Information and attention

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Abstract

This paper introduces a semantic framework in which the meaning of a sentence embodies both *informative* and *attentive* content. This framework allows for an improved implementation of the analysis of attentive *might* first presented in Ciardelli, Groenendijk, and Roelofsen (2009). This analysis sheds new light on the way in which *might* interacts with conjunction and disjunction, which is puzzling for the standard modal account of *might*, as well as the treatment of *might* in update semantics.

1   Introduction

Traditionally, semantic meaning is identified with informative content. Propositions are defined as sets of possible worlds, and in uttering a sentence φ, a speaker is taken to provide the information that the actual world is one of the worlds in $[\varphi]$. In this note, we will present a semantic framework in which the proposition expressed by a sentence does not only embody its informative content, but also its attentive content.[1]

In our view, such a framework will be useful in analyzing a wide range of constructions in natural language. To illustrate this, we will consider the behavior of *might* sentences in English, exemplified in (1).

(1)   John might be in London.

Traditionally, *might* is analyzed as an epistemic modal. However, this traditional analysis faces major challenges. For instance, Zimmermann (2000, p.258–259) observed that (7), (8), and (9) are intuitively all equivalent.[2]

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[1]Evidently, this enrichment is similar to the one pursued in *inquisitive semantics* (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011, among others). The aim of inquisitive semantics is to develop a notion of meaning that embodies both informative and inquisitive content. The present paper is concerned with attentive rather than inquisitive content. Ciardelli, Groenendijk, and Roelofsen (2009) develop a framework in which meaning encompasses all three types of content, informative, inquisitive, and attentive. However, as will be discussed in more detail below, this framework faces some foundational problems. The present paper shows that these problems can be avoided if we just focus on informative and attentive content.

[2]These type of examples have also often been discussed in relation to the phenomenon of *free choice permission*, which involves deontic modals (cf. Geurts, 2005; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007; Fox, 2007; Klinedinst, 2007; Chemla, 2009).

(2)   John might be in Paris or in London. $\diamond(p \vee q)$

(3)   John might be in Paris or he might be in London. $\diamond p \vee \diamond q$

(4)   John might be in Paris and he might be in London. $\diamond p \wedge \diamond q$

Notice that *might* behaves differently here from clear-cut epistemic modals; certainly (5) is not equivalent with (6).

(5)   It is consistent with my beliefs that John is in London or
           it is consistent with my beliefs that he is in Paris.

(6)   It is consistent with my beliefs that John is in London and
           it is consistent with my beliefs that he is in Paris.

Ciardelli, Groenendijk, and Roelofsen (2009) suggest that the equivalence of (7)-(9) can be explained if the main semantic contribution of *might* sentences is taken to lie in their potential to draw attention to certain possibilities. This idea will also be implemented here, in a way that avoids some of the problems of the original implementation.

    The paper is organized as follows. In section 2, we define a semantics for the language of propositional logic, in which the proposition expressed by each sentence captures both its informative and its attentive content. Section 3 shows how this framework can be used to give a semantic account of attentive *might*. Section 4 discusses how the new perspective on semantic meaning adopted in this paper also leads to a new perspective on pragmatics, and explores the consequences of this new pragmatic perspective for the interpretation of *might* sentences. Section 5 concludes.

2   Information and attention

In order to obtain a notion of meaning that embodies both informative and attentive content, we will take propositions to be sets of *possibilities*, where each possibility in turn is a sets of possible worlds. In uttering a sentence φ, a speaker will be taken to:

1. Draw attention to all the possibilities in $[\varphi]$

2. Provide the information that the actual world is included in at least one of the possibilities in $[\varphi]$

In this way, the proposition expressed by a sentence captures both the informative and the attentive content of the sentence.

In the remainder of this section, we will develop a concrete semantics for the language of propositional logic that is based on this general conception of propositions and the effect of utterances.

2.1 Attentive propositional logic

Let $\mathcal{P}$ be a set of atomic sentences, and let $\mathcal{L}_{\mathcal{P}}$ be the set of sentences that are built up from the elements of $\mathcal{P}$ using the Boolean connectives, $\wedge$, $\vee$, and $\neg$ in the usual way.[3]

Definition 1 (Possible worlds, possibilities, and propositions).

- A possible world is a function from $\mathcal{P}$ to $\{0,1\}$

- A possibility is a set of possible worlds

- A proposition is a non-empty set of possibilities

The proposition expressed by a sentence φ will be denoted by $[\varphi]$, and the possibilities in $[\varphi]$ will be referred to as the possibilities *for* φ. The set of all possible worlds will be denoted by ω, and the set of all propositions will be denoted by Σ.

Since in uttering a sentence φ a speaker is taken to provide the information that the actual world must be one that is included in at least one of the possibilities for φ, we will refer to the union of all the possibilities for φ, $\bigcup[\varphi]$, as the *informative content* of φ.

Definition 2 (Informative content). $\mathsf{info}(\varphi) = \bigcup[\varphi]$

For any proposition $[\varphi]$ and any set of possible worlds α, $[\varphi]_{\alpha}$ will denote the *restriction* of $[\varphi]$ to α, which is obtained by intersecting all the possibilities in $[\varphi]$ with α.

Definition 3 (Restricted propositions). $[\varphi]_{\alpha} = \{\alpha \cap \beta \mid \beta \in [\varphi]\}$

We are now ready to state the recursive semantics for $\mathcal{L}_{\mathcal{P}}$.

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[3]We do not include implication as a connective in our basic language, because it involves certain complexities that are orthogonal to the main issues addressed here.

Definition 4 (Semantics for $\mathcal{L}_\mathcal{P}$).

1. $[p] = \{\ \{w \mid w(p) = 1\}\ \}$ if p is atomic

2. $[\neg\varphi] = \{\ \overline{\bigcup[\varphi]}\ \}$

3. $[\varphi \vee \psi] = [\varphi] \cup [\psi]$

4. $[\varphi \wedge \psi] = [\varphi]_{\mathsf{info}(\psi)} \cup [\psi]_{\mathsf{info}(\varphi)}$

We will refer to this system as *attentive propositional logic*, APL. We will briefly go through the clauses one by one. In doing so we will speak of sentences as 'providing information' and 'drawing attention to possibilities'. Strictly speaking, sentences themselves do not provide information or draw attention to possibilities. Rather, this is done by *speakers* in uttering these sentences. However, the explanation of the semantics will be more illuminating if we allow ourselves to be somewhat sloppy in this respect.

Atoms.   The atomic clause says that an atomic sentence p draws attention to a single possibility, namely the possibility that consists of all worlds where p is true. It thereby provides the information that the actual world must be one where p is true.

Negation.   The clause for negation says that a negated sentence $\neg\varphi$ draws attention to a single possibility, which is the complement of the union of all the possibilities for φ itself. This means that $\neg\varphi$ provides the information that the actual world is not included in any of the possibilities that φ itself draws attention to. In other words, $\neg\varphi$ provides the information that the actual world is not contained in $\mathsf{info}(\varphi)$.

Disjunction.   The clause for disjunction says that a disjunctive sentence $\varphi\vee\psi$ draws attention to all the possibilities that φ draws attention to, plus all the possibilities that ψ draws attention to. This means that $\varphi\vee\psi$ provides the information that the actual world is included in at least one of the possibilities that φ draws attention to and/or in at least one of the possibilities that ψ draws attention to. In other words, $\varphi \vee \psi$ provides the information that the actual world must be one that is included in $\mathsf{info}(\varphi)$ or in $\mathsf{info}(\psi)$.

