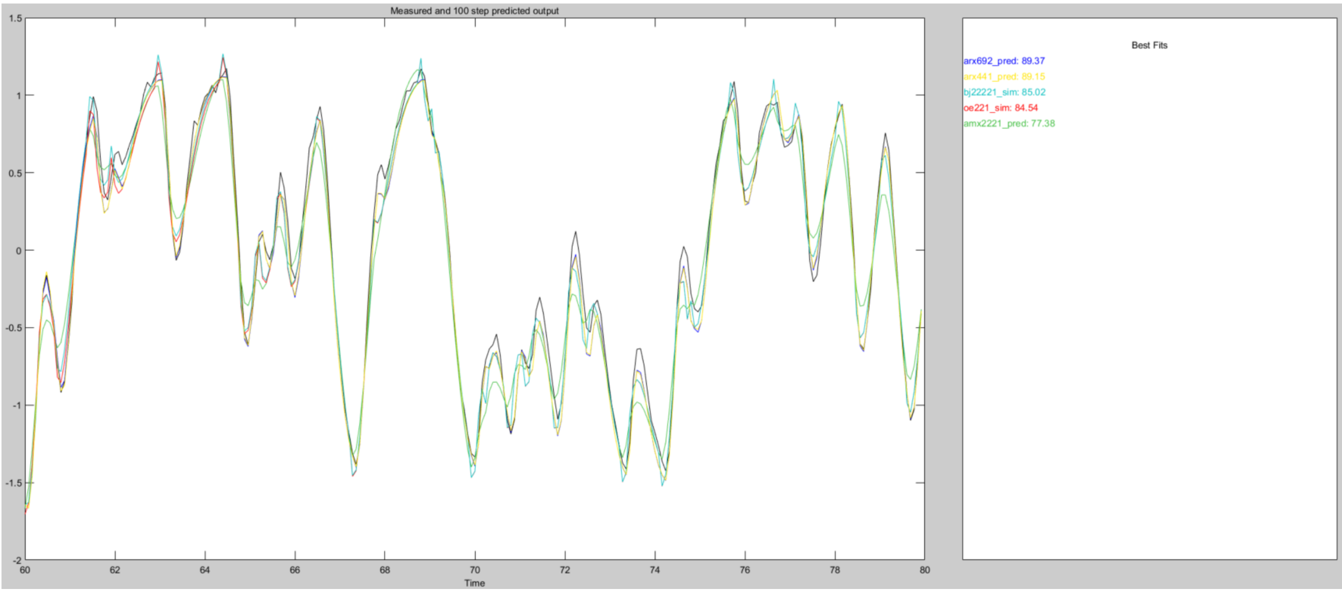

Figure 9: 100 Step Ahead Predicted Outputs of 5 Models

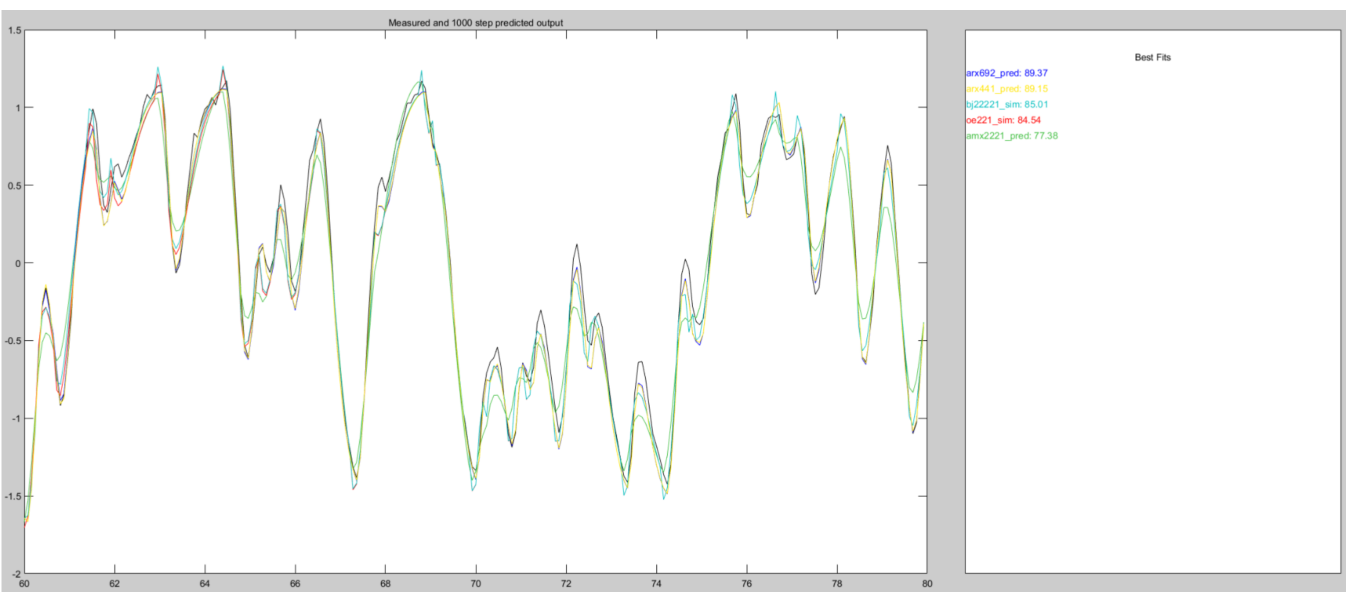

Figure 10: 1000 Step Ahead Predicted Outputs of 5 Models

estimation when simulation focus is preferred. When ARMAX is considered, in addition to the large difference (around 6%) in training accuracy as it is seen from Figure 3, there is also a huge gap of around 7% in validation accuracy.

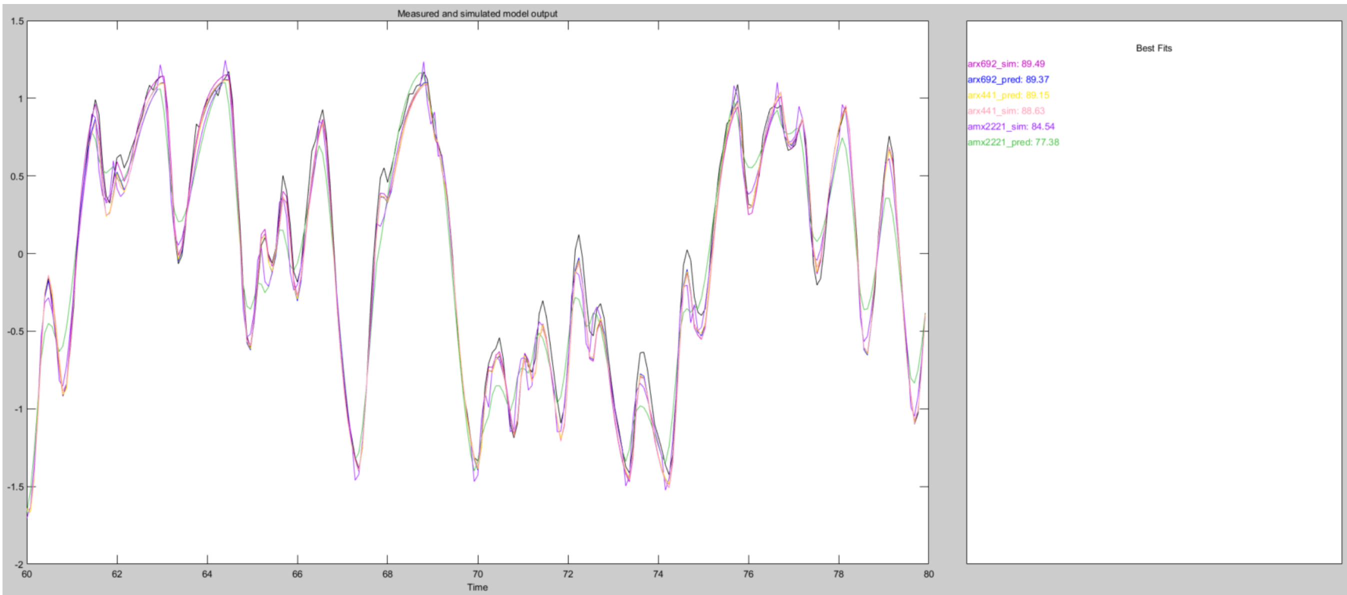

Figure 11: Simulation and Prediction Focus Comparison for ARX and ARMAX Models

5.4 OE and BJ Models with Simulation Focus

Since the last version of Matlab is used for the experiments, there is no prediction focus option for Output-Error models in System Identification Toolbox. That's why only BJ models will be compared. As it is seen in Figure 12, there is a huge difference of around 11% between models with prediction and simulation focus for BJ type of model in terms of validation accuracy. Moreover, when the training accuracy for BJ models is considered as in Figure 3, there is again a big difference of around 6%.

