

Inquisitive Semantics and Dialogue Pragmatics

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Preliminaries

This is a DRAFT, which still needs a lot of work. I concentrate on presenting the theory. Important references to related work still have to be included, like many other things, such as thanking a substantial number of colleagues and students.

I presented earlier subsequent versions of the paper in talks at ZAS-Berlin on January 15, at LeGO Amsterdam on February 1, and at Rutgers Linguistics, March 25.

1 Overview

The main result of the paper is a new analysis of implicatures of disjunction and alternative questions. The analysis arises from an enrichment of the semantics of disjunction: a disjunction may not only provide information, but may also raise an issue.

Concentrating on disjunction, I will expose an inquisitive semantics for a minimal propositional logical language, where disjunction is the sole source of inquisitiveness in the language.

In the logic that comes with the semantics, entailment has no role to play. The logic is not oriented towards the validity of argumentation, but towards the coherence of dialogue.

The central logical notion is called *compliance*, which is build from the two notions of homogeneity and relatedness. Homogeneity underlies the inquisitive version of Quantity, with the crucial feature that it prefers more informativeness but less inquisitiveness.

Comparative compliance is at the heart of the pragmatics that comes with the semantics: it is the source of implicatures. The implicatures we will deal with are fundamentally conversational in that they are effectuated — or cancelled — by the *interaction* of the participants in a dialogue.

At the same time it could be argued that these implicatures are conventional, in that they are calculated within a general interpretation process, called *dialogue management*, which is steered by a set of *rules*. More than half of the paper is devoted to explicating these rules.

I will consider dialogues, in the logical language, between two participants, called the *stimulator* and the *responder*. The dialogue management rules concern the *uptake* of an utterance made by the one participant in the *common ground*, together with the *absorption of the reaction* to the utterance by the other participant.

A distinctive feature of the inquisitive dialogues we consider is that we welcome critical moves in the dialogue. One of the three reactions to utterances that we distinguish is to call for *cancellation*. To call for cancellation can be essential to *maintain a common ground*, which is the first principle of our dialogue pragmatics.

The dialogue management rules will operate on the common ground in such a way that the uptake of an utterance made by the one participant will cause a *hypothetical update* of the common ground, which has to await a reaction of acceptance or support from the other participant before the information the utterance may contain is absorbed in the common ground. If the reaction of the other participant is to call for cancellation, the hypothetical update is undone.

In the semantics we interpret sentences as updates on states. To be able to deal with hypothetical updates we use the standard tool of stacks. The common ground is modeled as a stack of states.

Since our semantics is an inquisitive semantics, states should not just be ordinary information states. Next to information, states may contain issues. In the common ground, we will always be able to discern the current issue in the dialogue. It is part of the uptake rules for utterances to calculate the effect the utterance has relative to the current issue.

To be compliant to the the common ground, as is preferred by dialogue pragmatics, means to focus on the current issue. There are basically two ways to do this: provide information that is relevant for resolving the current issue, or to zoom in on a subissue of the current issue.

Obviously — changing terminology a bit is not intended to hide this — the main lines of my approach to dialogue and pragmatics are *not new*. I think

it is most akin in general architecture and spirit to Roberts (1996). There is also a wave of more recent articles and dissertations, in which analyses of implicatures and several other semantic and pragmatic phenomena are proposed, which one way or the other are linked to the semantics of questions. When we turn to the details of my approach correspondences with these analyses will abound.

I am not going to be able in this paper, lengthy as it already is, to point at all such correspondences, let alone to properly discuss them. The way I like to see the contribution of this paper, is that it provides a new, principled and simple, logical foundation for these current lines of research.

2 Minimal Inquisitive Language

Since in this paper, I will be mainly concerned with the pragmatics of disjunction, I opt for the following minimal version of an inquisitive propositional language.¹

Definition 1 (Inquisitive Propositional Syntax) Let \wp be a finite set of propositional variables. The sentences of L_\wp is the smallest set such that:

1. If $p \in \wp$, then $p \in L_\wp$
2. If $\varphi \in L_\wp$, then $\neg\varphi \in L_\wp$
3. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \vee \psi) \in L_\wp$

The syntax makes no distinction between questions and assertions. We will characterize questions in semantic terms as sentences which are not informative, and assertions as sentences which are not inquisitive. A third semantic category is formed by hybrid sentences, sentences which are neither assertions nor questions, i.e., sentences which are both inquisitive and informative. Simple disjunctions like $p \vee q$ turn out to be such hybrids.