Conjunction. The clause for conjunction says that a conjunctive sentence $\varphi \wedge \psi$ draws attention to all the possibilities for φ restricted to $\mathsf{info}(\psi)$, and to all the possibilities for ψ restricted to $\mathsf{info}(\varphi)$. This means that it provides the information that the actual world must be one that lies both in $\mathsf{info}(\varphi)$ and in $\mathsf{info}(\psi)$.

2.2 Comparison with classical propositional logic

Let us briefly compare APL with classical propositional logic, CPL. For any sentence $\varphi \in \mathcal{L}_{\mathcal{P}}$, let $|\varphi|$ denote the proposition expressed by φ in CPL. APL is of course richer than CPL, because it captures both informative and attentive content. It is important to note, however, that as far as informative content is concerned, APL coincides with CPL. That is, for every $\varphi \in \mathcal{L}_{\mathcal{P}}$, we have that $\mathsf{info}(\varphi) = |\varphi|$. In this sense, APL is a conservative extension of CPL.

Fact 1 (APL and CPL)**.** For every φ: $\mathsf{info}(\varphi) = |\varphi|$

Note that, since $[\neg \varphi]$ is defined as $\{\overline{\bigcup [\varphi]}\}$, we always have that $[\neg\neg\varphi] = \{\bigcup [\varphi]\} = \{\mathsf{info}(\varphi)\} = \{|\varphi|\}$. So by taking the double negation of a sentence φ we always get a sentence that expresses a proposition consisting of a single possibility, which coincides with the classical meaning of φ.

Fact 2 (Double negation)**.** For every φ: $[\neg\neg\varphi] = \{|\varphi|\}$

Finally, note that our language is *functionally complete*, in the sense that for every proposition it is possible to find a sentence that expresses that proposition.

Fact 3 (Functional completeness)**.**
For every proposition $A \in \Sigma$, there is a sentence $\varphi \in \mathcal{L}_{\mathcal{P}}$ such that $[\varphi] = A$.

Proof. Recall that $\mathcal{P}$ is assumed to be finite. This means that for every world w, there is a sentence φ_w such that $[\varphi_w] = \{\{w\}\}$, namely:

$$\varphi_w = \bigwedge \{p \mid w(p) = 1\} \wedge \bigwedge \{\neg p \mid w(p) = 0\}$$

But then for every possibility α, there is a sentence φ_α such that $[\varphi_\alpha] = \{\alpha\}$, namely $\neg\neg \bigvee \{\varphi_w \mid w \in \alpha\}$. And this means that for every proposition A, there is a sentence φ_A such that $[\varphi_A] = A$, namely $\bigvee \{\varphi_\alpha \mid \alpha \in A\}$. $\square$

2.3 Entailment, homogeneity, and refinement

In CPL, sentences are ordered in terms of their informative content:

$$\varphi \geq_{info} \psi \text{ iff } |\varphi| \subseteq |\psi|$$

and entailment is defined in terms of this informativeness order:

$$\varphi \vDash_{\mathsf{CPL}} \psi \text{ iff } \varphi \geq_{info} \psi$$

In the present setting, sentences can be ordered in terms of their informative content, but also in terms of their attentive content. As in the classical setting, φ is at least as informative as ψ iff $\mathsf{info}(\varphi) \subseteq \mathsf{info}(\psi)$. As for attentiveness, it is natural to say that φ is at least as attentive as ψ iff φ draws attention to all the possibilities that ψ draws attention to, restricted to the informative content of φ.

Definition 5 (Informative and attentive orders)**.**

- $\varphi \geq_{info} \psi$ iff $\mathsf{info}(\varphi) \subseteq \mathsf{info}(\psi)$

- $\varphi \geq_{att} \psi$ iff $[\psi]_{\mathsf{info}(\varphi)} \subseteq [\varphi]$

These orders can be combined in several ways. In particular, we will say that φ *entails* ψ, $\varphi \vDash \psi$, iff φ is at least as informative and at least as attentive as ψ, $\varphi \geq_{info} \psi$ and $\varphi \geq_{att} \psi$. Besides entailment, we will also introduce a notion of *homogeneity*: φ is at least as homogeneous as ψ, $\varphi \rightleftharpoons \psi$ iff φ is at least as informative and at *most* as attentive as ψ, $\varphi \geq_{info} \psi$ and $\varphi \leq_{att} \psi$.[4] Thus, one sentence is more homogeneous than another if it (i) leaves fewer possible candidates for the actual world, and (ii) draws attention to fewer different possibilities.[5]

Definition 6 (Entailment and homogeneity)**.**

- $\varphi \vDash \psi$ iff $\varphi \geq_{info} \psi$ and $\varphi \geq_{att} \psi$

- $\varphi \rightleftharpoons \psi$ iff $\varphi \geq_{info} \psi$ and $\varphi \leq_{att} \psi$

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[4] A similar notion of homogeneity can be found in Groenendijk and Roelofsen (2009). There, φ is defined to be at least as homogeneous as ψ iff φ is at least as informative and at most as inquisitive as ψ.

[5] A similar notion of homogeneity exists in inquisitive semantis (see Groenendijk and Roelofsen, 2009).

Note that entailment and homogeneity are defined here as relations between sentences. It will be useful to also define them as relations between *propositions*.

Definition 7 (Propositional entailment and homogeneity)**.**
Let A and B be two propositions. Then:

- $A \geq_{info} B$ iff $\bigcup A \subseteq \bigcup B$

- $A \geq_{att} B$ iff $B_{\bigcup A} \subseteq A$

- $A \vDash B$ iff $A \geq_{info} B$ and $A \geq_{att} B$

- $A \rightleftharpoons B$ iff $A \geq_{info} B$ and $A \leq_{att} B$

Evidently, there is a straightforward correspondence between the sentential notions of entailment and homogeneity, and the propositional notions.

Fact 4 (Sentential and propositional entailment and homogeneity)**.**

- $\varphi \vDash \psi$ iff $[\varphi] \vDash [\psi]$

- $\varphi \rightleftharpoons \psi$ iff $[\varphi] \rightleftharpoons [\psi]$

However, some properties of entailment and homogeneity only hold at the level of propositions. For instance, as desired, entailment and homogeneity form *partial orders* on the set of all propositions Σ.[6]

Fact 5 (Partial orders)**.** $\vDash$ and $\rightleftharpoons$ form partial orders on Σ.

Proof. First consider entailment. We have to show that $\vDash$ is reflexive, transitive, and anti-symmetric. It is clear that $\vDash$ is reflexive. For transitivity, suppose that $A \vDash B$ and $B \vDash C$. Then clearly $A \geq_{info} C$. It remains to be shown that $A \geq_{att} C$. Let $\gamma \in C$. We have to show that $\gamma \cap \bigcup A \in A$. Now, since $B \geq_{att} C$, $\gamma \cap \bigcup B \in B$. Thus, since $A \geq_{att} B$, $\gamma \cap \bigcup B \cap \bigcup A \in A$. But since $A \geq_{info} B$, we have that $\bigcup A \subseteq \bigcup B$, which means that $\gamma \cap \bigcup B \cap \bigcup A = \gamma \cap \bigcup A$. Thus, indeed, $\gamma \cap \bigcup A \in A$.

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[6]Entailment and homogeneity do not form partial orders on the set of all sentences, since two different sentences may very well express exactly the same proposition and therefore be just as informative and just as attentive. This means that entailment and homogeneity, conceived of as relations between sentences, are not anti-symmetric.