So in general for ARX, ARMAX, BJ, there is a huge difference generally around 6% in the training accuracy. The reason is that the simulation is a harder problem than the prediction since it does not take previous outputs as an input to the model. When more independent models are used like BJ where there is no common term between transfer function of the dryer plant and disturbance, the difference between models with prediction and simulation focus in validation accuracy becomes bigger. This is most probably due to the nature of the plant or the data generating mechanism. Probably, the disturbance in Dryer plant is entering into the system earlier and going through that process. In my opinion, that's why BJ model with prediction and simulation focus has a slightly bigger difference in validation accuracy. So, the transfer function of Dryer plant and disturbance should share a common denominator. This supports my earliest hypothesis that Dryer data is generated from ARX model.

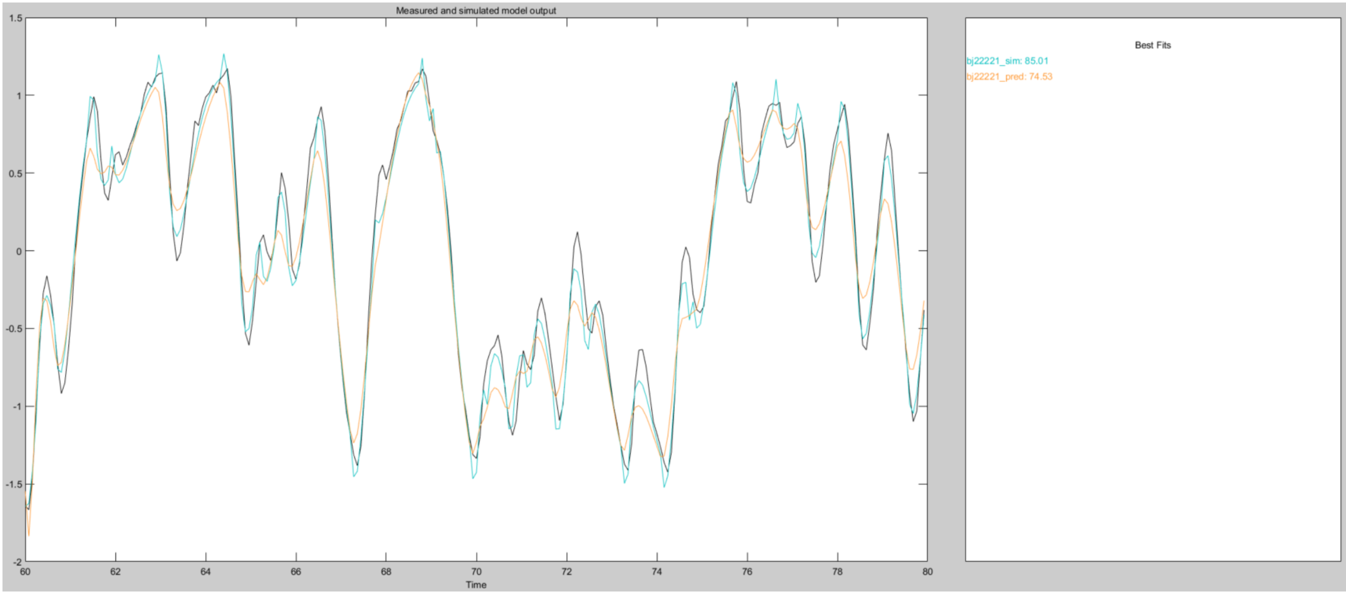

Figure 12: Simulation and Prediction Focus Comparison for BJ Models

6 Results & Discussion

To see how the best fits accuracy is changing with the horizon, all accuracy values for the same type of polynomial models are brought together into tables (Note that only BJ and OE models are put together into one table). By looking at colored cells in Figures 11, 12, and 13, it is seen that as the horizon (number of steps in prediction) goes into 1000, the validation accuracy for both prediction and simulation options becomes the same for each type of model. In conclusion, this observation coincides with the fact that prediction becomes simulation as the horizon becomes infinity. In other words, if the model is generated with the prediction focus, then this model should be used for the small horizon prediction purposes, not simulation purposes since as the horizon gets larger, the prediction accuracy becomes relatively small which is qual to simulation accuracy. The overall aim of the intended application is very crucial. If the intended application is a simulation, then do not use any model with the prediction focus because it will make the validation accuracy lower as seen from Figures 11, 12, and 13. If the application is a type of prediction, then use models with the prediction focus but keep the horizon as small as possible to get higher validation accuracy.

