Part 2

In section 2, the modified versions of the dijkstra and bellman ford algorithms are implemented, where each algorithm was allowed to perform at most k relaxations per node. Each implementation was then tested and compared against the original versions of the algorithms for accuracy, performance and memory usage. The results of the tests are shown in the tables below.

Accuracy

To measure the accuracy of the modified algorithms, we designed various experiments to determine how the size of the graph, density of the graph, and the number of relaxations per node (k value) affected the accuracy of the algorithms. The results of these experiments are shown in the graphs below.

The first experiment was designed to measure the accuracy of the modified algorithms with different k values. The results of this experiment are shown in Figure 1.

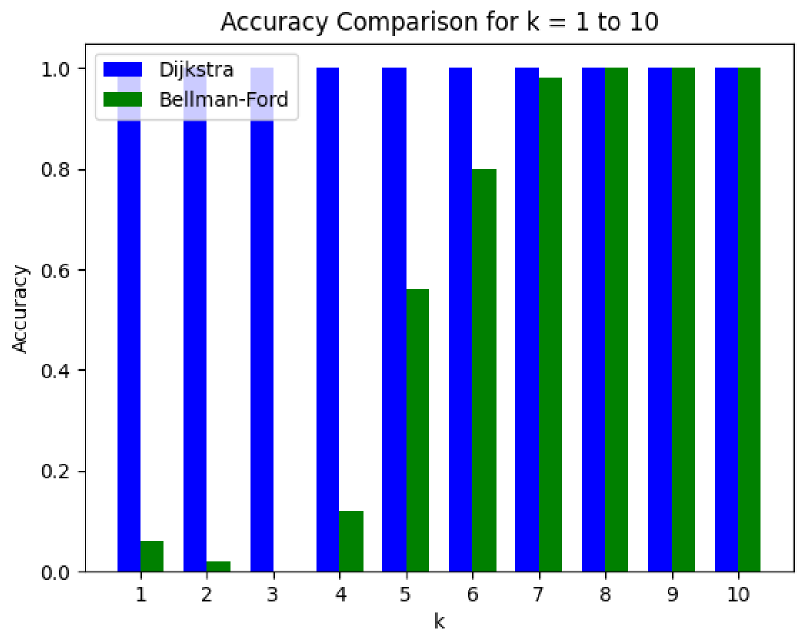

Figure 1: Experiment 1 - Accuracy of modified algorithms with different k values.

This experiment was performed across 50 trials, where each trial consisted of a graph with 100 nodes, 200 edges and a maximum weight of 10. The results of this experiment show that the accuracy of the modified Bellman-Ford algorithm increases as the k value increases. This is because the modified Bellman-Ford algortihm is allowed to perform at most k relaxations per node, which means that they may not be able to find the shortest path in the graph if the k value is too small.

However, the value of k does not seem to have any effect on the accuracy of the modified Dijkstra's algorithm, and the modified Dijkstra's algorithm is able to find the shortest path in the graph regardless of the value of k. This is because the modified Dijkstra's algorithm is able to find the shortest path in the graph by using a priority queue to keep track of the nodes that have been visited and the nodes that have not been visited. Therefore, the value of k does not affect the accuracy of the modified Dijkstra's algorithm.

The second and third experiments were designed to measure the accuracy of the modified algorithms with different graph sizes and densities. The results of these experiments are shown in Figure 2 and Figure 3.

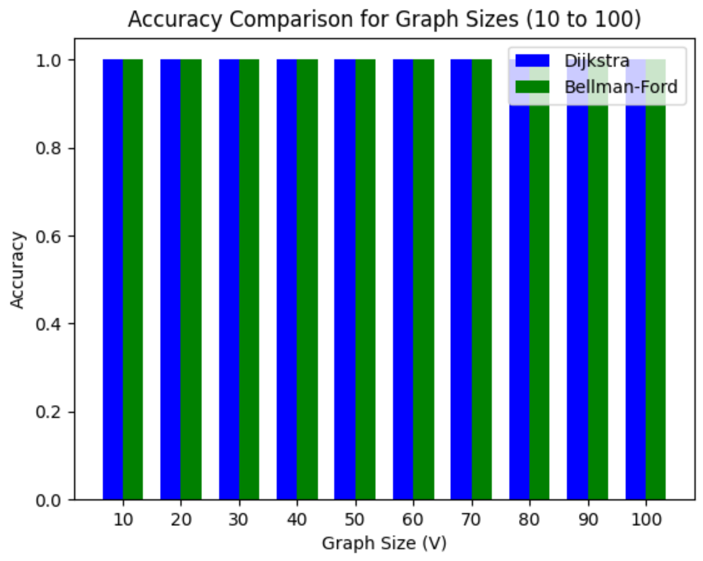

Figure 2: Experiment 2 - Accuracy of modified algorithms with different graph sizes.

