Inquisitive semantics
—NASSLLI 2012 lecture notes—

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About this document

These are lecture notes for a course on inquisitive semantics at NASSLLI 2012. Comments of any kind are of course more than welcome. We may still make some minor corrections in the weeks / months after the course. The most recent version of the notes will be posted on the course website:

https://sites.google.com/site/inquisitivesemantics/courses/nasslli-2012

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1 Introduction

The central aim of inquisitive semantics is to develop a notion of semantic meaning that captures both *informative* and *inquisitive* content. This enriched notion of meaning is intended to provide a new foundation for the analysis of linguistic discourse, in particular the type of discourse that is aimed at exchanging information.

The classical truth-conditional notion of meaning embodies the informative content of sentences, and thereby reflects the descriptive use of language. Stalnaker (1978) gave this notion a dynamic and conversational twist by taking the meaning of a sentence to be its potential to change the common ground, i.e., the body of shared information established in a conversation. The notion of meaning that resulted from this 'dynamic turn' reflects the active use of language in changing information. However, it does not yet reflect the *inter*active use of language in *ex*changing information. This requires yet another turn, an 'inquisitive turn', leading to a notion of meaning that directly reflects the nature of information exchange as a cooperative process of raising and resolving issues.

These lecture notes bring together and further expand on a number of results obtained in our recent work on inquisitive semantics Groenendijk and Roelofsen (2009); Ciardelli (2009); Ciardelli and Roelofsen (2011); Roelofsen (2011a); Ciardelli *et al.* (2012).[1] In particular, they provide a detailed exposition of what we currently see as the most basic implementation of the framework, which we refer to as InqB.

The notes are organized as follows. Section 2 introduces the new notion of meaning that forms the heart of inquisitive semantics. Section 3 identifies the algebraic structure of this new space of meanings, and section 4 presents a system that associates meanings with sentences in a first-order language. The basic logical properties of this system are characterized in section 5, and its relevance for natural language semantics is discussed in section 6. Finally, some possible extensions are outlined in section 7, and the role of inquisitive semantics in an overall theory of interpretation is discussed in section 8.

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[1]For more recent and ongoing work on inquisitive semantics we refer to www.illc.uva.nl/inquisitive-semantics.

2 Inquisitive meanings

A general scheme in which many notions of meaning can be naturally framed is the following. When a sentence is uttered in a certain *discourse context*, it expresses a *proposition*, which embodies a proposal to change the context in certain ways. This proposition is determined by the *meaning* of the sentence. The meaning of a sentence, therefore, can be thought of as something which embodies the potential of a sentence to express a proposition when uttered in a discourse context: formally, it is a function from contexts to propositions.

This general scheme gives rise to different notions of meaning depending on how the two parameters occurring in it, namely the notions of *discourse context* and *proposition*, are instantiated. In this section we will work our way to the inquisitive notion of meaning by motivating and illustrating the notion of context and proposition which are adopted in inquisitive semantics, as well as the particular restrictions imposed on meaning functions.

2.1 Information states and issues

In this section we introduce the two basic formal notions that play a role in the inquisitive picture of meaning: *information states* and *issues*.

Information states. We adopt the standard notion of an information state as a set of possible worlds. Throughout the discussion we assume a fixed set ω of possible worlds, whose nature will depend on the choice of a formal language.

Definition 1 (Information states).
An *information state* is a set $s \subseteq \omega$ of possible worlds.

We will often refer to an information state simply as a *state*. A state s is thought of as representing the information that the actual world lies in s. If $t \subseteq s$, then t locates the actual world at least as precisely as s: we may thus call t an *enhancement* of s.

Definition 2 (Enhancements).
A state t is called an *enhancement* of s just in case $t \subseteq s$.

Notice that this definition includes the *trivial* enhancement $t = s$. If an enhancement t is non-trivial, that is, if t is *strictly* more informed than s, then we call t a *proper* enhancement of s.
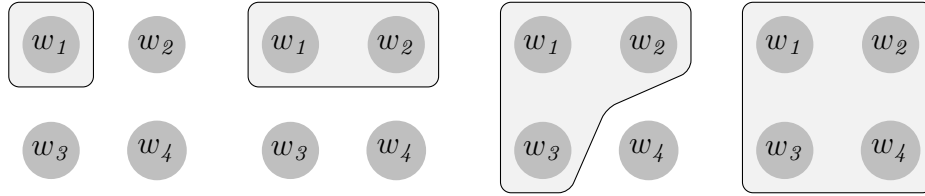
Figure 1: Information states.

The two extreme cases of the enhancement order are the empty state $\emptyset$, which is an enhancement of any state, and the set ω of all worlds, of which any state is an enhancement. The former models a state in which any possible world has been discarded as a candidate for the actual world, that is, the available information is inconsistent: we call $\emptyset$ the *inconsistent state*. The latter, on the contrary, models a state in which any possible world is taken to be a plausible candidate for the actual world, that is, we have no clue at all what the actual world is like: we call ω the *ignorant state*.

Figure 1 shows some examples of information states when our set of possible worlds consists of four worlds: w_1, w_2, w_3, w_4. Notice that these states are arranged according to the enhancement ordering from left to right, the last figure depicting the ignorant state ω.

Issues. An issue is meant to represent the semantic content of a request for information. Now, what does it mean to request information? If the information available in a certain context is represented by a state s, a request for information in s is a request to locate the actual world more precisely inside s. Thus, the content of the request consists in a specification of which enhancements of s locate the actual world with sufficient precision. Hence, an issue in s will be modeled as a non-empty set I of enhancements of s.

Importantly, we do not regard just *any* non-empty set I of enhancements of s as an issue. First, if I contains a certain enhancement t of s, and $t' \subseteq t$ is a further enhancement of t, then t' must also be in I. After all, if t locates the actual world with sufficient precision, then t' cannot fail to do so as well. So, I must be *downward closed*.

Second, the elements of I must together form a *cover* of s. That is, every world in s must be included in at least one element of I. To see why this is a natural requirement, suppose that w is a world in s that is not included in any element of I. Then the information available in s does not preclude w from
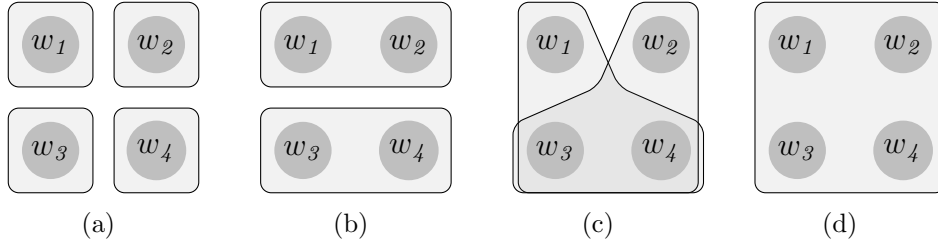
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Figure 2: Issues over the state $\{w1, w2, w3, w4\}$.

being the actual world. But if w is indeed the actual world, then it would be impossible to satisfy the request represented by I without discarding the actual world. Thus, in order to ensure that it is possible to satisfy the request represented by I without discarding the actual world, I should form a cover of s. This leads us to the following notion of an issue.

Definition 3 (Issues).
Let s be an information state, and I a non-empty set of enhancements of s. Then we say that I is an issue over s if and only if:

1. I is *downward closed*: if $t \in I$ and $t' \subseteq t$ then also $t' \in I$

2. I forms a *cover* of s: $\bigcup I = s$

Definition 4 (Settling an issue).
Let s be an information state, t an enhancement of s, and I an issue over s. Then we say that t settles I if and only if $t \in I$.

Notice that an issue I over a state s may contain s itself. This means that I does not request any information beyond the information that is already available in s. We call I a *trivial* issue over s in this case. Downward closure implies that for any state s there is precisely one trivial issue over s, namely $\wp(s)$. On the other hand, if $s \notin I$, then in order to settle I further information is required, that is, a proper enhancement of s must be established. In this case we call I a *proper* issue.

Two issues over a state s can be compared in terms of the information that they request: one issue I is at least as inquisitive as another issue J in case any state that settles I also settles J. Since an issue is identified with the set of states that settle it, we obtain the following definition.

Definition 5 (Ordering issues).
Given two issues I, J on a state s, we say that I is *at least as inquisitive* as J just in case $I \subseteq J$.

Among the issues over a state s there is always a *least* and a *most* inquisitive one. The least inquisitive issue over s is the trivial issue $\wp(s)$ which, as we saw, requests *no* information. The most inquisitive issue over s is $\{\{w\} \mid w \in s\} \cup \{\emptyset\}$, which can only be settled consistently by providing a complete description of what the actual world is like.

Figure 2 shows some issues over the information state $s = \{w_1, w_2, w_3, w_4\}$. In order to keep the figures neat, only the maximal elements of these issues are displayed. The issue depicted in (a) is the most inquisitive issue over s, which is only settled by specifying precisely which world is the actual one. The issue depicted in (b) is settled either by locating the actual world within the set $\{w_1, w_2\}$, or by locating it within $\{w_3, w_4\}$. The issue depicted in (c) is settled either by locating the actual world in $\{w_1, w_3, w_4\}$, or by locating it within $\{w_2, w_3, w_4\}$. Finally, (d) represents the trivial issue over s, which is *already* settled in s. Both (b) and (c) are less inquisitive than (a) and more inquisitive than (d), while they are incomparable with each other.

2.2 Discourse contexts and propositions

The notions of information states and issues introduced in the previous section constitute the basic ingredients of the inquisitive picture of meaning. In this section they will be put to use to define the notions of *discourse contexts* and *propositions*.

Discourse contexts. For our present purposes, a discourse context can be identified with the *common ground* of the discourse, that is, the body of information that is common knowledge among the conversational participants. A discourse context will thus be modeled simply as an information state.

Propositions. The proposition expressed by a sentence in a certain discourse context should embody the effect of an utterance of that sentence in that discourse context. We recognize two types of effects that an utterance may have. First, as is assumed in traditional accounts of meaning, a sentence can be used to *provide* information, that is, to enhance the information state

of the common ground. Thus, a proposition may specify an enhancement t of the current context s.

However, unlike those traditional accounts, we take sentences to have the potential to do *more* than just providing information. Namely, sentences may also be used to *request* information. Therefore, besides specifying a certain enhancement t of the current context s, a proposition may also specify a certain *issue* I.

Now, we might take the stance that there are two distinct types of propositions: *informative* propositions, which specify an enhancement of the current context, and *inquisitive* propositions, which specify an issue over the current context. However, we will take a more general and unifying standpoint, which assumes only one, richer type of proposition. Namely, we will define a proposition A over a context s as having both an informative component, consisting in an enhancement t of s, and an inquisitive component, consisting in an issue I over t.

Definition 6 (Propositions, to be simplified presently)**.**
A *proposition* over a state s is a pair $A = (t, I)$, where:

- t is an enhancement of s called the *informative content* of A;

- I is an issue over t called the *inquisitive content* of A.

Consider a proposition $A = (t, I)$. Since I is an issue over t, the union of all the states in I must coincide with t. This means that the informative component t of the proposition can be retrieved from I, and it need not appear explicitly in the representation of the proposition. Thus, our notion of propositions can be simplified as follows.

Definition 7 (Propositions, simplified)**.**

- A *proposition* A over a state s is an issue over an enhancement t of s.

- We denote by Π_s the set of all propositions over a state s.

Notice that if A is a proposition over s, it is also a proposition over any state $t \supseteq s$. In particular, any proposition is a proposition over the ignorant state ω: therefore, we simply write Π for Π_ω, and we call Π the *set of propositions*.

The informative content of a proposition A is embodied by $\bigcup A$. We will denote this set of worlds as $\mathsf{info}(A)$.
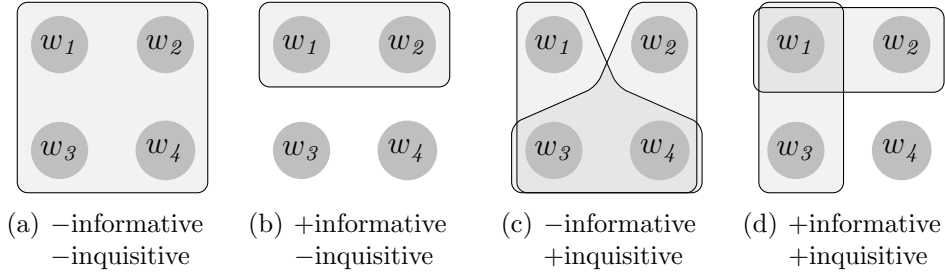
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Figure 3: Propositions over the state $\{w_1, w_2, w_3, w_4\}$.

Definition 8 (Informative content). For any $A \in \Pi$: $\mathsf{info}(A) := \bigcup A$

In expressing a proposition A, one provides the information that the actual world lies in $\mathsf{info}(A)$ and requests enough information from other participants to locate the actual world inside one of the states in A.

The fact that we take any proposition to have both an informative and an inquisitive component should not be taken to mean that we take any proposition to be both informative and inquisitive: for either, or even both, of these components may be trivial. We say that a proposition is *informative* only in case its informative content is non-trivial, and *inquisitive* only in case its inquisitive content is non-trivial.

Definition 9 (Informativeness and inquisitiveness).
Let A be a proposition over a state s.

- We say that A is *informative* in s just in case $\mathsf{info}(A) \subset s$;

- We say that A is *inquisitive* in s just in case $\mathsf{info}(A) \notin A$.

So, we still have *purely informative* propositions, which do not request any information, and *purely inquisitive* propositions, which do not provide any information. However, both are regarded as particular cases of a unique notion of proposition. Moreover, we are not committed to the assumption that every proposition is either purely informative or purely inquisitive: our approach leaves room for *hybrid* propositions, which are both informative and inquisitive at the same time.

In figure 3, several examples of propositions over $s = \{w_1, w_2, w_3, w_4\}$ are depicted, always with the convention that only the maximal elements are displayed. The proposition represented in figure (a) is not informative

in s, because its informative content coincides with s, and it is also not inquisitive in s, because its inquisitive content forms a trivial issue over s. The proposition represented in figure (b) is informative in s, because its informative content is a proper enhancement of s, but not inquisitive in s, since its inquisitive content is trivial. The proposition represented in figure (c) is not informative in s, because its informative content coincides with s, but it is inquisitive, because its inquisitive content is non-trivial. Finally, the proposition represented in figure (d) is both informative and inquisitive, since its informative content is a proper enhancement of s and its inquisitive content is non-trivial.

Propositions can be compared in terms of the information that they provide and the information that they request. We say that a proposition A is *at least as informative* as a proposition B if the informative content of A is an enhancement of the informative content of B.

Definition 10 (Informativeness ordering)**.** Let $A, B \in \Pi$.
We say that A is *at least as informative* as B just in case $\mathsf{info}(A) \subseteq \mathsf{info}(B)$.

If A and B are equally informative propositions, then we say that A is at least as inquisitive as B in case the inquisitive content of A is at least as demanding as the inquisitive content of B.

Definition 11 (Inquisitiveness ordering)**.** Let $A, B \in \Pi$, $\mathsf{info}(A) = \mathsf{info}(B)$. Then we say that A is *at least as inquisitive* as B just in case $A \subseteq B$.

Now we would like to say that a proposition A *entails* a proposition B just in case A is both at least as informative and at least as inquisitive as B. But there is a subtlety here. Namely, if A is strictly more informative than B, then A and B cannot be compared directly in terms of inquisitiveness. Thus, what we request is that A be at least as informative as B and moreover, that A be at least as inquisitive as *the restriction of B to* $\mathsf{info}(A)$.

Definition 12 (Restriction)**.**
If $A \in \Pi_s$ and $t \subseteq s$, the *restriction* of A to t is the proposition $A{\upharpoonright}t \in \Pi_t$ defined by:
$$A{\upharpoonright}t = \{t' \subseteq t \,|\, t' \in A\}$$

Intuitively, $A{\upharpoonright}t$ is a proposition over t that inherits the content of A. The informative content of $A{\upharpoonright}t$ amounts to $\mathsf{info}(A) \cap t$. Thus, $A{\upharpoonright}t$ provides the information that the actual world lies in $\mathsf{info}(A)$, which is precisely the

information provided by A itself. Moreover, the request expressed by $A{\restriction}t$ is to enhance $\mathsf{info}(A{\restriction}t)$ in such a way as to satisfy the issue A. Notice that if A itself is already a proposition over t, then $A{\restriction}t$ simply amounts to A.

The notion of restriction allows us to define entailment between propositions. A proposition A *entails* a proposition B in case (i) A is at least as informative as B: $\mathsf{info}(A) \subseteq \mathsf{info}(B)$; and (ii) A is at least as inquisitive as the restriction of B to $\mathsf{info}(A)$: $A \subseteq B{\restriction}\mathsf{info}(A)$. However, it is easy to see that these two conditions are satisfied if and only if $A \subseteq B$. Therefore, entailment between propositions can simply be defined as inclusion.

Definition 13 (Entailment between propositions)**.**
Let $A, B \in \Pi$. Then we say that A *entails* B just in case $A \subseteq B$.

2.3   Meanings

Equipped with formal notions of discourse contexts and propositions, let us now come back to our initial picture of meaning. In every discourse context, a sentence expresses a certain proposition. This proposition is determined by the meaning of the sentence. The meaning of a sentence, therefore, consists in the potential that the sentence has to express propositions in context. It can be modeled as a function f that maps each context s to a proposition $f(s) \in \Pi_s$. We call $f(s)$ the proposition *expressed* by f in s.

In principle, such a function f might express totally unrelated propositions in two states s and t. However, we expect our meanings to act in a *uniform* way across different contexts. The idea is that if the propositions $f(s)$ and $f(t)$ differ, the difference should be traceable to the initial difference in information between s and t. Once the information gap between s and t is filled, the difference between the two propositions should also vanish. This intuition is formalized by the compatibility condition specified below, which requires that when t is an enhancement of s, the proposition $f(t)$ expressed in t should coincide with the restriction of the proposition $f(s)$ to t.

Definition 14 (Compatibility condition)**.**
A function f which associates to any discourse context s a proposition $f(s) \in \Pi_s$ is called *compatible* just in case whenever $t \subseteq s$, $f(t) = f(s){\restriction}t$.

We can obtain an intuition by looking at figure 4. Here $s = \{w_1, w_2, w_3, w_4\}$ and $t = \{w_1, w_2, w_3\}$. Suppose the proposition $f(s)$ expressed by a function
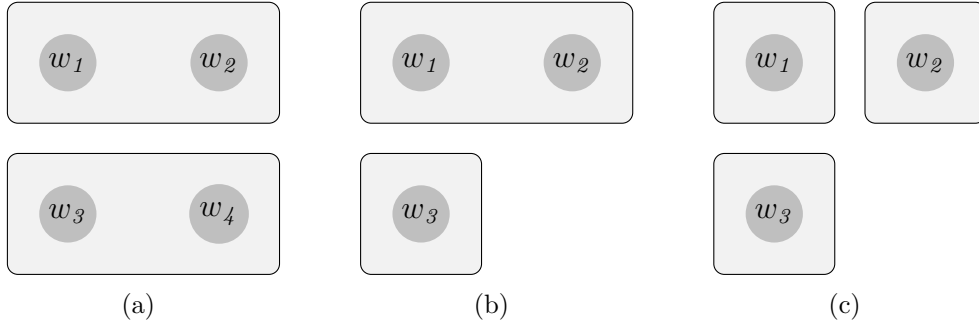
Figure 4: Illustrating the compatibility condition.

f on s is the one depicted in figure (a). Then in order for f to be compatible, the proposition $f(t)$ expressed by f on t should be the one depicted in figure (b), which is obtained by restricting the proposition in (a) to t, and not, say, the one depicted in figure (c).

Definition 15 (Meanings).

- A *meaning* is a compatible function that maps every discourse context s to a proposition over s.

- The set of all meanings is denoted by $\mathcal{M}$.

We say that a meaning is *informative* in case it has the potential to provide information, that is, if in some discourse contexts it expresses an informative proposition. Similarly, we say that a meaning is *inquisitive* if it has the potential to request information, that is, if in some contexts it expresses an inquisitive proposition.

Definition 16 (Informativeness and inquisitiveness). Let f be a meaning.

- We say that f is *informative* just in case there is a context s such that the proposition $f(s)$ expressed by f in s is informative in s.

- We say that f is *inquisitive* just in case there is a context s such that the proposition $f(s)$ expressed by f in s is inquisitive in s.

Meanings can be ordered in terms of the strength of the propositions they express: we will say that a meaning f *entails* a meaning g in case on any state s, the proposition $f(s)$ entails the proposition $g(s)$.

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Definition 17 (Entailment between meanings)**.**
If f and g are meanings, we say that f entails g, in symbols $f \leq g$, just in case $f(s) \subseteq g(s)$ for any context s.

Now, recall that any state s is a substate of the ignorant state ω consisting of all worlds. Thus, if f is a meaning, the compatibility condition yields that for all states s:

$$f(s) = f(\omega){\restriction}s$$

This shows that every meaning f is fully determined by a unique proposition, namely the proposition $f(\omega)$ that it expresses in the ignorant state. And vice versa, any proposition A uniquely determines a meaning f_A, which associates to each context s the proposition:

$$f_A(s) = A{\restriction}s$$

We have thus reached the following conclusion.

Fact 1.
There is a one-to-one correspondence between meanings and propositions.

The meaning of a sentence can thus be given by equipping it with a unique, *absolute* proposition A. The proposition expressed in a particular context s will then be obtained by restricting A to s. But there is more: the following facts ensure that all the properties of meanings we have seen so far, as well as the entailment ordering between them, can be recast in terms of properties of the corresponding propositions and of the entailment ordering between them.

Fact 2. Let f be a meaning.

- f is informative iff the proposition $f(\omega)$ is informative in ω.

- f is inquisitive iff the proposition $f(\omega)$ is inquisitive in ω.

Fact 3. For any two meanings f and g:

$$f \leq g \iff f(\omega) \subseteq g(\omega)$$

Combining facts 1 and 3 we obtain the following result.

Fact 4. The space $\langle \mathcal{M}, \leq \rangle$ of meanings ordered by entailment and the space $\langle \Pi, \subseteq \rangle$ of propositions ordered by entailment are isomorphic.