Disjunction is the source of inquisitiveness.² Questions can be obtained by taking the disjunction of a sentence with its negation. Negation only pertains to the informative content of a sentence, negation invariably leads

¹The standard version of the language starts from \perp , \rightarrow , \wedge , and \vee . Negation is defined standardly as $\varphi \rightarrow \perp$. See Groenendijk (2008), and Mascarenhas (2008).

²Strange enough, this feature of the language went unnoticed for a long time, and was discovered in 2007 by Salvador Mascarenhas.

to an assertion. We refer to the double negation of a sentence as its *assertive closure*.

Since questions have no informative content — they are informatively tautological — it makes sense to define the contradiction \perp as the negation of an arbitrary question. And if we define the tautology \top to be the negation of the contradiction, then the assertive closure of a question equals the tautology \top .

Definition 2 (Notation Conventions) Let $\varphi \in L_\varphi$.

1. $? \varphi = (\varphi \vee \neg \varphi)$
2. $! \varphi = \neg \neg \varphi$
3. $\perp = \neg ? \varphi$, for arbitrary φ
4. $\top = \neg \perp$

In classical propositional logic disjunction and negation form a functionally complete set of operators. All truth functions can be expressed in terms of them. Relative to the semantics to be presented below, the same holds for our inquisitive propositional logic. We will state the semantics for the language in update format.³ Sentences will be interpreted as update functions on states. In this setting, functional completeness means that starting from the minimal state, any state can be reached by a sequence of updates with sentences in the logical language.

What inquisitive propositional logic does not have in common with classical propositional logic, is that whereas in classical logic we can define conjunction and implication in terms of disjunction and negation, such definitions do not generally give appropriate results in the inquisitive logic. For example, given the way in which \top and \perp are defined, if we take $? \varphi \wedge ? \psi$ to be the same as $\neg(\neg ? \varphi \vee \neg ? \psi)$, the result would be the same as \top .

As for conditional questions, a question like $p \rightarrow ?q$ can be represented in terms of negation and disjunction by the formula $!(\neg p \vee q) \vee !(\neg p \vee \neg q)$, which is not the same as $\neg p \vee q \vee \neg q$, which would result under the standard definition of implication in terms of disjunction and negation.

³The logical framework of update semantics originates from Veltman (1996). Actually, the logical language at hand has none of the dynamic *semantic* features that normally motivate the use of update semantics. Our motivation to use the format of update semantics are the dynamic *pragmatic* features of dialogue.

So if we want to deal with conjunctions of questions and conditional questions, we should explicitly define operations of conjunction and implication that work uniformly and correctly for all categories of sentences. (See Groenendijk (2008)). But since in the present context we focus on disjunction, and given that it does not restrict the expressive power of the language, we stick to the simple syntax provided above.

With the aid of the notation conventions, we can form, e.g., atomic questions like $?p$ and — what will turn out to be — hybrid disjunctions like $p \vee q$, assertive disjunctions like $!(p \vee q)$, alternative questions like $?(p \vee q)$, and choice questions such as $?p \vee ?q$.

3 Inquisitive Semantics

We define the semantics in the format of an update semantics. Sentences are interpreted as functions from states to states. States embody information and issues. The basic ingredients of states are ordinary valuation functions for the propositional variables in the language, which I call *indices*, but you can think of them as possible worlds if you like.⁴

Definition 3 (States)

1. The set of *indices* I for L_\wp is the set of functions i such that for all $p \in \wp: i(p) \in \{0, 1\}$.
2. A *state* s is a reflexive and symmetric relation on a subset of I .

The relation on indices embodied by a state is a *relation of indifference*. When two indices i and j are related in s , i.e., $\langle i, j \rangle \in s$, the differences between i and j are not part of the issues embodied by s .⁵

⁴The main reason to use the more neutral term *index* rather than *possible world* has to do with how the predicate logical version of the system is designed. The indices then become richer, they correspond to a combination of a first order interpretation function and an assignment of values to variables. It is the latter that makes the use of the term *possible world* unsuitable. The term which then suggests itself is *possibility*. But as we shall soon see, I have another use for that term. By the way, needless to say that in inquisitive predicate logic, the existential quantifier inherits the inquisitive nature of disjunction.