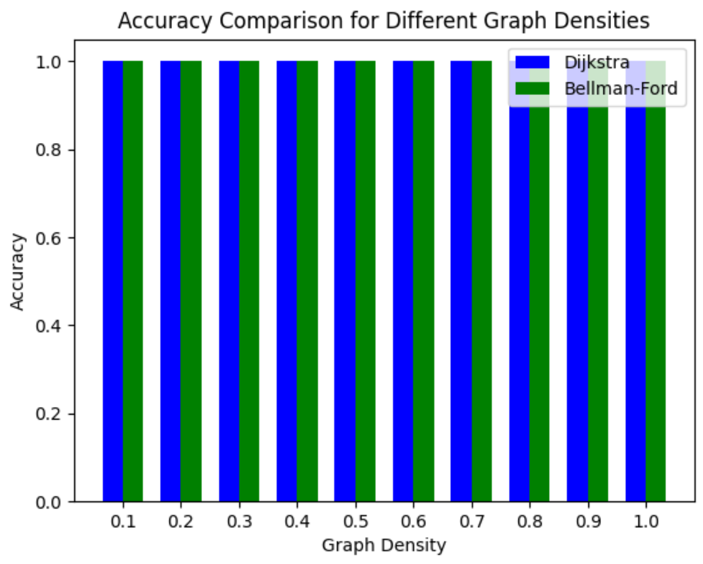

Figure 3: Experiment 3 - Accuracy of modified algorithms with different graph densities.

Experiment 2 was performed across 50 trials, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and a density D of 0.5. The experiment was conducted with $|V| = 10, ..., 100$ The number of vertices were calculated based on the number of edges in the graph using the following formula:

$$\frac{|E| = |V|(|V| - 1)0.5}{2}$$

where $|E|$ is the number of edges, $|V|$ is the number of vertices and D is the density of the graph.

Experiment 3 was performed across 50 trials, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and 20 vertices. The experiment was conducted with $D = 0.1, 0.2, ..., 1.0$. The density of the graph was calculated based on the number of edges in the graph using the following formula:

$$\frac{|E| = 20(20 - 1)D}{2}$$

where $|E|$ is the number of edges, $|V|$ is the number of vertices and D is the density of the graph.

The results of these experiments show that the accuracy of the modified Bellman-Ford and dijkstra's algorithms are not affected by the size of the graph or the density of the graph. We can see this in Figure 2 and Figure 3, where the accuracy of the modified Bellman-Ford and dijkstra's algorithms are 100% for all graph sizes and densities.

The results of these experiments show that the accuracy of the modified Bellman-Ford and dijkstra's algorithms are not affected by the size of the graph or the density of the graph, but Bellman-ford alone is affected by the number of relaxations allowed per node.

Time Complexity

To measure the time complexity/performance of the modified algorithms, we designed various experiments to determine how the size of the graphs, density of the graphs, and the number of relaxations per node (k value) affected the performance of the algorithms. The results of these experiments are shown below.

Experiment 4 was designed to measure the performance of the modified algorithms with different k values. This experiment was conducted with the same parameters as experiment 1, where each trial consisted of a graph with 100 nodes, 200 edges, a maximum edge weight of 10 and k values of $1, ..., 10$. The results of this experiment are shown in Figure 4.

