Ground Source Heat Pump
Coil Optimization

MATH 319 – Optimization

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Introduction:

Ground source heat pump systems have gained a lot of attention for their ability to produce high energy performance while maintaining sustainability and offering extremely low operation costs. For my project I chose to optimize a system that my Dad, an electrical engineering graduate of Walla Walla University, and I are installing in our home.

This paper presents the development of an optimization strategy for the control of a ground source heat pump coil. The optimization problem utilizes a model-based approach, in which the component models are used to estimate the system performance under various trial settings. Experimental data was collected to describe our specific system. An exhaustive search utilizing Excel's implementation of the method of feasible directions is used to identify the optimal settings under the search ranges defined. The variables optimized are the CFM (cubic feet per minute) of airflow through the coil and the water flow rate in GPM (gallons per minute). The overall objective of the optimization is to minimize the cost per BTU of cooling provided. An economic evaluation of the optimization results will be made, comparing the results to a state-of-the-art mini split with a high SEER rating.

System Characterization:

The system can be idealized through the airflow diagram located in Appendix 1. Our system used data from a YORK MV16CN21C blower assembly, a Flotec FP2212-12, ½ HP 10 GPM pump, and a First Company water coil. Specification sheets available on request.

The cost function we seek to optimize is expressed in equation (1). The optimal combination of the water flow rate (GPM) from the cold-water intake and the fan speed from the York conditioning unit which gives the minimal cost per BTU of output will be considered as the optimal control settings for the working condition. The fan speed is controlled by a microcontroller giving a constant CFM regardless of duct work: 716 to 1524 CFM in twelve different steps, thus, forming a feasible constraint on our variable. In addition, one has the option to reduce each setting by 15% by connecting the dehumidification contacts on the air blower. The pump flow rate has also been limited, to ensure that normal household water consumption, (i.e. taking a shower, watering the garden), will not be affected. A constraint on the BTU output of at least 18,000 BTU/hr will be placed. This value represents ½ of the typical cooling demands for a normal American household. Thus, formalizing our optimization problem:

$$\min \quad J(\overline{x}) = \frac{C_{fan}(\overline{x})+C_{pump}(\overline{x})}{Q_{coil}(\overline{x})} \tag{1}$$

$$\text{w.r.t.} \quad \overline{x} = \{CFM, GPM\}$$

$$\text{s.t} \quad Q_{coil}(\overline{x}) \geq 18{,}000$$

$$688.5 \leq CFM \leq 1524$$

$$0 < GPM \leq 5$$

where J is the cost function per BTU, C_{fan} and C_{pump} are cost functions in terms of (W), and Q is the energy transferred to the air by the coil in units of BTU/hr. As specified previously, CFM refers to the airflow through the fan and GPM is the rate of water flow through the pipe. (Note: Since power (W) is proportional to cost ($), I chose to leave my cost functions in units of power)

To determine the cost of the fan speed, the current required to drive the fan was measured while varying the fan speed using the microcontroller. The voltage was measured several times and was a relatively constant 245.3 V. The relationship between power and CFM was found to fit to a parabolic model and can be seen in Appendix 2(a). The results of our analysis produced a quadratic relationship between power and CFM seen below in equation (2):

$$C_{fan} = 0.0002152 \cdot (CFM)^2 - 0.033845 \cdot (CFM) \tag{2}$$

The cost of the pump was also determined experimentally by measuring the current and voltage at a flow rate of 10 GPM. Initially, we assumed that since the coil is currently hooked up to a pressurized system, the cost of operation should be directly proportional to the flow rate of the water. Theoretically, this is true. However, this assumption proved to make our optimization problem too simplistic. Thus, we additionally modeled the startup current of the pump motor and including this in our model increased the complexity and accuracy of our model. The cost of the pump is thereby described in equation (3). Details of the computation can be found in Appendix 2(b).

$$C_{pump} = 500 \times \frac{(GPM)\cdot(10-GPM)}{6.0383} + 2.070833 \cdot 60 \ \cdot (GPM) \tag{3}$$

