

Might and Free Choice in Inquisitive Semantics

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Overview. One prominent use of expressions in natural language is to *draw attention* to certain possibilities. We propose an extension of inquisitive semantics that captures this aspect of language use/meaning. As a concrete application of this general idea, we present an analysis of *might* that deals straightforwardly with certain free choice data that were particularly problematic for previous accounts. We also point at other potential applications.

States, stages, and possibilities. A *state* is a non-empty set of possible worlds. \mathcal{W} denotes the set of all possible worlds, and \mathcal{S} denotes the set of all states. A *stage* is a triple $\langle I, Q, A \rangle$ where I is a state representing the information acquired so far, Q is a set of states representing the current issue, and A is a set of states representing the current focus of attention. The \subseteq -maximal states in Q and A are called *possibilities*.

A stage should be thought of as a conversational *context*. The *initial stage* $\langle I_0, Q_0, A_0 \rangle$ is the stage in which no information has been acquired, no issues have been raised, and no states have been brought under attention yet: $\langle I_0, Q_0, A_0 \rangle = \langle \mathcal{W}, \mathcal{S}, \emptyset \rangle$.

Meaning as context change potential. We take the meaning of a formula to be its context change potential. The notion of a context is richer here than in standard frameworks: it does not only embody the information acquired so far, but also the current issue and the current focus of attention. The notion of meaning is therefore also richer than it normally is. A sentence

may not only provide information, but also raise an issue or draw attention to certain possibilities. Updating a stage $\langle I, Q, A \rangle$ with a formula φ may affect all three components. If φ provides information, then some possible worlds in I will be eliminated. If φ raises an issue, then Q will come to represent the new issue; and if φ draws attention to certain possibilities, then A will come to represent the new focus of attention.

Support and drawing attention. Updating a stage $\langle I, Q, A \rangle$ with a formula φ consists in manipulating the states in I , Q and A . How a state s in I , Q or A is manipulated depends on two things: whether s *supports* φ , and whether φ *draws attention to* s . Let us write $s \vDash \varphi$ if s supports φ , and $s \dashv \varphi$ if φ draws attention to s . In defining \vDash and \dashv , let us initially focus on atomic formulas, disjunction, conjunction, and *might* (the ingredients of the basic free choice puzzles). Negation and implication will be discussed later.

1. $s \vDash p \quad \text{iff} \quad \forall v \in s : v(p) = 1$
 $s \dashv p \quad \text{iff} \quad s \models p$
2. $s \vDash \varphi \wedge \psi \quad \text{iff} \quad s \vDash \varphi \text{ and } s \vDash \psi$
 $s \dashv \varphi \wedge \psi \quad \text{iff} \quad s \dashv \varphi \text{ or } s \dashv \psi$
3. $s \vDash \varphi \vee \psi \quad \text{iff} \quad s \vDash \varphi \text{ or } s \vDash \psi$
 $s \dashv \varphi \vee \psi \quad \text{iff} \quad s \dashv \varphi \text{ or } s \dashv \psi$
4. $s \vDash \Diamond \varphi \quad \text{always}$
 $s \dashv \Diamond \varphi \quad \text{iff} \quad s \models \varphi$

Notice that the standard inquisitive definition of support is preserved for atomic formulas, conjunction, and disjunction. The attention clauses for atoms, conjunction, and disjunction is, we think, as expected (we can't think of any sensible alternatives). As for *might*, $\Diamond \varphi$ is supported by any state, and draws attention to states that support φ . Here it is possible to think of several alternative definitions. One of them will be discussed below.

Meaning. Let $[\varphi]_s$ denote the set of states that support φ . This set of states represents the issue raised by φ . Let $[\varphi]_i$ denote the *union* of the set of states that support φ . This is a single state, which represents the information provided by φ . Finally, let $[\varphi]_a$ denote the set of states that φ draws attention to. The context change potential of φ will be defined in terms

of $[\varphi]_i$, $[\varphi]_s$, and $[\varphi]_a$. Let us therefore refer to the triple $\langle [\varphi]_i, [\varphi]_s, [\varphi]_a \rangle$ as the *meaning* of φ .

Possibilities. An important feature of \models is that it is *persistent*: if $s \models \varphi$ and $t \subseteq s$, then also $t \models \varphi$. The same holds for \dashv : if $s \dashv \varphi$ and $t \subseteq s$, then also $t \dashv \varphi$. This means that both $[\varphi]_s$ and $[\varphi]_a$ are characterized by their *maximal* elements. These maximal elements are called the *possibilities* in $[\varphi]_s$ and $[\varphi]_a$.