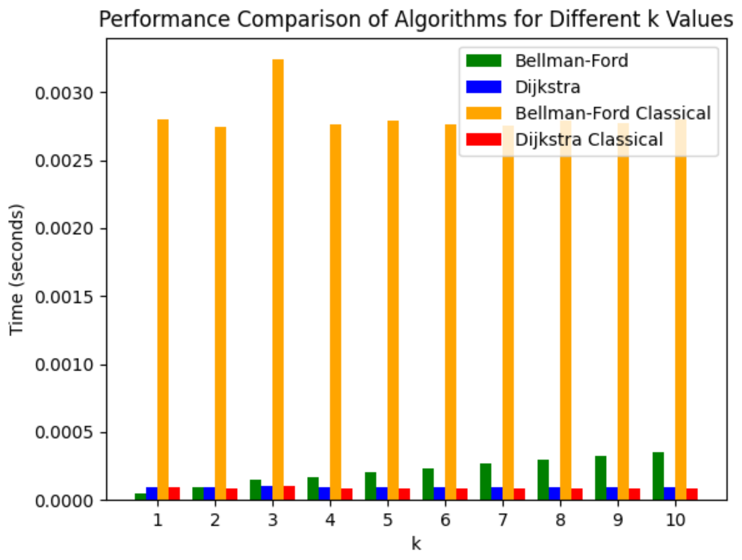

Figure 4: Experiment 4 - Performance of modified algorithms with different k values.

The results of this experiment varied for each algorithm. The modified Bellman-Ford algorithm was significantly faster than the original Bellman-Ford algorithm. However, the speed of the modified Bellman-Ford algorithm did increase as the value of k increased, with more of a linear increase. This shows that restricting the number of relaxations per node allows the Bellman-Ford algorithm to perform significantly faster. Furthermore, the modified dijkstra's algorithm performed almost identically to the original dijkstra's algorithm, showing that the modified dijkstra's algorithm is not affected by the value of k. Both versions of djikstra's algorithm performed faster than both versions of the Bellman-Ford algorithm, which is expected as dijkstra's algorithm is more efficient and has a better time complexity than Bellman-Ford ($O(E + V \log V)$ vs $O(VE)$).

Experiment 5 was designed to measure the performance of the modified algorithms with different graph sizes. This experiment was conducted with the same parameters as experiment 2, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and a density D of 0.5. The number of vertices were calculated based on the number of edges in the graph using the same formula as experiment 2. The results of this experiment are shown in Figure 5.

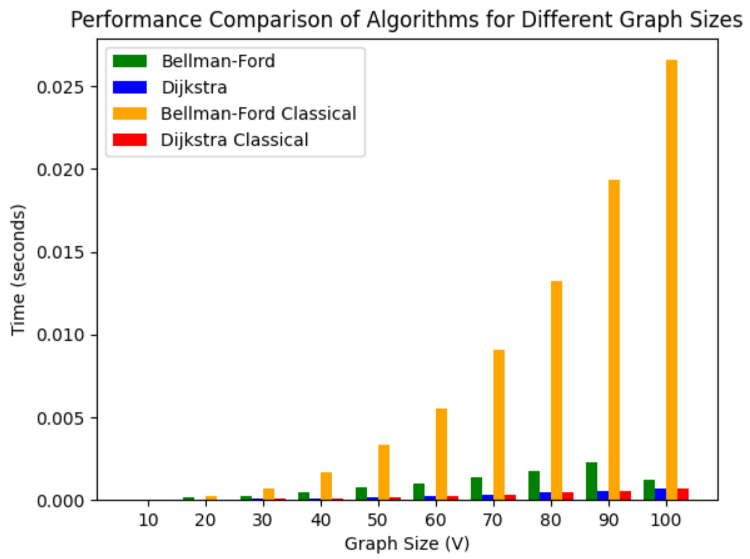

Figure 5: Experiment 5 - Performance of modified algorithms with different graph sizes.

Here, the results are similar to experiment 4, with the excpetion of the classical Bellman-Ford algorithm's performance. Both versions of bellman ford increased in time complexity as the number of vertices increased, with the classic Bellman-Ford algorithm decreasing in performance at a much faster rate than the modified Bellman-Ford algorithm. This is because the modified Bellman-Ford algorithm is able to perform at most k relaxations per node, which means that it is able to perform faster than the classical Bellman-Ford algorithm. Both version of djikstra's algorithm saw a decrease in performance as the numebr of vertices increased, but oth performed faster than each version of Bellman-Ford. The modified djikstra's algorithm performed very similarly to the classical dijkstra's algorithm, which is expected as the modified dijkstra's algorithm is not affected by the value of k. This is to be expected, since djikstra's algorithm is more efficient and has a better time complexity than Bellman-Ford ($O(E + V \log V)$ vs $O(VE)$).