In the next section, we turn to an investigation of the algebraic structure of the space of proposition. The above result guarantees that the results of this investigation will directly pertain to the structure of meanings as well.

3   Inquisitive algebra

In this section we will investigate the algebraic structure of the space $\langle \Pi, \subseteq \rangle$ of propositions ordered by entailment. This is of course interesting in its own right, but it will also play a crucial role in defining a concrete inquisitive semantics for the language of first-order logic, which we will turn to in the next section. The algebraic results to be presented here will suggest a particular way to deal with connectives and quantifiers. For instance, conjunction will be taken to behave semantically as a *meet* operator, yielding the greatest lower bound of the propositions expressed by its constituents, and other connectives and quantifiers will be associated with other basic algebraic operations, just as in classical logic.

To illustrate our approach, we will first briefly review the algebraic perspective on classical logic. After that, we will turn our attention to the algebra of propositions in inquisitive semantics.[2]

3.1   The algebraic perspective on classical logic

In the classical setting a proposition A is simply a set of possible worlds, which embodies the information that the actual world is located in A. Given this way of thinking about propositions, there is a natural *entailment order* between them: one proposition A entails another proposition B iff A is at least as informative as B, i.e., iff in uttering A, a speaker locates the actual world more precisely than in uttering B. This condition is fulfilled just in case $A \subseteq B$. Thus, the space of classical propositions ordered by entailment is the partially ordered set $\langle \wp(\omega), \subseteq \rangle$.

This space is equipped with a rich algebraic structure. To start with, for any set of propositions Σ, there is a unique proposition that (i) entails all the propositions in Σ, and (ii) is entailed by all other propositions that entail all propositions in Σ. This proposition is the *greatest lower bound* of Σ w.r.t. the entailment order, or in algebraic jargon, its *meet*. It amounts to $\bigcap \Sigma$ (given the stipulation that $\bigcap \emptyset = \omega$). Similarly, every set of propositions Σ also has a unique *least upper bound* w.r.t. the entailment order, which is called its *join*, and amounts to $\bigcup \Sigma$. The existence of meets and joins for arbitrary sets of classical propositions implies that the space of classical propositions ordered by entailment forms a *complete lattice*.

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[2]This section is based on Roelofsen (2011a). We present the main results here but omit the proofs.

This lattice is *bounded*. That is, it has a bottom element, $\bot := \emptyset$, and a top element, $\top := \omega$. Moreover, for every two propositions A and B, there is a unique weakest proposition C such that $A \cap C$ entails B. This proposition is called the *pseudo-complement of A relative to B*. It is denoted as $A \Rightarrow B$ and in the case of $\langle \wp(\omega), \subseteq \rangle$, it amounts to $\overline{A} \cup B$, where $\overline{A}$ denotes the set-theoretic complement of A, $\omega - A$. Intuitively, the pseudo-complement of A relative to B is the weakest proposition such that if we 'add' it to A, we get a proposition that is at least as strong as B. The existence of relative pseudo-complements implies that $\langle \wp(\omega), \subseteq \rangle$ forms a *Heyting algebra*.

If A is an element of a Heyting algebra, it is customary to refer to the pseudo-complement of A relative to the bottom element of the algebra, $A^* := (A \Rightarrow \bot)$, as the *pseudo-complement* of A. In the case of $\langle \wp(\omega), \subseteq \rangle$, A^* simply amounts to the set-theoretic complement $\overline{A}$ of A. By definition of pseudo-complements, we have that $A \cap A^* = \bot$ for any element A of any Heyting algebra. In the specific case of $\langle \wp(\omega), \subseteq \rangle$, we also always have that $A \cup A^* = \top$. This means that in $\langle \wp(\omega), \subseteq \rangle$, A^* is in fact the *Boolean complement* of A, and that $\langle \wp(\omega), \subseteq \rangle$ forms a *Boolean algebra*, a special kind of Heyting algebra.

Thus, the space of classical propositions is equipped with certain natural operations. Classical first-order logic is obtained by associating these operations with the logical constants. Indeed, the usual definition of truth can be reformulated as a recursive definition of the set $|\varphi|_g$ of models over a domain D in which φ is true relative to an assignment g. The inductive clauses then run as follows:

- $|\neg\varphi|_g = |\varphi|_g^*$

- $|\varphi \wedge \psi|_g = |\varphi|_g \cap |\psi|_g$

- $|\varphi \vee \psi|_g = |\varphi|_g \cup |\psi|_g$

- $|\varphi \to \psi|_g = |\varphi|_g \Rightarrow |\psi|_g$

- $|\forall x.\varphi|_g = \bigcap_{d \in D} |\varphi|_{g[x \mapsto d]}$

- $|\exists x.\varphi|_g = \bigcup_{d \in D} |\varphi|_{g[x \mapsto d]}$

Negation expresses complementation, conjunction and disjunction express binary meet and join, respectively, implication expresses relative pseudo-complementation, and quantified formulas, $\forall x.\varphi$ and $\exists x.\varphi$, express the infinitary meet and join, respectively, of $\{|\varphi|_{g[x \mapsto d]} \mid d \in D\}$.

Notice that everything starts with a certain notion of propositions and a natural entailment order on these propositions. This entailment order, then, gives rise to certain basic operations on propositions, and classical first-order logic is obtained by associating these basic semantic operations with the logical constants.

3.2   Algebraic operations on propositions in InqB

In exactly the same way, we may investigate the algebraic structure of the space of inquisitive propositions in order to determine which operations could be associated with the logical constants in inquisitive semantics. What kind of algebraic operations exist in the space $\langle \Pi, \subseteq \rangle$?

First, as in the classical setting, any set of meanings $\mathcal{F} \subseteq \Pi$ has a unique greatest lower bound (*meet*) and a unique least upper bound (*join*), which can be characterized as follows in terms of intersection and union.[3]

Fact 5 (Meet)**.** For any set $\mathcal{F} \subseteq \Pi$, $\bigcap \mathcal{F}$ is in Π and it is the meet of $\mathcal{F}$.

Fact 6 (Join)**.** For any set $\mathcal{F} \subseteq \Pi$, $\bigcup \mathcal{F}$ is in Π and it is the join of $\mathcal{F}$.

The existence of meets and joins for arbitrary sets of propositions implies that $\langle \Pi, \subseteq \rangle$ forms a complete lattice. And again, this lattice is bounded, i.e., there is a bottom element, $\bot := \{\emptyset\}$, and a top element, $\top := \wp(\omega)$. Finally, as in the classical setting, for every two propositions A and B, there is a unique weakest proposition C such that $A \cap C$ entails B. Recall that this proposition, which is called the pseudo-complement of A relative to B, can be characterized intuitively as the weakest proposition such that if we add it to A, we get a proposition that is at least as strong as B.

Definition 18. For any two propositions A and B:

$$A \Rightarrow B := \{s \mid \text{for every } t \subseteq s, \text{if } t \in A \text{ then } t \in B\}$$

Fact 7 (Relative pseudo-complement)**.**
For any $A, B \in \Pi$, $A \Rightarrow B$ is the pseudo-complement of A relative to B.

The existence of relative pseudo-complements implies that $\langle \Pi, \subseteq \rangle$ forms a Heyting algebra. This simple fact will be very useful in the investigation of

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[3]Given the convention that $\bigcap \emptyset = \wp(\omega)$ and $\bigcup \emptyset = \{\emptyset\}$.

the logic of our system (see section 5), since it immediately yields the fact that inquisitive logic is an extension of intuitionistic logic. Recall that in a Heyting algebra, $A^* := (A \Rightarrow \bot)$ is referred to as the *pseudo-complement* of A. In the specific case of $\langle \Pi, \subseteq \rangle$, pseudo-complements can be characterized as follows.

Fact 8 (Pseudo-complement). For any proposition $A \in \Pi$:

$$A^* = \{\beta \mid \beta \cap \alpha = \emptyset \text{ for all } \alpha \in A\} = \wp(\overline{\bigcup A})$$

Thus, A^* consists of all states that are disjoint from any element of A. This means that a piece of information settles A^* just in case it locates the actual world outside $\bigcup A$.

So far, then, everything works out just as in the classical setting. However, unlike in the classical setting, the pseudo-complement of a proposition is not always its *Boolean* complement. In fact, most propositions in $\langle \Pi, \subseteq \rangle$ do not have a Boolean complement at all. To see this, suppose that A and B are Boolean complements. This means that (i) $A \cap B = \bot$ and (ii) $A \cup B = \top$. Since $\top = \wp(\omega)$, condition (ii) can only be fulfilled if ω is contained in either A or B. Suppose $\omega \in A$. Then, since A is downward closed, $A = \wp(\omega) = \top$. But then, in order to satisfy condition (i), we must have that $B = \{\emptyset\} = \bot$. So the only two elements of our algebra that have a Boolean complement are $\top$ and $\bot$. Hence, the space $\langle \Pi, \subseteq \rangle$ of inquisitive propositions does not form a Boolean algebra.

Thus, starting with a new notion of propositions and a suitable entailment order on these propositions that takes both informative and inquisitive content into account, we have established an algebraic structure with two extremal elements and three basic operations, *meet*, *join*, and *relative pseudo-complementation*. The only algebraic difference with respect to the classical setting is that, except for the extremal elements, inquisitive propositions do not have Boolean complements. However, as in the classical setting, every proposition does have a pseudo-complement. This algebraic result gives rise to an inquisitive semantics for a first-order language, to which we turn now.

4 Inquisitive semantics

In this section we define an inquisitive semantics for a first-order language, motivated by the algebraic results presented in the previous section. We will

investigate the main properties of the system, and illustrate it with a range of examples.

4.1 Inquisitive semantics for a first-order language

We will consider a standard first-order language $\mathcal{L}$, with $\bot$, $\vee$, $\wedge$, $\rightarrow$, $\exists$, and $\forall$ as its basic logical constants. We will use $\neg\varphi$ as an abbreviation of $\varphi \rightarrow \bot$. We will also use $!\varphi$ as an abbreviation of $\neg\neg\varphi$ and $?\varphi$ as an abbreviation of $\varphi \vee \neg\varphi$. We refer to $!$ as the *declarative* operator and to $?$ as the *interrogative* operator. The precise role of these operators in the system will become clear below, especially in section 4.6.

In order to simplify matters, we consider a fixed domain D and a fixed interpretation of constants and function symbols: that is, we restrict our attention to the case in which the domain of discourse and the reference of proper names are common knowledge among the discourse participants, and the only uncertainty concerns the extension of predicates and relations.

Formally, we consider a fixed domain structure $\mathbb{D} = (D, I_{\mathbb{D}})$ which consists of a domain D and an interpretation function $I_{\mathbb{D}}$ that maps every individual constant c to an object in D and every $n-$ary function symbol f to a function from D^n to D. Our logical space consists of first-order models based on $\mathbb{D}$.

Definition 19 ($\mathbb{D}$-worlds)**.** A $\mathbb{D}-world$ is a structure $w = (\mathbb{D}, I_w)$, where I_w is a function interpreting each $n-$ary relation symbol R as a relation $I_w(R) \subseteq D^n$. The set of all $\mathbb{D}-$worlds is denoted $\omega_{\mathbb{D}}$.

Unless specified otherwise, the structure $\mathbb{D}$ will be considered fixed throughout our discussion and we shall drop reference to it whenever possible. In order not to have assignments in the way all the time, we will assume that for any $d \in D$, the language $\mathcal{L}$ contains an individual constant $\overline{d}$ such that $I_{\mathbb{D}}(\overline{d}) = d$: if this is not the case, we simply expand the language by adding new constants, and we expand $I_{\mathbb{D}}$ accordingly. In this way we can define our semantics for sentences only, and we can do without assignments altogether. This move is not essential, but it simplifies both notation and terminology.

If φ is a sentence of $\mathcal{L}$, we denote by $|\varphi|$ the set of all worlds where φ is classically true; we call $|\varphi|$ the *truth-set* of φ.

Definition 20 (Truth-set)**.** The truth set $|\varphi|$ of a formula φ is the set of worlds where φ is classically true.

We are now ready to recursively associate a proposition to each sentence of our first-order language. We start with atomic sentences: the proposition expressed by an atomic sentence $R(t_1, \ldots, t_n)$ is defined as the set of all states that consist exclusively of worlds where $R(t_1, \ldots, t_n)$ is true. This means that in uttering an atomic sentence, a speaker provides the information that the actual world is one where that sentence is true, and does not request any further information. Thus, atomic sentences are treated just as in the classical setting. The inductive clauses of the semantics are driven by the algebraic results established in the previous section. That is, the logical constants are taken to express the fundamental operations that we identified on our space of propositions.

Definition 21 (Inquisitive semantics for a first-order language).

1. $[R(t_1, \ldots, t_n)] := \wp(|R(t_1, \ldots, t_n)|)$

2. $[\bot] := \{\emptyset\}$

3. $[\varphi \wedge \psi] := [\varphi] \cap [\psi]$

4. $[\varphi \vee \psi] := [\varphi] \cup [\psi]$

5. $[\varphi \to \psi] := [\varphi] \Rightarrow [\psi]$

6. $[\forall x. \varphi(x)] := \bigcap_{d \in D} [\varphi(\bar{d})]$

7. $[\exists x. \varphi(x)] := \bigcup_{d \in D} [\varphi(\bar{d})]$

We refer to this first-order system as InqB, where B stands for *basic*. We refer to $[\varphi]$ as the *proposition expressed* by φ, and to the elements of $[\varphi]$ as the *possibilities* for φ. The clauses of InqB constitute a proper inquisitive semantics in the sense that they indeed associate every sentence $\varphi \in \mathcal{L}$ with a proposition in the sense of section 2.2.

Fact 9 (Suitability of the semantics).
For every $\varphi \in \mathcal{L}$, $[\varphi] \in \Pi$.

Since negation is defined as an abbreviation, its semantic behavior is determined by that of the basic connectives. The derived clause for negation is given below. Notice that $[\neg \varphi] = [\varphi]^*$, which means that negation expresses the pseudo-complementation operation.

Fact 10 (Derived clause for negation)**.**

- $[\neg\varphi] \;=\; [\varphi \to \bot] \;=\; [\varphi] \Rightarrow \{\emptyset\} \;=\; [\varphi]^{*} \;=\; \wp(\overline{\bigcup[\varphi]})$

The *informative content* of a sentence φ, $\mathsf{info}(\varphi)$, is defined as the informative content of the proposition it expresses, which amounts to $\bigcup[\varphi]$.

Definition 22 (Informative content)**.** For every $\varphi \in \mathcal{L}$: $\;\mathsf{info}(\varphi) := \bigcup[\varphi]$

A sentence φ *entails* another sentence ψ just in case the proposition expressed by φ entails the proposition expressed by ψ, and φ and ψ are *equivalent*, $\varphi \equiv \psi$, just in case they express exactly the same proposition.

Definition 23 (Entailment and equivalence)**.** Let $\varphi, \psi \in \mathcal{L}$. Then:

- φ entails ψ just in case $[\varphi] \subseteq [\psi]$

- φ and ψ are equivalent, $\varphi \equiv \psi$, just in case $[\varphi] = [\psi]$

4.2   Semantic categories

We call a sentence *informative* just in case it has an informative meaning, i.e., if it has the potential to express an informative proposition in some state, and *inquisitive* just in case it has an inquisitive meaning, i.e. if it has the potential to express an inquisitive proposition in some state. Fact 4 ensures that these properties can be recast in terms of properties of the proposition $[\varphi]$ as follows.

Definition 24 (Informativeness and inquisitiveness)**.**

- A sentence φ is informative if $\mathsf{info}(\varphi) \neq \omega$.

- A sentence φ is inquisitive if $\mathsf{info}(\varphi) \notin [\varphi]$.

These semantic properties allow us to distinguish several classes of sentences. First, we have sentences that are non-inquisitive, lacking the potential to request information. The meaning of such sentences consists exclusively in their informative potential, which means that, if their utterance in a context has any effect at all, then what it does is to provide information. We call such sentences *assertions*. Symmetrically, there are sentences that are non-informative, lacking the potential to provide information. Their meaning consists exclusively in their inquisitive potential, which means that, if

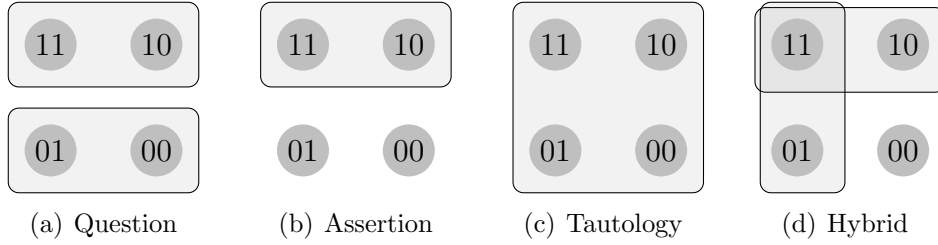(a) Question (b) Assertion (c) Tautology (d) Hybrid

Figure 5: Semantic categories.

their utterance in a context has any effect at all, then what it does is to request information. Such sentences are called *questions*. Thirdly, there are sentences that lack both informative and inquisitive potential. When uttered in a context, these sentences never have any effect at all. They are thus insignificant, and we call them *tautologies*. Finally, there are sentences which are both informative and inquisitive. Such sentences, which are capable both of providing and of requesting information, are called *hybrids*.

Definition 25 (Semantics categories). We say that a sentence φ is:

- an *assertion* iff it is non-inquisitive;

- a *question* iff it is non-informative;

- a *tautology* iff it is neither informative nor inquisitive;

- a *hybrid* iff it is both informative and inquisitive.

These four semantic categories are exemplified in figure 5. Spelling out what it means to be non-informative and non-inquisitive we obtain the following direct characterization of questions, assertions and tautologies.

Fact 11 (Direct characterization of questions, assertions and tautologies).

- φ is an assertion $\iff$ info$(\varphi) \in [\varphi]$.

- φ is a question $\iff$ info$(\varphi) = \omega$.

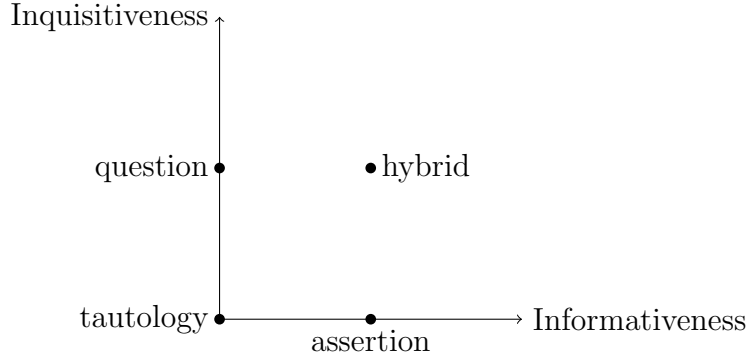- φ is a tautology $\iff$ $\omega \in [\varphi]$.

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Figure 6: Questions, assertions, tautologies, and hybrids in a two-dimensional space.

Notice that if $\mathsf{info}(\varphi) \in [\varphi]$ then, since $\mathsf{info}(\varphi) = \bigcup[\varphi]$, $\mathsf{info}(\varphi)$ must be the greatest element of $[\varphi]$. Vice versa, if $[\varphi]$ has a greatest element, it is easy to see that this greatest element must be precisely $\mathsf{info}(\varphi)$. We have thus established the following fact, which makes it particularly easy to say whether a sentence is an assertion, given a visualization of the proposition that it expresses.

Fact 12 (Alternative characterization of assertions)**.**

$$\varphi \text{ is an assertion} \quad \Longleftrightarrow \quad [\varphi] = \wp(\mathsf{info}(\varphi))$$
$$\Longleftrightarrow \quad [\varphi] \text{ has a greatest element.}$$

We can visualize sentences as inhabiting a two-dimensional space, as depicted in figure 6, arranged according to the strength of their informative and inquisitive content. The informative axis, where the inquisitive component is trivial, is inhabited by assertions, which are non-inquisitive. The inquisitive axis, where the informative component is trivial, is inhabited by questions, which are non-informative. The 'zero-point' of the space, where both components are trivial, is inhabited by tautologies, which are neither informative nor inquisitive. The rest of the space, where both components are non-trivial, is inhabited by hybrids, which are both informative and inquisitive.

4.3 Information and truth

Let us now look more closely at how informative content is treated in InqB. Recall that $\mathsf{info}(\varphi)$ is defined as $\bigcup[\varphi]$. Thus, $\mathsf{info}(\varphi)$ is a state. In uttering the sentence φ, a speaker provides the information that the actual world lies in this state. In classical logic, the informative content of a sentence φ is also embodied by a state, namely the truth-set $|\varphi|$, consisting of all worlds where φ is true. Now, the question that naturally arises is how these notions of informative content relate to each other. The following fact answers this question, establishing that the two notions always coincide.

Fact 13 (The treatment of information in InqB is classical)**.**
For any sentence φ, $\mathsf{info}(\varphi) = |\varphi|$.

This shows that InqB preserves the classical treatment of informative content. InqB only differs from classical logic in that it captures an additional aspect of meaning, namely inquisitive content.

Notice that in combination with facts 11 and 12, fact 13 entails the following characterization of assertions and questions in terms of classical truth.

Fact 14 (Questions, assertions, and classical truth)**.**

- φ is a question $\iff$ $|\varphi| = \omega$

- φ is an assertion $\iff$ $|\varphi| \in [\varphi] \iff [\varphi] = \wp(|\varphi|)$

Thus, questions are sentences that are classically true at any world. The proposition expressed by an assertion φ always amounts to $\wp(|\varphi|)$, which means that in uttering an assertion φ, a speaker is taken to provide the information that φ is true, and not to request any further information. Thus, assertions behave in InqB exactly as they do in classical logic. We will see in section 5 that this classical behavior of assertions is also reflected in the logic that InqB gives rise to.