Finally, to establish that $\vDash$ is anti-symmetric, assume that $A \vDash B$ and $B \vDash A$. We have to show that $A = B$. Let $\alpha \in A$. Then, since $B \vDash A$, $\alpha \cap \bigcup B$ must be in B. But since $A \geq_{info} B$, we have that $\bigcup A \subseteq \bigcup B$, which certainly means that $\alpha \subseteq \bigcup B$. Thus $\alpha \cap \bigcup B = \alpha$. It follows that $\alpha \in B$. We can conclude, then, that $A \subseteq B$, and in the same way we can establish that $B \subseteq A$. Thus, indeed, $A = B$.

As for homogeneity, $\rightleftharpoons$ is clearly reflexive, and transitivity and anti-symmetry are established in exactly the same way as for $\vDash$. $\square$

There is one proposition, namely $\{\varnothing\}$, which entails every other proposition. We will therefore refer to $\{\varnothing\}$ as the *absurd* proposition, and to sentences that express $\{\varnothing\}$ as *contradictions*. An example of a contradiction is the sentence $p \wedge \neg p$. We will use $\bot$ as an abbreviation of this sentence.

Perhaps unexpectedly, there is no proposition that is entailed by all other propositions. This means that *tautologies* cannot be defined in terms of entailment in the usual way, i.e., as sentences that are entailed by all other sentences. Instead, we will call a sentence a tautology just in case every other sentence is a *refinement* of it in the following sense.

Definition 8 (Refinement).

φ is a refinement of ψ, $\varphi \geq \psi$, if and only if:

1. $\varphi \geq_{info} \psi$ and

2. for all $\beta \in [\psi]$ there is a non-empty set $\Gamma \subseteq [\varphi]$ such that $\beta \cap \mathsf{info}(\varphi) = \bigcup \Gamma$

In order for φ to be a refinement of ψ, first of all φ must be at least as informative as ψ. However, φ does not have to draw attention to every possibility that ψ draws attention to (restricted to $\mathsf{info}(\varphi)$). Rather, for every possibility β that ψ draws attention to, φ must draw attention to a non-empty set of possibilities Γ such that the restriction of β to $\mathsf{info}(\varphi)$ coincides with $\bigcup \Gamma$. Again this notion of refinement can also be defined at the level of propositions (as opposed to sentences).

Definition 9 (Propositional refinement).

$A \geq B$ if and only if:

1. $A \geq_{info} B$ and

2. for all $\beta \in B$ there is a non-empty set $\Gamma \subseteq A$ such that $\beta \cap \bigcup A = \bigcup \Gamma$

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Clearly, there is a straightforward correspondence between sentential and propositional refinement.

Fact 6 (Sentential and propositional refinement)**.**

- $\varphi \geq \psi$ iff $[\varphi] \geq [\psi]$

Refinement is strictly weaker than entailment.

Fact 7 (Refinement and entailment)**.**

1. For every A and B: if $A \vDash B$ then also $A \geq B$

2. There are A and B such that $A \geq B$ but $A \nvDash B$

Proof. The first claim follows directly from the definitions. For the second claim, take A to consist of a single possibility α, and take B to consist of two mutually exclusive possibilities which are both contained in α. Then B is a refinement of A, but it does not entail A, because it does not draw attention to α restricted to $\mathsf{info}(B)$. □

Unlike entailment and homogeneity, the refinement relation does *not* form a partial order on the set of all propositions.

Fact 8 (No partial order)**.** $\geq$ does not form a partial order on Σ.

Proof. $\geq$ is clearly reflexive and transitive, but it is not anti-symmetric. To see this, consider the following two propositions:

- $A = [\top \vee (p \wedge q) \vee (p \wedge \neg q)]$

- $B = [\top \vee (p \wedge q) \vee (p \wedge \neg q) \vee p]$

These propositions are depicted in figures 1(a) and 1(b), respectively. In these figures, 11 is a world where both p and q are true, 10 is a world where p is true and q is false, etcetera. From inspecting the figures, it will be clear that $A \geq B$ and $B \geq A$, but $A \neq B$. So $\geq$ is not anti-symmetric. □

There is one proposition, namely $\{\omega\}$, which has the special property that every other proposition is a refinement of it. We will therefore refer to $\{\omega\}$ as the *trivial* proposition, and to sentences that express this trivial proposition as *tautologies*. An example of a tautology is the sentence $\neg\neg(p \vee \neg p)$. We will use $\top$ as an abbreviation of this sentence.
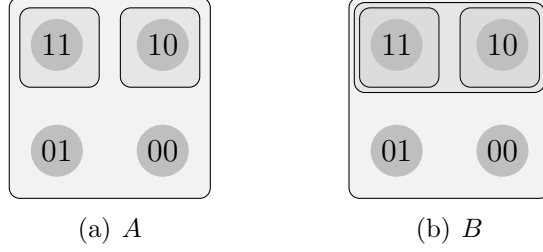
(a) A (b) B

Figure 1: Two propositions showing that refinement is not anti-symmetric.

2.4 Algebraic characterization of APL

Given the notions of entailment, homogeneity, and refinement, the semantic behavior of the Boolean connectives in APL can be characterized in algebraic terms. Recall that in CPL, conjunction behaves semantically as a *meet* operator, disjunction as a *join* operator, and negation as a Boolean *complementation* operator w.r.t. entailment. We will show that in APL, conjunction behaves again as a *meet* operator w.r.t. $\vDash$, while disjunction behaves as a *join* operator w.r.t. $\rightleftharpoons$, and negation behaves as a *pseudo-complementation* operator w.r.t. $\vDash$ and $\geq$. Let us first provide definitions of these algebraic notions.

Definition 10 (Meets, joins, and pseudo-complements).

Let A and B be two propositions. Then:

- The *meet* of A and B w.r.t. $\vDash$, if it exists, is the unique proposition M such that:

  1. $M \vDash A$ and $M \vDash B$
  2. For every proposition P, if $P \vDash A$ and $P \vDash B$, then $P \vDash M$.

  In other words, M is the *greatest lower bound* of A and B w.r.t. $\vDash$.

- The *join* of A and B w.r.t. $\rightleftharpoons$, if it exists, is the unique proposition J such that:

  1. $A \rightleftharpoons J$ and $B \rightleftharpoons J$
  2. For every proposition P, if $A \rightleftharpoons P$ and $B \rightleftharpoons P$, then $J \rightleftharpoons P$.

  In other words, J is the *least upper bound* of A and B w.r.t. $\rightleftharpoons$.

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- The *pseudo-complement*[7] of A w.r.t. $\vDash$ and $\geq$, if it exists, is the unique proposition C such that:

  1. The meet of A and C w.r.t. $\vDash$ is the absurd proposition, $\{\varnothing\}$.
  2. For every proposition P that satisfies 1. we have that $P \geq C$.

  In other words, C is the least refined proposition such that the meet of A and C is the absurd proposition.

Now let us show that the semantic behavior of the Boolean connectives in APL can be characterized in terms of these algebraic notions.

Fact 9 (Conjunction is meet w.r.t. $\vDash$).

For every φ and ψ, $[\varphi \wedge \psi]$ is the meet of $[\varphi]$ and $[\psi]$ w.r.t. $\vDash$.

