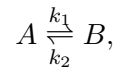


## 1. Solving ODEs in MATLAB: chemical reaction example

From the chemical reaction system



the following system of ODEs is obtained using the law of mass action:

$$\frac{d[A]}{dt} = -k_1[A] + k_2[B], \quad (1)$$

$$\frac{d[B]}{dt} = +k_1[A] - k_2[B]. \quad (2)$$

These ODEs can be implemented into MATLAB and solved numerically using the built-in ODE-solver ode15s, as is shown below.

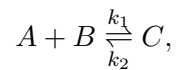
reaction\_ode\_main.m

```
1 %initial concentrations
2 - IC_A = 0.7;
3 - IC_B = 0.3;
4 - y0=[IC_A IC_B];
5
6 %rate constants
7 - k1 = 0.1;
8 - k2 = 0.01;
9
10 %timespan where we want to solve the ODE
11 - tspan = [0 100];
12
13 %use the ODEsolver. Note the ODE function takes 3 inputs: y,k,k2
14 - [t,y] = ode15s(@(t,y) chemicalreactions(y,k1,k2), tspan, y0);
15
16 %plot results
17 - figure
18 - plot(t,y,'LineWidth',2);
19 - xlabel('time')
20 - ylabel('concentration')
21 - legend('[A]', '[B]')
22 - title(['k1=', num2str(k1), ', k2=', num2str(k2)])
23
```

chemicalreactions.m

```
1 function yp = chemicalreactions(y,k1,k2)
2
3 - yp= [-k1*y(1) + k2*y(2)
4       +k1*y(1) - k2*y(2)];
5
6 - end
```

**Task:** Now formulate a system of ODEs for the following reaction system



and solve it using MATLAB. Try changing the initial conditions and the rate constants - what happens?

## 2. Finding evolutionary game theory outcomes in MATLAB

The MATLAB code below shows how to implement the hawk-dove game. In this game, hawks and doves in the system interact.

- Hawks are aggressive. If two hawks meet they will share resources and fight. This gives them a payoff  $V/2 - C/2$ . Here  $V$  denotes the benefit of a resource, and  $C$  denotes the cost of fighting.
- If a hawk meets a dove, the hawk will take the full resource and thus receive a payoff  $V$ . In this case the dove flees (does not fight) and receives no resource, so its payoff is 0.
- Doves can share a resources with other doves. So if a dove meets another dove, their payoff is  $V/2$ .

hawkdovegame.m

```

1  % close figures, clear variables
2 - close all;
3 - clear all;
4
5  % Set values v and c
6 - v = 1;
7 - c = 2;
8
9  % Define the Hawk-Dove game matrix
10 - A = ?
11
12  %%% This code computes the evolutionary game theory
13  %%% replicator equation ODEs based on the game matrix
14 - p = linspace(0,1,100);
15 - dp = zeros(1,length(p));
16 - for j = 1:length(p)
17 -     p_vec_temp = [p(j), 1-p(j)];
18 -     dp_vec_temp = p_vec_temp.*((A*p_vec_temp') - p_vec_temp*A*p_vec_temp');
19 -     dp(j) = dp_vec_temp(1);
20 - end
21  %%%
22  %%%
23
24  %the size of cooperators is plotted
25 - figure
26 - plot(p, dp, 'LineWidth', 2)
27 - xlabel('fraction of hawks (p)');
28 - ylabel('gradient of selection (dp)');
29 - %also plot a line at dp=0 (cosmetic)
30 - hold on
31 - plot(p, 0*p, 'k--', 'LineWidth', 2)
32
33

```

**Task:** What happens if the  $C$  (the cost of fighting) is larger than  $V$  (the benefit from resources)? What happens if  $V$  is larger than  $C$ ? What does this mean conceptually?