

Radical inquisitive semantics*

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1 Inquisitive semantics: propositions as proposals

Traditionally, the meaning of a sentence is identified with its *informative* content. In much recent work, this notion is given a dynamic twist, and the meaning of a sentence is taken to be its potential to change the ‘common ground’ of a conversation. The most basic way to formalize this idea is to think of the common ground as a set of possible worlds, and of a sentence as providing information by eliminating some of these possible worlds.

Of course, this picture is limited in several ways. First, when exchanging information sentences are not only used to provide information, but also—crucially—to *raise issues*, that is, to indicate which kind of information is desired. Second, the given picture does not take into account that updating the common ground is a *cooperative* process. One conversational participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other participants may react to such a proposal in several ways. In a cooperative conversation, changes of the common ground come about by mutual agreement.

In order to overcome these limitations, *inquisitive semantics* (Groenendijk, 2009; Mascarenhas, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009, among others) starts with a different picture. It views propositions as proposals to update the common ground. Crucially, these proposals do not always

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specify just one way of updating the common ground. They may suggest alternative ways of doing so, among which the addressee is then invited to choose. Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds and embodies a possible way to update the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites other participants to provide information in such a way that one or more of the proposed updates may be established. Inquisitive propositions raise an issue. They indicate which kind of information is desired. In this way, inquisitive semantics directly reflects the idea that information exchange consists in a cooperative dynamic process of raising and resolving issues.

A concrete implementation of inquisitive semantics for the language of propositional and first-order predicate logic has been specified in (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009; Ciardelli, 2009a,b, among other places). Here we will argue that this system, to which we will refer as *conservative* inquisitive semantics, only partially captures the central underlying conception of sentences as expressing proposals to update the common ground. Subsequently, an enriched implementation will be presented and illustrated in some detail.

2 Positive and negative responses

Conceiving of a proposition as a set of possibilities makes it possible to generate predictions as to what the positive responses to a given sentence are. For instance, (1) expresses a proposition consisting of two possibilities, corresponding with the two positive responses in (2):

- (1) Pete will play the piano, and Sue will sing or Mary will dance.
- (2)
 - a. Yes, Pete will play the piano and Sue will sing.
 - b. Yes, Pete will play the piano and Mary will dance.

So (1) is inquisitive, and this inquisitiveness stems from the second conjunct, which is a disjunction. We obtain the proposition expressed by a disjunction of two sentences φ and ψ by taking the union of the possibilities for φ and for ψ . For the disjunctive conjunct of (1) this means that we obtain two possibilities: the set of worlds where Sue sings, and the set of worlds where Mary dances.

As for conjunction, we obtain the proposition expressed by a conjunction of two sentences φ and ψ by taking the pairwise intersection of the possibilities for φ and the possibilities for ψ . In the case of (1), this means that we have to take

the pairwise intersection of the single possibility for the first conjunct, the set of worlds where Pete plays the piano, and each of the two possibilities we found for the disjunction in the second conjunct. This gives us two possibilities: one of them is the single possibility for (2a), the other is the single possibility for (2b). In this way, the semantics indeed captures the fact that (2a) and (2b) are positive responses to (1).

However, if sentences are taken to express proposals, then preferably our semantics should not only make predictions about positive responses, which accept the proposal in question, but also about *negative* responses, which *reject* the proposal. For instance, just as our semantics predicts that (1) licenses the positive responses in (2), we would like it to predict that (1) licenses the negative responses in (3):

- (3) a. No, Pete will not play the piano.
- b. No, Sue will not sing and Mary will not dance.

Conservative inquisitive semantics does not generate such predictions. It formally represents a proposal as a set of possibilities, and these possibilities correspond to positive responses. One way to bring negative responses into the picture as well is to represent a proposal not just as a set of possibilities, but rather as a set of possibilities plus a set of *counter-possibilities*, where possibilities correspond to positive responses and counter-possibilities to negative responses. This is indeed the approach that will be explored below. We will see that it deals with the above examples in a straightforward way, and also that it has some interesting further consequences, especially for the interpretation of conditional sentences.