4.4 Examples

In this section we will consider some simple sentences and examine the propositions that they express. We consider a language with just one unary predicate symbol, P, and two individual constants, a and b. Accordingly, we assume that the domain of discourse consists of just two objects, denoted by
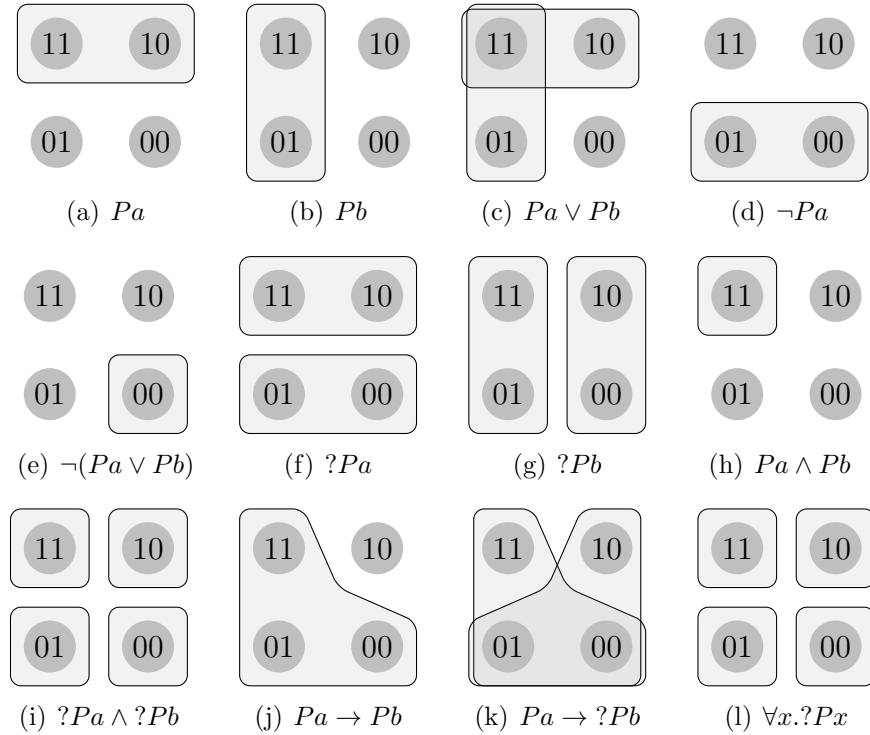
Figure 7: The propositions expressed by some simple sentences.

a and b, respectively. Our logical space, then, consists of four worlds, one in which both Pa and Pb are true, one in which Pa is true but Pb is false, one in which Pb is true but Pa is false, and one in which neither Pa nor Pb is true. In figure 7, these worlds are labeled 11, 10, 01, and 00, respectively. As usual, in order to keep the pictures orderly we display only the maximal elements of a proposition.

Atomic sentences. Let us first consider the proposition expressed by one of the two atomic sentences in our language, Pa. According to the clause for atomic sentences, $[Pa]$ consists of all states s such that every world in s makes Pa true: these states are $\{11, 10\}$, $\{11\}$, $\{10\}$, $\emptyset$. Thus, $[Pa]$ has a greatest element, namely the state $\{11, 10\}$ depicted in figure 7(a). Fact 12 therefore ensures that Pa is an assertion, and thus, according to fact 14, it behaves just like in the classical setting, providing the information that Pa is true and not requesting any further information.

Analogously, the other atomic sentence in our language, Pb, is an assertion which provides the information that Pb is true. The proposition expressed by Pb is depicted in figure 7(b).

Disjunction.   Next, we consider the disjunction $Pa \vee Pb$. According to the clause for disjunction, $[Pa \vee Pb]$ consists of those states that are either in $[Pa]$ or in $[Pb]$. These are $\{11, 10\}$, $\{11, 01\}$, $\{11\}$, $\{10\}$, $\{01\}$, and $\emptyset$.

Since $|Pa \vee Pb| = \{11, 10, 01\} \neq \omega$, the sentence $Pa \vee Pb$ is informative. More precisely, it provides the information that either Pa or Pb is true. However, unlike in the case of atomic sentences, in this case there is no greatest possibility that includes all the others. Instead, there are *two* maximal possibilities, $\{11, 10\}$ and $\{11, 01\}$, which together contain all the others. Thus, according to fact 12, $Pa \vee Pb$ is not an assertion, but it is inquisitive. In order to settle the issue raised by $Pa \vee Pb$, one has to establish either a state $s \subseteq \{11, 10\} = |Pa|$, or a state $s \subseteq \{11, 01\} = |Pb|$. In the former case, one establishes that Pa is true; in the latter, one establishes that Pb is true.

Thus, in inquisitive semantics the formula $Pa \vee Pb$ is a hybrid, which provides the information that at least one of the disjuncts is true, and requests enough information to establish for at least one of the disjuncts that it is true.

Negation.   Next, we turn to negation. According to the derived clause for negation, $[\neg Pa]$ consists of all states s such that s does not have any world in common with any state in $[Pa]$. Thus, $[\neg Pa]$ consists of all states that do not contain the worlds 11 and 10, which are $\{01, 00\}$, $\{01\}$, $\{00\}$, and $\emptyset$, as depicted in figure 7(d). Since this set of states has a greatest element, namely $\{01, 00\}$, fact 12 ensures that $\neg Pa$ is an assertion. And since the behavior of assertions is classical, $\neg Pa$ simply provides the information that Pa is false, without requesting any further information.

Now let us consider the negation of a non-atomic sentence, $\neg(Pa \vee Pb)$. According to the clause for negation, $[\neg(Pa \vee Pb)]$ consists of all states which do not have any world in common with any state in $[Pa \vee Pb]$. Thus, $[\neg(Pa \vee Pb)]$ consists of all states that do not contain the worlds 11, 10, and 01, which are $\{00\}$ and $\emptyset$, as depicted in figure 7(e). Again, there is a unique maximal possibility, namely $\{00\} = |\neg(Pa \vee Pb)|$. Thus, $\neg(Pa \vee Pb)$ is an assertion, which behaves classically, providing the information that both Pa and Pb are false, and not requesting any further information.

These cases of negative sentences exemplify a general fact which is ap-

parent from the semantic clause for negation (fact 10): for any sentence φ, the proposition $[\neg\varphi]$ always contains a greatest element, namely $\overline{\bigcup[\varphi]} = \overline{|\varphi|}$. Thus, a negative formula $\neg\varphi$ is always an assertion, which provides the information that φ is false. We will come back to this property of negation in section 4.5.

Interrogatives. Let us now consider another simple sentence involving both negation and disjunction, the interrogative $?Pa$, defined as $Pa \vee \neg Pa$. We have already seen what $[Pa]$ and $[\neg Pa]$ look like. According to the clause for disjunction, $[?Pa] = [Pa \vee \neg Pa]$ consists of all states that are either in $[Pa]$ or in $[\neg Pa]$. These states are $\{11, 10\}$, $\{01, 00\}$, and all substates thereof, as depicted in figure 7(f). Since $|?Pa| = |Pa \vee \neg Pa| = \omega$, the sentence $?Pa$ is not informative, that is, it is a question. However, since $[?Pa]$ does not have a unique greatest element, it is inquisitive. In order to settle the issue raised by $?Pa$, one has to establish either a state $s \subseteq \{11, 10\} = |Pa|$, or a state $s \subseteq \{01, 00\} = |\neg Pa|$. In the former case, one establishes that Pa is true; in the latter, one establishes that Pa is false. Hence, in order to settle the issue raised by $?Pa$, one has to establish whether Pa is true. Thus, while $?Pa$ is a shorthand for $Pa \vee \neg Pa$, perhaps the most famous classical tautology, this formula is not a tautology in InqB: instead, it corresponds to the polar question "whether Pa". Analogously, $?Pb$, depicted in figure 7(g), corresponds to the polar question "whether Pb".

Conjunction. Now let us consider conjunction. First, let us look at the conjunction of our two atomic sentences, Pa and Pb. According to the clause for conjunction, $[Pa \wedge Pb]$ consists of those states that are both in $[Pa]$ and in $[Pb]$. These are $\{11\}$ and $\emptyset$. Thus, $[Pa \wedge Pb]$ has a greatest element, namely $\{11\}$, and accordingly $Pa \wedge Pb$ is an assertion which, just like in the classical case, provides the information that both Pa and Pb are true.

Now let us look at the conjunction of two complex sentences, the polar questions $?Pa$ and $?Pb$. As depicted in figure 7(i), the proposition $[?Pa \wedge ?Pb]$ consists of the states $\{11\}$, $\{10\}$, $\{01\}$, $\{00\}$ and $\emptyset$. Since $|?Pa \wedge ?Pb| = \omega$, our conjunction is a question. Moreover, since there is no unique maximal possibility, this question is inquisitive. In order to settle the issue it raises, one has to provide enough information to establish at least one of $Pa \wedge Pb$, $Pa \wedge \neg Pb$, $\neg Pa \wedge Pb$, $\neg Pa \wedge \neg Pb$. Thus, our conjunction is a question which requests enough information to settle both the issue whether Pa, raised by

$?Pa$, and the issue whether Pb, raised by $?Pb$.

These two cases of conjunctive formulas exemplify a general fact: if φ and ψ are assertions, then the conjunction $\varphi \wedge \psi$ is itself an assertion which provides both the information provided by φ and the information provided by ψ; and if φ and ψ are questions, then the conjunction $\varphi \wedge \psi$ is itself a question, which requests the information needed to settle both the issue raised by φ and the issue raised by ψ.

Implication. Next, let us consider implication. Again, we will first consider a simple case, $Pa \to Pb$, where both the antecedent and the consequent are atomic. According to the clause for implication, $[Pa \to Pb]$ consists of all states s such that every substate $t \subseteq s$ that is in $[Pa]$ is also in $[Pb]$. These are precisely the states s such that $s \subseteq \{11, 01, 00\}$, as depicted in figure 7(j). So, $[Pa \to Pb]$ has a greatest element, $|Pa \to Pb| = \{11, 01, 00\}$, which means that the implication $Pa \to Pb$ is an assertion which, just like in the classical setting, provides the information that if Pa is true, then so is Pb.

Now let us consider a more complex case, $Pa \to ?Pb$, where the consequent is the polar question "whether Pb". As depicted in figure 7(k), the proposition $[Pa \to ?Pb]$ consists of the states $\{11, 01, 00\}$, $\{10, 01, 00\}$, and all substates thereof. Since $|Pa \to ?Pb| = \omega$, our implication is a question. Moreover, since there is no unique greatest possibility, this question is inquisitive. In order to settle the issue it raises, one must either establish a state $s \subseteq \{11, 01, 00\} = |Pa \to Pb|$, or a state $s \subseteq \{10, 01, 00\} = |Pa \to \neg Pb|$. In the former case one establishes that if Pa is true then so is Pb; in the latter case, that if Pa is true then Pb is false. So, in InqB the sentence $Pa \to ?Pb$ is a question which requests the information needed to establish whether Pb is the case under the assumption that Pa is the case.

Again, these two cases of conditional formulas exemplify a general feature of InqB: if ψ is an assertion, then $\varphi \to \psi$ is an assertion which provides the information that if φ is true, then so is ψ; and if ψ is a question, then $\varphi \to \psi$ is a question which requests the information needed to settle the issue raised by ψ assuming the information provided by φ and a resolution of the issue raised by φ.

Quantification. Finally, let us consider existential and universal quantification. As usual, existential quantification behaves essentially like dis-

junction and universal quantification behaves essentially like conjunction. In fact, since our current domain of discourse consists of only two objects, denoted by a and b, respectively, $\exists x.Px$ expresses exactly the same proposition as $Pa \lor Pb$, depicted in figure 7(c), and $\forall x.Px$ expresses exactly the same proposition as $Pa \land Pb$, depicted in figure 7(h). Finally, consider the proposition expressed by $\forall x.?Px$, depicted in figure 9(b). Notice that this proposition induces a partition on the logical space, where each block of the partition consists of worlds that agree on the extension of P. Thus, $\forall x.?Px$ is a question that asks for an exhaustive specification of the objects that have the property P. This concludes our illustration of the behavior or the logical constants in InqB.

4.5 Syntactic properties of questions and assertions

Assertions were defined in section 4.2 as sentences whose meaning consists exclusively in their informative potential, and questions as sentences whose meaning consists exclusively in their inquisitive potential. Notice that these characterizations are *semantic* in nature. In this section we provide *syntactic* conditions for sentences to be assertions or questions.

Let us start by examining assertions. The following fact provides some sufficient syntactic conditions, which generalize the particular observations made in the previous section.

Fact 15 (Sufficient conditions for assertionhood)**.**

1. An atomic sentence $R(t_1, \dots, t_n)$ is an assertion;

2. $\bot$ is an assertion;

3. if φ and ψ are assertions, then so is $\varphi \land \psi$;

4. if ψ is an assertion, then so is $\varphi \rightarrow \psi$ for any sentence φ;

5. if $\varphi(\overline{d})$ is an assertion for all $d \in D$, then so is $\forall x \varphi(x)$.

This fact immediately yields the following corollary, which shows that disjunction and the existential quantifier are the only sources of inquisitiveness in our logical language.

Corollary 1 (Sources of inquisitiveness)**.**
Any sentence that does not contain $\lor$ or $\exists$ is an assertion.

Also, since a negation $\neg\varphi$ is an abbreviation for $\varphi \to \bot$, items 2 and 4 combined yield the following corollary.

Corollary 2 (Negations are assertions)**.**
$\neg\varphi$ is an assertion for any φ.

Now let us turn to syntactic conditions for being a question, which again generalize our particular observations in the previous section.

Fact 16 (Sufficient conditions for questionhood)**.**

1. Any classical tautology is a question;

2. if φ and ψ are questions, so is $\varphi \wedge \psi$;

3. if ψ is a question, then for any φ so are $\varphi \vee \psi$ and $\varphi \to \psi$;

4. if $\varphi(\overline{d})$ is a question for all $d \in D$, then so is $\forall x\varphi(x)$;

5. if $\varphi(\overline{d})$ is a question for some $d \in D$, then so is $\exists x\varphi(x)$.

4.6   Projection operators

We proposed in section 4.2 to regard sentences of InqB as inhabiting a two dimensional space, where assertions lie on the horizontal axis and questions on the vertical axis. A natural question that arises, then, is whether we can define *projection operators* on this space, i.e., whether there are natural ways to turn any given sentence into an assertion, or into a question.

Suppose we add an operator $\mathcal{A}$ to our language, which is intended to behave as a non-inquisitive projection operator, turning every sentence into an assertion. How should the semantic contribution of $\mathcal{A}$ be defined in order for it to behave as a proper non-inquisitive projection operator? First of all, for any φ, $\mathcal{A}\varphi$ should be an assertion. Moreover, it is natural to require that, while trivializing inquisitive content, A should preserve informative content, that is, $\mathcal{A}\varphi$ should have the same informative content as φ.

Definition 26 (Non-inquisitive projection operator)**.**
We call an operator $\mathcal{A}$ a *non-inquisitive projection operator* just in case for any φ:

- $\mathcal{A}\varphi$ is an assertion

- $\mathsf{info}(\mathcal{A}\varphi) = \mathsf{info}(\varphi)$

Now, in section 4.2 we saw that the meaning of an assertion φ is completely determined by its informative component: if φ is an assertion, we must have that $[\varphi] = \wp(\mathsf{info}(\varphi))$. This means that the semantic behavior associated with a non-inquisitive projection operator is uniquely determined.

Fact 17 (Uniqueness of the non-inquisitive projection operator)**.**
$\mathcal{A}$ is a non-inquisitive projection operator if and only if $[\mathcal{A}\varphi] = \wp(\mathsf{info}(\varphi))$.

Now, recall that the declarative $!\varphi$ was defined as an abbreviation for $\neg\neg\varphi$. According to corollary 2, $!\varphi$ is an assertion for any φ. Moreover, using fact 13 we have that $\mathsf{info}(!\varphi) = |!\varphi| = |\neg\neg\varphi| = |\varphi| = \mathsf{info}(\varphi)$, which shows that $!$ preserves informative content. This means that the declarative operator $!$ is a non-inquisitive projection operator. Moreover, the previous fact guarantees that any non-inquisitive projection operator must be equivalent with it.

Fact 18 ($!$ is the non-inquisitive projection operator)**.**

- The declarative operator $!$ is a non-inquisitive projection operator;

- If $\mathcal{A}$ is a non-inquisitive projection operator, then $[\mathcal{A}\varphi] = [!\varphi]$ for all φ.

Identifying what requirements we should place on a non-informative projection operator $\mathcal{Q}$ is less straightforward. Obviously, we should require $\mathcal{Q}$ to trivialize informative content; that is, $\mathcal{Q}$ should turn any formula φ into a question $\mathcal{Q}\varphi$. But we cannot just require $\mathcal{Q}$ to preserve inquisitive content: for, if φ and $\mathcal{Q}\varphi$ do not have the same informative content, then the inquisitive content of φ and the inquisitive content of $\mathcal{Q}\varphi$ are issues over different states, and therefore they are necessarily different objects.

To solve this problem, we will associate to any formula an issue $D(\varphi)$ over ω, no matter what the informative content of φ is. This issue requests enough information to either settle the issue $[\varphi]$, thus locating the actual world in a possibility for φ, or to reject the informative content of φ altogether, thus locating the actual world outside of any possibility for φ and making the issue $[\varphi]$ insubstantial.

Definition 27 (Settling, contradicting, and deciding on a proposition)**.**
Let s be an information state and A a proposition. Then we say that:

- s *settles* A in case $s \in A$;

- s *contradicts* A in case $s \cap \mathsf{info}(A) = \emptyset$;

- s *decides* on A in case s settles A or s contradicts A.

Definition 28 (Decision set).
The *decision set* $D(\varphi)$ of a sentence φ is the set of states that decide on $[\varphi]$.

The decision set of a sentence can be characterized explicitly as follows.

Fact 19. For any φ, $D(\varphi) = [\varphi] \cup [\varphi]^*$

Notice that the decision set $D(\varphi)$ of a sentence φ is always an issue over ω, no matter what the informative content of ω is. Therefore, we can require of a non-informative projection operator $\mathcal{Q}$ that it preserve the decision set of the sentence it applies to.

Definition 29 (Non-informative projection operator).
We call an operator $\mathcal{Q}$ a *non-informative projection operator* just in case for any φ:

- $\mathcal{Q}\varphi$ is a question;

- $D(\mathcal{Q}\varphi) = D(\varphi)$.

Now suppose that Q is a non-informative projection operator. Then for any φ, $\mathcal{Q}\varphi$ should be a question, which means that the informative content of $\mathcal{Q}\varphi$ should be ω. But then $[\mathcal{Q}\varphi]^* = \wp(\overline{\mathsf{info}(\mathcal{Q}\varphi)}) = \{\emptyset\}$, and therefore $D(\mathcal{Q}\varphi) = [\mathcal{Q}\varphi] \cup [\mathcal{Q}\varphi]^* = [\mathcal{Q}\varphi]$. But since $\mathcal{Q}$ should preserve the decision set of φ, we must also have $D(\mathcal{Q}\varphi) = D(\varphi) = [\varphi] \cup [\varphi]^*$. Putting these things together, we obtain that we must have $[\mathcal{Q}\varphi] = [\varphi] \cup [\varphi]^*$. We have thus found that the requirements we placed on $\mathcal{Q}$ uniquely determine its semantic behavior.

Fact 20 (Uniqueness of the non-informative projection operator).
$\mathcal{Q}$ is a non-informative projection operator if and only if $[\mathcal{Q}\varphi] = [\varphi] \cup [\varphi]^*$.

Now recall that the interrogative operator ? was introduced by the convention that $?\varphi$ abbreviates $\varphi \vee \neg\varphi$. Spelling out the semantics of negation and disjunction, we have that $[?\varphi] = [\varphi \vee \neg\varphi] = [\varphi] \cup [\varphi]^*$. Thus, the interrogative operator ? is a non-informative projection operator. Moreover, the previous fact guarantees that any non-informative projection operator must be equivalent with it.

Fact 21 (? is the non-informative projection operator)**.**

- The interrogative operator ? is a non-informative projection operator;

- If $\mathcal{Q}$ is a non-informative projection operator, $[\mathcal{Q}\varphi] = [?\varphi]$ for all φ.
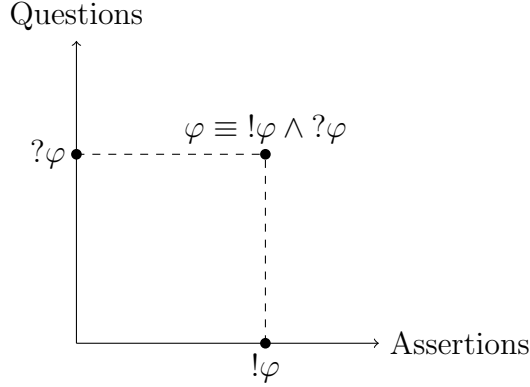

Figure 8: Non-informative and non-inquisitive projections.

Thus, for any sentence φ, the projection operators yield an assertion $!\varphi$ which has the same informative content as φ, and a question $?\varphi$ which has the same decision set as φ. The following fact says that the full meaning of φ can then be reconstructed as the conjunction of these "pure components". Thus, we obtain a representation of the meaning of φ in which the labor is divided between an assertion, that takes care of the informative content of φ, and a question, that takes care of the inquisitive content of φ.

Fact 22 (Division)**.** For any φ, $\varphi \equiv !\varphi \wedge ?\varphi$

4.7 Propositions expressed in a state and support

When a sentence φ is uttered in a context s, it expresses a proposition $[\varphi]_s$ which embodies a proposal to enhance the context s in certain ways. As discussed in section 2.3, this proposition is obtained by restricting the absolute proposition $[\varphi]$ expressed by φ to s.

Definition 30 (Proposition expressed by a sentence in a state)**.**
The proposition $[\varphi]_s$ expressed by a sentence φ in a state s is defined as:

$$[\varphi]_s = [\varphi]\!\restriction\! s = \{t \subseteq s \mid t \in [\varphi]\}$$

The informative content of this proposition is the set $\mathsf{info}([\varphi]_s) = \bigcup[\varphi]_s$. Using the definition of $[\varphi]_s$ and the fact that $\bigcup[\varphi] = |\varphi|$, it is easy to see that $\mathsf{info}([\varphi]_s) = |\varphi| \cap s$. We denote this set by $|\varphi|_s$ and we call it the *information provided by φ in s.*

Definition 31 (Information provided by a sentence in a state).
The information $|\varphi|_s$ provided by a sentence φ in a state s is the state $|\varphi| \cap s$.