Experiment 6 was designed to measure the performance of the modified algorithms with different graph densities. It was conducted with the same parameters as experiment 3, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and 20 vertices. The density of the graph was calculated based on the number of edges in the graph using the same formula as experiment 3. The results of this experiment are shown in Figure 6.

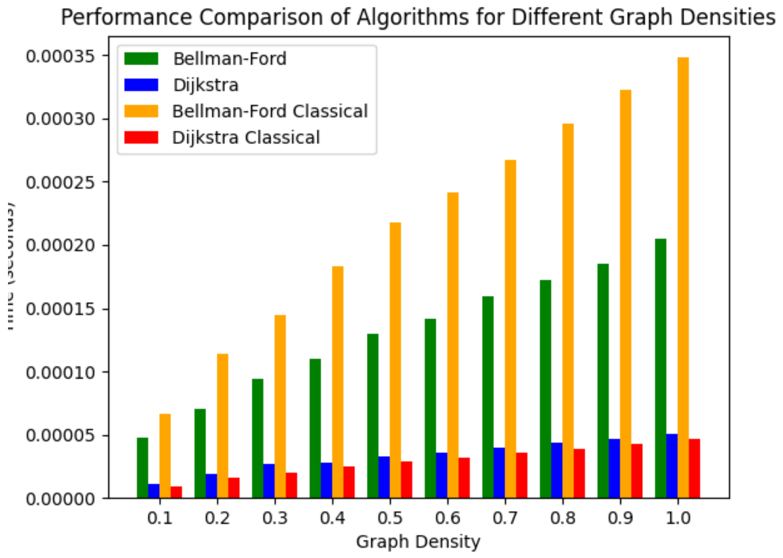

Figure 6: Experiment 6 - Performance of modified algorithms with different graph densities.

Here, we can see that the density of the graph affects the performance of all the algorithms in a similar way. Both versions of Bellman-Ford and dijkstra's algorithms saw a decrease in performance as the density of the graph increased, with the classical Bellman-Ford algorithm decreasing in performance at a much faster rate than the modified Bellman-Ford algorithm. Similar to experiment 5, this is because the restriction of k relaxations per node allows the modified Bellman-Ford algorithm to perform faster than the classical Bellman-Ford algorithm. The modified dijkstra's algorithm also performed similarly to the classical dijkstra's algorithm, however, the modified dijkstra's algorithm performed slightly slower than the classical dijkstra's algorithm. however, the difference in perfoance was very small. Both of these algorithms performed faster than both versions of Bellman-Ford, which is expected as dijkstra's algorithm is more efficient and has a better time complexity than Bellman-Ford.

indent Therefore, we can see that each parameter has different effects on the performance tiem of each algorithm.

Memory Usage

Similar to the previous experiments, to calculate the memory usage of the modified algorithms, we designed various experiments to determine how the size of the graphs, density of the graphs, and the number of relaxations per node (k value) affected the memory usage of the algorithms.

Experiment 7 has designed to measure the memory usage of the modified algorithms with different k values. The parameters of the experiment were the same as experimetn 1 and experiment 4, where each trial consisted of a graph with 100 nodes, 200 edges, a maximum edge weight of 10 and k values of $1, ..., 10$. The results of this experiment are shown in Figure 7.