3 Radical inquisitive semantics

For reasons of space, we consider a propositional language here. The extension to the first-order case seems straightforward.

Definition 1 (Language). We consider a language whose formulas are built up from a finite set of proposition letters \mathcal{P} , using the standard operators \neg , \wedge , \vee and \rightarrow , and an additional operator \div . We will refer to \div as *inversion*, and for any formula φ , we will refer to $\div\varphi$ as the *inverse* of φ . Finally, we will use $? \varphi$ as an abbreviation of $\varphi \vee \div\varphi$.

The basic ingredients of the semantics are worlds and possibilities.

Definition 2 (Worlds and possibilities).

A *world* is a function from \mathcal{P} to $\{0, 1\}$. A *possibility* is a set of worlds.

For any possibility α , $\bar{\alpha}$ will denote the *complement* of α , i.e., the set of all worlds not in α . For any formula φ , $|\varphi|$ will denote the possibility consisting of all worlds that make φ true in a classical setting. We will refer to $|\varphi|$ as the *truth-set* of φ .

Definition 3 below recursively defines, for every sentence φ in our language, the *proposition* $\lceil \varphi \rceil$ expressed by φ , and the *counter-proposition* $\lfloor \varphi \rfloor$ for φ . Both $\lceil \varphi \rceil$ and $\lfloor \varphi \rfloor$ will be sets of possibilities. We will refer to the elements of $\lceil \varphi \rceil$ as the *possibilities for* φ , and to the elements of $\lfloor \varphi \rfloor$ as the *counter-possibilities for* φ . The clauses of the definition will be illustrated right below.

Definition 3 (Radical inquisitive semantics).

1. $\lceil p \rceil := \{ |p| \}$
 $\lfloor p \rfloor := \{ \overline{|p|} \}$
2. $\lceil \neg \varphi \rceil := \{ \cap_{\alpha \in \lceil \varphi \rceil} \bar{\alpha} \}$
 $\lfloor \neg \varphi \rfloor := \lceil \varphi \rceil$
3. $\lceil \varphi \vee \psi \rceil := \lceil \varphi \rceil \cup \lceil \psi \rceil$
 $\lfloor \varphi \vee \psi \rfloor := \{ \alpha \cap \beta \mid \alpha \in \lfloor \varphi \rfloor \text{ and } \beta \in \lfloor \psi \rfloor \}$
4. $\lceil \varphi \wedge \psi \rceil := \{ \alpha \cap \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lceil \psi \rceil \}$
 $\lfloor \varphi \wedge \psi \rfloor := \lfloor \varphi \rfloor \cup \lfloor \psi \rfloor$
5. $\lceil \varphi \rightarrow \psi \rceil := \{ \gamma_f \mid f: \lceil \varphi \rceil \rightarrow \lceil \psi \rceil \}$ where $\gamma_f := \cap_{\alpha \in \lceil \varphi \rceil} \alpha \Rightarrow f(\alpha)$
 $\lfloor \varphi \rightarrow \psi \rfloor := \{ \alpha \Rightarrow \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lceil \psi \rceil \}$
6. $\lceil \div \varphi \rceil := \lfloor \varphi \rfloor$
 $\lfloor \div \varphi \rfloor := \lceil \varphi \rceil$

The final clause of the definition says that the proposition expressed by the *inverse* of a sentence φ is the *counter-proposition* for φ itself. And vice versa, the counter-proposition for the inverse of φ is the proposition expressed by φ itself.

The clause for implication is defined in terms of a two-place operator \Rightarrow , which remains to be specified. Notice that \Rightarrow takes two possibilities as its input and yields a third possibility as its output. For concreteness and simplicity, we

will define \Rightarrow as material implication here. But in principle, any more sophisticated existing analysis of non-inquisitive conditionals could be ‘plugged in’ here. We will return to this point in section 3.8.

Definition 4 (\Rightarrow). $\alpha \Rightarrow \beta := \bar{\alpha} \cup \beta$.

The remainder of this paper is entirely devoted to explaining and illustrating the semantics specified in definition 3. We start with the clauses for atomic sentences, disjunction and conjunction.