Now, we will say that a sentence φ is informative (resp. inquisitive) in s in case it expresses an informative (resp. inquisitive) proposition in s.

Definition 32 (Informativeness and inquisitiveness in a state).

- φ is *informative* in s iff the proposition $[\varphi]_s$ is informative in s.

- φ is *inquisitive* in s iff the proposition $[\varphi]_s$ is inquisitive in s.

The following fact provides an explicit characterization of informativeness and inquisitiveness of a sentence in a state in terms of the information it provides and the proposition it expresses in that state.

Fact 23 (Informativeness and inquisitiveness in a state).

- φ is *informative* in s iff $|\varphi|_s \neq s$.

- φ is *inquisitive* in s iff $|\varphi|_s \notin [\varphi]_s$.

If φ is neither informative nor inquisitive in s, then an utterance of φ in the context s has no effect at all. In this case, we say that s *supports* φ.

Definition 33 (Support).
A state s supports a sentence φ, in symbols $s \models \varphi$, in case s is neither informative nor inquisitive in s.

Spelling out the definition of informativeness and inquisitiveness in a state, we see that s supports φ if and only if $|\varphi|_s \in [\varphi]_s$ and $|\varphi|_s = s$, that is, if and only if $s \in [\varphi]_s$. And since $[\varphi]_s = \{\alpha \subseteq s \mid \alpha \in [\varphi]\}$, the condition $s \in [\varphi]_s$ is equivalent to $s \in [\varphi]$. Hence, we have found the following tight connection between support and the proposition expressed by a sentence.

Fact 24 (Propositions and support).
For any sentence φ and any state s:

$$s \models \varphi \iff s \in [\varphi]$$

Support is to meanings in InqB what truth is to classical meanings. Indeed, if we write $w \models^{\mathsf{cl}} \varphi$ for "φ is classically true in w", the connection between classical truth and classical meanings can be formulated as follows:

$$w \models^{\mathsf{cl}} \varphi \iff w \in |\varphi|$$

Just like the proposition expressed by φ in classical logic coincides with the set of worlds where φ is true, the proposition expressed by φ in InqB coincides with the set of states where φ is supported. As a consequence, just like classical logic can be characterized by means of a recursive definition of the truth conditions of the sentences in the language, InqB can be characterized by a recursive definition of the support conditions of the sentences in the language. These support conditions are as follows.

Fact 25 (Support).

1. $s \models R(t_1, \ldots, t_n)$ iff $s \subseteq |R(t_1, \ldots, t_n)|$

2. $s \models \bot$ iff $s = \emptyset$

3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$

5. $s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s : \text{if } t \models \varphi \text{ then } t \models \psi$

6. $s \models \forall x \varphi(x)$ iff   for all $d \in D$, $s \models \varphi(\overline{d})$

7. $s \models \exists x \varphi(x)$ iff   for some $d \in D$, $s \models \varphi(\overline{d})$

In much previous work on inquisitive semantics (e.g. Groenendijk and Roelofsen, 2009; Ciardelli, 2009; Ciardelli and Roelofsen, 2011), support is indeed presented as the basic semantic notion, with propositions and meanings as derived notions. One advantage of this approach is that it parallels the usual presentation of classical logic, with truth as the basic notion. Another advantage is that the support conditions immediately suggests a connection with intuitionistic logic. The set of non-empty states, ordered by the relation $\supseteq$, constitutes a Kripke frame for intuitionistic logic. The support clauses amount precisely to the usual Kripke semantics for intuitionistic logic on this frame, the particular valuation function being provided by the clause for atoms. This connection is explored in depth in Ciardelli (2009) and Ciardelli and Roelofsen (2011).

Here, we have chosen a different route. The main reason for this is that the current presentation of the semantics brings out more explicitly how the notion of meaning is reshaped according to our needs. The algebraic perspective presented in section 3, then, allows us to motivate the clauses of InqB in a solid way, relying only on the structure of our new space of meanings. Thus, unlike the support-based approach, which is only motivated *a posteriori*, this mode of presentation flows directly from the abstract motivations and the philosophical underpinnings of the system to its concrete implementation.

Moreover, given the intuitive interpretation of support in terms of insignificance (non-informativeness and non-inquisitiveness), the notion has a negative flavor to it, and its relation to the positive contribution of a sentence, as given in terms of its *potential*, is intuitively far from immediate. The current presentation focuses on this positive side, hopefully reflecting more directly how inquisitive semantics can be used in modeling conversation.

5   Inquisitive logic

In this section, we will be concerned with the logic that InqB gives rise to. We will show that this logic is an intermediate logic, i.e., a logic in between classical and intuitionistic logic, which is not closed under uniform substitution. While relatively little is known about the general first-order system, with only some preliminary results in Ciardelli (2009), propositional inquisitive logic has been investigated in detail, and much is known about it, including a range of axiomatizations and its precise relation to classical and intuitionistic logic. We will present a number of results that hold for the general first-order case in section 5.1 and then zoom in on the propositional case in section 5.2.[4]

5.1   First-order inquisitive logic

Recall from section 4.1 that a sentence φ entails another sentence ψ in InqB iff $[\varphi] \subseteq [\psi]$. We write $\varphi \models_{\mathsf{InqQL}} \psi$ in this case. We say that a formula φ is *valid* in inquisitive semantics iff it expresses a tautology, i.e., iff $[\varphi] = \wp(\omega)$.

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[4]This section is based on Ciardelli (2009) and Ciardelli and Roelofsen (2011). For proofs and more comprehensive discussion of the logical issues discussed here, the reader is referred to these sources.

The set of first-order formulas that are valid in inquisitive semantics is called *inquisitive first-order logic* and is denoted by InqQL.

Definition 34 (Logic). $\mathsf{InqQL} := \{\varphi \mid [\varphi] = \wp(\omega)\}$.

As usual, the relation between entailment and validity is provided by the deduction theorem.

Fact 26 (Deduction theorem). For any two formulas φ and ψ:

$$\varphi \models_{\mathsf{InqQL}} \psi \quad \Longleftrightarrow \quad \varphi \to \psi \in \mathsf{InqQL}$$

We will use IQL and CQL to denote intuitionistic and classical first-order logic respectively, and $\models_{\mathsf{IQL}}$ and $\models_{\mathsf{CQL}}$ to denote the corresponding entailment relations.

Now, if a formula φ expresses a tautology in inquisitive semantics, in particular it is not informative; therefore, by fact 13, it is a classical tautology. Formally, this simply amounts to the observation that if $[\varphi] = \wp(\omega)$, then $|\varphi| = \bigcup[\varphi] = \omega$. So, inquisitive logic in contained in classical first-order logic.

Fact 27. $\mathsf{InqQL} \subseteq \mathsf{CQL}$.

This inclusion is strict: for instance, in section 4 we saw that one of the most famous classical tautologies, $p \vee \neg p$, expresses a polar question in inquisitive semantics, not a tautology.

However, there is an important class of formulas on which inquisitive and classical logic coincide: assertions. This is to be expected, since assertions are those formulas whose meaning consist exclusively of informative content, just like in the classical case.

Fact 28 (Assertions behave classically). If φ and ψ are assertions,

$$\varphi \models_{\mathsf{InqQL}} \psi \iff \varphi \models_{\mathsf{CQL}} \psi$$

In particular, since any formula which does not contain disjunction or the existential quantifier is an assertion (corollary 1), inquisitive logic coincides with classical logic on the whole $\vee, \exists-$free fragment of the language.

There is also something interesting to say about the other most famous classical tautology, i.e., the law of double negation, $\neg\neg\varphi \to \varphi$.

Fact 29. $\neg\neg\varphi \to \varphi$ is in InqQL iff φ is an assertion.

This means that assertions are *precisely* the class of formulas on which inquisitive and classical entailment coincide.

Notice that the double negation law holds for atoms, which are assertions, but fails to hold in general. This simple fact displays a peculiar feature of InqQL: it is not closed under uniform substitution. That is, if $\varphi \in$ InqQL and we replace the atoms in φ with arbitrary formulas, we have no guarantee that the resulting formula will be valid as well.

This is due to the fact that, in inquisitive semantics, atomic formulas do not express generic propositions. They are assertions, and thus have the special property of being equivalent with their own double negation. Indeed, one can prove that substituting atoms by assertions in a valid formula always results in a valid formula.

In section 3 we have seen that the algebra $(\Pi, \subseteq)$ of inquisitive meanings forms a complete Heyting algebra. Our semantics simply amounts to evaluating formulas in this particular structure, interpreting the logical constants as the corresponding algebraic operations. In this perspective, InqQL is the theory of a particular Heyting-valued model. Since intuitionistic first-order logic can be characterized as the set of those formulas valid in all Heyting-valued models (Troelstra and van Dalen, 1988), we have the following result.

Fact 30. IQL $\subseteq$ InqQL.

This inclusion is also strict. For instance, we have seen that the double negation law holds for atoms in InqQL, whereas this is not the case in IQL.

Summing up the results in this section: InqQL is a logic lying in between intuitionistic and classical first-order logic, not closed under uniform substitution. A number of non intuitionistically valid principles are known to hold in InqQL (see Ciardelli, 2009). However, the axiomatic characterization of InqQL remains an open problem.

5.2   Propositional inquisitive logic

In this section, we will restrict our attention to inquisitive logic for a propositional language, which we denote as InqPL. Intuitionistic and classical propositional logic will be denoted by IPL and CPL respectively.

It is immediate that all the facts stated in the previous section for InqQL also hold for InqPL. In particular, InqPL is an intermediate logic that is not closed under uniform substitution. But what logic is it exactly? Can it be described by means of a natural axiomatization?

This time, we will be able to answer these questions in the positive. Let's start once again from classical logic. In that setting, all formulas are assertions. On the proof-theoretic side, this is witnessed by the fact that any formula is equivalent to a negation, for instance to its own double negation.

Fact 31. There exists a recursively defined map nt s.t. for any φ:

1. $\mathsf{nt}(\varphi)$ is a negation;

2. $\varphi \equiv_{\mathsf{CPL}} \mathsf{nt}(\varphi)$.

Once classical meanings are expressed by means of negations, any difference between classical and intuitionistic logic vanishes.

Fact 32. If φ and ψ are negations, then:

$$\varphi \models_{\mathsf{CPL}} \psi \iff \varphi \models_{\mathsf{IPL}} \psi$$

This means that, in a very precise sense, classical logic can be regarded as the negative fragment of intuitionistic logic, and the map nt—commonly known as *negative translation*—provides an embedding.

The situation is very similar for inquisitive logic. Now, formulas are no longer assertions in general. However, in the propositional setting, a formula is always equivalent to a *disjunction* of assertions, that is, to a disjunction of negations. This syntactic feature reflects the fact that inquisitive meanings are sets (disjunctions) of classical meanings (negations).

Fact 33. There exists a recursively defined map dnt s.t. for any φ:

1. $\mathsf{dnt}(\varphi)$ is a disjunction of negations;

2. $\varphi \equiv_{\mathsf{InqPL}} \mathsf{dnt}(\varphi)$.

Moreover, once an inquisitive meaning of a formula is expressed as a disjunction of negations, inquisitive and intuitionistic logic coincide.

Fact 34. If φ and ψ are disjunctions of negations, then

$$\varphi \models_{\mathsf{InqPL}} \psi \iff \varphi \models_{\mathsf{IPL}} \psi$$

This means that inquisitive logic can be regarded as the disjunctive-negative fragment of intuitionistic logic, and that dnt—which we refer to as the *disjunctive-negative translation*—provides an embedding.

Thus, the algebra of intuitionistic meanings[5] provides a very rich environment, in which certain particular meanings, those associated to disjunctions of negations, correspond to the meanings of inquisitive semantics; among these, in turn, the meanings associated to negations represent the spectrum of classical meanings, that is, of meanings of assertions.

The disjunctive-negative translation dnt provides the key to understanding the logical properties of propositional inquisitive logic. As we have seen, it clarifies the way in which InqPL sits in between IPL and CPL. But it does more than that: it also paves the way for a completeness result.

For, suppose that L is an extension of intuitionistic logic which "justifies the disjunctive negative translation", in the sense that for any φ, L proves that φ and $\mathsf{dnt}(\varphi)$ are equivalent. We write $\vdash_L$ for provability in L.

Consider a valid inquisitive entailment, $\varphi \models_{\mathsf{InqPL}} \psi$. Then we also have that $\mathsf{dnt}(\varphi) \models_{\mathsf{InqPL}} \mathsf{dnt}(\psi)$ and therefore by proposition 34, $\mathsf{dnt}(\varphi) \models_{\mathsf{IPL}} \mathsf{dnt}(\psi)$. But then, since L extends IPL, $\mathsf{dnt}(\varphi) \vdash_L \mathsf{dnt}(\psi)$.

Since L justifies dnt, we also have $\varphi \vdash_L \mathsf{dnt}(\varphi)$ and $\mathsf{dnt}(\psi) \vdash_L \psi$, whence, putting everything together, $\varphi \vdash_L \psi$. Therefore, L is a complete derivation system for InqPL.

We are only left with the task of identifying what is needed, on top of intuitionistic logic, to justify dnt. Analyzing the inductive definition of the map dnt, we see that two extra ingredients suffice.

1. Atomic double negation axioms $\neg\neg p \to p$, needed for the translation of atoms;

2. For any number n, any instance of the following scheme, needed to justify the translation of implication:

   $(\mathtt{ND}_n)$ $(\neg\varphi \to \bigvee_{1 \leq i \leq n} \neg\psi_i) \to \bigvee_{1 \leq i \leq n}(\neg\varphi \to \neg\psi_i)$

   The intermediate logic that is obtained by expanding IPL with the schemata $\mathtt{ND}_n$ for all natural numbers n is called ND. It has been investigated by Maksimova (1986).

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[5] Here we use the word *meanings* for equivalence classes of formulas; for classical and inquisitive logic, for which we have a semantical notion of meaning, there is a bijective correspondence between the two notions.

These considerations yield the following sound and complete axiomatization of InqPL.

Theorem 1 (Completeness theorem). ND augmented with double negation for atoms constitutes a sound and complete axiomatization of InqPL.

The infinite family of axioms $\{\mathtt{ND}_n \,|\, n \in \mathbb{N}\}$ may be substituted by a single, stronger axiom, known as the Kreisel-Putnam axiom.

$$(\mathtt{KP}) \qquad (\neg\varphi \to \psi \lor \chi) \to (\neg\varphi \to \psi) \lor (\neg\varphi \to \chi)$$

The Kreisel-Putnam logic KP is obtained by expanding IPL with the schema KP. The logic KP is strictly stronger than Maksimova's logic ND. However, when augmented with atomic double negation axioms, both logics amount to the same thing, namely InqPL.

Theorem 2 (Completeness theorem). KP augmented with double negation for atoms also constitutes a sound and complete axiomatization of InqPL.

We have thus reached what is perhaps the most elegant axiomatization of InqPL. However, it should be remarked that there is a whole range of intermediate logics Λ which, expanded with atomic double negation, yield InqPL. In Ciardelli (2009); Ciardelli and Roelofsen (2011), this range is precisely characterized as the set of logics included between ND and a logic called Medvedev's logic.

6 Relevance for natural language semantics

So far, we have motivated inquisitive semantics at a rather abstract level, specified a concrete system, InqB, and investigated the main features of this basic system. In this section, we discuss the relevance of inquisitive semantics, and in particular of InqB, for natural language semantics. We start with a very general discussion of the role that inquisitive semantics is intended to play in the semantic analysis of natural language. Then we discuss three different perspectives on inquisitiveness that are all in principle compatible with the basic philosophy behind the framework. Finally, we discuss the specific treatment of the logical connectives and the projection operators in InqB, and the possible significance thereof for natural language semantics, paying particular attention to disjunction and existential quantification as sources of inquisitiveness.
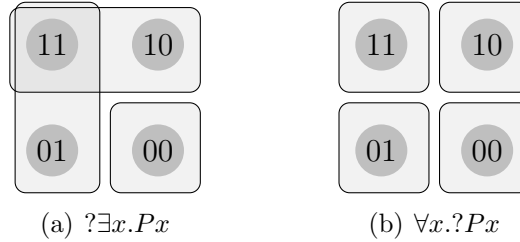
(a) $?\exists x.Px$ (b) $\forall x.?Px$

Figure 9: Two possible analysis of *wh*-interrogatives.

6.1 Inquisitive semantics as a semantic framework

Inquisitive semantics is, first and foremost, intended to serve as a *framework* for natural language semantics. This means that it does not constitute a specific *theory* of any particular construction in any particular language. Rather, it is intended to provide the formal tools that are necessary to formulate, compare, and further develop such theories.

This is perhaps best illustrated by means of an example. Consider the *wh*-interrogative in (1).

(1)   Who is coming to the party?

InqB provides a certain space of meanings, and associates these meanings in a particular way with formulas in a first-order language. But it leaves open how these meanings/formulas should be linked to sentences in natural language. For instance, the sentence in (1) may be associated with the formula $?\exists x.Px$ and the corresponding meaning, in line with Hamblin's (1973) and Karttunen's (1977) analysis of *wh*-interrogatives, illustrated in figure 9(a). But it may also be associated with the formula $\forall x.?Px$ and the corresponding meaning, in line with Groenendijk and Stokhof's (1984) partition theory of *wh*-interrogatives, illustrated in figure 9(b). Both type of theories may be formulated, compared, and possibly adapted or combined within the framework.

6.2 Three perspectives on inquisitiveness

The proposition $[\varphi]$ expressed by a sentence φ in InqB embodies the issue that is raised in uttering φ. In order to settle this issue, other participants must provide enough information to establish one of the states in $[\varphi]$. Moreover, we

have been assuming implicitly that, when raising an issue in a conversation, a speaker *requests* a response from other participants that settles the issue. However, the latter assumption is independent of the basic philosophy behind the framework, and is not reflected in any way by its formal implementation. Alternatively, we may just as well assume that a speaker, when raising an issue in a conversation, merely *invites* a response from other participants that settles the issue. Or we may adopt a more context-sensitive perspective, and assume that in uttering a sentence φ in a state s, a speaker requests a response that settles $[\varphi]$ just in case φ is purely inquisitive (not informative) in s. These three perspectives on inquisitiveness are listed below.

1. **The strong perspective.** In uttering a sentence φ in a state s, a speaker *requests* a response from other participants that provides enough information to establish one of the states in $[\varphi]_s$.

2. **The weak perspective.** In uttering a sentence φ in a state s, a speaker *invites* a response from other participants that provides enough information to establish one of the states in $[\varphi]_s$.

3. **The context-sensitive perspective.** In uttering a sentence φ in a state s, a speaker *invites* a response from other participants that provides enough information to establish one of the states in $[\varphi]_s$, and she *requests* such a response just in case φ is purely inquisitive (not informative) in s.

As mentioned above, we have assumed the strong perspective in these notes. But the other perspectives are equally viable, and have indeed been adopted elsewhere. For instance, the weak perspective can be found in Groenendijk (2009), and the context-sensitive perspective in AnderBois (2011).

This said, it is important when using inquisitive semantics as a framework for linguistic analysis to always explicitly choose a particular perspective on inquisitiveness. After all, the predictions of a theory formulated in the framework are partly determined by the perspective on inquisitiveness that is assumed. To illustrate this point, consider the following disjunctive declarative sentence.

(2)   Alf or Bea will play the piano tonight.

Suppose we have a theory that associates this sentence with the formula $Pa \vee Pb$ and the corresponding meaning. This means that the sentence is predicted

to be inquisitive. If we adopt the weak perspective on inquisitiveness, this prediction is reasonable. However, if we adopt the strong perspective on inquisitiveness, the prediction is wrong. After all, in uttering (2) a speaker does not request a response from other participants that establishes that Alf will play the piano or that Bea will play the piano. Under a strong perspective on inquisitiveness, (2) should not be associated with $Pa \vee Pb$ but rather with $!(Pa \vee Pb)$ and the corresponding meaning, which is not inquisitive.

Under the weak perspective, certain basic contrasts cannot be captured. Consider, for instance, the polar interrogative in (3), in comparison with the disjunctive declarative in (2).

(3)   Will Alf play the piano tonight?

There is a clear intuition that (3) is inquisitive in a strong sense: in uttering this sentence, a speaker does not just invite, but really *requests* a response from other participants that establishes whether Alf will play the piano or not. In this sense there is a clear contrast between (2) and (3). Under the strong perspective, this contrast can be captured straightforwardly. Under the weak perspective, it cannot be captured because sentences, even if inquisitive, are never predicted to request an informative response.

6.3   Basic operations on meanings

We identified certain basic operations on meanings in InqB: the algebraic *meet*, *join*, and *(relative) pseudo-complement* operators, as well that the non-informative and non-inquisitive projection operators. We defined a semantics for the language of first-order logic, in which the basic logical connectives, as well as ! and ?, are associated with these basic operations on meanings: conjunction behaves as a *meet* operator, disjunction behaves as a *join* operator, ! behaves as a non-inquisitive projection operator, etcetera.

Of course, natural languages are much more intricate than the language of first-order logic. However, if we take sentences in natural language to have the type of meanings considered here, then it is natural to expect that it is also possible in these language to express the basic operations on such meanings. In other words, it is to be expected that natural languages will generally have certain words or constructions whose semantic function (possibly among others) is to produce, say, the non-informative projection of a proposition, or the meet of two propositions.

It seems plausible to assume that in the specific case of English, the meet of two propositions is constructed using the word *and*, the join is constructed using *or*, the relative pseudo-complement is constructed using *if... then*, and the pseudo-complement is constructed using *not*. This is not to say that this is the only semantic function that these words may have. But the expectation that the language makes it possible to express the basic algebraic operations on meanings seems to be borne out in the case of English, and many other languages alike.