In Figure 16, the relation between bias and variance can be examined more compactly. Assume that the intended application is simulation. If the model with prediction focus is estimated, then there will be a small training error (*i.e* low bias) but huge variance. If the model with simulation focus is trained, then there exists less variance but high bias. If the intended application is 1-step prediction, it would be better to choose ARX model for Dryer plant. If the application requires 5-step prediction, then ARX, OE, and, BJ model (with simulation focus) can be chosen. When the focus of Box-Jenkins model is prediction, the model has very low validation accuracy. As it was mentioned earlier, this is due to the nature of the disturbance in the system.

Model Name	Validation Option	Validation Best Fits (%)
arx692_pred	Simulation	89.37
arx692_pred	1-Step Prediction	95.15
arx692_pred	5-Step Prediction	91.34
arx692_pred	10-Step Prediction	90.21
arx692_pred	20-Step Prediction	89.43
arx692_pred	1000-Step Prediction	89.37
arx692_sim	Simulation	89.49
arx692_sim	1-Step Prediction	92.7
arx692_sim	5-Step Prediction	79.93
arx692_sim	10-Step Prediction	78.03
arx692_sim	20-Step Prediction	84.06
arx692_sim	1000-Step Prediction	89.15
arx441_pred	Simulation	89.15
arx441_pred	1-Step Prediction	94.79
arx441_pred	5-Step Prediction	90.79
arx441_pred	10-Step Prediction	89.63
arx441_pred	20-Step Prediction	89.17
arx441_pred	1000-Step Prediction	89.15
arx441_sim	Simulation	89.63
arx441_sim	1-Step Prediction	88.93
arx441_sim	5-Step Prediction	69.58
arx441_sim	10-Step Prediction	81.5
arx441_sim	20-Step Prediction	88.6
arx441_sim	1000-Step Prediction	88.63

Figure 13: Validation Performance Comparison of All ARX Models

Model Name	Validation Option	Validation Best Fits (%)
amx2221_pred	Simulation	77.38
amx2221_pred	1-Step Prediction	88.58
amx2221_pred	5-Step Prediction	74.85
amx2221_pred	10-Step Prediction	76.44
amx2221_pred	20-Step Prediction	77.36
amx2221_pred	1000-Step Prediction	77.38
amx2221_sim	Simulation	84.54
amx2221_sim	1-Step Prediction	86.85
amx2221_sim	5-Step Prediction	84.46
amx2221_sim	10-Step Prediction	84.36
amx2221_sim	20-Step Prediction	84.53
amx2221_sim	1000-Step Prediction	84.54

Figure 14: Validation Performance Comparison of All ARMAX Models

Model Name	Validation Option	Validation Best Fits (%)
bj22221_pred	Simulation	74.53
bj22221_pred	1-Step Prediction	89.22
bj22221_pred	5-Step Prediction	75.29
bj22221_pred	10-Step Prediction	74.59
bj22221_pred	20-Step Prediction	74.53
bj22221_pred	1000-Step Prediction	74.53
bj22221_sim	Simulation	85.01
bj22221_sim	1-Step Prediction	87.64
bj22221_sim	5-Step Prediction	85.97
bj22221_sim	10-Step Prediction	85.64
bj22221_sim	20-Step Prediction	85.68
bj22221_sim	1000-Step Prediction	85.01
oe221_sim	Simulation	84.54
oe221_sim	1-Step Prediction	85.54
oe221_sim	5-Step Prediction	84.54
oe221_sim	10-Step Prediction	84.54
oe221_sim	20-Step Prediction	84.54
oe221_sim	1000-Step Prediction	84.54

Figure 15: Validation Performance Comparison of All BJ and OE Models

It was a bit surprising when I saw that arx692_sim model outperforms bj22221_sim model by 5% in terms of simulation perspective although BJ model seems more powerful/flexible with a high number of parameters. I guess this result also proves that Dryer data is generated with ARX model.

