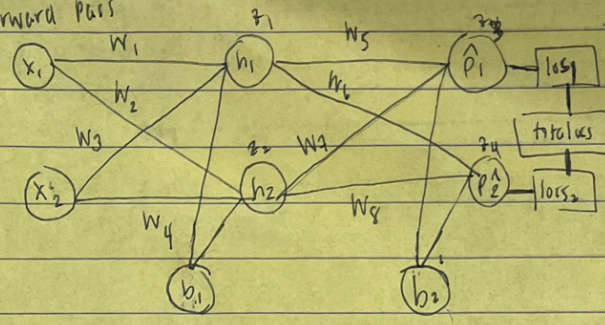
Olympia, Francherska L. BSCS-ML

Part 1: forward pass



Parameter	Value
w_1	0.15
w_2	0.20
w_3	0.25
w_4	0.30
w_5	0.40
w_6	0.45
w_7	0.50
w_8	0.55
b_1	.35
b_2	.60

x_1	x_2	y_1	y_2
0.05	0.10	.01	.99

1.) $h_1 = 0.5945$

2.) $h_2 = 0.5963$

3.) $\hat{p}_1 = 0.7569$

4.) $\hat{p}_2 = 0.7677$

5.) Total loss = 1.052

- $z_1 = w_1 x_1 + w_3 x_2 + b_1$
  $= 0.15(0.05) + 0.25(0.10) + 0.35$
  $\boxed{z_1 = 0.3825}$
- $h_1 = \frac{1}{1+e^{-x}}$ // z_1
  $= \frac{1}{1+e^{(-0.3825)}}$
  $\boxed{h_1 = 0.5945}$
- $z_2 = w_2 x_1 + w_4 x_2 + b_1$
  $= 0.20(0.05) + 0.30(0.10) + 0.35$
  $\boxed{z_2 = 0.39}$
- $h_2 = \frac{1}{1+e^{-x}}$ // z_2
  $= \frac{1}{1+e^{-0.39}}$
  $\boxed{h_2 = 0.5963}$
- $z_3 = h_1 w_5 + h_2 w_7 + b_2$
  $= 0.5945(0.40) + 0.5963(0.50) + 0.60$
  $\boxed{z_3 = 1.13595}$
- $z_4 = h_1 w_6 + h_2 w_8 + b_2$
  $= 0.5945(0.45) + 0.5963(0.55) + 0.60$
  $\boxed{z_4 = 1.19549}$
- $\hat{p}_1 = \frac{1}{1+e^{-x}}$ // z_3
  $= \frac{1}{1+e^{-1.13595}}$
  $\boxed{\hat{p}_1 = 0.7569}$
- $\hat{p}_2 = \frac{1}{1+e^{-x}}$
  $= \frac{1}{1+e^{-1.19549}}$
  $\boxed{\hat{p}_2 = 0.7677}$
- $\text{loss}_1 = \sum_{i=1}^{n=1} (y_1 - \hat{p}_1)^2$
  $= (0.01 - 0.7569)^2$
  $\boxed{\text{loss}_1 = 0.5579}$
- $\text{loss}_2 = \sum_{i=1}^{n=1} (y_2 - \hat{p}_2)^2$
  $= (0.99 - 0.7677)^2$
  $\boxed{\text{loss}_2 = 0.4941}$
- Total loss = $0.5579 + 0.4941$
  $\boxed{\text{Total loss} = 1.052}$