Proof. It follows immediately from the definition of $[\varphi \wedge \psi]$ that $[\varphi \wedge \psi] \vDash [\varphi]$ and $[\varphi \wedge \psi] \vDash [\psi]$. Now suppose that $[\xi]$ is another proposition that entails both $[\varphi]$ and $[\psi]$. Then we have to show that $[\xi]$ also entails $[\varphi \wedge \psi]$. First, since $[\xi] \vDash [\varphi]$ and $[\xi] \vDash [\psi]$, we have that $\mathsf{info}(\xi) \subseteq \mathsf{info}(\varphi)$ and $\mathsf{info}(\xi) \subseteq \mathsf{info}(\psi)$. So $\mathsf{info}(\xi) \subseteq \mathsf{info}(\varphi) \cap \mathsf{info}(\psi)$. But $\mathsf{info}(\varphi) \cap \mathsf{info}(\psi)$ coincides with $\mathsf{info}(\varphi \wedge \psi)$. So $\mathsf{info}(\xi) \subseteq \mathsf{info}(\varphi \wedge \psi)$. In other words, $[\xi] \geq_{info} [\varphi \wedge \psi]$. It remains to be shown that $[\xi] \geq_{att} [\varphi \wedge \psi]$. Let γ be a possibility in $[\varphi \wedge \psi]$. Then γ is either the intersection of some possibility $\alpha \in [\varphi]$ with $\mathsf{info}(\psi)$, or the intersection of some possibility $\beta \in [\psi]$ with $\mathsf{info}(\varphi)$. Suppose that it is the intersection of some possibility $\alpha \in [\varphi]$ with $\mathsf{info}(\psi)$. Then, since $[\xi] \geq_{att} [\varphi]$, $\alpha \cap \mathsf{info}(\xi)$ must be in $[\varphi]$. But since $\mathsf{info}(\xi) \subseteq \mathsf{info}(\psi)$, we have that $\alpha \cap \mathsf{info}(\xi) = \alpha \cap \mathsf{info}(\varphi) \cap \mathsf{info}(\xi) = \gamma \cap \mathsf{info}(\xi)$. So $\gamma \cap \mathsf{info}(\xi)$ is in $[\xi]$, which is exactly what we set out to show. If we assume that γ is the intersection of some possibility $\beta \in [\psi]$ with $\mathsf{info}(\varphi)$, we can show in a similar way that $\gamma \cap \mathsf{info}(\xi)$ is in $[\xi]$. So we may conclude that $[\xi] \geq_{att} [\varphi \wedge \psi]$. $\square$

Fact 10 (Disjunction is join w.r.t. $\rightleftharpoons$).

For every φ and ψ, $[\varphi \vee \psi]$ is the join of $[\varphi]$ and $[\psi]$ w.r.t. $\rightleftharpoons$.

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[7]The standard notion of a pseudo-complement comes from Heyting algebra, which is the algebra underlying intuitionistic logic. The notion we define here is non-standard because it is defined in terms of *two* relations, entailment and refinement. The standard notion is defined just in terms of entailment. In all other respects, the definition is the same.

Proof. It follows immediately from the definition of $[\varphi \vee \psi]$ that $[\varphi] \rightleftharpoons$ $[\varphi \vee \psi]$ and $[\psi] \rightleftharpoons [\varphi \vee \psi]$. Now suppose that $[\xi]$ is another proposition such that $[\varphi] \rightleftharpoons [\xi]$ and $[\psi] \rightleftharpoons [\xi]$. Then we have to show that $[\varphi \vee \psi] \rightleftharpoons$ $[\xi]$. First, since $[\varphi] \rightleftharpoons [\xi]$ and $[\psi] \rightleftharpoons [\xi]$, we have that $\mathsf{info}(\varphi) \subseteq \mathsf{info}(\xi)$ and $\mathsf{info}(\psi) \subseteq \mathsf{info}(\xi)$, which means that $\mathsf{info}(\varphi) \cup \mathsf{info}(\psi) \subseteq \mathsf{info}(\xi)$. But $\mathsf{info}(\varphi) \cup \mathsf{info}(\psi) = \mathsf{info}(\varphi \vee \psi)$. So $\mathsf{info}(\varphi \vee \psi) \subseteq \mathsf{info}(\xi)$, which means that $[\varphi \vee \psi] \geq_{info} [\xi]$. It remains to be shown that $[\xi] \geq_{att} [\varphi \vee \psi]$. Let α be a possibility in $[\varphi \vee \psi]$. Then α is either in $[\varphi]$ or in $[\psi]$. Suppose that it is in $[\varphi]$. Then, since $[\xi] \geq_{att} [\varphi]$, $\alpha \cap \mathsf{info}(\xi)$ must be in $[\xi]$. Similarly, if α is in $[\psi]$ then it follows from the fact that $[\xi] \geq_{att} [\psi]$ that $\alpha \cap \mathsf{info}(\xi)$ must be in $[\xi]$. So, indeed, $[\xi] \geq_{att} [\varphi \vee \psi]$. $\square$

Fact 11 (Negation is pseudo-complement w.r.t. $\vDash$ and $\geq$).

For every φ, $[\neg\varphi]$ is the pseudo-complement of $[\varphi]$ w.r.t. $\vDash$ and $\geq$.

Proof. First we show that the meet of $[\varphi]$ and $[\neg\varphi]$ is the absurd proposition, $\{\varnothing\}$. We already know that the meet of $[\varphi]$ and $[\neg\varphi]$ w.r.t. $\vDash$ is $[\varphi \wedge \neg\varphi]$. Now let γ be a possibility in $[\varphi \wedge \neg\varphi]$. Then γ is either the intersection of some possibility $\alpha \in [\varphi]$ with $\mathsf{info}(\neg\varphi)$, or the intersection of some possibility $\beta \in [\neg\varphi]$ with $\mathsf{info}(\varphi)$. However, $\mathsf{info}(\varphi)$ and $\mathsf{info}(\neg\varphi)$ are disjoint, so in both cases γ must be empty. So $[\varphi \wedge \neg\varphi] = \{\varnothing\}$. Now let $[\xi]$ be another proposition such that the meet of $[\varphi]$ and $[\xi]$ w.r.t. $\vDash$ is $\{\varnothing\}$. Then we have to show that $[\xi] \geq [\neg\varphi]$. First, since the meet of $[\varphi]$ and $[\xi]$ w.r.t. $\vDash$ is $\{\varnothing\}$, $\mathsf{info}(\xi)$ and $\mathsf{info}(\varphi)$ must be disjoint. But this means that $\mathsf{info}(\xi) \subseteq \mathsf{info}(\neg\varphi)$. So $[\xi] \geq_{info} [\neg\varphi]$. It remains to be shown that for every possibility $\beta \in [\neg\varphi]$ there is a set of possibilities $B \subseteq [\xi]$ such that $\alpha \cap \mathsf{info}(\xi) = B$. This is easy to see: there is in fact only one possibility $\beta \in [\neg\varphi]$, and since $\mathsf{info}(\xi) \subseteq \mathsf{info}(\neg\varphi)$, the restriction of β to $\mathsf{info}(\xi)$ is bound to coincide with the union of all the possibilities in $[\xi]$. So, indeed, $[\xi] \geq [\neg\varphi]$. $\square$

Now let us take a step back, and spell out what these results tell us about the semantic behavior of the Boolean connectives in APL.

First, fact 9 tells us that for any φ and ψ, $[\varphi \wedge \psi]$ is the weakest proposition that entails both $[\varphi]$ and $[\psi]$. In other words, $[\varphi \wedge \psi]$ is the unique proposition with the following properties:

1. $[\varphi \wedge \psi]$ is at least as informative as $[\varphi]$ and as $[\psi]$

2. $[\varphi \wedge \psi]$ is at least as attentive as $[\varphi]$ and as $[\psi]$

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3. Every proposition that is at least as informative and attentive as $[\varphi]$ and $[\psi]$ is also at least as informative and attentive as $[\varphi \wedge \psi]$.