As for the projection operators, it seems plausible to hypothesize that these are expressed in English and many other languages by declarative and interrogative *complementizers*. More specifically, it seems plausible to treat the declarative complementizer in English as !, the *wh*-interrogative complementizer as ?, and the polar interrogative complementizer as ?!. A detailed examination of this analysis is beyond the scope of these notes. Importantly, however, note that the framework also allows us to formulate alternative analyses. As emphasized above, the framework as such does not make any direct predictions about the semantic behavior of any specific constructions in any specific language. It mainly offers the logical tools that are necessary to formulate such analyses, and gives rise to the expectation that, in general, natural languages will have ways to express the basic algebraic operations and the basic projection operations on meanings.

6.4 Inquisitive disjunction and indefinites

Among the basic operations on meanings that we have considered, the *join* operator is the essential source of inquisitiveness: without applying this operator, it is impossible to produce inquisitive meanings from non-inquisitive ones. In InqB, disjunction, the existential quantifier, and the non-informative projection operator all behave as join operators: $[\varphi \vee \psi]$ is the join of $[\varphi]$ and $[\psi]$, $[\exists x.\varphi(x)]$ is the join of $\{[\varphi(d)] \mid d \in D\}$, and $[?\varphi]$ is the join of $[\varphi]$ and $[\varphi]^*$. Thus, there is a close connection between the non-informative projection operator, ?, which is naturally associated with interrogative complementizers in natural languages, and disjunction / existential quantification. All these constructions are sources of inquisitiveness. This fact may provide the basis for an explanation of the well-known observation that in many natural languages, interrogative pronouns/complementizers are homophonous with words for disjunction and/or indefinites (e.g., Japanese *ka*) (see Jayaseelan, 2001, 2008; Bhat, 2005; Haida, 2007; AnderBois, 2011, among others).

It is also interesting to note that there is a close connection between the treatment of disjunction and existential quantification in InqB, and their treatment in *alternative semantics* (Kratzer and Shimoyama, 2002; Simons, 2005a,b; Alonso-Ovalle, 2006, 2008, 2009; Aloni, 2007a,b; Menéndez-Benito, 2005, 2010, among others). In both cases, disjunction and existentials are taken to introduce sets of alternatives. In the case of alternative semantics, this treatment is motivated by a number of empirical phenomena, including free choice inferences, exclusivity implicatures, and conditionals with disjunctive antecedents. The analysis of disjunction and existentials as introducing sets of alternatives has made it possible to develop new accounts of these phenomena which improve considerably on previous accounts. However, alternative semantics does not provide any motivation for the alternative treatment of disjunction and indefinites *independently* of the linguistic phenomena at hand. Moreover, the treatment of disjunction in alternative semantics has been presented as a real *alternative* for the classical treatment of disjunction as a *join* operator. Thus, it appears that adopting the alternative treatment of disjunction forces one to give up the classical account.

The algebraically motivated inquisitive semantics presented here sheds new light on these two issues. First, it shows that, once inquisitive content is taken into consideration besides informative content, general algebraic considerations lead essentially to the treatment of disjunction that was proposed in alternative semantics, thus providing exactly the independent motivation that has so far been missing. Moreover, it shows that the 'alternative' treatment of disjunction is actually a natural generalization of the classical treatment: disjunction is still taken to behave semantically as a *join* operator, only now the meanings that this join operator applies to are more fine-grained in order to capture both informative and inquisitive content. Thus, we can have our cake and eat it: we can adopt a treatment of disjunction as introducing sets of alternatives, and still characterize it as a *join* operator.

7   Extensions

So far, we have discussed the general philosophical underpinnings of inquisitive semantics and explored in more detail the system InqB, which we consider to be the most basic implementation of the framework. In the present section, we discuss some extensions of InqB. In the extended systems, the notion of meaning is further enriched, in order to capture differences in meaning

that go beyond informative and inquisitive content, and can therefore not be captured in InqB. The extensions that we will discuss are *modular*. Thus, depending on the type of linguistic phenomenon under consideration, certain extensions may be adopted and others may be left out, in order to obtain a system that is just rich enough to deal with the phenomenon at hand.

We must note up front that the discussion in this section will be much less detailed and more speculative than in previous sections. The extensions that we will sketch are all 'work in progress', and none of them is as well-understood as the basic system InqB at this point.

7.1 Presuppositions

In section 2.3 we defined a meaning as a function f that determines, for any context s, a proposition $f(s) \in \Pi_s$, in accordance with the compatibility condition. That is, we took meanings to be *total* functions from contexts to propositions. As a consequence, in InqB sentence are always taken to express a well-defined proposition in any discourse context.

In natural language, sentences often only express a well-defined proposition in a restricted set of discourse contexts. Such sentences are said to have a *presupposition*. A sentence with a presupposition only expresses a proposition in those discourse contexts that satisfy its presupposition. Formally, this means that the meaning of a sentence with a presupposition should be modeled as a *partial* function from contexts to propositions, which is defined on a context s whenever s satisfies the presupposition. Thus, in order to incorporate presuppositions into our framework, we need to relax the totality requirement on meanings.

7.1.1 Presuppositional meanings

Presuppositions are known to be introduced by many types of constructions, such as definite descriptions (the king of France), factive verbs (know, realize), aspectual verbs (continue, stop) and adverbs (still, again), anaphoric pronouns (he, they), and temporal clauses, among others. Such constructions are called *presupposition triggers*.

We will focus here on sentences with a *factive* presupposition, that is, sentences which, in order to express a proposition, require a certain piece of information to be established in the context. The following are all examples

of sentences with factive presuppositions.[6]

(4)   Bea knows that John cheats on her.
       → John cheats on Bea.

(5)   Bea stopped smoking.
       → Bea used to smoke.

(6)   Bea is in Paris again.
       → Bea was in Paris before.

We thus model a presupposition as a piece of information $\pi \subseteq \omega$, and define a meaning with presupposition π as a compatible function that expresses a proposition precisely in those contexts where π has indeed been established.

Definition 35 (Meanings)**.** Let π be a state. A *meaning* with a presupposition π is a function f that maps any state $s \subseteq \pi$ to a proposition $f(s) \in \Pi_s$, in accordance with the compatibility condition (see definition 14).

Notice that our former, total notion of meaning can be recovered as the particular case in which the presupposition is trivial, i.e., $\pi = \omega$. In general it will no longer be possible to identify a meaning f with the proposition it expresses in the ignorant state ω. Indeed, $f(\omega)$ need not even be well-defined. However, the compatibility condition still ensures that if $f(s)$ is well-defined and $t \subseteq s$, then $f(t)$ is completely determined by $f(s)$. As before, $f(t)$ can be obtained by restricting $f(s)$ to t in this case: $f(t) = f(s){\restriction}t$.

Now, if f is a meaning with presupposition π, then *any* state s on which f is defined is a subset of π, and so the proposition $f(s)$ is determined by $f(\pi)$. This means that the meaning f is jointly determined by its presupposition π and the proposition $f(\pi)$ expressed on π:

$$f(s) = \begin{cases} f(\pi){\restriction}s & \text{if } s \subseteq \pi \\ \text{undefined} & \text{otherwise} \end{cases}$$

Vice versa, any pair $\langle \pi, A \rangle$, where π is a state and A is a proposition over π determines a meaning f_A with presupposition π, obtained from A by restriction:

$$f_{\langle \pi, A \rangle}(s) = \begin{cases} A{\restriction}s & \text{if } s \subseteq \pi \\ \text{undefined} & \text{otherwise} \end{cases}$$

---

[6]Not all presuppositions are factive. For instance, anaphoric pronouns do not presuppose that a certain piece of information has been established, but rather that a suitable antecedent has been made available.

We have thus reached the following analogue of fact 1 for the presuppositional case.

Fact 35. There is a one-to-one correspondence between meanings and pairs $\langle \pi, A \rangle$, where π is a state and A is a proposition over π.

So, now that presuppositions have been brought into the picture, we can no longer identify a meaning with a unique proposition, but we can still identify it with a unique static object, namely a pair consisting of a state (the presupposition) and a proposition over that state.

The notions of informativeness and inquisitiveness introduced in section 2.3, definition 16, still make perfect sense for presuppositional meanings: we simply call a meaning *informative* if it has the potential to provide information, and *inquisitive* if it has the potential to request information.

Definition 36 (Informativeness and inquisitiveness). Let f be a meaning.

- f is *informative* if for some s, the proposition $f(s)$ is informative in s.

- f is *inquisitive* if for some s, the proposition $f(s)$ is inquisitive in s.

These properties of a meaning f can be recast in terms of properties of the associated presupposition π and proposition $f(\pi)$.

Fact 36. Let f be a meaning with presupposition π. Then:

- f is informative iff $f(\pi)$ is informative in π.

- f is inquisitive iff $f(\pi)$ is inquisitive in π.

Besides calling certain meanings informative and/or inquisitive, we will also call certain meanings *presuppositional*. The natural requirement for presuppositional meanings is of course that they have a non-trivial presupposition.

Definition 37.
We say that a meaning f with presupposition π is *presuppositional* if $\pi \neq \omega$.

7.1.2 Inquisitive semantics with presuppositions

The notion of meaning as a partial function forms the basis for an inquisitive semantics in which sentences may have presuppositions. If we consider the usual first-order language we dealt with so far, the natural thing to do is to modify the semantics to allow for partial worlds, where:

1. definite descriptions and proper names may lack a referent, i.e., a closed term t only denotes an individual in certain worlds;

2. predicates may have selectional restrictions, i.e., their interpretation in a world is a *partial* function from the domain to $\{0, 1\}$.

We can then denote by $\lfloor R(t_1, \ldots, t_n) \rfloor$ the set of worlds where all terms $t_1, \ldots, t_n$ denote individuals of the domain, and moreover the predicate associated with R is defined for these individuals, resulting in a determinate truth-value. Of course, this should all be made more precise, but since our goal is only to sketch the general features of an inquisitive semantics with presuppositions here, we do not to insist on these formal details for now.

Now, in view of fact 35, to associate a sentence φ with a certain meaning it is no longer sufficient to equip it with a proposition $[\varphi]$: we also need to specify a presupposition $\pi(\varphi) \subseteq \omega$. For the proposition $[\varphi]$, we will simply keep the clauses of InqB, as specified by definition 21.

As for presuppositions, we will, for now, adopt one of the classical accounts of presupposition projection, due to Karttunen (1974). There are of course many other accounts of presupposition projection in the literature. We do not take a stance on which of these accounts is empirically most adequate. In principle, other existing accounts can be plugged into our system as well, and the inquisitive perspective may also give rise to new approaches to the projection problem. This line of investigation, however, has not yet been pursued in much detail.

We formulate Karttunen's account by recursively defining a *presupposition satisfaction* relation $\Vdash$ between states and sentences (the clauses for the connectives are taken directly from Karttunen (1974), the others are added).

Definition 38 (Presupposition satisfaction).

1. $s \Vdash R(t_1, \ldots, t_n)$ iff $s \subseteq \lfloor R(t_1, \ldots, t_n) \rfloor$

2. $s \Vdash \bot$ iff $s \subseteq \omega$

3. $s \Vdash \varphi \wedge \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\varphi) \Vdash \psi$

4. $s \Vdash \varphi \vee \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\neg\varphi) \Vdash \psi$

5. $s \Vdash \varphi \rightarrow \psi$ iff $s \Vdash \varphi$ and $s \cap \mathsf{info}(\varphi) \Vdash \psi$

6. $s \Vdash \forall x.\varphi(x)$ iff $s \Vdash \varphi(\overline{d})$ for all $d \in D$

7. $s \Vdash \exists x.\varphi(x)$ iff $s \Vdash \varphi(\overline{d})$ for some $d \in D$

These clauses may be read as follows. The presupposition of an atomic sentence $R(t_1, \ldots, t_n)$ is satisfied in a state s just in case the terms $t_1, \ldots, t_n$ are known to denote individuals, and these individuals are known to match the selectional restrictions of the relation R. The proposition of $\bot$ is always satisfied. The presupposition of a conjunction $\varphi \wedge \psi$ is satisfied in a state s just in case the presupposition of φ is satisfied in s, and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\varphi)$; in other words, when evaluating the second conjunct, the information provided by the first may be assumed. The presupposition of a disjunction $\varphi \vee \psi$ is satisfied in a state s just in case the presupposition of φ is satisfied in s, and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\neg\varphi)$; so, when evaluating the second disjunct, the negation of the first disjunct may be assumed. The presupposition of an implication $\varphi \rightarrow \psi$ is satisfied in s just in case the presupposition of φ is satisfied in s and the presupposition of ψ is satisfied in the state $s \cap \mathsf{info}(\varphi)$; so, when evaluating the consequent of an implication, the information provided by the antecedent may be assumed. Finally, the presupposition of $\forall x\varphi(x)$ is satisfied in s just in case the presupposition of $\varphi(\overline{d})$ is satisfied in s for all elements d, and the presupposition of $\exists x\varphi(x)$ is satisfied in s just in case the presupposition of $\varphi(\overline{d})$ is satisfied in s for some element d.

Now, we define $\pi(\varphi)$ as the set of worlds that are included in some state s such that $s \Vdash \varphi$. This means that $\pi(\varphi)$ embodies the information that the actual world is located in a state that satisfies the presupposition of φ.

Definition 39 (The presupposition of a sentence). $\pi(\varphi) \coloneqq \bigcup\{s \mid s \Vdash \varphi\}$

Thus, for every sentence φ, we have a way to derive the presupposition of φ, $\pi(\varphi)$, and the proposition expressed by φ, $[\varphi]$. Recall that in order for a presupposition-proposition pair $\langle \pi, A \rangle$ to determine a meaning, A should be a proposition over π. The following fact ensures that this is indeed the case for all pairs $\langle \pi(\varphi), [\varphi] \rangle$ yielded by our semantic clauses.

Fact 37 (Suitability of the semantics).
For any $\varphi \in \mathcal{L}$, $[\varphi]$ is a proposition over the state $\pi(\varphi)$.

We refer to the system defined here as InqP. Notice that in InqP, conjunction and disjunction are no longer commutative in general, since in order to determine the presupposition of a conjunction or disjunction, the order of the constituents matters.

   We say that a sentence is informative (resp. inquisitive) if the associated meaning is. Using fact 35 and spelling out the definition explicitly, we obtain the following characterization.

Fact 38 (Informativeness and inquisitiveness).

- φ is informative iff $\mathsf{info}(\varphi) \subset \pi(\varphi)$

- φ is inquisitive iff $\mathsf{info}(\varphi) \notin [\varphi]$

Thus, φ is informative if it provides strictly more information than it presupposes, and inquisitive if the issue it raises is non-trivial. We can then identify assertions, questions, tautologies and hybrids in terms of informativeness and inquisitiveness, just like we did in section 4. Notice that not only assertions, but also questions, tautologies, and hybrids may have presuppositions.

7.1.3   Open and closed interrogatives

Questions are defined as non-informative sentences. By making explicit what it means to be non-informative, we get the following characterization.

Fact 39 (Explicit characterization of questions in InqP).

- φ is a question iff $\mathsf{info}(\varphi) = \pi(\varphi)$

In other words, φ is a question if and only if the proposition $[\varphi]$ forms a cover of the presupposition $\pi(\varphi)$. Now, take a generic sentence φ. There are two natural strategies to make the meaning of φ non-informative, i.e., to turn φ into a question. We may add possibilities to the proposition $[\varphi]$ in order to obtain a cover of $\pi(\varphi)$, or we may leave the proposition $[\varphi]$ untouched, and shrink the presupposition $\pi(\varphi)$ to coincide with $\mathsf{info}(\varphi)$.

   The interrogative operator ? defined in section 4 as $?\varphi := \varphi \vee \neg\varphi$, implements the first strategy in the simplest possible way, adding to $[\varphi]$ the possibilities $\wp(\pi(\varphi) - \mathsf{info}(\varphi))$ to yield a cover of $\pi(\varphi)$.

But we may also implement the second strategy by expanding our language with a second interrogative operator, which we will refer to as the *closed interrogative operator*, and denote $?_c$. The semantics of this operator is specified by the following definition.

Definition 40 (Closed interrogative operator)**.**

- $[?_c\varphi] = [\varphi]$

- $s \Vdash ?_c\varphi \quad$ iff $\quad s \subseteq \mathsf{info}(\varphi)$

To contrast the two interrogative operators, we will call the one introduced in section 4 the *open* interrogative operator, and we will denote it by $?_o$. To illustrate the behavior of the two operators, figure 10 displays their effect on a simple disjunction $P(a) \vee P(b)$, where the disjuncts are assumed for simplicity to be non-presuppositional. Figure 10(a) shows the familiar meaning of the disjunction $Pa \vee Pb$ in inquisitive semantics. As before, the maximal states in the proposition expressed by the sentence are depicted with solid borders. The presupposition of the sentence is depicted with a dashed border.

Applying the open interrogative operator we obtain the meaning depicted in figure 10(b). The resulting formula, $?_o(Pa \vee Pb)$, is a non-presuppositional question, which requests sufficient information to locate the actual world in $|Pa|$, $|Pb|$, or $|\neg(Pa \vee Pb)|$. Applying the closed interrogative operator, on the other hand, results in the meaning depicted in figure 10(c). The formula $?_c(Pa \vee Pb)$ is a question which presupposes $\mathsf{info}(Pa \vee Pb)$, i.e., it presupposes that at least one of Pa and Pb is the case, and it requests additional information in order to locate the actual world in $|Pa|$ or in $|Pb|$.

In general, a sentence φ specifies a certain set $[\varphi]$ of states. The question $?_o\varphi$ requests other participants to establish one of these states, if this is possible, or to establish a state that is incompatible with all the states in $[\varphi]$. Thus, the question $?_o\varphi$ is *open* in the sense that it leaves room for rejecting the possibilities specified by φ. The question $?_c\varphi$, on the other hand, *presupposes* that the actual world is indeed located in one of the states in $[\varphi]$. Thus, $?_c\varphi$ is *closed* in the sense that it does not leave room for rejecting the possibilities specified by φ.

Relevance for natural language semantics. The distinction between open and closed interrogative operators seems useful in analyzing the semantics of interrogatives in natural language. To illustrate this, consider (7) and (8) below, where $^\uparrow$ and $^\downarrow$ indicate rising and falling intonation, respectively.
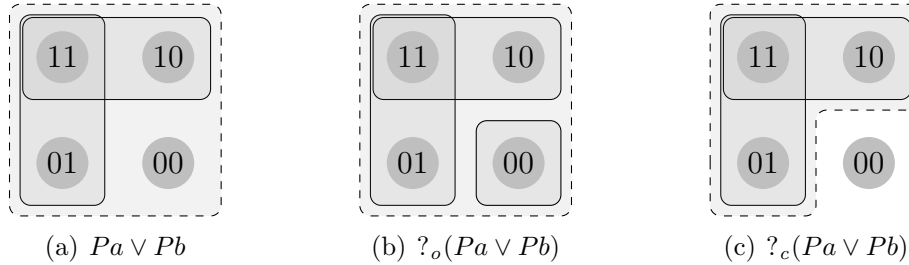
Figure 10: Open and closed interrogatives.

(7)   Did you call Andrew$^\uparrow$, or Mark$^\uparrow$?

(8)   Did you call Andrew$^\uparrow$, or Mark$^\downarrow$?

We may associate (7) with the open interrogative $?_o(Pa \vee Pb)$ depicted in 10(b), and (8) with the closed interrogative $?_c(Pa \vee Pb)$ depicted in 10(c). Then, the prediction is that (7), unlike (8), may be settled by a response that rejects both possibilities specified by the disjunction, such as (9).

(9)   No, I didn't call Andrew or Mark.

Question (8), on the other hand, is analyzed as *presupposing* that the hearer called Andrew or Mark; thus, a response like (9) would not count as settling the issue raised by the question, but rather as going against the presupposition of the question. Another way to characterize the difference between (7) and (8) is that in uttering (8), a speaker signals that he takes the two disjuncts to provide an exhaustive list of available options, whereas this is not the case in uttering (7). Of course, this treatment extends straightforwardly to alternative questions with more than two disjuncts.

The analysis of closed alternative questions sketched here is in line with several existing accounts, both in the formal semantics literature (e.g. Rawlins, 2008; Biezma, 2009; Haida, 2010; AnderBois, 2011; Biezma and Rawlins, 2012) and in the philosophical logic literature (Hintikka, 1999, 2007; Wiśniewski, 1996, 2001; Aloni *et al.*, 2009; Aloni and Égré, 2010). Of course, there is much more to say about the meaning of such questions. For instance, they may have to be analyzed as involving an exclusive strengthening operation, like the one described in Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011). This would give rise to an analysis in which alternative questions do not just presuppose that *at least* one of the proposed possibilities

holds, but rather that *exactly* one of them holds.

Besides alternative questions, the closed interrogative operator may also be relevant for the analysis of wh-questions. We have seen in section 6 that several analyses of wh-questions may be formulated in InqB. Of course, these analyses can also be articulated in InqP. However, it has sometimes been argued that wh-interrogatives such as (10) come with an existential presupposition, of the kind expressed by (11) (see, e.g., Belnap, 1969; Keenan and Hull, 1973; Prince, 1986; Rullmann and Beck, 1998; Haida, 2007).

(10)   Who did Ann go out with yesterday?

(11)   Ann went out with someone yesterday.

In such analyses, answers to (10) are taken to be sentences of the form (12), where d denotes a specific individual, and the question is taken to presuppose that at least one of these answers is true.

(12)   Ann went out with d yesterday.

Such approaches can be implemented straightforwardly in InqP by taking a wh-interrogative like (10) to correspond to a closed existential interrogative $?_c \exists x P x$. Indeed, the sentence $?_c \exists x P x$ is a question that presupposes that some entity has the property P, and requests sufficient information to establish for at least one entity d that it has the property P. Notice that if the domain of discourse consists of just two individuals, a and b, then $?_c \exists x P x$ is equivalent with $?_c (Pa \vee Pb)$, whose meaning is depicted in figure 10(c). Thus, under this analysis, wh-questions are a generalized form of alternative question.

This is not to say that the presuppositional analysis yields the correct account of all (or even some particular type of) wh-interrogatives. The main point here is just that such an analysis can be formulated naturally in InqP, in addition to the non-presuppositional analyses we discussed in section 6.