The relationship between amount of heat transferred from the coil and the CFM and GPM is given via the specific heat formula:

$$Q_{water} = 500 \cdot \mathrm{GPM} \cdot \Delta T_{d,water} \tag{4}$$

$$Q_{air} = 1.08 \cdot CFM \cdot \Delta T_{d,air} \tag{5}$$

where ΔT_d is the temperature difference between entering temperature and leaving temperature of the water or air. In an ideal system, the heat lost by the air is equal and opposite to the heat absorbed by the water, thus, I focused solely on equation (4) and used equation (5) to cross check my answers. Note that ΔT_d depends heavily on specific coil parameters. For my project, I was unable to feasibly measure these coil parameters, however, I partnered with Baltimore Air and Coil (BAC) to design a coil that best represented the ground source cooling coil that I was using. BAC graciously provided me with coil rating spec sheets that calculated ΔT_d for a variety of GPM and CFM. An example of one of these spec sheets can be seen in Appendix 3(a).

It should be noted that when modeling the 1 GPM case, BAC's software failed to produce results, due to the change from turbulent flow to laminal flow and the consequential reduction in heat output. Based on rough estimates, the heat output would drop by more than a factor of 10, which renders low flow rates below 1 GPM to be infeasible solutions. ("Comparison of Laminar & Turbulent Flow.") We attempted to model this, however, it proved to be out of the scope of this project.

Using the data, I fitted a surface using MATLAB's curve fitting app. A table of data used to fit the surface can be seen in Appendix 3(b). The result gave me $Q_{coil}(\bar{x})$, the amount of Btu's extracted as a function of GPM and CFM.

$$\begin{aligned} Q_{coil}(\overline{x}) = & -0.102(CFM)^2(GPM) - 0.0005(CFM)(GPM)^2 + 1.7 \cdot 10^{-6}(GPM)^3 \\ & -63.21(CFM)^2 + 2.948(CFM)(GPM) - 0.0846(GPM)^2 \\ & +15.766(GPM) + 2328.35 \end{aligned} \tag{6}$$

A graph of $Q_{coil}(\bar{x})$ can be seen in Appendix 4. The corresponding MATLAB code can be seen in the pdf entitled "Froelich_MATH319_Project_MATLAB.pdf". Note that MATLAB normalizes the variables, thus, the equation presented there may not look equivalent, however, it is identical to equation (6).

Optimization Algorithm Selection:

Despite its length, our objective function is relatively simplistic. Thus, we have a wide variety of optimization algorithms to choose from. We are limited slightly by the added constraint on $Q_{coil}(\bar{x})$, however, from the algorithms that we learned about in class, more than half of them would apply to our problem. The three main criteria used when selecting our optimization algorithm are detailed below:

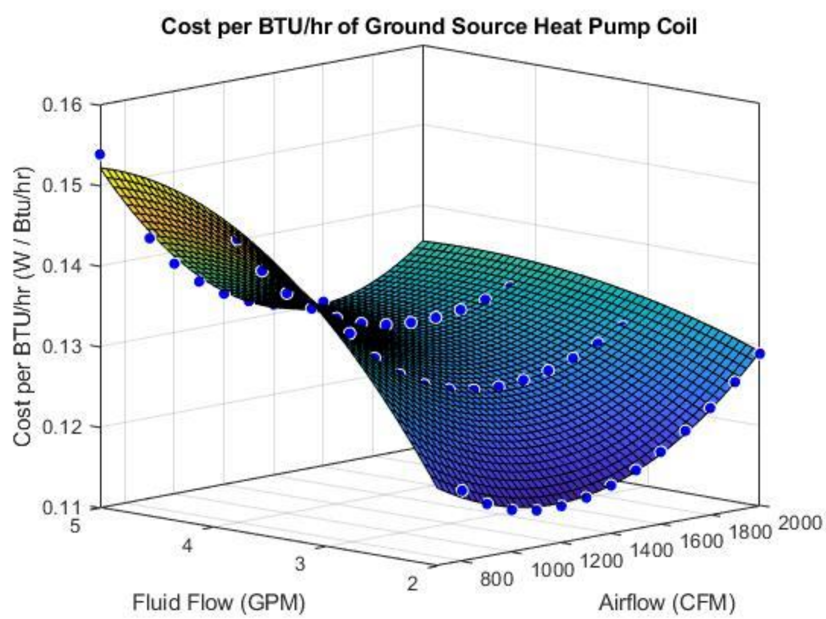
- Convergence to optimum value: The ideal algorithm would almost-always converge to optimal solutions.
- Ease of use: The effort required to set up the algorithm for the problem would be minimized. (another optimization problem!)
- Computational demand: The resources needed to compute the minimum, i.e., number of function evaluations or iterations, should be minimized in order for a quality solution to be found in a timely manner.