Fact 10 tells us that for any φ and ψ, $[\varphi \vee \psi]$ is the most homogeneous proposition that is at most as homogeneous as $[\varphi]$ and at most as homogeneous as $[\psi]$. In other words, $[\varphi \vee \psi]$ is the unique proposition with the following properties:

1. $[\varphi \vee \psi]$ is at most as informative as $[\varphi]$ and as $[\psi]$

2. $[\varphi \vee \psi]$ is at least as attentive as $[\varphi]$ and as $[\psi]$

3. Every proposition that is at most as informative and at least as attentive as $[\varphi]$ and $[\psi]$ is also at most as informative and at least as attentive as $[\varphi \vee \psi]$.

And finally, fact 11 tells us that for any φ, $[\neg \varphi]$ is the least refined proposition whose meet with $[\varphi]$ is $\{\varnothing\}$.

These algebraic characterizations give us a general understanding of the semantic behavior of the Boolean connectives in APL, and thereby provide suitable foundations for the framework.[8] We now turn to an illustration of how the framework may be used in natural language semantics.

3 Attentive *might*

In this section we implement the semantic analysis of attentive *might* presented in Ciardelli, Groenendijk, and Roelofsen (2009) in APL, and compare this implementation with the original one. In section 4 we will turn to pragmatic aspects of attentive *might*, and compare the present account with the more standard modal and dynamic accounts of *might*.

3.1 *Might* as an attentive operator

We will add an operator $\diamondsuit$ to our formal language, representing *might*, and define the proposition expressed by $\diamondsuit\varphi$ as follows.

Definition 11 (Might).
For any φ, $[\diamondsuit\varphi] = [\varphi] \cup \{\omega\}$

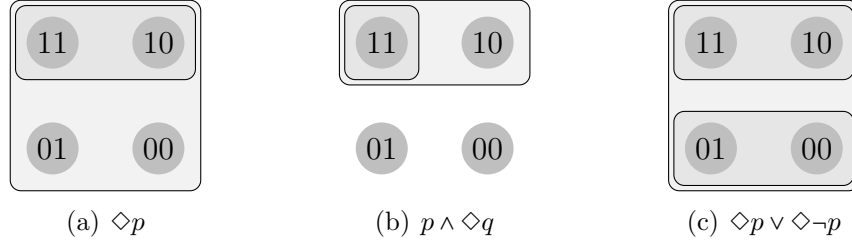[8]For analogous algebraic results in inquisitive semantics, see Roelofsen (2011a).

Figure 2: Three simple *might* sentences.

Thus, in uttering $\Diamond\varphi$, a speaker draws attention to all the possibilities for φ (and to the 'trivial possibility' ω) without providing any information.

To get a first impression of what this attentive treatment of *might* amounts to, let us consider three examples. First consider the proposition depicted in figure 2(a). This proposition consists of two possibilities: one possibility consisting of all worlds where p is true, and one possibility containing all possible worlds, i.e., the trivial possibility, ω. Together, these two possibilities make up the proposition expressed by $\Diamond p$. Thus, in uttering $\Diamond p$, a speaker draws attention to the possibility that p and to the trivial possibility, without providing any information.

Next, consider the proposition depicted in figure 2(b). This is the proposition expressed by $p \wedge \Diamond q$. It consists of two possibilities: $|p|$ and $|p \wedge q|$. Thus, in uttering $p \wedge \Diamond q$, a speaker provides the information that p holds, and draws attention to the possibility that q may hold as well.

The proposition depicted in figure 2(c) is the proposition expressed by $\Diamond p \vee \Diamond \neg p$. In uttering this sentence, a speaker draws attention to the possibility that p, the possibility that $\neg p$, and the trivial possibility, again without providing any information.

3.2 *Might* meets the propositional connectives

It is well-known that *might* interacts with the propositional connectives in peculiar ways. We will consider two specific observations here, one concerning the interaction of *might* with disjunction and conjunction, and one concerning the interaction of *might* with negation. Both these observations are puzzling for the standard modal account of *might*.
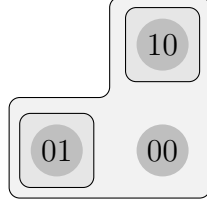
Figure 3: $[\diamond(p \vee q)] = [\diamond p \vee \diamond q] = [\diamond p \wedge \diamond q]$

Disjunction and conjunction. As mentioned in the introduction, Zimmermann (2000, p.258–259) observed that (7), (8), and (9) are intuitively all equivalent.

(7) John might be in Paris or in London. $\diamond(p \vee q)$

(8) John might be in Paris or he might be in London. $\diamond p \vee \diamond q$

(9) John might be in Paris and he might be in London. $\diamond p \wedge \diamond q$

Notice that *might* behaves differently here from clear-cut epistemic modals; clearly (10) is not equivalent with (11).

(10) It is consistent with my beliefs that John is in London or
      it is consistent with my beliefs that he is in Paris.

(11) It is consistent with my beliefs that John is in London and
      it is consistent with my beliefs that he is in Paris.

In APL, the equivalence between (7)–(9) is straightforwardly accounted for: all these sentence express exactly the same proposition, which is depicted in figure 3. Notice that, since p stands for 'John is in London' and q stands for 'John is in Paris' in this example, it is impossible for p and q to hold at the same time. Thus, our logical space consists of three worlds in this case: one world where John is in London (10), one where John is in Paris (01), and one where John is neither in London nor in Paris (00). The proposition expressed by (7)–(9) consists of three possibilities, the possibility that John is in London, the possibility that he is in Paris, and the trivial possibility. Thus, in uttering (7), (8), or (9), a speaker draws attention to the possibility that John is in London and to the possibility that John is in Paris, without

providing any information.[9]

Negation. Now let us consider how *might* interacts with negation. One striking observation is that in English, standard sentential negation cannot take wide scope over *might*. For instance, (12) can only be taken to draw attention to the possibility that John is not in London.

(12)   John might not be in London.

Notice, again, that *might* behaves differently from clear-cut epistemic modals here, which can very well occur in the scope of negation:

(13)   It is not consistent with my beliefs that John is in London.

The fact that *might* cannot occur in the scope of negation is explained in APL by the fact that $\neg\Diamond\varphi$ is always a contradiction. On the other hand, $\Diamond\neg p$ seems to be a suitable representation of (12) in our logical language. In uttering $\Diamond\neg p$, a speaker draws attention to the possibility that $\neg p$, without providing any information.

3.3   Comparison with the original implementation

As mentioned before, the analysis of attentive *might* presented here was, in essence, proposed originally in Ciardelli, Groenendijk, and Roelofsen (2009). However, it was implemented in a different system, to which we will refer as CGR-09. There are some important differences between APL and CGR-09, both at the general architectural level and in terms of concrete predictions about *might*. We will first consider the general architectural differences, and then turn to some concrete predictions about *might*.

General architectural differences.   Just as in APL, propositions are defined in CGR-09 as non-empty sets of possibilities. However, in CGR-09 propositions are not only taken to embody informative and attentive content, but also inquisitive content. That is, in uttering a sentence φ, a speaker is not only taken to draw attention to all the possibilities in $[\varphi]$ and to provide the information that the actual world lies in at least one of the possibilities

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[9]In section 3.3 we will consider variants of Zimmermann's examples where p and q are not mutually exclusive.

in $[\varphi]$, but also to *request* information from other participants in order to establish of at least one possibility in $[\varphi]$ that it indeed contains the actual world.

