1. Solving ODEs in MATLAB: chemical reaction example

From the chemical reaction system

$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B,$$

the following system of ODEs is obtained using the law of mass action:

$$\frac{d[A]}{dt} = -k_1[A] + k_2[B], \tag{1}$$

$$\frac{d[B]}{dt} = +k_1[A] - k_2[B]. \tag{2}$$

These ODEs can be implemented into MATLAB and solved numerically using the built-in ODE-solver ode15s, as is shown below.

reaction_ode_main.m

```
%initial concentrations
IC_A = 0.7;
IC_B = 0.3;
y0=[IC_A IC_B];

%rate constants
k1 = 0.1;
k2 = 0.01;

%timespan where we want to solve the ODE
tspan = [0 100];

%use the ODEsolver. Note the ODE function takes 3 inputsL y,k,k2
[t,y] = ode15s(@(t,y) chemicalreactions(y,k1,k2), tspan, y0);

%plot results
figure
plot(t,y,'LineWidth',2);
xlabel('time')
ylabel('concentration')
legend('[A]','[B]')
title(['k1=',num2str(k1),', k2=',num2str(k2)])

```

chemicalreactions.m

```
function yp = chemicalreactions(y,k1,k2)

yp= [-k1*y(1) + k2*y(2)
    +k1*y(1) - k2*y(2)];

end
```

Task: Now formulate a system of ODEs for the following reaction system

$$A + B \underset{k_2}{\overset{k_1}{\rightleftharpoons}} C,$$

and solve it using MATLAB. Try changing the initial conditions and the rate constants - what happens?

2. Finding evolutionary game theory outcomes in MATLAB

The MATLAB code below shows how to implement the hawk-dove game. In this game, hawks and doves in the system interact.

- Hawks are aggressive. If two hawks meet they will share resources and fight. This gives them a payoff $V/2 - C/2$. Here V denotes the benefit of a resource, and C denotes the cost of fighting.
- If a hawk meets a dove, the hawk will take the full resource and thus receive a payoff V. In this case the dove flees (does not fight) and receives no resource, so its payoff is 0.
- Doves can share a resources with other doves. So if a dove meets another dove, their payoff is $V/2$.

hawkdovegame.m

```
% close figures, clear variables
close all;
clear all;

% Set values v and c
v = 1;
c = 2;

% Define the Hawk-Dove game matrix
A= ?

%%% This code computes the evolutionary game theory
%%% replicator equation ODEs based on the game matrix
p=linspace(0,1,100);
dp = zeros(1,length(p));
for j = 1:length(p)
    p_vec_temp = [p(j), 1-p(j)];
    dp_vec_temp = p_vec_temp.*((A*p_vec_temp') - p_vec_temp*A*p_vec_temp');
    dp(j) = dp_vec_temp(1);
end
%%%
%%%

%the size of cooperators is plotted
figure
plot(p, dp, 'LineWidth', 2)
xlabel('fraction of hawks (p)');
ylabel('gradient of selection (dp)');
%also plot a line at dp=0 (cosmetic)
hold on
plot(p, 0*p, 'k--', 'LineWidth', 2)


```

Task: What happens if the C (the cost of fighting) is larger than V (the benefit from resources)? What happens if V is larger than C? What does this mean conceptually?