With these three criteria in mind, I chose to use the method of feasible directions, i.e. the generalized reduced gradient (GRG). Since it is gradient based, the method of feasible directions converges quickly to the optimal solution. However, that speed comes with compromises. The GRG method has a high dependency on initial conditions and requires that the objective function and constraints be smooth, continuous functions. These two issues do not pose any concerns for our optimization problem as our functions are continuous and do not have multiple relative minimums. Thus, in the context of our problem, the GRG method has the capability to always converge towards near-optimal solutions. Additionally, Excel's implementation of the GRG method made it easy to integrate previously calculated data with data necessary to run the optimization algorithm.

Description and Validation of Analysis Tool:

The two primary analysis tools used were MATLAB and Excel. MATLAB was utilized to generate a surface fit of the experimental data. An example of the code can be seen in Appendix 4. Plots of the function in 3D using MATLAB, verified the 2D plots made with Excel.

Graphically, the results were reasonably intuitive. A graph of our objective function is seen to the right.



Optimization Results and Validation:

Our optimization led to a constrained minimum of:

$$J(\overline{x}) = 0.1197\ \text{W/(Btu/hr)} \qquad \text{located at:}$$

$$\overline{\text{x}} = \{1488, 2.261\}$$

While we optimized our system assuming the flow rate as a continuous variable, as mentioned before it is discrete, thus, minimizing the cost function at the two closest fan settings produced a new minimum of:

$$J(\overline{x}) = 0.1198\ \text{W/(Btu/hr)} \qquad \text{located at:}$$

$$\overline{\text{x}} = \{1518.95, 2.308\}$$

Thus, we should control our coil with 1518.95 CFM and 2.308 GPM.

The feasibility of our solution is highly dependent on the accuracy of our estimates. While we tried to use hard data whenever possible, due to the nature of our problem we had to make two critical estimates to develop a mathematical model. These were:

1. The coil parameters needed to create a model with BAC's software, and;
2. The starting current of the pump.

While the coil parameters and thermal behavior were computed based of commercial software, the design of the coil utilizing its physical aspects had to be roughly approximated. Additionally, the software did not allow for the correct shape of coil, thus, I used the professional advice of the technician I spoke with to come up with a similar coil structure.

Our assumption for the starting power of the pump (500W – for details see Appendix 2(b)) was not made with as much scientific rigor. We based our estimate off the startup current for a brushless induction motor, however, the efficiency of smaller, low flow water pumps is notably much *much* higher than our pump and could significantly sway the results of our optimization.

In truth, the only way to completely verify the validity of our solution is to take more experimental data. This would be relatively straightforward with the right equipment. Measuring the temperature drop across the piping would be sufficient information to compute the Btu/hr extracted from the coil utilizing equation (4).

The economic implications of our solutions are startling. Initially, I assumed the system performance to be comparable with other low-cost HVAC solutions. For the purpose of this project, I chose to compare the results with a Fujitsu 9RLS3 mini split as we have installed several of these units in our home. The Fujitsu 9RLS3 is rated at 33 SEER and has a sensible heat of 8.67 kBtu/hr with an input power of 0.45 kW. Thus, the cost per Btu / hr for a Fujitsu is 0.051903 W / (Btu/hr), half the cost of our system! Our results are however, on par with a traditional HVAC system. A low-cost ENERGY STAR system has a SEER rating of 14.5 which translates approximately to a cost of 0.118124 W / (Btu/hr). ("What Does SEER Mean?")