The fact that meanings in CGR-09 have this additional inquisitive dimension makes it more difficult to spell out a well-behaved recursive definition of the propositions expressed by complex sentences, even if such complex sentences can only be formed with disjunction, conjunction and negation. Indeed, the definition provided by CGR-09 is problematic in several ways. The general problem is that CGR-09 was obtained by *ad hoc* measures from the basic implementation of inquisitive semantics, in which propositions only embody informative and inquisitive content (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011). The clauses of the system lack a proper motivation. For instance, it is unclear how the semantic behaviour of the connectives in CGR-09 can be characterized algebraically in terms of inquisitive/attentive entailment. One concrete problem that can be seen as a result of this general deficiency is that conjunction is not idempotent. For instance, $(p \vee q) \wedge (p \vee q)$ is not equivalent to $(p \vee q)$.[10]

There have been several attempts to develop more principled implementations of inquisitive semantics with attentive content. In fact, the system developed in Roelofsen (2011c) seems to be satisfactory for the language of propositional logic (without implication). However, it is not clear yet how to extend this system to the first-order case (see Roelofsen, 2011b). APL on the other hand, can be extended to the first-order case in a straightforward way—existentially and universally quantified sentences can be taken to behave semantically as infinite disjunctions and conjunctions, respectively. Of course, ideally we would like to have a system in which every sentence has a single semantic value, which embodies informative, inquisitive, and attentive content all at the same time. However, this may turn out to be impossible. In that case, APL could be adopted alongside a basic implementation of inquisitive semantics, which is only concerned with informative and inquisitive content. Each sentence would then be associated with two semantic values, one in APL and one in inquisitive semantics, and these two semantic values together would capture the informative, inquisitive, and attentive content of the sentence.

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[10]This can be verified straightforwardly, given that disjunction is defined as union, and conjunction is defined as pointwise intersection in CGR-09.

(a) $[\diamondsuit p \wedge \diamondsuit q]_{\mathsf{APL}}$ (b) $[\diamondsuit p \wedge \diamondsuit q]_{\mathsf{CGR\text{-}09}}$
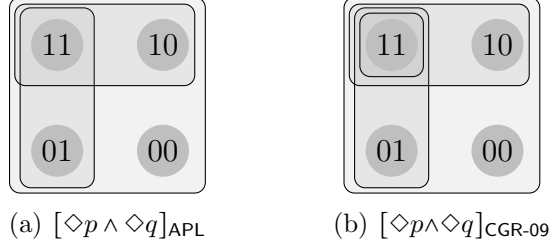
Figure 4: A difference between APL and CGR-09.

Predictions about *might*. All the examples of *might* sentences that we have seen so far express exactly the same proposition in CGR-09 as they do in APL. However, there are other sentences involving *might* that express a different proposition in CGR-09 than in APL.

Consider the following variants of Zimmermann's examples, where, unlike in the original examples, p and q are not mutually exclusive.

(14)   John might speak English or French. $\diamondsuit(p \vee q)$

(15)   John might speak English or he might speak French. $\diamondsuit p \vee \diamondsuit q$

(16)   John might speak English and he might speak French. $\diamondsuit p \wedge \diamondsuit q$

In APL, these three sentences are still equivalent; they all express the proposition depicted in figure 4(a). In CGR-09 however, these three sentences are *not* equivalent. The first two express the proposition depicted in figure 4(a), while the third, involving conjunction, expresses the proposition depicted in figure 4(b).

Ciardelli *et al.* (2009) argue that this prediction is in fact desirable, based on a scenario suggested by Anna Szabolcsi. The scenario is one in which someone is looking for an English-French translator, i.e., someone who speaks *both* English and French. In that context, (16) would be perceived as a useful recommendation, while (14) and (15) would not. Now, in CGR-09, $\diamondsuit p \wedge \diamondsuit q$, unlike $\diamondsuit(p \vee q)$ and $\diamondsuit p \vee \diamondsuit q$, draws attention to the possibility that $p \wedge q$, that is, the possibility that John speaks both English and French. This, then, could explain the observation that (16) is perceived as a useful recommendation in the translator-scenario, unlike (14) and (15).

In APL, this explanation is no longer available. However, there are reasons to be skeptical about the prediction made by CGR-09. For instance, as noted by Luis Alonso-Ovalle (p.c.), $\diamondsuit p \wedge \diamondsuit q$ is predicted to express exactly the

same proposition as $\Diamond(p \vee q \vee (p \wedge q))$. However, in English there are clear differences between these sentences:

(17)   ✓John might speak English and he might speak French,
       but of course he doesn't speak both.

(18)   #John might speak English or French, or both,
       but of course he doesn't speak both.

This contrast is consistent with APL, where $\Diamond(p \vee q \vee (p \wedge q))$ draws attention to the possibility that $p \wedge q$, but $\Diamond p \wedge \Diamond q$ doesn't. However, in CGR-09 the contrast cannot be explained, since both $\Diamond p \wedge \Diamond q$ and $\Diamond(p \vee q \vee (p \wedge q))$ draw attention to the possibility that $p \wedge q$.

Thus, in Szabolsci's scenario CGR-09 seems to make better predictions than APL, but for Alonso-Ovalle's examples the predictions of APL seem to be more appropriate. Below we will suggest two ways to ameliorate the predictions of APL in Szabolsci's scenario.

Concord.   One option would be to assume that *might*, in English, makes no direct semantic contribution. Instead it signals that there is some operator—call it *Op*—higher up in the syntactic tree, which is interpreted as $\Diamond$, and stands in a syntactic agreement relation with *might*. Crucially, we may assume that *Op* can agree with multiple occurrences of *might* in its scope. Similar proposals have been made in the literature for negation and modals, under the heading of *negative* and *modal concord*, respectively (see, for instance Zeijlstra, 2004, 2007, 2008). Under these assumption about *might*, sentences like (19) are structurally ambiguous: depending on their underlying syntactic structure, they could be translated into our logical language either as (19-a) or as (19-b).

(19)   John might speak English and he might speak French.
         a. $\Diamond p \wedge \Diamond q$
         b. $\Diamond(p \wedge q)$

Of course, sentences like (20) would also be structurally ambiguous, with (20-a) and (20-b) as possible translations depending on the underlying syntactic structure.

(20)   John might speak English or he might speak French.

a. $\diamondsuit p \vee \diamondsuit q$
b. $\diamondsuit (p \vee q)$

However, we have seen that (21-a) and (21-b) are semantically equivalent. So in the case of (21), the presumed structural ambiguity does not give rise to a semantic ambiguity.

The analysis sketched here would allow us to explain Szabolsci's observation: if (19) is interpreted as in (19-b), it draws attention to the possibility that John speaks both English and French, which makes it a useful recommendation in Szabolsci's scenario. This does not hold for (20), or for the variant where disjunction takes low scope under *might*, because these sentences do not draw attention to the possibility that John speaks both English and French, no matter what their underlying syntactic structure is.

At the same time, the analysis is compatible with Alonso-Ovalle's observation, since (19), on one of its syntactic analyses, does *not* draw attention to the possibility that John speaks both English and French, and therefore differs semantically in the relevant respect from *John might speak English or French or both*.

Subordination.   Another option would be to enrich the semantic apparatus, rather than the syntactic assumptions. In particular, we could assume that *might* sentences, besides drawing attention to certain possibilities, also make these possibilities available as hypothetical contexts relative to which subsequent sentences may be evaluated. This phenomena is known as *modal subordination* (see, for instance Roberts, 1989; Kaufmann, 2000; Brasoveanu, 2007). A detailed implementation is beyond the scope of this paper, but it can be expected that such an enrichment of the semantic framework would naturally lead to an account of Szabolsci's and Alonso-Ovalle's observations.