3.1 Atoms, disjunction, and conjunction

Consider example (1), which was used in section 2 to motivate our move to a more radical inquisitive semantics. A translation of (1) into our formal language is:

$$(4) \quad p \wedge (q \vee r)$$

According to the atomic clause, the proposition expressed by the first conjunct p is $\{ |p| \}$, and similarly for the atomic sentences q and r . The clause for disjunction tells us that:

$$(5) \quad [q \vee r] = [q] \cup [r] = \{ |q|, |r| \}$$

And the clause for conjunction yields:

$$(6) \quad [p \wedge (q \vee r)] = \{ |p \wedge q|, |p \wedge r| \}$$

So there are two possibilities for $p \wedge (q \vee r)$. The sentence is inquisitive, and its inquisitiveness can be resolved by providing the information that p and q are the case or the information that p and r are the case. This explains the fact that (2a) and (2b) are positive responses to (1).

Now let us turn to the counter-proposition for (4). First, the atomic clause says that the counter-proposition for p is $\{ |\bar{p}| \}$, which is the same as $\{ |\neg p| \}$, and similarly for q and r . The clause for disjunction tells us that:

$$(7) \quad [q \vee r] = \{ |\neg q| \cap |\neg r| \} = \{ |\neg q \wedge \neg r| \}$$

And the clause for conjunction yields:

$$(8) \quad [p \wedge (q \vee r)] = [p] \cup [q \vee r] = \{ |\neg p|, |\neg q \wedge \neg r| \}$$

Thus, there are two counter-possibilities for (4), corresponding exactly to the two negative responses in (3a) and (3b).

3.2 Negation

Next, consider a simple example involving negation:

(9) Sue will not sing or dance.

a. *Positive response:*

That's right, she will not sing or dance.

b. *Negative responses:*

That's not right, she will sing.

That's not right, she will dance.

We translate (9) as $\neg(q \vee r)$. The proposition expressed by a negated sentence $\neg\varphi$ always consists of a single possibility, which is the intersection of the complements of all the possibilities for φ (or, equivalently, the complement of the union of all the possibilities for φ). Thus, for the particular case of $\neg(q \vee r)$, we get:

$$(10) \quad [\neg(q \vee r)] = \{ |\neg q| \cap |\neg r| \} = \{ |\neg q \wedge \neg r| \}$$

That is, the proposition expressed by $\neg(q \vee r)$ consists of a single possibility, which corresponds with the positive response in (9a).

Now let us turn to the counter-proposition for $\neg(q \vee r)$. The counter-clause for negation tells us that, in general, the counter-proposition for $\neg\varphi$ is the proposition expressed by φ . Thus for the case of $\neg(q \vee r)$ we get:

$$(11) \quad [\neg(q \vee r)] = [q \vee r] = \{ |q|, |r| \}$$

This means that there are two counter-possibilities for $\neg(q \vee r)$, which correspond to the negative responses in (9b).

3.3 Atomic questions

Next, let us consider the interpretation of an atomic polar question $?q$. By definition, $?q$ abbreviates $q \vee \div q$. So $[?q] = [q \vee \div q]$. The clause for disjunction tells us that $[q \vee \div q] = [q] \cup [\div q]$. By the clause for inversion, we have that $[q] \cup [\div q] = [q] \cup [q]$, which, by the atomic clause, amounts to $\{ |q| \} \cup \{ \overline{|q|} \}$, and that is the same as $\{ |q|, |\neg q| \}$. So:

$$(12) \quad [?q] = \{ |q|, |\neg q| \}$$

This means that there are two positive responses to the atomic polar question $?q$, corresponding to *yes* and *no*.

As for the counter-proposition expressed by $?q$, we have, by definition of $?q$ as an abbreviation of $q \vee \div q$, that $[?q] = [q \vee \div q]$. By the counter-clause for disjunction, we have that $[q \vee \div q] = \{ \alpha \cap \beta \mid \alpha \in [q] \text{ and } \beta \in [\div q] \}$, which reduces to $\{ \alpha \cap \beta \mid \alpha \in \{ |\neg q| \} \text{ and } \beta \in \{ |q| \} \}$. So, what we end up with, not surprisingly, is:

$$(13) \quad [?q] = \{ \emptyset \}$$

This means that there are no non-contradictory negative responses to $?q$.

