Inquisitive Semantics

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Lecture 3: Algebraic Foundations

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Outline

- The semantics presented so far was mainly motivated by linguistic examples
- Today we will develop a semantics entirely motivated by algebraic considerations
- We will focus on informative and inquisitive content only

Outline

- The algebraically motivated semantics will turn out to be equivalent with the linguistically motivated semantics presented on Monday
- Thus, we will not develop a new semantics here, but rather provide a more solid foundation for the existing system

- The approach is currently being extended to the setting where attentive content is taken into account as well
- In that setting, we do expect to obtain a new semantics (the current system still has some undesirable features)

Plan

- Review of algebraic foundations of classical logic
- Algebraically motivated inquisitive semantics
- Comparison with the support-based system presented on Monday
- Outlook

Algebraic foundations of classical logic

Classical propositions

- Sets of possible worlds
- Embody informative content

Ordering propositions

- Propositions are ordered in terms of informative content
- $A \leq B$ iff A provides at least as much information as B
- Formally: $A \leq B \iff A \subseteq B$

Algebraic foundations of classical logic

Join and meet

- Relative to $\leq$, every two classical propositions have
  - a greatest lower bound (aka their meet)
  - a least upper bound (aka their join)
- The meet of two propositions amounts to their intersection

$$\textsc{meet}(A, B) = A \cap B$$

- The join of two propositions amounts to their union

$$\textsc{join}(A, B) = A \cup B$$

- The existence of meets and joins implies that the set of all propositions, Σ, together with $\leq$, forms a lattice

Algebraic foundations of classical logic

Top and bottom

- The lattice has a bottom element, $\emptyset$, and a top element, W
- That is, for every proposition A, we have that:

$$\emptyset \leq A \leq W$$

- Thus, $\langle \Sigma, \leq \rangle$ forms a bounded lattice

Algebraic foundations of classical logic

Complementation

- For every propositions A, there is another proposition $C(A)$ such that:
  - The meet of A and $C(A)$ is the bottom element of the lattice, $\emptyset$
  - The join of A and $C(A)$ is the top element of the lattice, W
- $C(A)$ is called the complement of A
- For every A, $\quad C(A) = \{w \mid w \notin A\}$
- The existence of complements implies that $\langle \Sigma, \leq \rangle$ forms a complemented lattice

Algebraic foundations of classical logic

Distributivity

- The meet and join operators distribute over each other:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- This means that $\langle \Sigma, \leq \rangle$ is a distributive complemented lattice
- Such lattices are also called Boolean algebras

Algebraic foundations of classical logic

Classical logic

- The semantic operators we considered can be associated with syntactic operators:
  - $[\neg\varphi] = C([\varphi]) = W - [\varphi]$
  - $[\varphi \wedge \psi] = M([\varphi], [\psi]) = [\varphi] \cap [\psi]$
  - $[\varphi \vee \psi] = J([\varphi], [\psi]) = [\varphi] \cup [\psi]$
- This is how classical propositional logic is obtained
- The approach can be extended to first-order logic as well

Algebraic inquisitive semantics

Propositions

- Non-empty sets of possibilities
- Intuition:
  $\alpha \in A$ iff establishing α resolves the issue that A raises
- Consequence:
  For every proposition A and every two possibilities α and β:

  \- If $\alpha \in A$ and $\beta \subset \alpha$, then it must also be the case that $\beta \in A$
- So propositions are persistent non-empty sets of possibilities

Algebraic inquisitive semantics

Ordering propositions

- $A \leq B$ if and only if:
  - A provides at least as much information as B:

$$\text{info}(A) \subseteq \text{info}(B)$$

  - A requests at least as much information as B:

$$A \subseteq B$$

Simplification

- If $A \subseteq B$ then also $\text{info}(A) \subseteq \text{info}(B)$
- So $A \leq B$ if and only if $A \subseteq B$

Joins and meets

- As before, relative to $\leq$, every two propositions have
  - a greatest lower bound (aka their meet)
  - a least upper bound (aka their join)
- The meet of A and B still amounts to their intersection:

$$\textsc{meet}(A, B) = A \cap B$$

- The join of A and B still amounts to their union:

$$\textsc{join}(A, B) = A \cup B$$

- Conjunction and disjunction can still be seen as the syntactic counterparts of these semantic operators

$\langle \Sigma, \leq \rangle$ is not a Boolean algebra

- The existence of meets and joins implies that the set of all propositions Σ, together with the order $\leq$, forms a lattice

- Moreover, $\langle \Sigma, \leq \rangle$ has:
  - a top element, $\top = \wp(W)$
  - a bottom element, $\bot = \{\emptyset\}$

- This means that $\langle \Sigma, \leq \rangle$ forms a bounded lattice

- However, $\langle \Sigma, \leq \rangle$ does not form a Boolean algebra

- That is, not every $A \in \Sigma$ has a complement B such that:

$$\begin{aligned} \textsc{meet}(A, B) &= \top \\ \textsc{join}(A, B) &= \bot \end{aligned}$$

$\langle \Sigma, \leq \rangle$ is a Heyting algebra

- We do have that for every two propositions A, B there is a unique weakest proposition C such that $\textsc{meet}(A, C) \leq B$
- This proposition C is called the relative pseudo-complement of A with respect to B, and is denoted as:

$$A \Rightarrow B$$

- The existence of relative pseudo-complements implies that $\langle \Sigma, \leq \rangle$ forms a Heyting algebra
- The (non-relative) pseudo-complement of A is defined as:

$$A^* \quad := \quad A \Rightarrow \bot$$

- Implication and negation can be seen as the syntactic counterparts of $\Rightarrow$ and *, respectively