4   Attentive pragmatics

Standard Gricean pragmatics generally assumes a classical, truth-conditional semantics, where the meaning of a sentence is identified with its informative content. In APL, semantic meaning is not identified with informative content; rather, it encompasses both informative and attentive content. This shift in our conception of the notion of semantic meaning also changes our

perspective on pragmatics.[11] This new perspective on pragmatics, and the particular consequences that it has for the interpretation of *might*, were already explored in Ciardelli, Groenendijk, and Roelofsen (2009). We will make the ideas presented there slightly more explicit, and add an account of certain constructions involving embedded *might* sentences, in terms of pragmatic strengthening.[12]

4.1   Sincerity and transparency

Consider a conversation in which the participants' main purpose is to exchange information in order to resolve a given issue as effectively as possible. In such a cooperative effort, each participant must be *sincere*. In the present setting, this sincerity requirement has an informative and an attentive component. On the one hand, a speaker who utters a sentence φ must believe that the actual world lies in $\mathsf{info}(\varphi)$. We will call this *informative sincerity*. On the other hand, a speaker who draws attention to a certain possibility must consider this possibility a 'live possibility:' it must be consistent with her information state. This we will call *attentive sincerity*.

Participants must also be *transparent*. That is, if one participant draws attention to a certain possibility, and this possibility is inconsistent with the information state of another participant, then this other participant must publicly announce this inconsistency, so that other participants will refrain from considering the possibility in question. Notice that the sincerity requirement is *speaker* oriented, while the transparency requirement is *hearer* oriented.

Besides these qualitative sincerity and transparency requirements, there are also certain *quantitative preferences*. In particular, among all the sentences that could be sincerely uttered and that would be relevant for resolving the given issue under discussion, there is a general quantitative preference for *more informative* sentences—the more relevant information one provides, the more likely it is that the given issue will be resolved.

Without going into the more subtle details, let us lay out the basic repercussions of a pragmatic theory along these lines for the interpretation of *might*.

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[11]This is analogous to the way in which inquisitive semantics, by giving an inquisitive twist to the notion of semantic meaning, changes our perspective on pragmatics (see Groenendijk and Roelofsen, 2009).

[12]This section is also included, in more or less the same form, in Ciardelli *et al.* (2010).

4.2 Quality implicatures

There are two empirical observations about *might* that we have not discussed at all so far, even though each of them has given rise to one of the two 'classical' semantic theories of *might*. Both observations can be illustrated by means of our initial example:

(21)   John might be in London.

The first observation, perhaps the most basic one, is that if someone utters (21) we typically conclude that she considers it *possible* that John is in London. This observation has given rise to the analysis of *might* as an epistemic modal operator.

The second observation is that if someone hears (21) and already knows that John is not in London, she will typically object, pointing out that (21) is inconsistent with her information state. In this sense, even though *might* sentences do not provide any information about the state of the world, they can be 'inconsistent' with a hearer's information state. One classical account of this observation is that of Veltman (1996). Veltman's update semantics specifies for any given information state σ and any given sentence φ, what the information state $\sigma[\varphi]$ is that would result from updating σ with φ. The update effect of $\Diamond\varphi$ is defined as follows:

$$\sigma[\Diamond\varphi] = \left\{ \begin{array}{ll} \varnothing & \text{if } \varphi \text{ is inconsistent with } \sigma \\ \sigma & \text{otherwise} \end{array} \right.$$

The idea is that, if φ is inconsistent with a hearer's information state, then updating with $\Diamond\varphi$ leads to the absurd state. To avoid this, the hearer must make a public announcement signaling the inconsistency of φ with her information state. As a result, whoever uttered $\Diamond\varphi$ in the first place may also come to discard the possibility that φ holds.

Our semantics does not directly explain these observations. However, we believe that this is rightly so. In our view, both observations should be explained pragmatically. And they can be. It follows from the attentive sincerity requirement that if a cooperative speaker utters a sentence φ and α is a possibility in $[\varphi]$, then α must be consistent with the speaker's information state. In particular, a cooperative speaker who utters (21) must consider it possible that John is in London.[13]

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[13]It must be noted that the attentive sincerity requirement is sometimes 'neutralized' by other pragmatic factors. To see this, consider the sentences in (i-a-b) and (ii-a-b):

On the other hand, it follows from the transparency requirement that if a hearer is confronted with a sentence φ, and one of the possibilities for φ is inconsistent with her information state, then she must signal this inconsistency, in order to prevent other participants from considering the possibility in question a 'live option.'

Thus, both observations are accounted for. And this pragmatic account, unlike the mentioned semantic analyses, extends straightforwardly to more involved cases. Consider for instance:

(22)   John might be in London or in Paris.

This sentence is problematic for both semantic accounts just mentioned. The epistemic modality account predicts that the speaker considers it possible that John is in London or in Paris. But note that this is compatible with the speaker knowing perfectly well that John is not in London. What (22)

(i)   a.   John might be in London or in Paris.
        b.   John is in London or in Paris.

(ii)   a.   John is somewhere in Europe.
        b.   Where is John?

The sentences in (i-a-b) license the inference that the speaker considers it possible that John is in London and that she considers it possible that John is in Paris. The sentences in (ii-a-b) however, do not license this inference: a cooperative speaker who utters these sentences may very well know that John is not in London or in Paris. This is surprising under the assumption that indefinites and constituent questions draw attention to possibilities, just like disjunction, polar questions, and *might* sentences, and that the attentive sincerity requirement applies to each of these possibilities.

There are at least two possible ways to explain the contrast between (i) and (ii). First, the indefinite in (ii-a) and the question in (ii-b) are quantifying operators, and the domain that they quantify over is generally understood to be implicitly restricted. Thus, we cannot tell from the surface form of these sentences whether or not the intended domain of quantification contains Paris and/or London. Hence, the relevant inference does not arise. Notice that the constructions in (i-a-b) do not involve quantification. Thus, in these cases the inference cannot be blocked by uncertainty regarding the domain of quantification.

Another factor that plausibly plays a role is *efficiency*. Consider a speaker who knows that John must be somewhere in Europe, but not in Paris, Barcelona, Rome, Prague, Vienna, or Berlin. Such a speaker could choose to ask the question in (ii-b) without explicitly stating that she already knows that John is not in any of the mentioned cities. Strictly speaking, this move is not fully cooperative. However, this is outweighed by the fact that the fully cooperative alternative move is highly inefficient. This is different for, say, (i-b). In this case, the more cooperative alternative, which is just to state that John is in London, would also be more efficient.

implies is something stronger, namely that the speaker considers it possible that John is in London *and* that she considers it possible that John is in Paris. This follows straightforwardly on our pragmatic account.

Now consider a hearer who is confronted with (22) and who knows that John is possibly in Paris, but certainly not in London. We expect this hearer to object to (22). But Veltman's update semantics does not predict this: it predicts that an update with (22) has no effect on the hearer's information state. Our pragmatic account on the other hand, does urge the hearer to object.

The only task of our semantics is to specify which propositions are expressed by which sentences. The pragmatics, then, specifies when a speaker is licensed to utter a certain sentence, and how a hearer is supposed to react to a given utterance. Together, these two components account for the basic features of *might* that classical semantic theories take as their point of departure. Shifting some of the weight from semantics to pragmatics evades problems with more involved cases, like (22), in a straightforward way. But, of course, the necessary pragmatic principles can only be stated if the underlying semantics captures more than just informative content.

4.3   Quantity implicatures

If someone says that John might be in London, we typically do not only conclude that she considers it possible that John is in London, but also that she considers it possible that he is *not* in London. In short, we infer that she is *ignorant* as to whether John is in London or not.