Finally, notice that in the case of wh-questions, just as in the case of alternative questions, one may assume that there is (in some cases) an additional exclusive strengthening operation at work; this would yield an additional uniqueness presupposition for questions like (10).

The closed interrogative operator as a projection operator.   In section 4.6, we defined a non-inquisitive projection operator as an operator $\mathcal{A}$

that turns any sentence φ into an assertion $\mathcal{A}\varphi$ having the same informative content as φ. We then argued that a non-informative projection operator could not be defined in a similar fashion, as an operator $\mathcal{Q}$ that takes any formula φ into a question $\mathcal{Q}\varphi$ having the same inquisitive content as φ. For, in InqB the inquisitive content of a question is always an issue over ω, while the inquisitive content of a generic sentence φ is an issue over $\mathsf{info}(\varphi)$, and therefore the two can only coincide if φ itself is already a question. This problem no longer arises in InqP, since the inquisitive content of a question is not required to be a cover of the whole logical space ω, but rather a cover of the presupposition $\pi(\varphi)$, which might be different from ω. Therefore, in InqP we can meaningfully adopt the following notion of a non-informative projection operator.

Definition 41 (Non-informative projection operator in InqP)**.**
An operator $\mathcal{Q}$ is a *non-informative projection operator* just in case for any φ:

- $\mathcal{Q}\varphi$ is a question

- $[\mathcal{Q}\varphi] = [\varphi]$

Now, suppose that $\mathcal{Q}$ is a non-informative projection operator, and consider any sentence φ. By definition, the proposition $[\mathcal{Q}\varphi]$ must coincide with $[\varphi]$. Moreover, since $\mathcal{Q}\varphi$ has to be a question, by fact 39 the presupposition $\pi(\mathcal{Q}\varphi)$ must coincide with $\mathsf{info}(\mathcal{Q}\varphi)$ and thus with $\mathsf{info}(\varphi)$, since $[\mathcal{Q}\varphi] = [\varphi]$. Hence, the semantics of a non-informative projection operator is uniquely determined by the above two requirements.

Fact 40 (Uniqueness of the non-informative projection operator in InqP)**.**
$\mathcal{Q}$ is a non-informative projection operator iff $[\mathcal{Q}\varphi] = [\varphi]$ and $\pi(\mathcal{Q}\varphi) = \mathsf{info}(\varphi)$.

But this is precisely the behavior that we assigned to the closed interrogative operator. Thus, $?_c$ is the non-informative operator in InqP.

Fact 41 ($?_c$ is the non-informative projection operator in InqP)**.**

- The interrogative operator $?_c$ is a non-informative projection operator.

- If $\mathcal{Q}$ is a non-informative projection operator, then $[\mathcal{Q}\varphi] = [?_c\varphi]$ and $\pi(\mathcal{Q}\varphi) = \pi(?_c\varphi)$ for all φ.

In fact, $?_c$ is in certain ways more suitably regarded as a non-informative projection operator than $?_o$. For instance, as we might expect, projecting first along one component and then along the other always lead us to the zero point of the space. In other words, trivializing first one component and then the other always results in a tautology.

Fact 42. For any sentence φ,

- $[?_c!\varphi] = [!?_c\varphi]$

- $?_c!\varphi$ and $!?_c\varphi$ are tautologies.[7]

This does not hold for $?_o$. While $!?_o\varphi$ is always a tautology, $?_o!\varphi$ may be inquisitive: for instance, $?_o!p$ is equivalent with $?_op$, which is not a tautology but rather corresponds with a polar question.

The division fact, which says that the full meaning of a sentence φ can always be reconstructed as the conjunction of the two projections, also holds for the operator $?_c$, provided that we pay attention to the ordering: φ is always equivalent with $!\varphi \wedge ?_c\varphi$, but, due to the way in which the presuppositions of a conjunction project, φ is not always equivalent with $?_c\varphi \wedge !\varphi$.

Fact 43 (Division in InqP)**.** For any φ,

$$[\varphi] = [!\varphi \wedge ?_o\varphi] = [!\varphi \wedge ?_c\varphi]$$

7.2   Impositions

The proposition expressed by a sentence φ in a state s captures the ways in which a speaker proposes to enhance that state s. Both in InqB and in InqP, the meaning of a sentence determines the proposition expressed by a sentence in a state, the only difference being that in InqP, the meaning of a sentence does not necessarily deliver a well-defined proposition for every state, thereby capturing the presupposition of the sentence. Thus, in InqP the meaning of a sentence does not only capture the ways in which a speaker proposes to enhance the common ground in uttering that sentence, but also the presuppositions that he makes concerning the current state of the common ground.

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[7]Notice that tautologies are conceived of here as sentences that are neither informative nor inquisitive. They are not necessarily non-presuppositional. Indeed, $?_c!\varphi$ and $!?_c\varphi$ may very well be presuppositional.

Given this picture, it is natural to distinguish a third aspect of meaning as well. Namely, besides making certain presuppositions concerning the current state of the common ground, and proposing one or more ways to enhance the common ground, a speaker may sometimes also *impose* certain enhancements on the common ground.

The linguistic relevance of this three-way distinction between presuppositions, propositions, and *impositions* has been argued for by Murray (2010), AnderBois *et al.* (2010), and Pruitt and Roelofsen (2011).[8] In particular, it has been suggested that there is a wide range of linguistic constructions, including evidentials and appositives, whose usage typically imposes a certain enhancement on the common ground.

To illustrate this, consider the following example:

(13)   Jane, who damaged her car yesterday, called Ben to ask for money.

The idea is that in uttering this sentence, a speaker presupposes that Jane has a car, imposes that she damaged it yesterday, and proposes that she called Ben to ask for money.

There are at least two empirical differences between 'what is proposed' on the one hand, and 'what is presupposed' and 'what is imposed' on the other hand (see, e.g., AnderBois *et al.*, 2010; Pruitt and Roelofsen, 2011). One difference is that an existing question under discussion is naturally addressed by what is proposed but not by what is presupposed or imposed.

(14)   a.   Did Jane call Ben to ask for money?
        b. ✓Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

(15)   a.   Does Jane have a car?
        b. #Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

(16)   a.   Did Jane damage her car yesterday?
        b. #Yes, Jane, who damaged her car yesterday, called Ben to ask for money.

Second, there is a difference, at least in English, in how a disagreeing re-

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[8]The notion of impositions is also closely related to Horn's (2002) notion of *assertorically inert* implications, and to the notion of *suggestions* in (Groenendijk, 2008; Groenendijk and Roelofsen, 2009; Balogh, 2009; Roelofsen and van Gool, 2010).
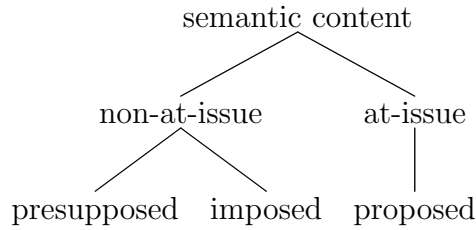
Figure 11: High-level taxonomy of semantic content.

sponse to a given utterance is marked, depending on whether it disagrees with what is proposed, or with what is presupposed or imposed. In particular, a response that disagrees with what is proposed is naturally marked with the particle *no*, while a response that disagrees with what is presupposed or imposed is more naturally marked with a weaker particle like *actually*.

(17)   Jane, who damaged her car yesterday, called Ben to ask for money.
    a.   No, she didn't call Ben to ask for money.
    b.   (#No / ✓Actually), she doesn't have a car.
    c.   (#No / ✓Actually), she didn't damage her car yesterday.

In light of these empirical differences, we say that proposed updates constitute *at-issue* content, while presuppositions and imposed updates constitute *non-at-issue* content. This terminology is in line with much other recent literature on at-issue and non-at-issue content (e.g. Potts, 2005; Roberts *et al.*, 2009), even though the three-way distinction between presuppositions, imposed updates, and proposed updates is not generally accepted in this literature. The overall high-level taxonomy of the different types of semantic content that arises is depicted in figure 11.

A simple way to extend any of the systems considered so far, InqB, InqA, or InqP, in order to incorporate the notion of impositions would be to define meanings as functions that map a state s to a pair (i, A), where $i \subseteq s$ is an imposition over s, modeled as a set of worlds, and A is a proposition over i. Two such imposition-proposition pairs are depicted in figure 12. Impositions are represented by shapes with dashed borders, while proposition are represented by shapes with solid borders.

First consider the imposition-proposition pair depicted in figure 12(a). Let 11 be a world where Jane damaged her car yesterday and called Ben to
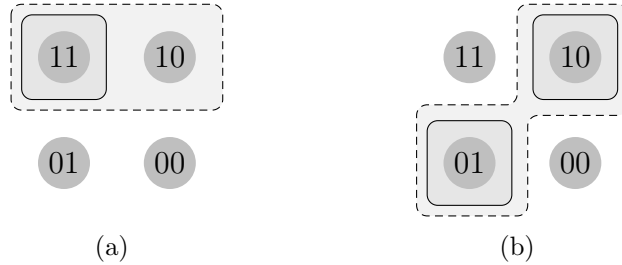
Figure 12: Two imposition-proposition pairs.

ask for money, 10 a world where Jane damaged her car yesterday but did not call Ben to ask for money, etcetera. Then the imposition-proposition pair depicted in figure 12(a) is the one expressed by our example sentence (13) in the state $s = \{11, 10, 01, 00\}$ (assuming that this state satisfies the presupposition of the sentence, i.e., that Jane has a car in all these worlds). In uttering (13) in s, a speaker imposes an update on s that restricts it to $\{11, 10\}$, i.e., worlds where Jane damaged her car yesterday, and proposes a further update that would restrict the state to $\{11\}$, i.e., the world where Jane called Ben to ask for money.

Now consider the imposition-proposition pair depicted in figure 12(b). Let 11 be a world where Jane damaged her car and her bike yesterday, 10 a world where she only damaged her car, 01 a world where she only damaged her bike, and 00 a world where she did not damage either. Then the imposition-proposition pair depicted in figure 12(b) can be taken to be the one expressed in the state $s = \{11, 10, 01, 00\}$ by the alternative question in (18) (where $\uparrow$ and $\downarrow$ represent rising and falling pitch, respectively).

(18)   Did Jane damage her car$\uparrow$ or her bike$\downarrow$ yesterday?

In uttering this sentence in s, a speaker imposes an update on s which eliminates the worlds 11 and 00, leaving only those worlds where Jane damaged either her car or her bike, but not both. Moreover, the speaker proposes two further updates, eliminating either world 10 or world 01, and thus establishing whether it was Jane's car or bike that she damaged. An account along these lines of the 'exclusive component' of alternative questions has been proposed in Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011), and is also closely related to the account of Karttunen and Peters (1976).

As we have seen, the exclusive component of alternative questions can also

be treated as a presupposition, rather than an imposition, and indeed it has often been treated as such in the literature (e.g. Rawlins, 2008; Aloni *et al.*, 2009; Aloni and Égré, 2010; Biezma, 2009; AnderBois, 2010; Haida, 2010; Biezma and Rawlins, 2012). It is difficult to decide on an empirical basis which treatment is more appropriate. Strictly speaking, the presupposition account predicts that an alternative question like (18) is *uninterpretable* in a state that does not yet contain the information that Jane damaged either her car or her bike. This prediction is clearly too strong. However, it can be avoided by assuming that presuppositions may be *accommodated* in case they are not directly satisfied by the input context. With this additional assumption, however, it becomes difficult, if not impossible, to distinguish presuppositions empirically from impositions.

One theoretical option would be to reserve the notion of presuppositions for real requirements on the input context which, if not met, give rise to uninterpretability, and to re-conceptualize accommodable presuppositions as imposed updates. But we do not take a strong stance on this issue here.

Of course, once the notion of impositions is adopted, one of the main issues that arises is how they should be derived compositionally. We refer to Murray (2010), AnderBois *et al.* (2010), and Pruitt and Roelofsen (2011) for discussion of this issue, but also note that many aspects in this area are in need of further exploration.

7.3 Attentive content

In InqB, a proposition A over a state s is a non-empty, downward closed set of enhancements of s. The elements of A are states where the information provided by A has been accepted and the issue raised by A has been settled. In this way, propositions embody both informative and inquisitive content.

However, there is a natural way to further enrich this notion of propositions, in such a way that they capture even more than just informative and inquisitive content. Namely, suppose that we define a proposition A over a state s simply as a non-empty set of enhancements of s, without requiring downward closedness. Then we can still think of A as capturing informative content. Namely, as before, we could think of A as providing the information that the actual world is located in $\bigcup A$, and we could think of A as requesting sufficient information to locate the actual world inside one of the elements of A.

Thus, the elements of A are still states where the information provided

by A has been accepted and the issue raised by A has been settled. However, the difference is that A no longer necessarily contains *all* such states; it may contain just some of them. This, then, allows us to think of A as capturing another aspect of meaning, which we will refer to as *attentive content*. The idea is that the states in A are enhancements of s that A draws particular attention to. They are in some sense 'priviliged' among all the states that accept the informative content and settle the inquisitive content of A.

This alternative notion of propositions gives rise to a different implementation of inquisitive semantics, which we refer to as InqA. In this setting, it is of course no longer justified to require that propositions be downward closed. After all, they may very well draw attention to a particular enhancement t of s, without drawing attention to any further enhancement t' of t. Thus, propositions are defined as arbitrary non-empty sets of enhancements.

Definition 42 (Propositions in InqA).
A proposition over a state s is a non-empty set of enhancements of s.

7.3.1 Informative, inquisitive, and attentive content

As in InqB, the informative content of a proposition A is embodied by the union of all the states in A.

Definition 43 (Informative content). $\mathsf{info}(A) := \bigcup A$

The inquisitive content of a proposition A is now embodied by its *downward closure*, i.e., $A^{\downarrow} := \{t \subseteq s \mid s \in A\}$.

Definition 44 (Inquisitive content). $\mathsf{inq}(A) := A^{\downarrow}$

In InqB, the proposition expressed by a sentence is completely determined by its informative and inquisitive content. This is not the case in InqA. To see this, consider the two propositions depicted in figure 13. In both cases, we have depicted all the states that the proposition consists of (not just the maximal states, as we did in InqB). These two propositions are clearly different, but in terms of informative and inquisitive content they are equivalent: in both cases the union of all the possibilities is $\{11, 10\}$, and the downward closure is $\{\{11, 10\}, \{11\}, \{10\}, \emptyset\}$.

The reason that the informative and inquisitive content of a sentence do not completely determine the proposition expressed by that sentence in InqA is precisely that in uttering a sentence that expresses a proposition A, a
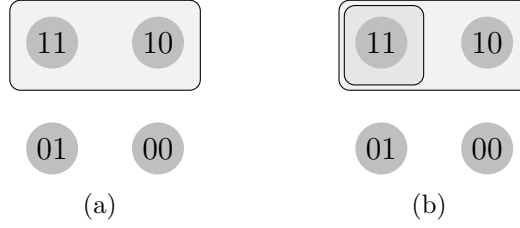
Figure 13: Two propositions in InqA that are equivalent in terms of informative and inquisitive content.

speaker is not just taken to provide the information embodied by $\mathsf{info}(A) = \bigcup A$ and raise the issue embodied by $\mathsf{inq}(A) = A^{\downarrow}$, but also to draw attention to some specific states in $A^{\downarrow}$. Thus, propositions in InqA do not only embody informative and inquisitive content, but also *attentive content*.

Definition 45 (Attentive content). $\mathsf{att}(A) := A$

7.3.2 A first-order system

Propositions in InqA may be associated with sentences in a first-order language. There are several ways in which this could be done. Below we specify one particular system, originally presented in Ciardelli (2009); Ciardelli, Groenendijk, and Roelofsen (2009).[9]

Definition 46 (A first-order system with propositions as proposals).

1. $[R(t_1 \ldots t_n)] := \{|Rt_1 \ldots t_n|\}$

2. $[\bot] := \{\emptyset\}$

3. $[\varphi \wedge \psi] := [\varphi] \sqcap [\psi]$

4. $[\varphi \vee \psi] := [\varphi] \cup [\psi]$

5. $[\varphi \to \psi] := \{\bigcap_{s \in A} s \Rightarrow f(s) \mid f \in [\psi]^{[\varphi]}\}$

6. $[\forall x.\varphi(x)] := \prod_{d \in D} [\varphi(\bar{d})]$

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[9]In this definition, $[\varphi] \sqcap [\psi]$ denotes the pointwise intersection of $[\varphi]$ and $[\psi]$, i.e., $\{s \cap t \mid s \in [\varphi] \text{ and } t \in [\psi]\}$, $s \Rightarrow f(s)$ denotes the pseudo-complement of s relative to $f(s)$, i.e., $\bar{s} \cup f(s)$, and $[\psi]^{[\varphi]}$ denotes the set of all functions from $[\varphi]$ to $[\psi]$.

7. $[\exists x.\varphi(x)] \coloneqq \bigcup_{d \in D} [\varphi(\overline{d})]$

We refer to Ciardelli (2009); Ciardelli *et al.* (2009) for detailed discussion and illustration of this system, and to Westera (2012) for discussion of a closely related system, which differs from the present one in its treatment of implication. It should be noted that an algebraic motivation for a particular treatment of the logical constants in InqA is not available at this point. This is partly due to the fact that, unlike in InqB, it is not so straightforward to say when one proposition *entails* another. In InqB, propositions are compared in terms of their informative and inquisitive content. In InqA, they should also be compared in terms of their attentive content. When two propositions have exactly the same informative and inquisitive content, it is easy to say whether one is more attentive than the other. However, in other cases this is not always clear. There are several possible routes to take, but none of the ones considered so far has lead to a satisfactory algebraic motivation for a particular treatment of the logical constants in InqA.

7.3.3 Linguistic relevance

Propositions are more fine-grained in InqA than in InqB. This can be useful for several linguistic purposes. Broadly speaking we see two main advantages that InqA may have over InqB. First, it provides a more suitable basis for defining formal notions of *relatedness*, in particular answerhood and subquestionhood. And second, it allows for a new semantic perspective on what may be called *attentive operators* in natural language, i.e., operators whose semantic contribution is mainly concerned with attentive content, rather than informative or inquisitive content. Below we will say a bit more about these two potential areas of application for InqA.

Formal notions of relatedness. Consider the following contrast (see also Westera, 2012):

(19) a. Alf: Sally will bring wine or juice.
      b. Bea: (Actually,) she will bring both.

(20) a. Alf: Sally will bring wine or juice, or both.
      b. Bea: (*Actually,) she will bring both.

Note that Bea's response may be preceded by *actually* in (19), but not in (20). Intuitively, what *actually* seems to indicate is that the given response
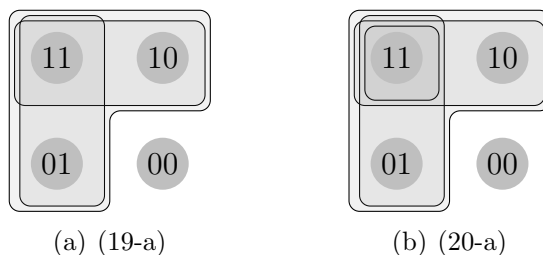
(a) (19-a)   (b) (20-a)

Figure 14: Possible treatment of (19-a) and (20-a) in InqA.

is possibly unexpected. It can be used by Bea in (19) because given Alf's initiative, the response is indeed possibly unexpected. In (20) on the other hand, we cannot conclude from Alf's initiative that Bea's response is possibly unexpected. Quite on the contrary, Bea's response is one of the expected responses. And therefore *actually* cannot be used in this case.

In order to turn this intuitive assessment of this particular example into a general theory of the use of *actually* (and other discourse particles / intonation patterns), we need to be able to distinguish in a systematic way between responses to a given initiative that are expected and responses that are possibly unexpected. InqB is not fine-grained enough for this purpose. In particular, since propositions are downward closed in InqB, it is impossible to assign two different propositions to the two sentences that Alf utters in (19) and (20), respectively. Thus, these two sentences come out as semantically equivalent, and there is no way of capturing the fact that Bea's response is possibly unexpected as a reaction to one but not the other.

In InqA on the other hand, the sentences that Alf utters in (19) and (20) may be assigned different propositions. The exact treatment of these sentences depends on the perspective that we take on inquisitiveness, and possibly also on the intonation pattern with which the sentences are pronounced. If we assume a strong perspective on inquisitiveness and an intonation pattern which indicates that Alf invites, but does request an informative response in uttering (19-a) and (20-a), then these sentences may be associated with the propositions depicted in figure 14.[10]

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[10]These propositions may be derived compositionally by assuming that the declarative complementizer functions as a non-inquisitive projection operator !, whose semantic contribution in InqA is defined as follows: $[!\varphi] := [\varphi] \cup \{\bigcup[\varphi]\}$. See Ciardelli, Groenendijk, and Roelofsen (2010).

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Once our semantics is fine-grained enough to distinguish between (19-a) and (20-a), we are in a position to try to capture the distinction between expected and possibly unexpected responses to a given sentence. One hypothesis, for instance, would be that ψ is an expected response to φ if $[\varphi] \subseteq [\psi]$, and that it is a possibly unexpected response otherwise. This hypothesis would correctly predict that (19-b) is a possibly unexpected response to (19-a) but not to (20-a).

This is just to illustrate that it is possible in InqA to characterize certain notions of relatedness, in this case a particular notion of answerhood, that are impossible to capture in InqB. Even if the hypothesis formulated here is too simplistic, the framework allows for alternative, more sophisticated hypotheses as well. And besides the particular notion of answerhood illustrated here, other notions of relatedness may of course be of interest as well.

Attentive operators. A second area of application is the analysis of operators in natural language whose semantic contribution is mainly concerned with attentive content, rather than informative or inquisitive content. In Ciardelli *et al.* (2009, 2010) it is argued that English *might*, at least on some of its usages, can be seen as such an operator. Usually, *might* is analyzed as an epistemic possibility modal. However, it is well-known that *might* interacts with the propositional connectives in peculiar ways. In particular, it behaves differently in this respect from expressions like 'it is possible that' or 'it is consistent with my beliefs that', which is problematic for any account that analyzes *might* as an epistemic modal. Its analysis as an attentive operator sheds new light on this issue.