Model Name	Training Fit Accuracy (%)	Validation Best Fits (%) for 1-Step Prediction	Validation Best Fits (%) for 5-Step Prediction	Validation Best Fits (%) for Simulation (infinite horizon prediction)
arx692_pred	95.7	95.15	91.34	89.37
arx441_pred	95.46	94.79	90.79	89.15
arx692_sim	89.19	92.7	79.93	89.49
arx441_sim	88.89	88.93	69.58	89.63
amx2221_pred	90.16	88.58	74.85	77.38
amx2221_sim	84.84	86.85	84.46	84.54
bj22221_pred	90.58	89.22	75.29	74.53
bj22221_sim	84.84	87.64	85.97	85.01
oe221_sim	84.84	85.54	84.54	84.54

Figure 16: Training and Validation Accuracy Comparison of All Models in Compact Form

In Figure 17, there are two graphs. The first one is the auto-correlation of residuals for output temperature. Residual simply means errors in the model. If we manage successful modeling, then auto-correlation of residuals should be very close to auto-correlation of white-noise. Because the auto-correlation function is an even function, symmetric behavior can be observed at the top graph in Figure 17. Since there are nine models, the residual graph seems a bit complicated. That's why four models with fewer residuals are plotted in Figure 17. These are arx692_pred (in blue), arx441_pred (in yellow), bj22221_pred (in orange), and bj2222_sim (in turquoise). Since the blue line is more close to 0 auto-correlation, it can be inferred that arx692_pred model is more accurate than the other three models. That deduction also agrees with the fact that arx692_pred model has a higher validation Best Fits accuracy than other models as seen in Figure 16. Ideally, it is desired to have 0 as cross-correlation for the input and output residuals. If there is a correlation between the input and output, then there may be some unexplained component of the input in residuals. That behavior is not desired. In Figure 17 at below graph, bj22221_pred (in orange) and bj2222_sim (in turquoise) are not very close to 0. It means that there is a correlation between input and output. BJ type model has suffered from the unexplained component of the input in residuals. Since both cross-correlation plots of arx692_pred (in blue) and arx441_pred (in yellow) are very close to 0 value at the bottom graph in Figure 17, these two ARX typed models have a good performance.

7 Appendix

Overall view of SID toolbox with corresponding data and models can be seen below.