It is perhaps worth emphasizing that questions are defined in terms of inversion here. This is a novel concept, so our treatment of questions is bound to diverge from previous analyses. In conservative inquisitive semantics for instance, questions are defined in terms of negation rather than inversion. That is, $?q$ is defined as an abbreviation of $q \vee \neg q$ rather than $q \vee \div q$. This happens to give exactly the same results for *atomic* polar questions. But the two theories start to make different predictions as soon as we go beyond the atomic case. Take, for instance, a simple conjunctive polar question:

$$(14) \quad \text{Will Sue sing and dance?}$$

We translate (14) into our logical language as $?(q \wedge r)$. In radical inquisitive semantics, there are three possibilities for this sentence, $|q \wedge r|$, $|\neg q|$, and $|\neg r|$, which correspond to the following responses:

- (15)
 - a. Yes, she will sing and dance.
 - b. No, she won't sing.
 - c. No, she won't dance.

In conservative inquisitive semantics, the proposition expressed by (14) consists of just two possibilities, $|q \wedge r|$ and $|\neg(q \wedge r)|$. This means that (15b) and (15c) are not predicted to be congruent responses to (14) in any straightforward way. This is clearly a shortcoming, which, for all we know, is shared by any other previous account of questions. And it is just the tip of an iceberg.

The treatment of questions in terms of inversion also has consequences for the analysis of conditional questions, which will be discussed in some detail below.

3.4 Conditionals with disjunctive consequents

The first example that we will use to illustrate the clause for implication is the conditional in (16). Notice that the consequent of this conditional is disjunctive (and therefore inquisitive). We want to derive that the positive and negative responses to (16) are the ones specified in (16a) and (16b), respectively.

(16) If Pete plays the piano, then Sue will sing or Mary will dance.

a. *Positive responses:*

Yes, if Pete plays the piano, Sue will sing.

Yes, if Pete plays the piano, Mary will dance.

b. *Negative response:*

No, if Pete plays the piano, Sue won't sing and Mary won't dance.

We translate (16) into our formal language as (17), and we will show that the proposition expressed by (17) and the counter-proposition for (17) are the ones in (17a) and (17b), respectively, which correspond exactly with the positive and negative responses specified in (16a) and (16b).

(17) $p \rightarrow (q \vee r)$

a. $\lceil p \rightarrow (q \vee r) \rceil = \{ |p \rightarrow q|, |p \rightarrow r| \}$

b. $\lfloor p \rightarrow (q \vee r) \rfloor = \{ |p \rightarrow (\neg q \wedge \neg r)| \}$

First consider the proposition $\lceil p \rightarrow (q \vee r) \rceil$. For convenience, let us repeat the clause for implication:

(18) $\lceil \varphi \rightarrow \psi \rceil := \{ \gamma_f \mid f: \lceil \varphi \rceil \rightarrow \lceil \psi \rceil \} \quad \text{where } \gamma_f := \bigcap_{\alpha \in \lceil \varphi \rceil} \alpha \Rightarrow f(\alpha)$

The idea behind this clause is the following. The proposal expressed by a sentence can in general be *realized* in one or more ways. That is, if a proposal consists of just one possibility, then it proposes just one update, and it can be realized in exactly one way, namely by establishing that update. If a proposal consist of several possibilities, it proposes several possible updates, and this means that it can be realized in several ways, namely by establishing either one (or more) of the proposed updates. What we take to be the ‘positive responses’ to a given proposal are sentences that do exactly this: they realize one of the proposed updates.