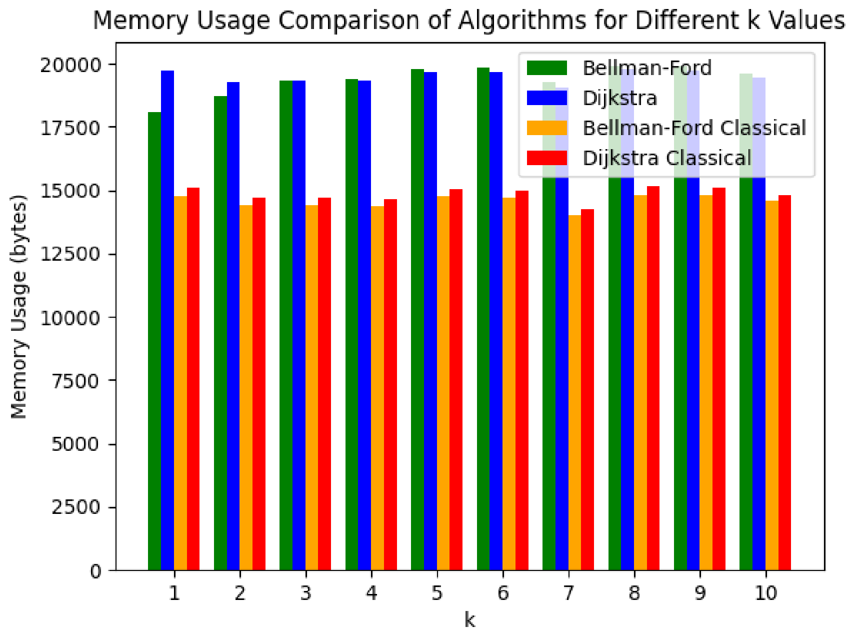

Figure 7: Experiment 7 - Memory usage of modified algorithms with different k values.

Experiment 7 shows that the memory usage of every algorithm does not seem to be affected by the value of k, as the memory usage of each algorithm is relatively the same for all values of k. This is expected as the memory usage of each algorithm is determined by the number of vertices and edges in the graph, rather than the value of k. Both the modified version of the Bellman-Ford algorithm and the modified version of dijkstra's algorithm used very similar amounts of memory as the classical versions of the algorithms. Both of the modified algorithms also use more memory than the classical versions of the algorithms, which is expected as the modified algorithms are more complex and have more variables to store and modify.

Therefore, we can see that the memory usage of each algorithmn is only affected by the number of vertices, and the density and size of the graph do not affect the memory usage of each algorithm.

Experiment 8 was designed to measure the memory usage of the modified algorithms with different graph sizes. The parameters of the experiment were the same as experiment 2 and experiment 5, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and a density D of 0.5. The number of vertices were calculated based on the number of edges in the graph using the same formula as experiment 2. The results of this experiment are shown in Figure 8.

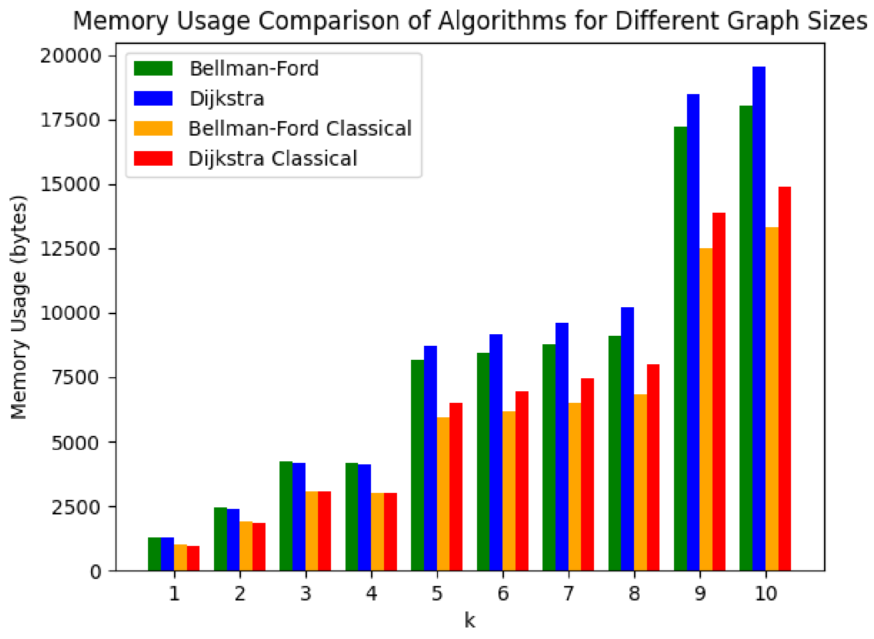

Figure 8: Experiment 8 - Memory usage of modified algorithms with different graph sizes.