Let us illustrate this with some concrete examples. Consider the sentences in (21), (22), and (23). In order to deal with such sentences we enrich our logical language with an operator, $\diamond$, which is intended to correspond to *might* in English. With the addition of this operator, each English sentence in (21)–(23) has a straightforward translation into our logical language, which is given to its right.

(21)   John might be in Paris or in London. $\diamond(p \vee q)$

(22)   John might be in Paris or he might be in London. $\diamond p \vee \diamond q$

(23)   John might be in Paris and he might be in London. $\diamond p \wedge \diamond q$

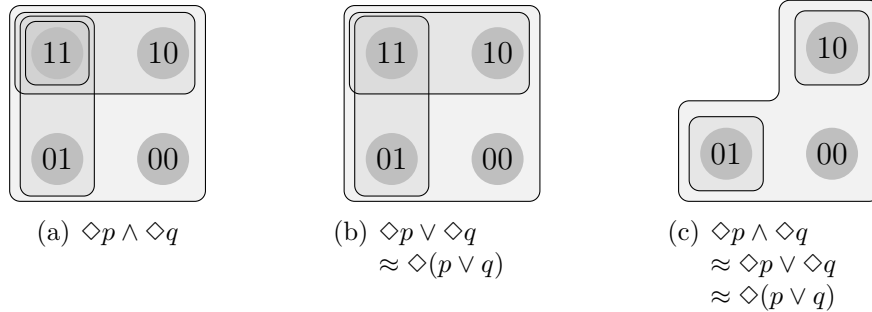Zimmermann (2000, p.258–259) observed that (21), (22), and (23) are all

Figure 15: *Might* interacting with conjunction and disjunction.

equivalent.[11] This is not the case for similar sentences with clear-cut epistemic modalities. For instance, (24) is clearly not equivalent with (25).

(24)   It is consistent with my beliefs that John is in London or it is consistent with my beliefs that he is in Paris.

(25)   It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

This contrast is problematic for modal accounts of *might*. A further subtlety is that Zimmermann's observation seems to crucially rely on the fact that 'being in London' and 'being in Paris' are mutually exclusive. To see this, consider the following examples:

(26)   John might speak English or French. $\Diamond(p \vee q)$

(27)   John might speak English or he might speak French. $\Diamond p \vee \Diamond q$

(28)   John might speak English and he might speak French. $\Diamond p \wedge \Diamond q$

'Speaking English' and 'speaking French' are not mutually exclusive, unlike 'being in London' and 'being in Paris'. As a result, the equivalence partly breaks down: (26) and (27) are still equivalent with each other, but not with (28). To see this, consider a situation, suggested to us by Anna Szabolcsi, in which someone is looking for an English-French translator, i.e.,

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[11]These type of examples have also often been discussed in the recent literature in relation to the phenomenon of *free choice permission*, which involves deontic modals (cf. Geurts, 2005; Simons, 2005a; Alonso-Ovalle, 2006; Aloni, 2007b; Fox, 2007; Klinedinst, 2007; Chemla, 2009; Aher, 2012).

someone who speaks *both* English and French. In that context, (28) would be perceived as a useful recommendation, while (26) and (27) would not.

These patterns can be accounted for quite straightforwardly if *might* is treated as an operator that trivializes the informative and inquisitive content of its complement, but preserves its attentive content. This is achieved by the following treatment of $\diamondsuit$ (Ciardelli *et al.*, 2009, 2010).

Definition 47 (Might in InqA)**.**

- $[\diamondsuit\varphi] \coloneqq [\varphi] \cup \{\omega\}$

For any φ, the proposition expressed by $\diamondsuit\varphi$ consists of all states in $[\varphi]$ plus the 'trivial' state ω. This means that the proposition expressed by $\diamondsuit\varphi$ relative to a particular state s consists of all states in $[\varphi]_s$ plus the state s itself. Thus, in uttering $\diamondsuit\varphi$ in s, a speaker proposes exactly the same enhancements of s that he would have proposed in uttering φ, with the addition of the trivial enhancement, which amounts to leaving s unchanged.

Notice that for any φ, $\mathsf{info}(\diamondsuit\varphi) = \omega$ and $\mathsf{inq}(\diamondsuit\varphi) = \wp(\omega)$. This means that $\diamondsuit\varphi$ is never informative or inquisitive. Thus, $\diamondsuit$ indeed trivializes the informative and inquisitive content of its complement, while preserving the attentive content.

Now let us return to the examples above. The proposition expressed by $\diamondsuit p \wedge \diamondsuit q$ is depicted in figure 15(a), and the proposition expressed by $\diamondsuit(p \vee q)$ and $\diamondsuit p \vee \diamondsuit q$ is depicted in figure 15(b). Notice that $\diamondsuit p \wedge \diamondsuit q$, unlike $\diamondsuit(p \vee q)$ and $\diamondsuit p \vee \diamondsuit q$, draws attention to the state $\{11\}$, which embodies the information that John speaks both English and French. This explains the observation that (28) is perceived as a useful recommendation in the translator-situation, unlike (26) and (27).

In Zimmermann's original example, p stands for 'John is in London' and q for 'John is in Paris'. It is impossible for John to be both in London and in Paris, so in dealing with this particular example, we should assume a logical space that does not contain worlds where p and q are both true, i.e., a logical space consisting of the worlds 10, 01, and 00, but not 11. Relative to this logical space, $\diamondsuit(p \wedge q)$, $\diamondsuit p \vee \diamondsuit q$, and $\diamondsuit p \wedge \diamondsuit q$ all express exactly the same proposition, as depicted in figure 15(c). Thus, the intuition that Zimmermann's original examples are all equivalent is also accounted for.

There is of course much more to say about the treatment of *might* as an attentive operator, about the relation between this account and the modal account, as well as the dynamic account of Veltman (1996), and about other

operators in natural language that may be treated (partially) as attentive operators. We refer to Ciardelli *et al.* (2009, 2010) for further discussion.

7.4   Highlighting

Evidently, one of the empirical domains where inquisitive semantics is intended to be put to use is the semantics of *interrogatives*. We emphasized in section 6 that inquisitive semantics should not be taken to constitute a particular *theory* of interrogatives, but rather a general framework in which several such theories may be formulated and compared. We mentioned in particular the classical theories of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) as ones that may be formulated in inquisitive semantics.

However, there is also a number of existing approaches to the semantics of interrogatives that cannot be articulated in inquisitive semantics, at least not in the most basic implementation of the framework. This holds in particular for the structured meaning approach of von Stechow (1991) and Krifka (2001), the dynamic approach of Aloni and van Rooij (2002); Aloni *et al.* (2007), and the orthoalgebraic approach of Blutner (2012).

Proponents of these approaches have explicitly argued against the classical 'proposition set' approach of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984). One of the main arguments is based on a contrast in the interpretation of polarity particle responses to 'opposing' polar interrogatives (see especially Krifka, 2001; Blutner, 2012):

(29)   Is the door open?

    a.   Yes $\Rightarrow$ the door is open
    b.   No $\Rightarrow$ the door is closed

(30)   Is the door closed?

    a.   Yes $\Rightarrow$ the door is closed
    b.   No $\Rightarrow$ the door is open

In terms of inquisitive content, (29) and (30) are entirely equivalent: each of these polar interrogatives elicits a choice between two possibilities, the possibility that the door is open, and the possibility that the door is closed.[12] However, there is a clear difference between the two interrogatives in terms

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[12]It is assumed here that *open* and *closed* are antonyms, but nothing hinges on this assumption—parallel examples could be constructed with, e.g., *even* and *odd*.

of polarity particle responses: in response to (29), *yes* means that the door is open, and *no* means that the door is closed, whereas in response to (30), *yes* means that the door is closed, and *no* means that the door is open.[13]

A further contrast is noted in Roelofsen and van Gool (2010): while (31) below is also equivalent with (29) and (30) in terms of inquisitive content, it does not license polarity particle responses at all:

(31)   Is the door open or closed?
         a. #Yes.
         b. #No.

A semantic account of this contrast is beyond the reach of theories that are formulated in InqB, or in any of the more fine-grained systems considered so far. However, in our view these observations do not constitute an argument against the framework in general. Rather, they show that there is an aspect of meaning that the systems considered so far do not yet capture.

In particular, while these systems do suitably capture informative and inquisitive potential, they do not yet capture the *anaphoric potential* of sentences, i.e., the potential to set up discourse referents that may serve as antecedents for subsequent anaphoric expressions.

To overcome this limitation, and more specifically to account for the contrast in (29)–(31), Roelofsen and van Gool (2010), Pruitt and Roelofsen (2011), and Farkas and Roelofsen (2012) develop an extension of InqA in which a sentence may *highlight* some of the possibilities that make up the proposition that it expresses. When a sentence is uttered, these highlighted possibilities become available as antecedents for subsequent anaphoric expressions. Polarity particles, then, can be analyzed as such anaphoric expressions: they either confirm or reject the highlighted antecedent possibilities.

Intuitively, the possibilities that are highlighted are the ones that are *explicitly mentioned.* The idea is that, in virtue of being mentioned explicitly, these possibilities are made more salient than other possibilities, and therefore more readily accessible for subsequent anaphoric elements.[14]

Consider for instance the polar question in (29). There is an intuitive

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[13]It should be noted that Groenendijk and Stokhof (1984, pp.321–323) actually provide an explicit account of the interpretation of *yes* and *no* that captures the difference between (29) and (30).

[14]This formulation should of course not be taken all too literally—strictly speaking, possibilities do not get 'mentioned' by expressions in the object-language.
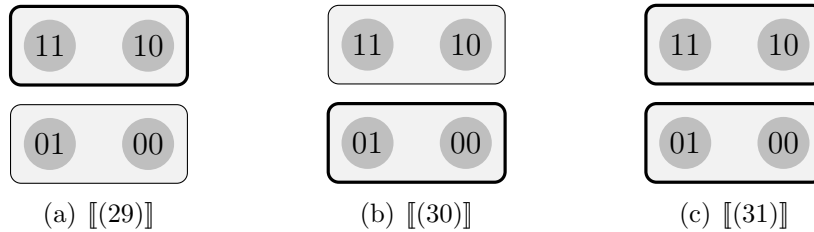
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Figure 16: Highlighted and non-highlighted possibilities for (29)–(31).

sense in which the possibility that the door is open is explicitly mentioned by this question, while the possibility that the door is closed is not. So (29) highlights the possibility that the door is open, while (30) highlights the possibility that the door is closed, and (31) highlights both possibilities.[15] This is depicted in figure 16, where 11 and 10 are worlds where the door is open, 01 and 00 are worlds where the door is closed, and highlighted possibilities are displayed with a thick border.

Now, if we assume that *yes* presupposes a unique highlighted possibility, and confirms this possibility in case the presupposition is met, while *no* presupposes one or more highlighted possibilities, and rejects all of these possibilities in case its presupposition is met, we obtain a straightforward account of the observed contrast in (29)–(31). In the case of (29), there is exactly one highlighted possibility, so both *yes* and *no* are licensed: *yes* confirms the highlighted possibility, conveying that the door is open, while *no* rejects the highlighted possibility, conveying that the door is closed. In the case of (30), there is again exactly one highlighted possibility, only now this is the possibility that the door is closed. So, again, both *yes* and *no* are licensed, only now *yes* conveys that the door is closed, while *no* conveys that the door is open. Finally, in the case of (31) there are two highlighted possibilities. This means that *yes* is not licensed because its presupposition fails, while *no* is contradictory, since the two highlighted possibilities together cover the entire logical space.

For further details and extension of the account of polarity particles sketched here, as well as further motivation for the notion of highlighting, we refer to Pruitt and Roelofsen (2011) and Farkas and Roelofsen (2012).

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[15]For details on how highlighted possibilities are computed compositionally we refer to Roelofsen and van Gool (2010); Pruitt and Roelofsen (2011); Farkas and Roelofsen (2012).

8   Inquisitive pragmatics and discourse

In this section we outline the role that inquisitive semantics is intended to play in an overall theory of interpretation and information flow in discourse. In section 8.1 we discuss how inquisitive semantics, in enriching the basic notion of semantic meaning, also gives rise to a richer perspective on pragmatics. In section 8.2 we show how inquisitive semantics can be integrated with dynamic epistemic logic (van Ditmarsch *et al.*, 2007; van Benthem, 2011, among others), obtaining a formal framework which does not only allow us to explicitly model the semantic interpretation of sentences, but also the flow of information that takes place when these sentences are uttered in discourse.

8.1   Inquisitive pragmatics

The main objective of Gricean pragmatics (Grice, 1975, and much subsequent work) is to explain aspects of interpretation which are not directly dictated by semantic content, in terms of general features of rational human behaviour. Since inquisitive semantics enriches the basic notion of semantic content, it gives rise to a new perspective on pragmatics as well.

The Gricean maxims specify what it means for the participants of a conversation to behave rationally. However, the theory as it has been developed so far has two important limitations. First, it is exclusively *speaker-oriented*, and second, it is only concerned with what it means for speakers to behave rationally in *providing* information, and not, for instance, in requesting information.

To illustrate this point, consider Grice's Quality and Quantity maxims. The Quality maxim says that a speaker should only provide information that is supported by his own information state, and the Quantity maxim says that a speaker should provide as much information as possible, as long as the information is relevant for the current purposes of the conversation. Clearly, both maxims are speaker-oriented, and only concerned with what it means to behave rationally in providing information.

Inquisitive semantics gives rise to a pragmatics which is both speaker-oriented and hearer-oriented, and which is not only concerned with what it means to behave rationally in providing information, but more generally with what it means to behave rationally in *exchanging* information.

What it means for a particular participant to behave rationally in exchanging information partly depends, of course, on the overall goals of that

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participant. For instance, her intention may be to obtain a certain piece of information from other participants, while concealing other pieces of information that she herself already has access to. We will focus here, however, on the case in which all participants try to resolve a given issue as effectively as possible in a fully cooperative way. Below we discuss two qualitative requirements that all participants should adhere to in such a cooperative effort, *sincerity* and *transparency*. For discussion of quantitative preferences and a formal notion of relevance that play a role in this setting, we refer to Groenendijk and Roelofsen (2009).

Sincerity. First of all, participants must be *sincere*. This requirement is comparable to Grice's Quality maxim. However, in the present setting, several types of sincerity can be distinguished. First, if a speaker utters a sentence φ, she must believe that the actual world is located in at least one of the states in $[\varphi]$. This means that the speaker's information state should be contained in $\mathsf{info}(\varphi)$. We refer to this requirement as *informative sincerity*.

Second, if φ is inquisitive relative to the common ground, then it should also be inquisitive relative to the speaker's own information state. Otherwise, the speaker would be raising an issue that she could just as well have settled herself. We refer to this requirement as *inquisitive sincerity*.

And third, if a speaker draws attention to a particular enhancement of the common ground, then that enhancement should be compatible with her information state. Formally, this means that the proposition that φ expresses in InqA, which captures the attentive content of φ, should only contain states that have at least one world in common with the speaker's information state. We refer to this requirement as *attentive sincerity*.

To illustrate these requirements and the implicatures that they give rise to, consider the following examples.

(32)   Does John speak French? $?p$

(33)   John speaks English or French. $!(p \vee q)$

The propositions that we take these sentences to express in InqA are depicted in figure 17. First consider (32). Suppose that a speaker S with information state σ utters this sentence, and suppose that no common information has been established in the conversation yet, which means that the common ground amounts to ω. First note that S cannot fail to be informatively sincere in uttering (32), since the informative content of the sentence is trivial. Next,
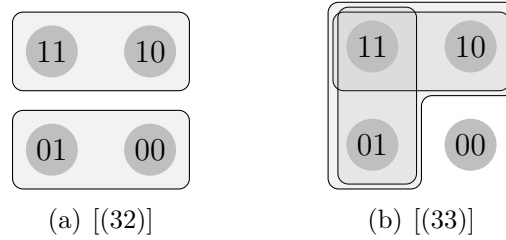
(a) [(32)]   (b) [(33)]

Figure 17: The propositions expressed by (32) and (33) in InqA.

note that (32) is inquisitive in the given common ground. Thus, in order for S to be inquisitively sincere, (32) should also be inquisitive in σ. This means that both states in [(32)] should overlap with σ. Thus, we derive as sincerity implicatures that S should consider it possible that John speaks French and that S should consider it possible that John does not speak French. Notice that this is also precisely what is needed for S to be attentively sincere in uttering (32). So the attentive sincerity requirement does not give rise to additional implicatures in this case. Notice that the ignorance implicature that arises here is inherently linked to inquisitiveness and attentiveness, and cannot be derived straightforwardly from the standard Gricean maxims.

Next, consider (33), which we translate into our formal language as $!(p \vee q)$. Again, suppose that a speaker S with information state σ utters this sentence, and suppose that the common ground amounts to ω. In this case, the informative sincerity requirement is not trivially met. Rather, σ should be contained in $\mathsf{info}(!(p \vee q))$, which means that S should believe that John speaks English or French. Second, note that $!(p \vee q)$ is not inquisitive in the given common ground (or in any other state for that matter), which means that the inquisitive sincerity requirement is trivially satisfied. However, the attentive sincerity requirement is only satisfied if every state in $[!(p \vee q)]$ has a non-empty overlap with σ. Thus, in this case we derive as sincerity implicatures that S should consider it possible that John speaks English and that S should consider it possible that John speaks French.

We should note that the attentive sincerity requirement is sometimes outweighed by efficiency considerations. To see this, consider the sentences in (34):

(34)   a.   John speaks a European language.
         b.   Which European languages does John speak?

Presumably, the propositions expressed by these two sentences both contain the state consisting of all worlds where John speaks French. However, intuitively, a speaker who utters these sentence does not implicate that she considers it possible that John speaks French. Strictly speaking, a speaker who utters these sentences knowing that John does not speak French, violates the attentive sincerity requirement. However, there is a tradeoff between sincerity and efficiency in these cases. For instance, instead of uttering (34-b), the speaker may explicitly list all the European languages of which she considers it possible that John speaks them, but this is likely to be a less efficient move (depending on how many such languages there are). Thus, in uttering sentences like (34-a-b), a speaker does implicate that she does not know exactly which European languages John speaks, but she does not implicate that for every European language she does not know whether John speaks that language.

Notice that this tradeoff does not exist in the case of (32) and (33). More generally, the attentive sincerity requirement may be outweighed by efficiency considerations in the case of indefinites and *wh*-interrogatives, but not in the case of disjunctions and polar interrogatives.

Transparency. Sincerity is a speaker-oriented qualitative requirement. Its hearer-oriented counterpart is *transparency*. If a speaker draws attention to a particular enhancement of the common ground which is inconsistent with the hearer's information state, then the hearer must publicly signal this inconsistency, to make sure that the enhancement is indeed not established. On the other hand, if one participant makes a certain proposal and no other participant objects, then each participant must incorporate the informative content of the proposal into her own information state and into her representation of the common ground.

To illustrate this requirement consider the disjunctive statement in (33) above. Suppose that this sentence is uttered and that one of the participants knows that John does not speak English. Then, in order to satisfy the transparency requirement, she should publicly announce that John does not speak English, even if she does not know whether or not John speaks French. On the other hand, if none of the participants objects to the proposal that is made in uttering (33), then all participants should update their own information state and their representation of the common ground with the information that John speaks English or French.

Sincerity, transparency, and attentive *might*.   To end this subsection, we will briefly consider the basic repercussions of the qualitative sincerity and transparency requirements just discussed for the treatment of *might* as an attentive operator sketched in section 7.3.

There are two basic empirical observations concerning *might* that we did not discuss at all in section 7.3, even though each of them has given rise to one of the two 'classical' semantic theories of *might*. Both observations can be illustrated by means of the following example:

(35)   John might speak English.

The first observation is that if someone utters (35) we typically conclude that she considers it possible that John speaks English. This observation has given rise to the classical analysis of *might* as an epistemic modal operator.

The second observation is that if someone hears (35) and already knows that John does not speak English, she will typically object, pointing out that (35) is inconsistent with her information state. In this sense, even though *might* sentences do not provide any information about the state of the world, they can be 'inconsistent' with a hearer's information state. The standard account of this observation is that of Veltman (1996). Veltman's update semantics specifies for any given information state σ and any given sentence φ, what the information state $\sigma[\varphi]$ is that would result from updating σ with φ. The update effect of $\Diamond\varphi$ is defined as follows:

$$\sigma[\Diamond\varphi] = \left\{ \begin{array}{ll} \emptyset & \text{if } \varphi \text{ is inconsistent with } \sigma \\ \sigma & \text{otherwise} \end{array} \right.$$

The idea is that, if φ is inconsistent with a hearer's information state, then updating with $\Diamond\varphi$ leads to the absurd state. To avoid this, the hearer must make a public announcement signaling the inconsistency of φ with her information state. As a result, the participant who uttered $\Diamond\varphi$ in the first place may also come to discard the possibility that φ holds.

Our semantic treatment of *might* as an attentive operator does not directly explain these two observations. However, both observations can be explained pragmatically. On the one hand, it follows from the attentive sincerity requirement that a cooperative speaker who utters (35) must consider it possible that John speaks English. On the other hand, it follows from the transparency requirement that if a hearer is confronted with (35), and one of the possibilities for φ is inconsistent with her information state, then she must signal this inconsistency.

Thus, both observations are accounted for. And this pragmatic account, unlike the semantic analyses just mentioned, extends straightforwardly to more involved cases. Consider for instance:

(36)   John might speak English or French.

This sentence is problematic for both semantic accounts. The epistemic modality account predicts that the speaker considers it possible that John speaks English or French. But note that this is compatible with the speaker knowing perfectly well that John does not speak English. What (36) implies is something stronger, namely that the speaker considers it possible that John speaks English *and* that she considers it possible that John speaks French. This follows straightforwardly on our pragmatic account.

Now consider a hearer who is confronted with (36) and who knows that John possibly speaks French, but certainly not English. We expect this hearer to object to (36). But Veltman's update semantics does not predict this: it predicts that an update with (36) has no effect on the hearer's information state. Our pragmatic account on the other hand, does urge the hearer to object.