Obviously, our estimates have a large effect on the economics of our system. If we had neglected the startup current requirements of the pump as we initially assumed, the system would have produced a solution 40% cheaper than the Fujitsu. That solution would be: J(x) = 0.031965 W / (Btu/hr). Additionally, if we used a different, lower-cost, water pump, the system would be more efficient. Back calculating, we find that the breakeven point is a startup pump power of 78.42 W. At this wattage, the cost per Btu/hr of our ground source heat pump is the same as the Fujitsu 9RLS3. (For more information on these calculations see the attached Excel sheet entitled “Froelich_MATH319_Project_Excel.xlsx”). While this solution is ideal for those interested in adding a ground source heat pump to replace an older less-efficient system, it is not as practical in our situation, since we already have several efficient mini-splits installed. A more sensible solution would be to reroute the piping such that all water from the well goes through the coil. A simple microcontroller (Arduino Nano, Raspberry Pi, etc.) could then be used to start the fan whenever the well pump is turned on and/or when the water temperate is significantly cooler than the air temperate. Arguably, the cost of the pump would then be negligible, as the cost was already incurred for other usage.

Reflecting on this project, I have learned about the application of optimization to real world problems. I have learned how to apply knowledge obtained in class towards the development of mathematic models and minimize those models using an appropriate optimization algorithm. Our analysis has been significant in aiding our ground source heat pump design. The results obtained above are not trivial, my Dad and I fully plan on using this project as a resource to better our ground source heat pump design.

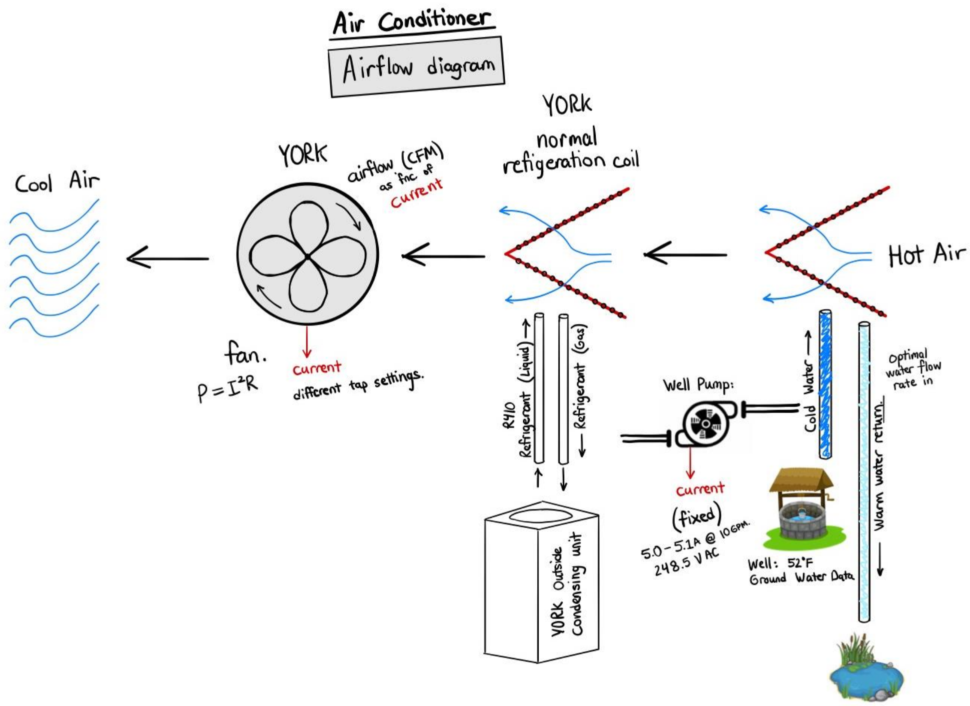
Bibliography:

Berg, Carl E. “BACK TO BASICS: USEFUL EQUATIONS WHEN SIZING COILS WITH COILPRO .” Colmac Coil Manufacturing, www.colmaccoil.com/literature/technical-bulletins/back-to-basics-useful-equations-when-sizing-coils-with-coilpro.aspx.

“Comparison of Laminar & Turbulent Flow.” HRS Heat Exchangers, www.hrs-heatexchangers.com/us/resource/comparison-of-laminar-and-turbulent-flow/.