Update. If $\langle I, Q, A \rangle$ is a stage, then $\langle I, Q, A \rangle[\varphi]$ denotes the stage that results from updating $\langle I, Q, A \rangle$ with φ . $\langle I, Q, A \rangle[\varphi]$ is defined in terms of $\langle [\varphi]_i, [\varphi]_s, [\varphi]_a \rangle$:

$$(1) \quad \langle I, Q, A \rangle[\varphi] = \langle I', Q', A' \rangle \quad \text{where } \begin{cases} I' &= [\varphi]_i \cap I \\ Q' &= [\varphi]_s \\ A' &= [\varphi]_a \end{cases}$$

Notice that information is accumulated: I' contains only states that were also in I . Issues and attention are not accumulated: the issue represented by Q' may have nothing to do with the one represented by Q , and the focus of attention represented by A' may have nothing to do with the one represented by A .

Meaning defined directly. Above, $[\varphi]_i$, $[\varphi]_s$, and $[\varphi]_a$ are defined in terms of \models and \dashv . But they can also be defined directly. $[\varphi]_s$ and $[\varphi]_a$ are defined recursively below. $[\varphi]_i$ is defined as $\bigcup [\varphi]_s$ for all φ .

1. $[p]_s = \{s \mid \forall v \in s : v(p) = 1\}$
 $[p]_a = [p]_s$
2. $[\varphi \wedge \psi]_s = [\varphi]_s \cap [\psi]_s$
 $[\varphi \wedge \psi]_a = [\varphi]_a \cup [\psi]_a$
3. $[\varphi \vee \psi]_s = [\varphi]_s \cup [\psi]_s$
 $[\varphi \vee \psi]_a = [\varphi]_a \cup [\psi]_a$
4. $[\Diamond \varphi]_s = \mathcal{S}$
 $[\Diamond \varphi]_a = [\varphi]_s$

Entailment. A natural way to define entailment in this setup is as follows:

$$(2) \quad \varphi \vDash \psi \quad \text{iff} \quad \begin{cases} [\varphi]_s \subseteq [\psi]_s \\ [\varphi]_a \supseteq [\psi]_a \end{cases}$$

The pre-theoretical intuition is that φ entails ψ iff uttering ψ after φ is felt to be redundant. Everything that ψ communicates is already communicated by φ . The standard formalization of this intuition is that ψ should not provide any information that φ does not provide: whenever ψ excludes a possible world, φ excludes it as well. In our setting, a sentence does not just exclude possible worlds. Rather, it excludes and highlights possibilities. Thus, φ entails ψ iff (i) every possibility that is excluded by ψ is also excluded by φ , and (ii) every possibility that is highlighted by ψ is also highlighted by φ .

As usual, equivalence is defined as mutual entailment.

Free choice. A basic observation concerning free choice is that the following three sentences are equivalent:

- (3) John might be in Paris or in London.
 $\Diamond(p \vee q)$
- (4) John might be in Paris or he might be in London.
 $\Diamond p \vee \Diamond q$
- (5) John might be in Paris and he might be in London.
 $\Diamond p \wedge \Diamond q$

This observation is straightforwardly accounted for. First, all three relevant formulas are supported by all states:

- (6) $[\Diamond(p \vee q)]_s = \mathcal{S}$
- (7) $[\Diamond p \vee \Diamond q]_s = \mathcal{S}$
- (8) $[\Diamond p \wedge \Diamond q]_s = \mathcal{S}$

Second, all three formulas draw attention to exactly the same states:

- (9) $[\Diamond(p \vee q)]_a = [p \vee q]_a = [p]_a \cup [q]_a = [p]_s \cup [q]_s$
- (10) $[\Diamond p \vee \Diamond q]_a = [\Diamond p]_a \cup [\Diamond q]_a = [p]_s \cup [q]_s$
- (11) $[\Diamond p \wedge \Diamond q]_a = [\Diamond p]_a \cup [\Diamond q]_a = [p]_s \cup [q]_s$

An alternative for *might* that does not work. The following alternative definition for *might* may seem attractive:

$$(12) \quad s \models \diamond\varphi \text{ iff } s \models \varphi$$

The corresponding direct definition of $[\diamond\varphi]_a$ is:

$$(13) \quad [\diamond\varphi]_a = [\varphi]_a$$

The original definition says that $\diamond\varphi$ draws attention to the states that support φ . The alternative definition says that $\diamond\varphi$ draws attention to states that φ itself draws attention to.