Now, under this perspective, a conditional sentence $\varphi \rightarrow \psi$ can be thought of as expressing a proposal to establish a certain *implicational dependency* between the ways in which φ may be realized and the ways in which ψ may be realized, or,

in more neutral terms, between the possibilities for φ and the possibilities for ψ . Such a dependency links every possibility $\alpha \in [\varphi]$ to some possibility $f(\alpha) \in [\psi]$, in such a way that for all $\alpha \in [\varphi]$, $\alpha \Rightarrow f(\alpha)$ holds.¹

How many potential implicational dependencies there are depends on the number of possibilities for φ and ψ . If there are m possibilities for φ and n possibilities for ψ then there are n^m functions from $[\varphi]$ to $[\psi]$. Each of these functions f links every possibility $\alpha \in [\varphi]$ to some possibility $f(\alpha) \in [\psi]$. Thus, each of these functions corresponds with a potential implicational dependency between the possibilities for φ and the possibilities for ψ .²

In order to establish the implicational dependency corresponding to some function f from $[\varphi]$ to $[\psi]$, we have to establish that $\alpha \Rightarrow f(\alpha)$ holds for all $\alpha \in [\varphi]$. This means that we have to establish $\bigcap_{\alpha \in [\varphi]} \alpha \Rightarrow f(\alpha)$.³ Notice that this intersection is a possibility, which is called γ_f in the clause for implication. For each function $f: [\varphi] \rightarrow [\psi]$, then, there is a corresponding possibility γ_f , and together, these possibilities make up the proposition expressed by $\varphi \rightarrow \psi$.

Now let us return to our example, $p \rightarrow (q \vee r)$. We have already seen that $[p] = \{ |p| \}$ and $[q \vee r] = \{ |q|, |r| \}$. Thus, there are two functions from $[p]$ to $[q \vee r]$, one that maps $|p|$ to $|q|$, and another one that maps $|p|$ to $|r|$. Call the first one f_q and the second one f_r . Then, the clause for implication tells us that $[p \rightarrow (q \vee r)]$ consists of two possibilities, γ_{f_q} and γ_{f_r} , where $\gamma_{f_q} = |p| \Rightarrow |q| = |p \rightarrow q|$ and $\gamma_{f_r} = |p| \Rightarrow |r| = |p \rightarrow r|$. Thus, we obtain the desired result:

$$(19) \quad [p \rightarrow (q \vee r)] = \{ |p \rightarrow q|, |p \rightarrow r| \}$$

Next we turn to the counter-proposition for $p \rightarrow (q \vee r)$. Again, let us pause one moment to repeat the counter-clause for implication, and briefly explain the intuition behind it.

$$(20) \quad [\varphi \rightarrow \psi] := \{ \alpha \Rightarrow \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi] \}$$

¹Notice that the implicational dependencies in question do not necessarily reveal any ‘inherent dependency’ between the possibilities for φ and the possibilities for ψ . In fact, the type of dependency that is involved is completely determined by the interpretation of \Rightarrow . As mentioned above, there are many ways in which \Rightarrow may be defined. If it is defined as material implication, as we did above, then the implicational dependency that $\varphi \rightarrow \psi$ proposes to establish has nothing to do with any kind of inherent dependency. This is why we use the term ‘implicational dependency’.

²In case the antecedent is not inquisitive, as in the example (18), $[\varphi]$ contains just one possibility, and the number of functions from $[\varphi]$ to $[\psi]$ equals the number of possibilities in $[\psi]$.

³In case the antecedent is not inquisitive, as in the example (18), $[\varphi]$ contains just one possibility α and $\bigcap_{\alpha \in [\varphi]} \alpha \Rightarrow f(\alpha)$ boils down to $\alpha \Rightarrow f(\alpha)$.

As specified above, we think of $\varphi \rightarrow \psi$ as expressing a proposal to establish a certain implicational dependency between the possibilities for φ and the possibilities for ψ . Rejecting such a proposal, then, amounts to saying that none of the potential dependencies could possibly be established. This means that there must be some way of realizing φ that leads to the *rejection* of ψ . Thus, to reject $\varphi \rightarrow \psi$, we must point out that the realization of some possibility α for φ implies the realization of some counter-possibility β for ψ . This is why for every $\alpha \in \lceil \varphi \rceil$ and every $\beta \in \lfloor \psi \rfloor$, $\alpha \Rightarrow \beta$ is a counter-possibility for $\varphi \rightarrow \psi$, corresponding to a negative response. So if there are m possibilities for φ and n counter-possibilities for ψ then there are (at most) $m \times n$ counter-possibilities for $\varphi \rightarrow \psi$.