Part 2: Back Propagation

1.) Find the derivative of $\frac{d\text{loss}}{d\hat{p}_1}$

$\frac{d\text{loss}}{d\hat{p}_1} = \text{loss}_1 (y_1 - \hat{p}_1)^2$

$= \frac{d}{d\hat{p}_1} (y_1 - \hat{p}_1)^2$

$= 2(y_1 - \hat{p}_1)(-1)$

$\boxed{\frac{d\text{loss}}{d\hat{p}_1} = -2(y_1 - \hat{p}_1)}$

$= \frac{d\text{loss}}{d\hat{p}_1} = -2(0.01 - 0.7569)$

$\boxed{\frac{d\text{loss}}{d\hat{p}_1} = 1.4938}$

2.) Find the derivative of $\frac{d\text{loss}}{d\hat{p}_2}$

$\text{loss}_2 = (y_2 - \hat{p}_2)^2$

$= \frac{d}{d\hat{p}_2} (y_2 - \hat{p}_2)^2$

$= 2(y_2 - \hat{p}_2)(-1)$

$\boxed{\frac{d\text{loss}}{d\hat{p}_2} = -2(y_2 - \hat{p}_2)}$

$\therefore \frac{d\text{loss}}{d\hat{p}_2} = -2(0.99 - 0.7677)$

$\boxed{\frac{d\text{loss}}{d\hat{p}_2} = -0.4446}$

3.) Find the derivative of $\frac{d\text{loss}}{db_2}$

$\frac{d\text{loss}}{db_2} = \frac{d\text{loss}_1}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{db_2} + \frac{d\text{loss}_2}{d\hat{p}_2} \times \frac{d\hat{p}_2}{dz_4} \times \frac{dz_4}{db_2}$

- $\frac{d\text{loss}_1}{d\hat{p}_1} = -2(y_1 - \hat{p}_1) = 1.4938$
- $\frac{d\hat{p}_1}{dz_3} = \hat{p}_1(1 - \hat{p}_1) = 0.1840$
- $\frac{dz_3}{db_2} = 1$
- $\frac{d\text{loss}_2}{d\hat{p}_2} = -2(y_2 - \hat{p}_2) = -0.4446$
- $\frac{d\hat{p}_2}{dz_4} = \hat{p}_2(1 - \hat{p}_2) = 0.1783$
- $\frac{dz_4}{db_2} = 1$

$\therefore \frac{d\text{loss}_1}{db_2} = -2(y_1 - \hat{p}_1) \cdot \hat{p}_1(1 - \hat{p}_1)$

$\therefore \frac{d\text{loss}_2}{db_2} = -2(y_2 - \hat{p}_2) \cdot \hat{p}_2(1 - \hat{p}_2)$

$\therefore \boxed{\frac{d\text{loss}}{db_2} = -2(y_1 - \hat{p}_1) \cdot \hat{p}_1(1 - \hat{p}_1) + -2(y_2 - \hat{p}_2) \cdot \hat{p}_2(1 - \hat{p}_2)}$

// Applying values:

$\frac{d\text{loss}}{db_2} = -2(0.01 - 0.7569) \cdot 0.7569(1 - 0.7569) \times -2(0.99 - 0.7677) \cdot 0.7677(1 - 0.7677)$

$\therefore \boxed{\frac{d\text{loss}}{db_2} = 0.1956}$

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4.) Find the derivative of $\frac{dloss}{dw_5}$

$$\frac{dloss}{dw_5} = \frac{dloss}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{dw_5}$$

- $\frac{dloss}{d\hat{p}_1} = -2(y_1 - \hat{p}_1) = 1.4938$
- $\frac{d\hat{p}_1}{dz_3} = \hat{p}_1(1-\hat{p}_1) = 0.1840$
- $\frac{dz_3}{dw_5} = h_1 = 0.5945$

$\therefore \frac{dloss}{dw_5} = -2(y_1 - \hat{p}_1) \cdot \hat{p}_1(1-\hat{p}_1) \cdot h_1$

// with values:

$= (1.4938)(0.1840)(0.5945)$

$\therefore \frac{dloss}{dw_5} = 0.1634$

5.) Find the derivative of $\frac{dloss}{dw_7}$

$$\frac{dloss_1}{dw_7} = \frac{dloss_1}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{dw_7}$$

- $\frac{dloss}{d\hat{p}_1} = -2(y_1 - \hat{p}_1) = -0.4446$
- $\frac{d\hat{p}_1}{dz_3} = \hat{p}_1(1-\hat{p}_1) = 0.1840$
- $\frac{dz_3}{dw_7} = h_2 = 0.5963$

$\therefore \frac{dloss}{dw_7} = -2(y - \hat{p}_2) \cdot \hat{p}_2(1-\hat{p}_2) \cdot h_2$

// with values $= (-0.4446)(-0.1783)(0.5963)$

$\therefore \frac{dloss}{dw_5} = -0.0473$

6.) Find the derivative of $\frac{dloss}{dw_6}$

$$\frac{dloss}{dw_6} = \frac{dloss_2}{d\hat{p}_2} \times \frac{d\hat{p}_2}{dz_4} \times \frac{dz_4}{dw_6}$$