The problem with this alternative definition is that we get:

$$(14) \quad \diamond(p \wedge q) \equiv \diamond p \wedge \diamond q$$

In particular, what goes wrong is that $\diamond(p \wedge q)$ draws attention to states that support p and to states that support q , whereas it should only draw attention to states that support both p and q .

With the original definition this problem does not arise. We get:

$$(15) \quad [\diamond(p \wedge q)]_a = [p \wedge q]_s = [p]_s \cap [q]_s$$

$$(16) \quad [\diamond p \wedge \diamond q]_a = [p]_a \cup [q]_a = [p]_s \cup [q]_s$$

So $\diamond(p \wedge q)$ only draws attention to states that support both p and q , while $\diamond p \wedge \diamond q$ draws attention to states that support p and to states that support q , as desired.

Implication. Let us now turn to implication. We preserve the standard inquisitive definition of support:

$$(17) \quad s \vDash \varphi \rightarrow \psi \text{ iff } \forall t \subseteq s : \text{if } t \vDash \varphi \text{ then } t \vDash \psi$$

There are several ways we could define the attention clause. We need to establish some basic data in order to decide which definition is most appropriate.

One relevant observation (which we think is new and quite striking) is that the following sentences are semantically equivalent:

$$(18) \quad \begin{aligned} &\text{If John got suspicious, he might be in Paris now.} \\ &p \rightarrow \diamond q \end{aligned}$$

- (19) It might be that John got suspicious and is in Paris now.
 $\Diamond(p \wedge q)$

A second emperical criterion comes from conditionalized free choice data (which we think have not been considered before either). We think the following sentences should all come out semantically equivalent:

- (20) If John got suspicious, he might be in Paris or in London now.
 $p \rightarrow \Diamond(p \vee q)$
- (21) If John got suspicious, he might be in Paris or he might be in London now.
 $p \rightarrow \Diamond p \vee \Diamond q$
- (22) If John got suspicious, he might be in Paris and he might be in London now.
 $p \rightarrow \Diamond p \wedge \Diamond q$

A third observation that should be accounted for is that (20)–(22) all entail (18) and (19).

Proposal. These data can be accounted for if we assume that $\varphi \rightarrow \psi$ draws attention to a state s iff φ is supported by s and ψ draws attention to it:

$$(23) \quad s \models \varphi \rightarrow \psi \text{ iff } s \vDash \varphi \text{ and } s \models \psi$$

Equivalently:

$$(24) \quad [\varphi \rightarrow \psi]_a = [\varphi]_s \cap [\psi]_a$$

We think this makes sense. When uttering $\varphi \rightarrow \psi$, a speaker suggests, first, to focus on states that support φ , and then highlights among these states the ones that ψ draws attention to.

Evaluation. Let us first check that (18) and (19) are predicted to be semantically equivalent. Clearly, any state supports $\Diamond(p \wedge q)$. Moreover, if the consequent of an implication is supported by any state, then the implication as a whole is also supported by any state. So $p \rightarrow \Diamond q$ is supported by any state as well. Furthermore, $\Diamond(p \wedge q)$ and $p \rightarrow \Diamond q$ draw attention to exactly the same states:

$$\begin{aligned}
(25) \quad & [\diamond(p \wedge q)]_a \\
&= [p \wedge q]_s \\
&= [p]_s \cap [q]_s \\
&= [p \wedge q]_s
\end{aligned}$$

$$\begin{aligned}
(26) \quad & [p \rightarrow \diamond q]_a \\
&= [p]_s \cap [\diamond q]_a \\
&= [p]_s \cap [q]_s \\
&= [p \wedge q]_s
\end{aligned}$$

Next, consider the conditionalized free choice data. First, notice that (20)–(22) are supported by all states:

$$(27) \quad [p \rightarrow \diamond(q \vee r)]_s = \mathcal{S}$$

$$(28) \quad [p \rightarrow (\diamond q \vee \diamond r)]_s = \mathcal{S}$$

$$(29) \quad [p \rightarrow (\diamond q \wedge \diamond r)]_s = \mathcal{S}$$

Moreover, (20)–(22) draw attention to exactly the same states:

$$\begin{aligned}
(30) \quad & [p \rightarrow \diamond(q \vee r)]_a \\
&= [p]_s \cap [\diamond(q \vee r)]_a \\
&= [p]_s \cap [q \vee r]_s \\
&= [p]_s \cap ([q]_s \cup [r]_s) \\
&= ([p]_s \cap [q]_s) \cup ([p]_s \cap [r]_s) \\
&= [p \wedge q]_s \cup [p \wedge r]_s
\end{aligned}$$