Experiment 8 shows that the memory usage of each algorithm increases as the number of vertices in the graph increases. The classical versions of both algorithms perform faster than the modified versions of the algorithms and use less memory than the modified versions of the algorithms. However, there is a clear trend among all algorithms, where the memory usage of each algorithm increases as the number of vertices in the graph increases. This is expected as the number of vertices in the graph increases, as we know that the space complexity of both algorithms is $O(V)$, where V is the number of vertices in the graph.

Experiment 9 was designed to measure the memory usage of the modified algorithms with different graph densities. The parameters of the experiment were the same as experiment 3 and experiment 6, where each trial consisted of a graph with $k = 10$, a maximum edge weight of 10 and 20 vertices. The density of the graph was calculated based on the number of edges in the graph using the same formula as experiment 3. The results of this experiment are shown in Figure 9.

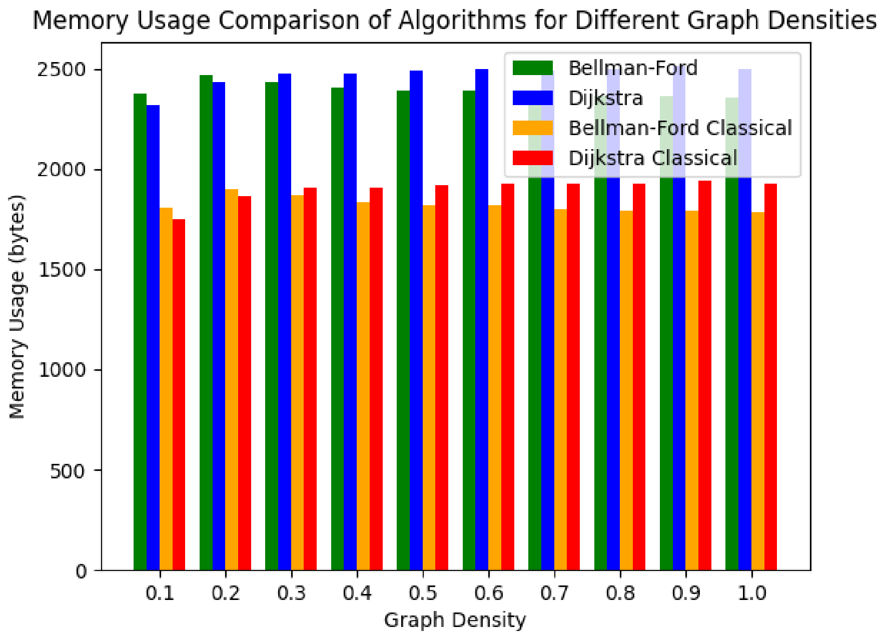

Figure 9: Experiment 9 - Memory usage of modified algorithms with different graph sizes.

Experiment 9 shows that the memory usage of each algorithm is not affected by the density of the graph, as the memory usage of each algorithm is relatively the same for all values of density. This is expected as the memory usage of each algorithm is determined by the number of vertices and edges in the graph, rather than the density of the graph. Hence, the results of this experiment are very similar to the reulsts of experiment 7, where both the modified version of the Bellman-Ford algorithm and the modified version of dijkstra's algorithm used very similar amounts of memory as the classical versions of the algorithms.

Part 3

In section 3, an algorithm to find the shortest path in a graph from all nodes to a single node were implemented. This was implemented in two different ways, one using dijkstra's algorithm and the other using bellman ford.

For dense graphs, the single-source implementation of dijkstra's algorithm has a time complexity of $O(V^2)$, where V is the number of vertices in the graph, while it's time complexity for sparse graphs is $O(E + V \log V)$, where E is the number of edges in the graph. The implementation of the shortest path algorithm (for all nodes) using dijkstras has a time complexity of $O(V^3)$ for dense graphs and $O(E^2 + EV^2 \log V)$. This is because dijkstra's algorithm is run V times, once for each node in the graph.

The same logic applies to the implementation of the shortest path algorithm using bellman ford. The time complexity for dense graphs is $O(V^3)$ and for sparse graphs is $O(E^2 + EV^2)$. The implementation of the shortest path algorithm using bellman ford has a time complexity of $O(V^4)$ for dense graphs and $O(E^2 + EV^2)$ for sparse graphs. This is because bellman ford is run V times, once for each node in the graph.