Returning to our concrete example, the counter-possibilities for $p \rightarrow (q \vee r)$ are possibilities of the form $\alpha \Rightarrow \beta$, where $\alpha \in \lceil p \rceil$ and $\beta \in \lfloor q \vee r \rfloor$. Recall that $\lfloor q \vee r \rfloor = \{ |\neg q \wedge \neg r| \}$. So, since there is only one possibility for the antecedent p and only one counter-possibility for the consequent $q \vee r$, there is also only one counter-possibility for the implication as a whole, which indeed corresponds with the negative response in (16b):

$$(21) \quad \lfloor p \rightarrow (q \vee r) \rfloor = \{ |p \rightarrow (\neg q \wedge \neg r)| \}$$

3.5 Conditionals with disjunctive antecedents

If we reverse the antecedent and the consequent of example (16) we arrive at (22). Notice that the number of the positive and negative responses is also reversed.

(22) If Sue sings or Mary dances, then Pete will play the piano.

a. *Positive response:*

Yes, if Sue sings, Pete will play, and if Mary dances, he'll play too.

b. *Negative responses:*

No, if Sue sings Pete will not play.

No, if Mary dances Pete will not play.

We translate (22) as (23). The proposition expressed by (23) is (23a) and the counter-proposition for (23) is (23b), which correspond exactly with the positive and negative responses specified in (22a) and (22b).

$$(23) \quad (q \vee r) \rightarrow p$$

a. $\lceil (q \vee r) \rightarrow p \rceil = \{ |q \rightarrow p| \cap |r \rightarrow p| \}$

b. $\lfloor (q \vee r) \rightarrow p \rfloor = \{ |q \rightarrow \neg p|, |r \rightarrow \neg p| \}$

Since there is only a single possibility $|p|$ for the consequent of (23), there is only one function f that maps both possibilities $|q|$ and $|r|$ for the inquisitive antecedent of (23) to $|p|$. Thus, the single possibility γ_f for (23) is $(|q| \Rightarrow |p|) \cap (|r| \Rightarrow |p|)$, which, as (23a) reports, is the same as $|q \rightarrow p| \cap |r \rightarrow p|$.

The counter-possibilities for (23) are of the form $\alpha \Rightarrow \beta$, where α is a possibility for the antecedent, $q \vee r$, and β is a counter-possibility for the consequent, p . The possibilities for $q \vee r$ are $|q|$ and $|r|$, and the only counter-possibility for p is $|\neg p|$. So there are two counter-possibilities for (23): $|q| \Rightarrow |\neg p|$ and $|r| \Rightarrow |\neg p|$, which, as (23b) reports, can also be written as $|q \rightarrow \neg p|$ and $|r \rightarrow \neg p|$.

3.6 Conditional questions

Next consider the conditional question in (24). The positive responses that we would like to derive are listed in (24a), and the negative response, which is a denial of the antecedent of the conditional question, is given in (24b).⁴

(24) If Pete plays the piano, will Sue sing?

a. *Positive responses:*

Yes, if Pete plays the piano, then Sue will sing.

No, if Pete plays the piano, then Sue will not sing.

b. *Negative response:*

Well, Pete will not play the piano.

We translate (24) into our logical language as $p \rightarrow ?q$, which expresses the proposition specified in (25a) and has the counter-proposition in (25b):⁵

(25) $p \rightarrow ?q \equiv p \rightarrow (q \vee \div q) \equiv p \rightarrow (q \vee \neg q)$

a. $[p \rightarrow ?q] = \{ |p| \Rightarrow |q|, |p| \Rightarrow |\neg q| \} = \{ |p \rightarrow q|, |p \rightarrow \neg q| \}$

⁴There is an ongoing controversy in the literature about the exact status of responses that deny the antecedent of a conditional question, such as (24b) (see, for instance, Isaacs and Rawlins, 2008). The common intuition is that such responses do not really resolve the given question, but that they do have, in the words of Isaacs and Rawlins, an *issue-dispelling* effect. This is perfectly in line with their treatment here as negative responses: as such, they do not resolve the given question, but rather reject the given proposal altogether. It is perhaps worth pointing out that conservative inquisitive semantics, as well as many other frameworks, has nothing to say about ‘denial of the antecedent’ responses.