$$\begin{aligned}
(31) \quad & [p \rightarrow (\diamond q \vee \diamond r)]_a \\
&= [p]_s \cap [\diamond q \vee \diamond r]_a \\
&= [p]_s \cap ([\diamond q]_a \cup [\diamond r]_a) \\
&= [p]_s \cap ([q]_s \cup [r]_s) \\
&= ([p]_s \cap [q]_s) \cup ([p]_s \cap [r]_s) \\
&= [p \wedge q]_s \cup [p \wedge r]_s
\end{aligned}$$

$$\begin{aligned}
(32) \quad & [p \rightarrow (\diamond q \wedge \diamond r)]_a \\
&= [p]_s \cap [\diamond q \wedge \diamond r]_a \\
&= [p]_s \cap ([\diamond q]_a \cup [\diamond r]_a) \\
&= [p]_s \cap ([q]_s \cup [r]_s) \\
&= ([p]_s \cap [q]_s) \cup ([p]_s \cap [r]_s)
\end{aligned}$$

$$= [p \wedge q]_s \cup [p \wedge r]_s$$

So (20)–(22) are indeed predicted to be semantically equivalent.

Finally, it is also predicted that (20)–(22) entail (18)–(19). Clearly, every state that supports (20)–(22) also supports (18)–(19). Moreover, the above computations show that every state that is highlighted by (18)–(19) is also highlighted by (20)–(22).

Thus, the proposed clause for implication successfully accounts for some basic (but non-trivial) data. However, there may be alternatives, and we should consider further data.

Persistence. Notice that \doteq remains persistent given the above definition for implication. For suppose that $s \doteq \varphi \rightarrow \psi$ and $t \subseteq s$. This means that $s \models \varphi$, $s \doteq \psi$, and $t \subseteq s$. Then, by persistence of \models , we have that $t \models \varphi$, and by the induction hypothesis, we have that $t \doteq \psi$. Thus, $t \doteq \varphi \rightarrow \psi$.

Disjunctions and questions. Attention could capture other interesting phenomena as well. For instance, in pure inquisitive semantics $?p$ is defined as $p \vee \neg p$. But there is an intuitive difference between the two. This difference may be captured in terms of attention. We could define, for example:

$$(33) \quad \begin{aligned} s \models ?\varphi &\text{ iff } s \models \varphi \vee \neg \varphi \\ s \doteq ?\varphi &\text{ iff } s \models \varphi \end{aligned}$$

Thus, $?p$ and $p \vee \neg p$ raise the same issue, but they highlight different possibilities. If $?p$ is defined in this way, the difference between positive and negative questions is captured straightforwardly.

Disjunctions and indefinites. If we move to predicate logic, we could capture a difference between disjunctions and indefinites, analogous to the one between disjunctions and questions in the propositional case. To see what is at stake, consider the following sentences, assuming that the domain of discourse consists of John, Sue, and Mary:

$$(34) \quad \begin{aligned} \text{John, Sue, or Mary called this morning.} \\ Pa \vee Pb \vee Pc \end{aligned}$$

- (35) Someone called this morning.
 $\exists xPx$

In pure inquisitive semantics, we have that:

$$(36) \quad Pa \vee Pb \vee Pc \equiv \exists xPx$$

Intuitively, however, there is a difference: (34) explicitly draws attention to the three disjuncts, while (35) does not. This is exactly what we would get.

Attention and ignorance implicatures. There are further observations that support the line of reasoning just described. Disjunctions notoriously give rise to ignorance implicatures. For instance, if I utter (34) under normal circumstances, you will conclude that I don't know whether John called, that I don't know whether Sue called, and that I don't know whether Mary called. However, if I utter (34), you will not draw this conclusion. You will still draw the conclusion that I do not know who called of course, but this is a weaker level of ignorance. I may not know who called even if I do know that it wasn't John, for instance.

This indicates that the Gricean explanation of ignorance implicatures is problematic. The strong ignorance implicature triggered by disjunction may be explained in terms of the following principle:¹

- (37) If a cooperative speaker draws attention to a certain possibility without affirming or denying it, then she does not have sufficient information to do so.

Notice that this principle, unlike the Gricean account, avoids the consideration of alternative utterances, and all the controversies that come with selecting those alternatives.

The simplest kind of sentence that is used to draw attention to a certain possibility without affirming or denying it is of course $\Diamond p$. (37) predicts that $\Diamond p$ implicates that the speaker does not know whether p is the case.

Attention, relevance, and questions. Drawing attention to a possibility also triggers a relevance implicature, roughly along the following lines:

¹It is also possible that this principle may be used to explain Emmanuel Chemla's symmetry principle, or something very similar.