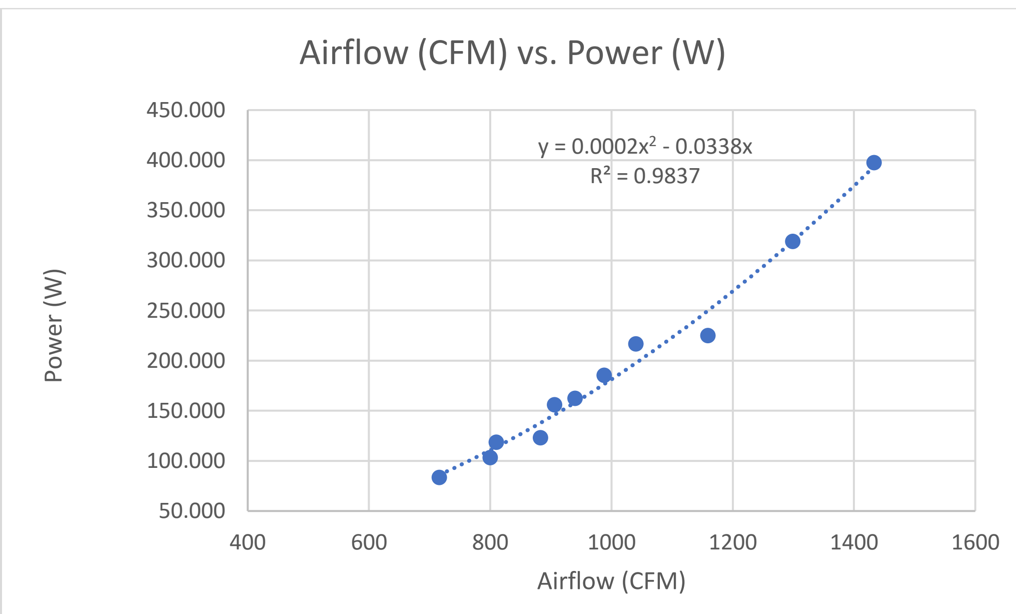
“What Does SEER Mean?” Trane, www.trane.com/residential/en/resources/glossary/what-is-seer/.

Appendix 1: System airflow diagram.



Appendix 2: Details regarding measured data used in the mathematical description of the system.

2(a): The relationship between power and CFM was determined experimentally. A graph of the results is seen below:



Cost equation: Power (W) as a function of CFM.

$$C_{fan} = 0.0002152 \cdot (CFM)^2 - 0.033845 \cdot (CFM)$$

2(b): We built two models of the starting cost of the pumps. The simple model only has a running cost involved. That was found by measuring the running current and voltage and calculating the power.

At 10 GPM = 600 GPH (Gallons per hour)		
Current =	5	A
Voltage =	248.5	V
Power =	1242.5	W

Thus, the linear relationship is:

Cost equation: Power (W) as a function of GPH.

$$C_{pump} = 2.070833 \cdot 60 \cdot (GPM)$$

The second model included a startup cost for the pump motor. To find the affect of the starting current, we need to find out how many starts/hr of the pump. We begin by assuming that the storage tank is initally full. If the flow in GPM through the coil is 0, the starts per hour is also 0. Similarly, since the well pump is rated at 10 GPM, if the flow through the coil is 10 GPM the starts per hour is also 0. Thus, our function will have roots at GPM = 0 and GPM = 10. Since time = volume / rate, we can divide the capacity of our tank by (GPM) to obtain the time in minutes needed to drain the tank. The time to fill the tank is found by dividing the capacity of the tank by (10 - GPM). Summing the results gives us the total time of one start of the pump. We simplify to obtain:

$$\text{time of start} = \frac{724.6}{(GPM)(10-GPM)}$$

Note that we didn't want all of the water to be used, thus, we assumed that we use only 50% of the capacity of the tank. Now note that starts = 1/time. Thus, the number of starts / hr is:

$$\text{starts/hr} = \frac{2 \cdot 60 \cdot (GPM)(10-GPM)}{724.6}$$

Assuming the starting power is 500 W, we get a cost for starting of:

$$\text{cost/hr} = 500 \times \frac{2 \cdot 60 \cdot (GPM)(10-GPM)}{724.6}$$

Thus, our total cost is:

$$C_{pump} = 500 \times \frac{(GPM)(10-GPM)}{6.0383} + 2.070833 \cdot 60 \cdot (GPM)$$

Appendix 3: Data provided from Baltimore Air and Coil.