- $\frac{dloss}{d\hat{p}_2} = -2(y_2 - \hat{p}_2) = -0.4446$
- $\frac{d\hat{p}_2}{dz_4} = \hat{p}_2(1-\hat{p}_2) = 0.1783$
- $\frac{dz_4}{dw_6} h_1 = 0.5945$

$\therefore \frac{dloss}{dw_6} = -2(y_2 - \hat{p}_2) \cdot \hat{p}_2(1-\hat{p}_2) \cdot h_1$

$= (-0.4446)(0.1783)(0.5945)$

$\therefore \frac{dloss}{dw_6} = -0.0471$

7.) Find the derivative of $\frac{dloss}{dw_8}$

$$\frac{dloss}{dw_8} = \frac{dloss_2}{d\hat{p}_2} \times \frac{d\hat{p}_2}{dz_4} \times \frac{dz_4}{dw_8}$$

- $\frac{dloss}{d\hat{p}_2} = -2(y_2 - \hat{p}_2) = -0.4446$
- $\frac{d\hat{p}_2}{dz_4} = \hat{p}_2(1-\hat{p}_2) = 0.1783$
- $\frac{dz_4}{dw_8} = h_2 = 0.5963$

$\therefore \frac{dloss}{dw_8} = -2(y_2 - \hat{p}_2) \cdot \hat{p}_2(1-\hat{p}_2) \cdot h_2$

$(-0.4446)(0.1783)(0.5963)$

$\therefore \frac{dloss}{dw_8} = -0.0473$

8.) Find the derivative of $\frac{dloss}{dh_1}$

$$\frac{dloss}{dh_1} = \frac{dloss_1}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{dh_1} + \frac{dloss}{d\hat{p}_2} \times \frac{d\hat{p}_2}{dz_4} \times \frac{dz_4}{dh_1}$$

- $\frac{dloss_1}{d\hat{p}_1} = -2(y_1 - \hat{p}_1) = 1.4938$
- $\frac{d\hat{p}_1}{dz_3} = \hat{p}_1(1-\hat{p}_1) = 0.1840$
- $\frac{dz_3}{dh_1} = w_5 = 0.40$
- $\frac{dloss_2}{d\hat{p}_2} = -2(y_2 - \hat{p}_2) = -0.4446$
- $\frac{d\hat{p}_2}{dz_4} = \hat{p}_2(1-\hat{p}_2) = 0.1783$
- $\frac{dz_4}{h_1} = w_6 = 0.45$

$\therefore \frac{dloss}{dh_1} = -2(y_1 - \hat{p}_1) \cdot \hat{p}_1(1-\hat{p}_1) \cdot w_5 + -2(y_2 - \hat{p}_2) \cdot \hat{p}_2(1-\hat{p}_2) w_6$

$= (1.4938)(0.1840)(0.40) + (-0.4446)(0.1783)(0.45)$

$\therefore \frac{dloss}{dh_1} = 0.0743$

9.) Find the derivative of $\frac{dloss}{dh_2}$

$\frac{dloss}{dh_2} = \frac{dloss_1}{dh_2} + \frac{dloss_2}{dh_2}$ or $\frac{dloss_1}{dh_2} = \frac{dloss_1}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{dh_2} + \frac{dloss_2}{dh_2} =$

$= \frac{dloss}{dh_2} = 0.0743 + (-0.0436)$

$\therefore = 0.0307$

10.) $\frac{dloss}{b_1} = \frac{dloss_1}{d\hat{p}_1} \times \frac{d\hat{p}_1}{dz_3} \times \frac{dz_3}{db_1}$

$= (1.4938)(0.1840)(1)$

$\frac{dloss}{b_1} = 0.2749$

11.) $\frac{dloss}{dw_1} = \frac{dloss}{dw_1} = \frac{dloss}{dh_1} \times \frac{dh_1}{dz_1} \times \frac{dz_1}{dw_1}$

$(0.0743)(0.2410)(0.03)$

$\therefore \frac{dloss}{dw_1} = 0.0009$