This implicature is straightforwardly derived. We have already seen how to establish the inference that the speaker considers it *possible* that John is in London and that John is in Paris. Moreover, it follows from the quantitative preference for more informative sentences that whenever a cooperative speaker S utters a sentence φ and α is a possibility in $[\varphi]$ such that $\alpha \subseteq \mathsf{info}(\varphi)$, we can conclude that S does not have sufficient information to sincerely utter a sentence φ' expressing the proposition $\{\alpha\}$. After all, assuming that φ is strictly related to the given question under discussion, φ' would also be strictly related to the given question under discussion (under any sensible notion of relatedness). Thus, the only possible reason why S did not directly utter φ' instead of the less informative φ is that her information state does not support the informative content of φ'. In other words, she is not certain whether the actual world is contained in α.

4.4 Epistemic re-interpretation

In certain embedded environments, $\diamond p$ really seems to be interpreted as saying that p is consistent with some contextually given body of information (usually, but not necessarily, the information state of the speaker). For instance, (23) univocally conveys that the speaker believes that John will not go to London.

(23)   It is not true that John might go to London.

If the sentence is analyzed as $\neg\diamond\varphi$, then it is predicted to be a contradiction in APL, which is evidently not the right prediction.

  One may be tempted to conclude that this simply shows that *might* is ambiguous, permitting both an 'epistemic use' and an 'attentive use,' and possibly other usages as well. However, it may be worth trying to avoid such a conclusion, at least in its strongest form. For, if *might* were simply ambiguous between an attentive use and an epistemic use, then we would lose our explanation for the fact that *might* obligatorily takes wide scope over standard negation, unlike modal operators like 'it is consistent with my beliefs that.' Recall the relevant example:

(24)   John might not go to London.

In section 3.2, we hypothesized that negation cannot take wide scope in (24) because $\neg\diamond p$ is a contradiction. But of course this explanation only goes through if the semantic contribution of $\diamond p$ indeed univocally lies in its potential to draw attention to the possibility that p. If $\diamond p$ were ambiguous, and could also be interpreted semantically as saying that p is consistent with some contextually determined body of information, then there would be no reason anymore why negation should obligatorily take narrow scope. After all, we saw that negation is perfectly happy with wide scope in sentences like (25):

(25)   It is not consistent with my beliefs that John will go to London.

Thus, rather than assuming plain ambiguity, we would like to offer a more nuanced account of the epistemic interpretation of *might* in (23). We will argue that in this particular case there is a specific *reason* not to adopt the standard interpretation of $\diamond p$, and that this triggers *re-interpretation* of $\diamond p$

in terms of the implicatures that it typically triggers when not embedded.[14]

   More specifically, we hypothesize that (23) is interpreted as a denial of one or more implicatures of the embedded clause. It is in fact a common use of 'it is not true that' constructions to deny pragmatic inferences or presuppositions of their complement clause. For example, in (26) the implicature of the embedded clause is denied, and in (27) the presupposition of the embedded clause is denied:

(26)   It is not true that John has four children. He has five.

(27)   It is not true that the king of France is bald. There is no king of France.

Notice that (23) is not necessarily interpreted as denying that it is *possible* that John will go to London. It may also be interpreted as denying the stronger implicature that it is *unknown* whether John will go to London or not. For, someone who utters (23) may continue as in (28), but also as in (29) (where SMALLCAPS indicate contrastive stress).[15]

(28)   It is not true that John might go to London. He will go to PARIS.

(29)   It is not true that John might go to London. He WILL go to London.

Notice that a similar pattern arises with disjunction:

(30)   It is not true that John speaks English or French. He speaks NEITHER.

(31)   It is not true that John speaks English or French. He speaks BOTH.

These observations support the idea that 'it is not true that' constructions can be interpreted as denying pragmatic inferences that the embedded clause gives rise to, and thus lend support to a re-interpretation analysis of examples like (23).

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[14]The proposal made here is in line with recent observations by Levinson (2000) and Chierchia, Fox, and Spector (2008), among others, that the semantic contribution of certain expressions is sometimes strengthened 'locally', i.e., before it enters the semantic composition process. Construing this process as 're-interpretation' is especially in line with Geurts' (2009) take on such phenomena.

[15]In (29) and (31), it is strongly preferred, perhaps even necessary, to not only place contrastive stress on *will* and *both*, but also on *might* and *or*. This observation does not seem to affect our argument however. See (Fox and Spector, 2009) for relevant discussion.

One may ask, of course, why this same re-interpretation strategy could not be applied in (24). We would argue that re-interpretation only occurs if it is triggered. In (24), negation can take narrow scope, and the interpretation of $\diamond \neg p$ is unproblematic. Thus, there is no need for re-interpretation. In (23) however, negation is forced to take wide scope, and $\neg \diamond p$ is, at face value, a contradiction. This is what triggers re-interpretation in this case.

In Ciardelli *et al.* (2010) it is argued that this explanation also extends to the interpretation of *might* clauses embedded under a question operator or in the antecedent of a conditional. We hypothesize, therefore, that non-attentive readings of *might* are generally the result of re-interpretation. More work is needed, of course, to solidify this claim. But we think this is a direction worth pursuing.[16]

5   Final remarks

The idea that the core semantic contribution of *might* sentences lies in their potential to draw attention to certain possibilities has been entertained before. For instance, Groenendijk, Stokhof, and Veltman (1996) wrote that "in many cases, a sentence of the form *might-φ* will have the effect that one becomes aware of the possibility of φ."[17] However, it was thought that capturing this aspect of the meaning of *might* would require a more complex notion of possible worlds and information states, and a different way to think about growth of information. Thus, immediately following the above quotation, Groenendijk *et al.* (1996) write that their own framework "is one in which possible worlds are total objects, and in which growth of information about the world is explicated in terms of elimination of possible worlds. Becoming aware of a possibility cannot be accounted for in a natural fashion in such an eliminative approach. It would amount to extending partial worlds, rather than eliminating total ones. To account for that aspect of the meaning of *might* a constructive approach seems to be called for."

The present paper can be taken to show that attentive content can in fact be dealt with in an eliminative setup. In the framework developed

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[16]A weaker hypothesis that may be worth considering is that the attentive use of *might* is historically primary, and that non-attentive usages are derivative, though (partly) grammaticized (in the general spirit of, e.g., Levinson, 2000).

[17]See also the more recent work of Swanson (2006), Franke and de Jager (2008), Brumwell (2009), and de Jager (2009).

here, possible worlds are still total objects, and growth of information is still explicated in terms of eliminating worlds. Nonetheless, the meaning of a sentence embodies both informative and attentive content.

It is perhaps worth emphasizing that, even though part of this paper was focused on giving a systematic account of the possibilities that *might* sentences draw attention to, we certainly do not think that this is all there is to the meaning of *might*. Drawing attention to possibilities may have several side-effects. We discussed how ignorance implicatures typically enter the picture through (possibly grammaticized) pragmatic reasoning. Another potential side-effect of drawing attention to a certain possibility, also briefly mentioned at the end of section 3, is the introduction of a *hypothetical context*, an idea that is familiar from the literature on modal subordination (Roberts, 1989; Kaufmann, 2000; Brasoveanu, 2007, among others).

Finally, we would like to emphasize that the primary purpose of this paper was not so much to propose a novel analysis of *might*, but rather to develop a formal framework that can be used to capture attentive content more generally. The analysis of *might* was intended to illustrate the usefulness of the framework. One other domain where attentive content seems to play a crucial role is that of *evidentials*. For instance, certain types of evidentials have been reported to 'present' a certain proposition, without establishing whether that proposition holds or not (see, for instance, Faller, 2002; Murray, 2010). These are precisely the type of empirical findings that the framework developed in this paper could help to elucidate.

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