The only task of our semantics is to specify the proposition expressed by each sentence, and thus the proposal that would be made in uttering that sentence. The pragmatics, then, specifies what a context—in particular, the common ground and the information state of the speaker—must be like in order for a certain proposal to be made, and how a hearer is supposed to react to a given proposal, depending on the common ground and her own information state. In the case of *might* sentences, shifting some of the weight from semantics to pragmatics evades problems with more involved cases, like (36), in a straightforward way. But, of course, the necessary pragmatic principles can only be stated if the underlying semantics captures more than just informative content.

For further discussion of the interplay of semantics and pragmatics in the interpretation of *might* sentences, especially in embedded contexts, we refer to Ciardelli *et al.* (2009, 2010); Roelofsen (2011b).

8.2   Modeling inquisitive discourse

Inquisitive semantics provides a framework which allows us to formally capture the semantic interpretation of a sentence, and we have just sketched how

certain further pragmatic inferences may be accounted for as well. However, eventually all this should be embedded in a formal framework that does not only allow us to specify the semantic interpretation of a sentence and derive further pragmatic inferences, but which also allows us to explicitly model the flow of information that results from uttering a sentence in discourse.

Such a framework is provided by dynamic epistemic logic (DEL) (van Ditmarsch *et al.*, 2007; van Benthem, 2011, among others), a system that is designed to model explicitly how utterances change the epistemic states of the participants in a conversation. Dynamic epistemic logic builds on ordinary (static) epistemic logic (EL) (Hintikka, 1962), which in turn is an extension of classical propositional logic (CPL). Below we provide a brief overview of epistemic logic and dynamic epistemic logic, and then indicate how inquisitive semantics can be integrated with these formalisms, drawing on Roelofsen (2011c) and Ciardelli (2012).

8.2.1   Epistemic logic

The language of EL is obtained from the language of CPL by adding a knowledge operator K_a for each relevant agent a.[16] Intuitively, $K_a\varphi$ means that agent a knows that φ holds. The semantics of EL is given in terms of Kripke models. A Kripke model for a set of agents $\mathcal{A}$ and a set of proposition letters $\mathcal{P}$ is a tuple:

$$\langle W, w_0, R, V \rangle$$

where W is a set of possible worlds, $w_0 \in W$ is a possible world representing the actual world, $R : \mathcal{A} \to W \times W$ is a function that assigns to every agent $a \in \mathcal{A}$ a binary relation on W capturing, in a way to be made precise presently, the agent's epistemic state in all possible worlds, and $V : \mathcal{P} \to \wp(W)$ is a valuation function which assigns to every atomic sentence a set of worlds in W, i.e., the set of worlds in which that sentence is true.

Intuitively, $\langle w, v \rangle \in R(a)$ means that in world w, agent a considers world v possible. The *epistemic state* $R(a, w)$ of agent a in world w is represented by the set of all worlds that a considers possible in w:

$$R(a, w) := \{v \in W \mid \langle w, v \rangle \in R(a)\}$$

A sentence $K_a\varphi$ is true in a world w just in case φ holds in all worlds in $R(a, w)$.

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[16]The term *knowledge* is used here as a placeholder; many subtly different notions of knowledge and belief can in fact be modeled in epistemic logic.

Definition 48 (Interpretation of knowledge operators in EL).

- $M, w \models K_a \varphi$ iff for all $v \in R(a, w)$, $M, v \models \varphi$

Kripke models do not only capture the agents' knowledge about the configuration of the world, but also their knowledge about other agents' knowledge, and about those other agents' knowledge about yet other agents' knowledge, etcetera. In short, epistemic states in Kripke models embody *higher-order information*. Moreover, epistemic logic is not only concerned with the knowledge of individual agents, but also with various notions of group knowledge. In particular, the logical language is often further extended with an operator C, which stands for *common knowledge*. Intuitively, $C\varphi$ means that it is common knowledge among all agents that φ holds, i.e., everyone knows that φ holds, and everyone knows that everyone knows that φ holds, etcetera. Formally, $C\varphi$ is true in a world w just in case φ holds in any world v such that $\langle w, v \rangle$ is in the transitive closure of $\bigcup_{a \in \mathcal{A}} R(a)$. We will denote the latter relation as R^*, and we will write $R^*(w)$ for $\{v \mid \langle w, v \rangle \in R^*\}$. Then the clause for the common knowledge operator can be formulated as follows.

Definition 49 (Interpretation of the common knowledge operator in EL).

- $M, w \models C\varphi$ iff for all $v \in R^*(w)$, $M, v \models \varphi$

8.2.2 Dynamic epistemic logic

DEL extends EL in such a way that it becomes possible to formally specify and reason about the effects of certain types of speech acts on the epistemic states of the conversational participants. Most work in the DEL tradition has so far focused on one particular type of speech act, namely that of making an *assertion*. To model the effect of an assertion, the logical language of EL is expanded with expressions of the form $[a\!:\!!\varphi]\psi$. Intuitively, $[a\!:\!!\varphi]\psi$ means that an assertion of φ by agent a leads to a state where ψ holds. Formally, $[a\!:\!!\varphi]\psi$ is defined to be true in a world w in a model M just in case ψ is true in w in the model $M^{a:!\varphi}$, which is obtained from M by means of a procedure that captures the effect of an assertion of φ by agent a. There are several ways to define $M^{a:!\varphi}$. For concreteness, we adopt the following definition.

Definition 50 (The effect of an assertion in DEL).

- $M^{a:!\varphi} = \begin{cases} \langle W, w_0, R^{a:!\varphi}, V \rangle & \text{if } M, w_0 \models K_a\varphi \\ \text{undefined} & \text{otherwise} \end{cases}$

where:

- $R^{a:!\varphi}(a) \coloneqq R(a) \cap \{\langle w, v \rangle \mid M, v \models \varphi\}$
- $R^{a:!\varphi}(b) \coloneqq R(b)$ for all $b \neq a$

According to this definition, an assertion of φ by a is only felicitous if in the actual world a knows that φ is the case. If this condition is met, then the effect of the assertion is to make $K_a\varphi$ common knowledge. Otherwise, the effect of the assertion is undefined.

Fact 44. For any M, w such that $M, w_0 \models K_a\varphi$: $M^{a:!\varphi}, w \models CK_a\varphi$

Now that we know how assertions change the model of evaluation, we can specify precisely how sentences of the form $[a:!\varphi]\psi$ are interpreted.

Definition 51 (Interpretation of assertion operators in DEL).

- $M, w \models [a:!\varphi]\psi$ iff $M^{a:!\varphi}, w \models \psi$

Notice that the basic DEL system presented so far is truth-conditional. This means that in this system the semantic meaning of a sentence is identified with its informative content. Moreover, the speech act of making an assertion is completely characterized by the information that is provided in performing that speech act. Thus, inquisitiveness is not yet part of the picture, neither at the level of semantic content, nor at the level of speech acts. Evidently, inquisitiveness does play a crucial role in the process of exchanging information. This has been recognized in recent work within the DEL tradition, in particular by Van Benthem and Minică (2012). In order to bring inquisitiveness into the picture, Van Benthem and Minică enrich the logical language with a second dynamic speech act operator, $[a:?\varphi]$. This operator is used to describe the effects of a speech act of asking whether φ holds, performed by agent a. Intuitively, $[a:?\varphi]\psi$ means that the speech act of asking whether φ holds, performed by agent a, leads to a state where ψ holds. We will refer to the resulting system as DELQ.

A crucial feature of DELQ is that the basic static fragment of the logical language (which can be thought of as the language that the agents in the conversation speak) does not contain any sentences that are interrogative in any syntactic sense, or sentences that are inquisitive in any semantic sense. A question is seen as a specific kind of *speech act* that may be performed by an agent. But in terms of syntactic form and semantic content, sentences that

are used in asking questions are not taken to be any different from sentences that are used in making assertions. In particular, they are not interrogative or inquisitive in any sense.

An alternative approach would be to actually enrich the semantics of the basic static fragment of the logical language, in such a way that the proposition expressed by every sentence in this fragment already captures both its informative and its inquisitive content. This enrichment is precisely what is provided by inquisitive semantics. On this alternative approach, the static fragment of the language could be taken to contain interrogative sentences of the form $?\varphi$, and such sentences could be taken to express inquisitive propositions, embodying the issue of whether φ is the case. The dynamic part of the language could then be simplified: instead of having separate assertion and question operators, $[a\!:\!!\varphi]$ and $[a\!:\!?\varphi]$, we could have a single *utterance operator* $[a\!:\!\varphi]$, where φ could be syntactically indicative or interrogative, and semantically informative and/or inquisitive. Intuitively, $[a\!:\!\varphi]\psi$ would then mean that an utterance of φ by agent a leads to a state where ψ holds. Thus, on this approach, inquisitiveness does not enter the picture at the speech act level, but rather already at the level of the syntax and semantics of the basic static language.

In Roelofsen (2011c) it is argued that this alternative approach has some crucial advantages. Most importantly, it allows us to deal with *embedded* questions, such as conditional questions (e.g., *If John goes to the party, will Mary go as well?*) and questions embedded under knowledge operators (e.g. *John knows whether Mary will go*). This is impossible if questions only enter the picture at the speech act level, because in such a setup the logical language does not contain sentences of the form $p \to\ ?q$ or $K_a?q$.[17]

This second approach gives rise to an *inquisitive epistemic logic* (IEL) and an *inquisitive dynamic epistemic logic* (IDEL). Below we specify and discuss the main features of these systems. For further motivation and discussion we refer to Roelofsen (2011c) and Ciardelli (2012).

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[17]The point that a proper treatment of (embedded) questions requires inquisitiveness to enter the picture at the semantic level, and not just at the speech act level, was also made in Groenendijk and Stokhof (1997). At the time, it was directed mostly at the speech act treatment of questions proposed by Searle (1969) and Vanderveken (1990), and at the imperative-epistemic treatment of questions proposed by Åqvist (1965) and Hintikka (1976, 1983). The argument in Roelofsen (2011c) is essentially the same, but now redirected at the speech act treatment of questions in DEL proposed by Van Benthem and Minică (2012).

8.2.3 Inquisitive epistemic logic

We will start by presenting the system IEL, an inquisitive semantics for the language of epistemic logic, with ! and ? as additional operators. As in InqB, we define $!\varphi$ as an abbreviation of $\neg\neg\varphi$ and $?\varphi$ as an abbreviation of $\varphi \vee \neg\varphi$.

The semantics of IEL is stated in terms of Kripke models, just like the semantics of EL. The only difference is that now, sentences are not evaluated relative to worlds, but relative to *states*, which are sets of worlds, as in InqB. We formulate the semantics of IEL in terms of support (see section 4.7).

Definition 52 (Support)**.** Let M be a model and s a state.

1. $M, s \models p$ iff $s \subseteq V(p)$

2. $M, s \models \bot$ iff $s = \emptyset$

3. $M, s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

4. $M, s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$

5. $M, s \models \varphi \rightarrow \psi$ iff $\forall t \subseteq s :$ if $t \models \varphi$ then $t \models \psi$

6. $M, s \models K_a\varphi$ iff   for all $w \in s$, $R(a, w) \models \varphi$

7. $M, s \models C\varphi$ iff   for all $w \in s$, $R^*(w) \models \varphi$

As in InqB, support is persistent, i.e., if $M, s \models \varphi$ and $t \subseteq s$, then $M, t \models \varphi$ as well. As usual in inquisitive semantics, support is persistent. The atomic clause and the clauses for the Boolean connectives are just as in InqB. The clause for individual knowledge operators says that $K_a\varphi$ is supported by a state s just in case for every world $w \in s$, φ is supported by the epistemic state of agent a in w. The clause for common knowledge says that $C\varphi$ is supported in a state s just in case for every $w \in s$, $R^*(w)$ supports φ. Notice that as a result of this definition, $K_a\varphi$ and $C\varphi$ are never inquisitive, regardless what φ is.

Example 1. IEL provides a unified treatment of knowledge-*that* and knowledge-*whether* constructions, assuming that *that* is translated into our logical language as !, and *whether* as ?. Consider the following sentences:

(37) a. Alex knows that Peter is coming. $K_a!p$
      b. Alex knows that Peter or Quinten is coming. $K_a!(p \vee q)$

c.   Alex knows whether Peter is coming. $K_a?p$

For a state s to support the first sentence, every w in s must be such that $R(a, w)$ supports $!p$, which means that every world in $R(a, w)$ must be one where p holds. Similarly, for s to support the second sentence, every w in s must be such that $R(a, w)$ supports $!(p \vee q)$, which means that every world in $R(a, w)$ must be one where either p or q holds. Finally, for s to support the third sentence, every w in s must be such that $R(a, w)$ supports $?p$, which means that we must either have that every world in $R(a, w)$ is one where p holds, or that every world in $R(a, w)$ is one where $\neg p$ holds. These are precisely the desired predictions for these sentences.[18]

We define the proposition $[\varphi]_M$ expressed by φ in M as the set of all states supporting φ, and the informative content of φ in M, $\mathsf{info}_M(\varphi)$, as $\bigcup[\varphi]_M$. We could also define informative and inquisitive sentences, as well as questions, assertions, and hybrids, exactly as we did in InqB. Finally, the notion of entailment also directly carries over from InqB to IEL.

Thus, the extension of InqB to the language of epistemic logic is rather straightforward. The next step is to add dynamic speech act operators to the system.

8.2.4   Inquisitive dynamic epistemic logic

Recall that in DELQ there are two speech act operators, one for assertions and one for questions. This is necessary because in DELQ the proposition expressed by a sentence only embodies the informative content of that sentence. In IEL, the proposition expressed by a sentence captures both its informative and its inquisitive content. This means that we no longer need to introduce two distinct speech act operators for questions and assertions. Instead we can have a single operator for utterances more generally. In addition to this, we will introduce an *acceptance* operator, which is used to model the speech act of accepting the informative content of a previously uttered sentence.

Thus, the language of IDEL is obtained from the language of IEL by adding expressions of the form $[a{:}\varphi]\psi$ and $[a{:}ok]\psi$. Intuitively, $[a{:}\varphi]\psi$ means

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[18]The present system may be further refined in order to account for embedded disjunctive questions (incorporating ideas from Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2011) and to deal with one of Gettier's famous puzzles concerning the notion of knowledge as justified true belief, which involves sentences with disjunctive clauses embedded under knowledge operators (see Uegaki, 2011).

that an utterance of φ by agent a leads to a state that supports ψ, while $[a\!:\!ok]\psi$ means that acceptance by agent a of the informative content of the previously uttered sentence leads to a state that supports ψ.

Speech acts are taken to change the discourse context. We have seen that in DEL, the discourse context is represented by a Kripke model, which captures the epistemic states of all the conversational participants. In order to capture the effect of an acceptance speech act, this simple notion of a discourse context needs to be extended somewhat. In particular, we need to keep track of the sentences that have been uttered so far, since the effect of an acceptance speech act depends on the informative content of the previously uttered sentence. Thus, we define a discourse context X as a pair $\langle M, T \rangle$, where M is a Kripke model and T is a stack of sentences, i.e., those sentences that have been uttered so far. Following Farkas and Bruce (2010) and Farkas and Roelofsen (2011), we refer to T as the *Table* of the conversation.

Definition 53 (Stacks)**.**

- For any $n \in \mathbb{N}$, a stack of length n is a tuple with n elements.

- If T is a stack of length $n \geq 1$, then for every $0 \leq m \leq n$, T_m denotes the mth element of T.

- If T is a stack of length $n \geq 1$, then $\mathsf{top}(\mathrm{T})$ denotes the nth element of T.

- If T is a stack of length n, and x an object, then $T + x$ is a stack T' of length $n+1$, such that $T'_m = T_m$ for all $1 \leq m \leq n$, and $T'_{n+1} = x$.

Definition 54 (Discourse contexts)**.** A discourse context is a pair $\langle M, T \rangle$, where M is a Kripke model and T a stack of sentences.

Now we are ready to specify the effect of an utterance on the discourse context. We take the effect of an utterance of φ by an agent a to be twofold: first, φ is put on the Table, and second, the epistemic state of agent a in every world in the current model is restricted to the informative content of φ. Thus, it becomes common knowledge that a's epistemic state supports the informative content of φ. This idea is captured by the following definitions.

Definition 55 (Restricting epistemic states)**.** Let $M = \langle W, w_0, R, V \rangle$ be a Kripke model, a an agent, and φ a sentence. Then we define $R^{a:\varphi}$ as follows:

- $R^{a:\varphi}(a) := R(a) \cap \{\langle w, v \rangle \mid v \in \mathsf{info}_M(\varphi)\}$

- $R^{a:\varphi}(b) := R(b)$ for all $b \neq a$

Definition 56 (The effect of an utterance on the discourse context).
Let $X = \langle M, T \rangle$ be a discourse context, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\mathsf{IDEL}}$. Then:

$$X^{a:\varphi} := \langle M^{a:\varphi}, T^{a:\varphi} \rangle$$

where:

1. $M^{a:\varphi} := \begin{cases} \langle W, w_0, R^{a:\varphi}, V \rangle & \text{if } X, \{w_0\} \models K_a!\varphi \\ \text{undefined} & \text{otherwise} \end{cases}$

2. $T^{a:\varphi} := T + \varphi$

The speech act of acceptance has a simpler effect than that of uttering a sentence: it does not put a new proposal on the Table, but only eliminates worlds in which the epistemic state of the agent of the speech act does not support the informative content of the proposition that is on top of the Table. Thus, in making an acceptance move, a speaker publicly commits to the informative content of the previously uttered sentence.

Definition 57 (The effect of acceptance on the discourse context).
Let $\langle M, T \rangle$ be a discourse context, $a \in \mathcal{A}$, and $\varphi \in \mathcal{L}_{\mathsf{IDEL}}$. Then:

$$X^{a:ok} = \langle M^{a:ok}, T^{a:ok} \rangle$$

where:

1. $M^{a:ok} = \begin{cases} \langle W, w_0, R^{a:\mathsf{top}(T)}, V \rangle & \text{if } T \neq \emptyset \\ \text{undefined} & \text{otherwise} \end{cases}$

2. $T^{a:ok} = T$

Notice that $\langle M, T \rangle^{a:ok}$ is only well-defined if T contains at least one element. This reflects the anaphoric nature of acceptance: an acceptance move is appropriate only if there is at least one previously uttered sentence on the Table.

Having specified how utterances and acceptance moves affect the discourse context, we are now ready to define when a state supports a sentence in IDEL, given a certain discourse context. The first seven clauses are essentially the same as those for IEL. The two additional clauses deal with constructions involving speech act operators.

Definition 58 (Support in IDEL).

Let $X = \langle M, T \rangle$ be a discourse context, $p \in \mathcal{P}$, $a \in \mathcal{A}$, and $\varphi, \psi \in \mathcal{L}_{\mathsf{IDEL}}$.

1. $X, s \models p$ iff $s \subseteq V(p)$

2. $X, s \models \bot$ iff $s = \emptyset$

3. $X, s \models \varphi \wedge \psi$ iff $X, s \models \varphi$ and $X, s \models \psi$

4. $X, s \models \varphi \vee \psi$ iff $X, s \models \varphi$ or $X, s \models \psi$

5. $X, s \models \varphi \rightarrow \psi$ iff   for all $s' \subseteq s$: if $X, s' \models \varphi$ then $X, s' \models \psi$

6. $X, s \models K_a \varphi$ iff   for all $w \in s$: $X, R_a[w] \models \varphi$

7. $X, s \models C\varphi$ iff   for all $w \in s$: $X, R^*[w] \models \varphi$

8. $X, s \models [a\text{:}\varphi]\psi$ iff $X^{a:\varphi}, s \models \psi$

9. $X, s \models [a\text{:}ok]\psi$ iff $X^{a:ok}, s \models \psi$

The proposition expressed by a sentence φ in a discourse context X can be defined as the set of all states that support φ in X. All the other by now familiar notions, such as the informative content of a sentence, informative and inquisitive sentences, questions, assertions, hybrids, and entailment carry over straightforwardly from InqB via IEL to IDEL.

Moreover, several discourse related notions can be defined in IDEL. For instance, we could say that a discourse context $\langle M, T \rangle$ is *stable* if and only if none of the sentences in T are inquisitive in M. Intuitively, this means that all the issues that have been raised so far are settled. Similarly, we could say that a sentence φ has the potential to *resolve* a discourse context $\langle M, T \rangle$ just in case an utterance of φ by one of the agents, and subsequent acceptance by all the other agents, would lead to a stable discourse context. We could also add operators corresponding to these notions to the object language. For instance, we could add an operator R to the language, and say that a sentence $R\varphi$ is supported by s relative to a discourse context X if and only if φ has the potential to resolve X. A detailed exploration of such notions will be left for another occasion.

For now, we note one particular feature of the system, which seems especially natural and desirable. Namely, if a sentence φ is uttered by an agent a and subsequently accepted by all other agents $b \neq a$, its informative content cannot fail to become common knowledge.

Fact 45. If $\mathcal{A} = \{a_1, \ldots, a_n\}$, then for any discourse context X, any state s, and any sentence φ:

$$X, s \models [a_1 : \varphi][a_2 : ok] \ldots [a_n : ok]C!\varphi$$

Finally, we would like to mention two possible ways to further enhance the system discussed here. First, it would be possible to enrich Kripke models in such a way that they do not just associate an information state with every agent in every world, but rather an information state plus an issue over that state, representing not just the information that the agent currently has, but also the information that she would like to acquire. This approach is explored in Ciardelli (2012).

Second, it is possible to think of Kripke models not only as capturing the agents' knowledge, but also as capturing their *discourse commitments*. This makes it possible to model situations in which an agent commits to a certain piece of information without really knowing whether this piece of information actually holds. Under this perspective, it is also natural to go one step further, and to introduce *conditional commitments*. It is natural to think of certain speech acts as involving such conditional commitments. Consider for instance the tag-question in (38):

(38)   John is coming, isn't he?

Arguably, in uttering (38) a speaker conditionally commits to the possibility that John is coming, i.e., on the condition that the responder commits to this possibility, the speaker commits to it as well. An approach along these lines is explored in Farkas and Roelofsen (2012).

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