⁵Incidentally, we could just as well have translated (24) as $?(p \rightarrow q)$, which is equivalent with $p \rightarrow ?q$ in the present system (in the sense that it expresses exactly the same proposition, and has exactly the same counter-proposition).

$$\text{b. } \lfloor p \rightarrow ?q \rfloor = \{ |p| \Rightarrow (|q| \cap |\neg q|) \} = \{ |p| \Rightarrow \emptyset \} = \{ |\neg p| \}$$

As is to be expected, $p \rightarrow ?q$ is inquisitive: $\lceil p \rightarrow ?q \rceil$ consists of two possibilities, which correspond to the two positive responses in (24a).

Perhaps more surprisingly, whereas we saw earlier that atomic questions do not license any sensible negative response, we now see that conditional questions do. In particular, the counter-proposition for $p \rightarrow ?q$ contains exactly one possibility, which corresponds with the negative response in (24b).

3.7 Denying the antecedent of a conditional assertion

The present framework also offers a new perspective on responses that deny the antecedent of a conditional assertion, like our earlier example (16). Such responses do not count as negative responses to the conditional itself, but they do count as negative responses to the ‘question behind’ the conditional. If we take the question behind any sentence φ to be $?_\varphi$, then the question behind (16) is (26), which is translated into our logical language as (27). The first two positive responses to (26) are also positive responses to (16), the third positive response to (26) is a negative response to (16), and the negative response to (26) denies the antecedent of (16).

(26) Will Sue sing or Mary dance, if Pete plays the piano?

a. *Positive responses:*

Yes, if Pete plays the piano, Sue will sing.

Yes, if Pete plays the piano, Mary will dance.

No, if Pete plays the piano, Sue won’t sing and Mary won’t dance.

b. *Negative response:*

Well, Pete will not play the piano.

(27) $?(p \rightarrow (q \vee r))$

a. $\lceil ?(p \rightarrow (q \vee r)) \rceil = \{ |p \rightarrow q|, |p \rightarrow r|, |p \rightarrow (\neg q \wedge \neg r)| \}$

b. $\lfloor ?(p \rightarrow (q \vee r)) \rfloor = \{ |\neg p| \}$

In general we may distinguish three types of responses to an informative sentence φ . First, there are the positive and negative responses specified directly by the proposition expressed by φ and the counter-proposition for φ . Together these responses also form the positive responses to the question $?_\varphi$ behind φ . The third

class of responses to φ are the negative responses to $? \varphi$.⁶

3.8 Unconditionals

As a final example, consider (28), in a sense the reverse of (24):⁷

- (28) Whether Pete plays the piano or not, Sue will sing.

Sentences of this kind are referred to as *concessive conditionals*, or *unconditionals*. Rawlins (2008) argues that they are conditional sentences whose antecedent is a question. This suggests translating (28) as $?p \rightarrow q$, which abbreviates $(p \vee \neg p) \rightarrow q$. In our system, this is equivalent with $(p \vee \neg p) \rightarrow q$ and with $(p \rightarrow q) \wedge (\neg p \rightarrow q)$. From these equivalences, it should immediately be clear that we predict (28) to license the following positive and negative responses:

- (29) a. *Positive response*:
 Yes, if Pete plays the piano Sue will sing,
 and if he doesn't play, she will sing too.
 b. *Negative responses*:
 No, if Pete plays the piano, Sue won't sing.
 No, if Pete doesn't play the piano, Sue won't sing.

In principle, these predictions are correct. However, there is a subtlety to note here. We defined \Rightarrow as material implication, and as a result of this the proposition expressed by $(p \rightarrow q) \wedge (\neg p \rightarrow q)$ is actually the same as the one expressed by q . In a classical setting, and in conservative inquisitive semantics, these two formulas are in fact completely equivalent. That is not the case here, because the two formulas are assigned different counter-propositions. But we do predict that (28) expresses exactly the same proposition as (30), and this is clearly a problematic prediction.