- (38) If a cooperative speaker draws attention to a certain possibility, she considers this possibility relevant for the purposes of the conversation.

This principle, together with the ignorance principle, explains the conversational effects of questions! To ask the question $?p$ is to draw attention to the possibility that p without affirming or denying it. Thus, such a question indicates (i) that the speaker does not know whether p , and (ii) that he considers p relevant, i.e. that he would be interested to learn whether p . This seems to be exactly what a question conveys.

Attention, relevance, and conditionals. It is often observed that conditionals normally convey that there is a *correlation* between the truth of the antecedent and the truth of the consequent (see especially the relevance logic literature on conditionals). Someone who utters a conditional conveys that, according to his beliefs, the truth of the consequent depends on the truth of the antecedent. Frank Veltman argued in his work on Data Semantics that this effect can be derived from the fact that a speaker who utters a conditional is normally taken to be ignorant with respect to both the antecedent and the consequent of the conditional. He leaves open, however, where these ignorance effects come from. Quite plausibly, this gap could be filled now that attention is taken into account. The ignorance effects could be seen as attention related implicatures.

Attention-related effects at the discourse level. There will be at least two kinds of attention-related phenomena cropping up at the discourse level. First, it may be that in some contexts, an attempt to draw attention to a certain possibility is *infelicitous*. For example, if I say:

- (39) It is not raining outside.

then it is infelicitous to continue with:

- (40) It might be raining outside.

According to the semantics proposed above, these two sentences are not contradictory. So the infelicity must be explained in some other way. One plausible explanation would be that the following principle is at work here:

- (41) It is infelicitous to draw attention to a possibility that has already been excluded.

Another kind of highlighting effect at the discourse level has to do with the relatedness between one utterance and the next. In pure inquisitive semantics, the central notion of relatedness is *compliance*. It is predicted, for instance, that p is compliant with $?p$ and that $p \rightarrow q$ is compliant with $p \rightarrow ?q$. However, there are certain cases of relatedness that are not captured by compliance. For instance, $\neg p$ is not compliant with $p \vee q$, nor with $p \wedge q$, nor with $p \rightarrow q$.

This problem may be overcome by making the notion of compliance sensitive to attention. One straightforward implementation of this idea would be the following:

- (42) φ is compliant with ψ if φ affirms or denies a possibility that ψ draws attention to.

Notice that this notion of compliance is intended as complementary to the original inquisitive notion of compliance, not as a replacement thereof. Also notice that the notions of affirming and denying have not been specified explicitly. This should not be very difficult to do, but I'll leave the details for later.

(42) straightforwardly predicts that $\neg p$ is compliant with $p \vee q$ and with $p \wedge q$. It also predicts that $\neg p$ is compliant with $p \rightarrow q$, because $p \rightarrow q$ draws attention to the possibility that $p \wedge q$. To deny this possibility, one would normally say $\neg(p \wedge q)$, but in a context in which $p \rightarrow q$ has been established, $\neg(p \wedge q)$ is equivalent to $\neg p$. Thus, the attention-based notion of compliance proposed here seems to resolve some of the problems that a purely inquisitive notion of compliance gives rise to.

Deontic *may*. The analysis above should carry over to deontic *may* with minor adjustments. Stages should get one more component, which keeps track of permissible *options*. Deontic *may* is used to add options to the stack of permissible options.

Negating *might*. One observation that also seems relevant here, and which seems to support our approach, is that *might* cannot be negated standardly:

- (43) John might not be in London.

This sentence does not mean that it is inconsistent with the speaker's belief state that John is in London (this is what it could mean on Tikitū's account). On our account, this sentence draws attention to the possibility that John is not in London, and implicates that the speaker does not know whether John is in London. This seems correct.

There is a complication however. Consider:

- (44) It is not true that John might be in London.

This sentence *does* imply that it is inconsistent with the speaker's belief state that John is in London. If the sentence would be analyzed as $\neg\Diamond\varphi$, then it would be a contradiction on our account, which would be wrong. But what is going on here, I think, is that the sentence is interpreted as a denial of the *implicature* of the embedded clause. This is a common use of *it is not true that* constructions. For example, in (45) the implicature of the embedded clause is denied, and in (46) the presupposition of the embedded clause is denied:

- (45) It is not true that John has four children. He has five.

- (46) It is not true that the king of France is bald. There is no king of France.

Notice that it is possible to say:

- (47) It is not true that John might be in London. He is in *Paris*.

- (48) It is not true that John might be in London. He *is* in London.

This observation seems to provide further support for our account.