3(a): Example coil rating sheet provided by Baltimore Air and Coil.

Water Coil	Coil Qty: 1	Model: 40x23 - 3R - 0.375/144
Input		
Airside Requirements:		
Airside Fouling:	(hr·ft²·F)/Btu	0.00010
Coil Application:		Cooling
Air Flow:	SCFM	1,400
Capacity:	Btu/Hr.	24,000
Entering Air Dry-Bulb:	°F	80.0
Entering Air Wet-Bulb:	°F	69.0
Leaving Air Wet-Bulb:	°F	64.0
Air Pressure:	PSIA	14.696
Coil Hand:		Left Hand
Tubeside Requirements:		
Tubeside Fouling:	(hr·ft²·F)/Btu	0.00010
Tubeside Fluid:		Water
Flow Rate:	GPM	4.0
Entering Fluid Temperature:	°F	52.0
Output		
Coil Selection:		
Model Number:		40x23 - 3R - 0.375/144
Tube Size:	In.	0.375
Arrangement:		1 x 0.866 Staggered
Fin Surface:		Corrugated
Face Area / Coil:	ft²	6.4
Face Velocity / Coil:	Ft/Min. (STD)	219.1
Number Of Circuits:	Qty	20
Tube Velocity:	Ft/Second	0.6
Reynolds Number:		1,474
Circuitry Flow:		Thermal Counter Flow
Tube Material:		Copper
Tube Wall:	In.	0.014
Fin Material:		Aluminum
Fin Thickness:	In.	0.0055
Header OD:	In.	0.625
Header Material:		Std.Type 'L' Copper
Connection OD:	In.	0.625
Casing Material:		16 Ga. Galv. Steel (Std.)
Casing Depth:	In.	4.625
Dry Weight:	Lbs./Coil	58
Coil Rating		
Capacity:		
Capacity / Coil:	Btu/Hr.	21,806
Leaving Air Dry-Bulb:	°F	66.8
Leaving Air Wet-Bulb:	°F	64.5
Sensible Cap. / Coil:	Btu/Hr.	20,241
Air Friction:	In.H2O/Coil	0.08
Surface Condition:		Wet
Leaving Fluid Temperature:	°F	62.9
Fluid Pressure Drop:	Ft.H2O/Coil	3.59

er Radiator Coil

3(b): Example table of data used in MATLAB to fit a surface to $Q_{coil}(\bar{x})$. This data was calculated based off the data sheets provided by Baltimore Air and Coil. *(Not all data is shown.)*

GPM	CFM	$ / BTU	BTU
5	700	0.153917	18017
5	900	0.14233	19920
5	1000	0.138598	20727
5	1100	0.135786	21464
5	1200	0.133699	22144
5	1300	0.132197	22777
5	1400	0.131189	23369
5	1500	0.130602	23926
5	1600	0.130377	24453
4	800	0.145213	17870
4	900	0.140701	18679
4	1000	0.137334	19410
4	1100	0.134841	20079
4	1200	0.133049	20696
4	1300	0.131835	21269
4	1400	0.131098	21806
4	1500	0.130782	22310
4	1600	0.130834	22785
4	1700	0.131211	23235
4	1800	0.131884	23662
4	1900	0.132827	24068
4	2000	0.134018	24455
3	800	0.135903	16352
3	900	0.13224	17056
3	1000	0.129599	17693
3	1100	0.127773	18273
3	1200	0.126591	18808
3	1300	0.125963	19302
3	1400	0.125793	19763
3	1500	0.126037	20193
3	1600	0.126646	20596
3	1700	0.127582	20975
3	1800	0.128819	21332
3	1900	0.130334	21669
2	800	0.118794	14176
2	900	0.116548	14734
2	1000	0.1152	15232
2	1100	0.114567	15681

Appendix 4: Graphs of the relationship between amount of Btu's extracted as a function of GPM and CFM. For more details see "Froelich_MATH319_Project_MATLAB.pdf".