- (30) Sue will sing.

⁶We conjecture that a denial of the presupposition of a sentence will always belong to this third class of responses. In turn this may lead to the conjecture that the presupposition, or rather the *supposition* of a sentence φ coincides with the possibility $\gamma = \{v \mid \exists \alpha \in [\varphi] \& \exists \beta \in [\varphi] : v \in \alpha \cap \beta\}$.

⁷The benefit of using inquisitive semantics to analyze this type of sentences was pointed out to us by Stefan Kaufmann, and the particular analysis to be presented below follows, in essence, his insight. See Kaufmann (2009) for a slightly different account, largely in the same spirit.

However, identifying the source of the problem, and fixing it in the obvious way, leads to a promising analysis. The source of the problem is that \Rightarrow is defined as material implication. And the obvious fix is to redefine it along the lines of a more sophisticated existing analysis of conditionals. Suppose for instance, that we make the standard assumption, originating in the work of Lewis and Stalnaker, that \Rightarrow is sensitive to a *similarity order* between worlds:

$$(31) \quad \alpha \Rightarrow \beta := \{w \mid \text{MIN}_w(\alpha) \subseteq \beta\}$$

where $\text{MIN}_w(\alpha)$ is the set of worlds that belong to α and do not differ more from w than any other world in α .

Under this assumption, $\lceil q \rceil$ and $\lceil ?p \rightarrow q \rceil$ differ in exactly the right way. The former still contain a single possibility consisting of all worlds where q holds. $\lceil ?p \rightarrow q \rceil$, however, becomes stronger:

$$(32) \quad \lceil ?p \rightarrow q \rceil = \{ \gamma \}$$

where $\gamma = \{w \mid \text{MIN}_w|p| \subseteq |q| \text{ and } \text{MIN}_w|\neg p| \subseteq |q| \}$

To see whether w belongs to γ we should not just check whether q holds at w , but rather we should look at all p -worlds that minimally differ from w and all $\neg p$ -worlds that minimally differ from w , and check whether q holds in all those worlds. In the terms of our original natural language example, we should not just check whether Sue sings at w , but we should look at all worlds minimally different from w where Pete plays the piano, and at all worlds minimally different from w where Peter doesn't play the piano, and check whether Sue sings in all those worlds. This indeed appears to be the correct analysis of (28).

The approach also seems to lead to a suitable account of other types of unconditionals, such as:

$$(33) \quad \text{Whoever plays the piano, Sue will sing.}$$

Spelling out such an account, however, would require us to move to a first-order setting. This is left for another occasion.

3.9 Ramsey on conditionals

As a final note, we would like to point out that in the present framework, the simple conditional assertions in (34) and (35) contradict each other in a sense: (35) is a negative response to (34), i.e., it rejects the proposal expressed by (34).

And vice versa, (34) rejects the proposal expressed by (35).

- (34) If Alf goes to the party, then Bea will go as well.
- (35) If Alf goes to the party, then Bea will not go.

This corresponds exactly to what Frank Ramsey wrote in his famous footnote in 1929, which is generally referred to as the *Ramsey test* for conditionals:

If two people are arguing “If p will q ?” and are both in doubt as to p , they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ; so that in a sense “If p , q ” and “If p , $\neg q$ ” are contradictories.

In fact, in line with Ramsey’s footnote, radical inquisitive semantics takes (34) and (35) to be the two opposing answers to the conditional question in (36) (which is the ‘question behind’ both (34) and (35)):

- (36) If Alf goes to the party, will Bea go as well?

4 Conclusion

We have provided a more radical implementation of inquisitive semantics, which better captures the central idea that sentences express proposals to update the common ground of a conversation. The key difference between conservative and radical inquisitive semantics is that the former only captures the positive responses to the proposal that a sentence expresses, whereas the latter also captures negative responses. The proposed framework raises many issues that need to be investigated in more detail, but we nevertheless hope to have demonstrated that it facilitates an insightful formal characterization of certain aspects of linguistic interaction that were previously beyond reach.

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