Are You The One? Probabilistic Stable Matching

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1 Introduction

Are You the One? is a reality television dating show where a house of approximately 20 contestants are secretly paired up by a match-making algorithm. Over the course of approximately 10 weeks, the contestants complete various physical and emotional challenges to win dates in search of their perfect match. Every week, the contestants form pairs. Additionally each week, the contestants vote on a couple to send into the "truth booth" which reveals the answer about the chosen pair. After all pairs are locked in, the contestants learn how many pairs they guessed correctly (but not which pairs are correct), and the contestants must get at least one pair correct every week or else they lose 25% of the final prize: $1,000,000.

Every season, there is a clear divide between "head versus heart" or "strategy versus emotion", and the show emphasizes a healthy mixture of the two approaches in order for the contestants to win. The following seeks to understand this game probabilistically and to make it playable by AI agents.

- First, we translate the show's dynamics into a playable environment.
- Second, we analyze the probabilistic structure of the game's action and observation spaces.
- Third, we describe a learning agent that develops a strategy for winning the game.
- Last, we evaluate learning over randomly generated games with respect to number of contestants and number of guesses.

2 Stable Marriage

The Stable Marriage Problem is a common constraint satisfaction problem in graph theory that seeks to find pairs of nodes that maximizes the preferences in a weighted bipartite graph. First, here are some definitions in graph theory.

Definition 2.1 (Weighted Bipartite Graph)**.** *Let $G = (A, B, E, W)$ be a weighted bipartite graph with node set $V = A \cup B$, edges $E \in A \times B$ which cross between A and B, and a matrix of weights W indexed as $W_{a,b}$ containing the weight of the edge between $(a, b) \in E$.*

For consistency in notation, let $a, a', \ldots \in A$ and similarly $b, b', \ldots \in B$.

Definition 2.2 (Matching)**.** *A matching in a bipartite graph B is a subset of edges, $M \subset E$, such that:*

- *all vertexes in one of A or B are covered by an edge in M, and*
- *no node in V belongs to more than one edge in M.*

A perfect matching further restricts the matching so that it covers all of V.

Given a weighted bipartite graph, G, a stable marriage is an edge $(a, b) \in E$ such that $W_{a,b} \geq W_{a',b}$, $\forall a' \in A$. The objective of the stable marriage problem is to find this preferential assignment. The most common algorithm for finding a stable marriage is the Gale-Shapely algorithm given below.

Algorithm 1 Gale-Shapely Stable Marriage

procedure STABLE-MATCHING(A, B, E, W)
  Initialize an empty matching, M
  while $\exists a \in A \;\&\; a \notin M$ **do**
    $b \leftarrow E.Neighbors(a).SortBy(W).First()$
    if $\exists(a', b) \in M$
      if $W[a, b] > W[a', b]$
        $M.Remove(a', b)$
        $M.Add(a, b)$
    else
      $M.Add(a, b)$
  return M

This algorithm iteratively builds up a matching by adding an edge to the matching at each step of the while loop, sometimes replacing an existing edge with a more preferred edge; however, note that the algorithm really only constrains on the preferences of the second vertex partition, B. Ultimately, this algorithm terminates when all $a \in A$ have chosen a partner, and it is guaranteed to terminate because we are bounded by a finite set of edges to test. Additionally, this yields the best possible matching for the elements of B, and the worst possible matching for elements of A. In order to counteract this

In the game of *Are You The One¿?* We assume that the cast of contestants were paired up according to this algorithm using some hidden weight matrix which we hope to approximate as the game progresses.

3 *Are You The One?*

Given the appropriate foundations in graph theory, we can formally describe the game of *Are You The One?* with the following dynamics:

- The observation space, S, is the set of sequences of bipartite graphs representing possible matches remaining to be explored, plus the number of beams revealed by the previous action. The reason we observe a sequence of graphs is to track changes as a result of choices.
- The initial observation is the fully connected bipartite graph of contestants in the game with 0 beams.
- The action space, A, contains the set of possible matchings on the bipartite graph, crossed with the edge set. This represents a guessed match plus a guessed truth booth every week.
- The terminal reward R_h is a hidden variable tracking the remaining cash prize. Let @ be the terminal success state and R_{max} be the maximum possible prize pool, then $R(@) = R_h$. A blackout occurs when an action yields 0 correct pairs. Whenever a blackout, occurs the prize is updated to $R_h = R_h - 0.25 * R_{max}$.
- Normally, the game would end when $R_h = 0$; however, this results in a sparse reward signal, so we allow the agent to receive negative rewards and force it to learn to find perfect matches.
- The game terminates when the agent has discovered the random initial perfect matching.

4 Milestones

For the first milestone, we needed to implement a playable game for the agent to play. This has been accomplished with the implementation of the AreYouTheOne environmemt deacribed above.

For the second milestone, we needed to construct a brute force search agent to collect experiences for the agent so I can measure the effect of the hyperparameters of the game against the rewards for playing. For example, how does the number of matches affect the constraint satisfaction problem? How about the number of weeks?

For the third milestone, we needed to implement a probabilistic approach to solving the game that incorporates match-up ceremonies probabilistically. The best way we found to approach this was relying on a Naive Bayes approach that evaluates stable matchings on the basis on the combined likelihoods that pairs belong or do not belong in the target matching based on the "beam" dynamics in the game. By maintaining incremental likelihood matrices for the match versus no match assumptions, we encorporate observations at each guessing step and seek a stable matching that incorporates them.

Lastly, we were unable to measure the agent against number of guesses and prize money to over the course of training on randomly generated experiences.

Ultimately, we were only able to design and implement the environment due to unforeseen problems interacting with graph-based state and action spaces.

The current state of the project is posted to:

https://github.com/frndlytm/areyoutheone