Middle East Technical University
Department of Computer Engineering

CENG 222
STATISTICAL METHODS FOR CENG
Spring 2019
TAKE HOME EXAM 2

1) Suppose you choose a *real number* X randomly from the interval $[10, 100]$.

a. Find the density function $f(x)$ and the probability of an event I for this experiment, where I is a subinterval $[a, b]$ of $[20, 120]$.
b. Find the expected value $E_X[x]$
c. From (i) find the probability of $X^2 - 110X + 2800 > 0$
d. What if we have chosen the real number with the density function, $f(x) = Cx$, rather than uniform, find C, and expected value $E_X[x]$.

2) Suppose a die producing a machine in Las Vegas produces dice whose probability of getting an even number is a random variable D with PDF

$$f_D(d) = \begin{cases} de^d & , \quad d \in [0,1] \\ 0 & , otherwise \end{cases}$$

Now, we select a die randomly and roll repeatedly (hence in independent fashion)

a. Find the probability that a die roll comes up even number.
b. Given the die roll shows up even, find the conditional PDF of D.
c. Assuming the very first die roll shows up even, find the conditional probability of getting an even number on the next roll.

3) Suppose we have a coin C with some bias p, i.e. $P(C = 1)$, probability of getting a head. We want to estimate the bias of this coin (*by repetitively tossing and calculating the mean*). Using proper approximation, compute the number of tosses in order to be at least 90% certain that the desired estimate is within 10% of its true value?

4) Let $X_1, X_2, X_3 \ldots X_n$ be random variables from $N(\mu_1, \sigma_1^2), N(\mu_2, \sigma_2^2), N(\mu_3, \sigma_3^2) \ldots\ N(\mu_n, \sigma_n^2)$. Let $X = \sum_{i=1}^{n} a_i X_i$, where a_is are any real valued constants. Use Method of Moments to show that X is normally distributed and find its mean and variance in terms of μ_i, σ_i^2, a_i s.

5) Prove that the MLE for univariate Poisson distribution for the parameter θ is given by the average of the samples in the given observation set of discrete random variable $X, \{x_1, \ldots\ x_n\}$ and

$$X \sim \frac{\theta^x}{x!} e^{-\theta} \quad , \ x = 0, 1, 2 \ldots$$

6) Determine whether the estimate found in question 5 is:

a. Unbiased
b. Consistent

or not.

REGULATIONS

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Cheating** : We have zero tolerance policy for cheating}. People involved in cheating will be punished according to the university regulations.
4. You must follow odtuclass for discussions and possible updates on a daily basis.
5. **Evaluation**: Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using ``black-box'' technique and manually by assistants, so make sure to obey the specifications.

SUBMISSION

Submission will be done via ODTUCLASS. Download the given template file, "the2.tex", when you finish your exam upload your "the2.tex" file to ODTUCLASS.