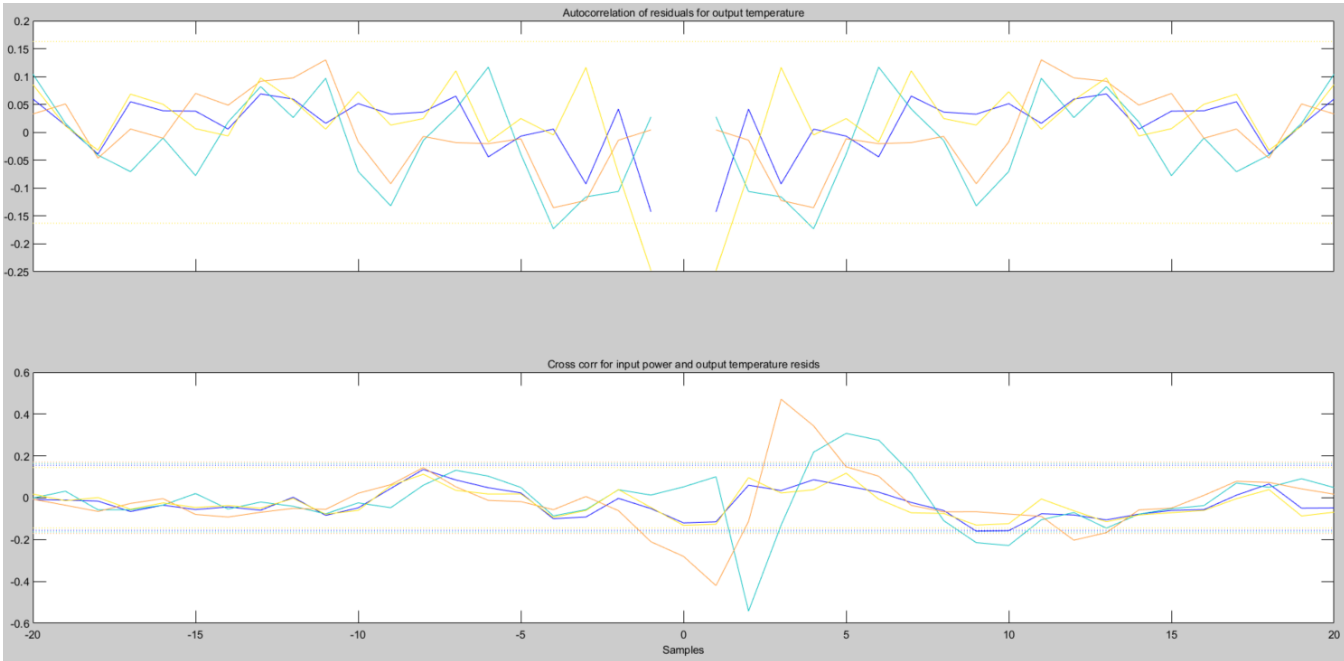

Figure 17: Residual Analysis of All Nine Models with Auto-Correlation and Cross-Correlation Functions

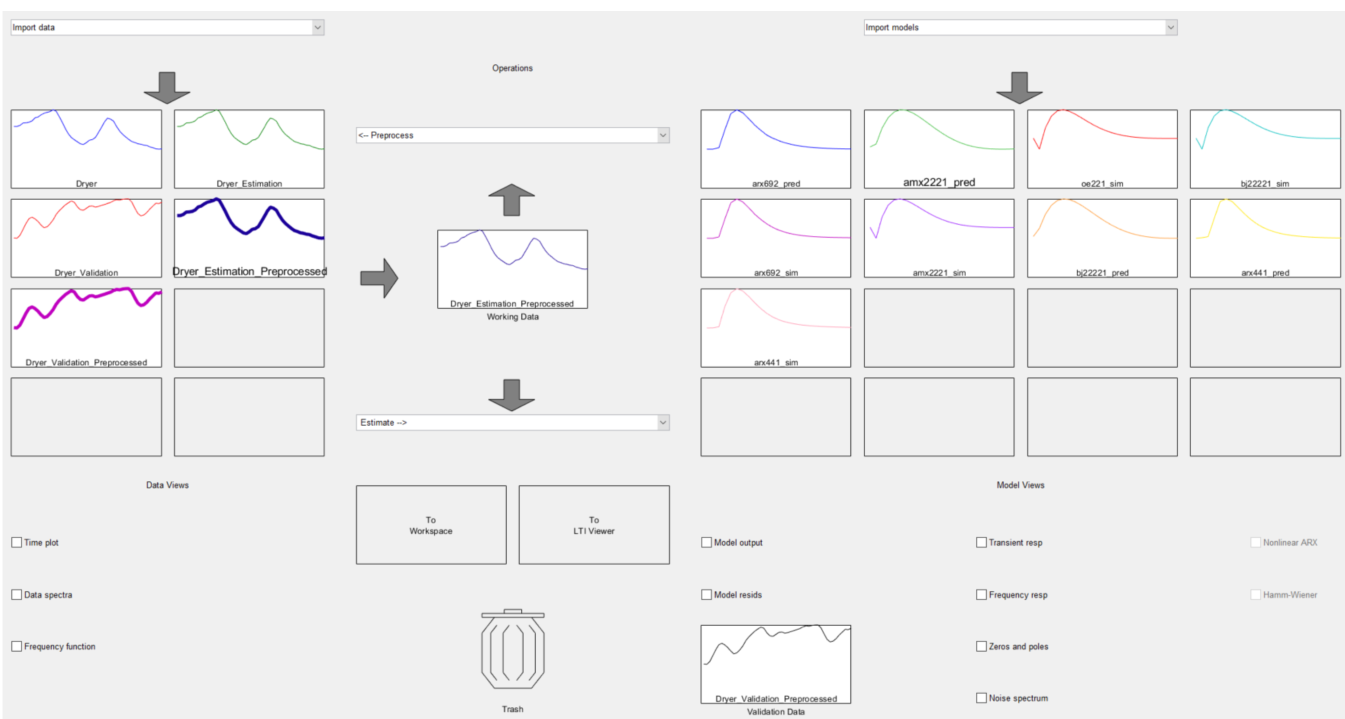

Figure 18: SID Toolbox with Corresponding Data Sets and Models