Algebraic inquisitive semantics

- $[p] \quad = \quad \{\alpha \mid \forall w \in \alpha.\ w(p) = 1\}$
- $[\neg\varphi] \quad = \quad [\varphi]^*$ pseudo-complement
- $[\varphi \wedge \psi] \quad = \quad [\varphi] \cap [\psi]$ meet
- $[\varphi \vee \psi] \quad = \quad [\varphi] \cup [\psi]$ join
- $[\varphi \rightarrow \psi] = \quad [\varphi] \Rightarrow [\psi]$ relative pseudo-complement

Relevance for natural language semantics

- Natural languages are, of course, much more intricate than the language of propositional logic
- However, it is reasonable to expect that natural languages generally also have connectives which behave semantically as meet, join, and complementation operators
- Just like it is reasonable to expect that natural languages generally have ways to express basic operations on quantities, like addition, substraction, and multiplication

Relevance for natural language semantics

- Disjunction (JOIN) is a source of inquisitiveness
- This provides the basis for an explanation of the disjunctive-interrogative affinity observed cross-linguistically

(1) We eten vanavond boerenkool of hutspot.
We eat tonight boerenkool or hutspot.
'We will eat boerenkool or hutspot tonight.'

(2) Maria weet of we vanavond hutspot eten.
Maria knows or we tonight hutspot eat.
'Maria knows whether we will eat hutspot tonight.'

- See AnderBois (2009, 2010) on Yukatec Maya and Haida (2009, 2010) on Chadic languages

Relevance for natural language semantics

- Disjunction (JOIN) is a source of inquisitiveness
- This facilitates a perspicuous account of sluicing

(3) Fred works for a big software company, I don't remember which.

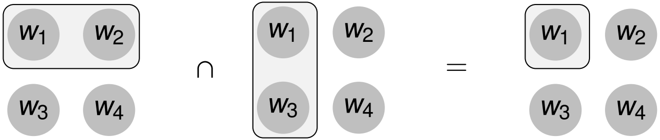
(4) Fred works for Oracle, IBM, or Adobe, I don't remember which.

- See AnderBois (2010)

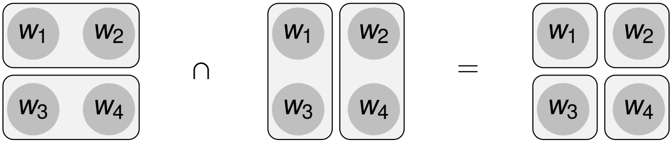
Relevance for natural language semantics

Conjunction (MEET) applies uniformly to questions and assertions

(5) John speaks Russian and he speaks French.



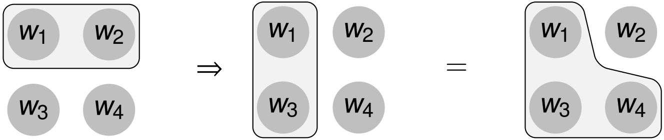
(6) Does John speak Russian, and does he speak French?



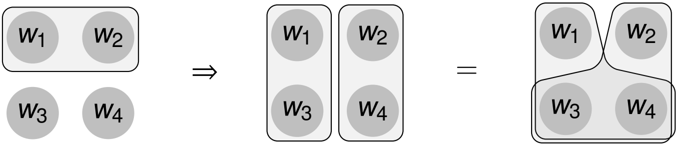
Relevance for natural language semantics

Implication ($\Rightarrow$) applies uniformly to questions and assertions

(7) If John will go to the party, Mary will go as well.



(8) If John will go to the party, will Mary go as well?



Relevance for natural language semantics

Conditional questions with disjunctive antecedents

(9) If John or Fred goes to the party, will Mary go as well?

There are four maximal possibilities for this sentence, corresponding to the following responses:

(10)
a. Yes, if John or Fred goes, Mary will go as well.
b. No, if John or Fred goes, Mary won't go.
c. If J goes, M will go as well, but if F goes, M won't go.
d. If F goes, M will go as well, but if J goes, M won't go.

Equivalence result

- The algebraic semantics given here is equivalent with the support-based semantics presented on Monday
- For any state s and any sentence φ:

$$s \models \varphi \quad \Longleftrightarrow \quad s \in [\varphi]$$

- So we have not established a new semantics, but rather a more solid foundation for the existing system

Extensions

Quantifiers

- Extension to the first-order setting is straightforward
- $[\forall x.\varphi]^g \; = \; \bigcap_{d \in D} \; [\varphi]^{g[x/d]}$
- $[\exists x.\varphi]^g \; = \; \bigcup_{d \in D} \; [\varphi]^{g[x/d]}$

Attentive content

- The given semantics only captures informative and inquisitive content
- We are also developing an algebraic semantics that captures attentive content, but there is still a problem in the current version: the MEET of two propositions does not always exist
- Hopefully, this issue can be resolved by adapting the attentiveness-ordering that we are currently assuming

Summary

- Inquisitive semantics can be motivated by general algebraic considerations, independently of specific linguistic examples
- Just as in the classical setting, connectives can be taken to behave semantically as join, meet, and complementation operators
- The only difference is the order that gives rise to these operations
- In the classical setting, propositions are ordered based on their informative content only
- In the inquisitive setting, propositions are ordered based on their informative and their inquisitive content