⁵The idea to view states as embodying a relation of indifference stems from Jäger (1996) and Hulstijn (1997). See also Groenendijk (1999). However, in these earlier approaches the relation of indifference was an equivalence relation, which — in the tradition of Groenendijk & Stokhof (1984) — means that states, and questions, are partitions of logical space. To my knowledge, Velissaratou (2000), was the first to drop transitivity as a

If for some $i \in I: \langle i, i \rangle \notin s$, this means that the index i , a possible assignment of values to the propositional variables, or a possible world if you like, is excluded by the information embodied in s .

In Figure 1 we have depicted a simple state which only concerns two propositional variables, say p and q . The nodes correspond to indices, where the value of p and q is indicated in alphabetical order. A white node corresponds to an index that is excluded by the information embodied in the state. The arrows between nodes represent the relation of indifference. When two nodes are connected by arrows, the difference between them is not an issue in the state.

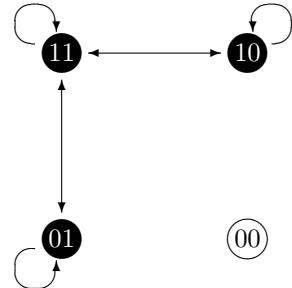


Figure 1: State for $\varphi = \{p, q\}$

We distinguish three properties of states.

Definition 4 (Ignorance, Indifference, and Consistency)

1. A state s is *ignorant* iff $\forall i \in I: \langle i, i \rangle \in s$.
2. A state s is *indifferent* iff $\forall i, j: \langle i, i \rangle \in s \ \& \ \langle j, j \rangle \in s \Rightarrow \langle i, j \rangle \in s$.
3. A state s is *consistent* iff $s \neq \emptyset$.

The universal relation on the set of indices is the only state which satisfies all three properties.

Definition 5 (The State of Ignorance and Indifference)

property of the relation of indifference, motivated by the semantic analysis of conditional questions.

The state of ignorance and indifference $\omega = I^2$.

In Figure 2 we have depicted the state of ignorance and indifference for a language with two propositional variables.

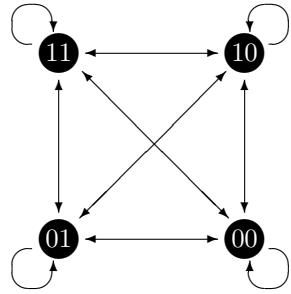


Figure 2: State of Ignorance and Indifference for $\varphi = \{p, q\}$

Every state is a substate of the state of ignorance and indifference.

Fact 1 (Substates) For all states $s: s \subseteq \omega$.

We can distinguish possibilities in a state, which are largest sets of indices such that all of them are related to each other in the state.

Definition 6 (Possibilities in States) Let s be a state.

ρ is a *possibility* in s iff ρ is a set of indices such that:

- (a) for all $i, j \in \rho: \langle i, j \rangle \in s$; and
- (b) there is no set of indices $\rho' \supset \rho$ such that ρ' satisfies (a)

In Figure 3 we have indicated the two possibilities that can be found in the state depicted in Figure 1. In the state of ignorance and indifference depicted in Figure 2 there is only a single possibility containing all four indices.

The properties of indifference, ignorance and consistency are reflected by the possibilities in a state in the following way.

Fact 2 (Indifference, Ignorance, Consistency, and Possibilities)

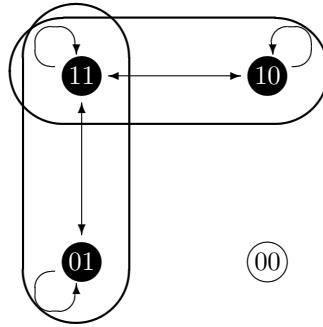


Figure 3: State with two possibilities for $\varphi = \{p, q\}$

1. A state s is indifferent iff there is a single possibility in s .
2. A state is ignorant iff the union of the possibilities in s equals I .
3. A state s is consistent iff the union of the possibilities in s is not \emptyset .

As illustrated in the state depicted in Figure 3, the possibilities in a state may overlap. States need not be partitions. States would be partitions if the relation of indifference embodied by a state would be required to be reflexive and euclidean, which implies symmetry and transitivity, and hence amounts to an equivalence relation. However, intuitively, to require the relation of indifference to be euclidean is too strong. One may not care about the difference between i and j , nor about the difference between i and k , whereas at the same time one is interested in the difference between j and k .⁶

Finally, we define an operation of indifferentiation on states, which clears a state of any issues.

Definition 7 (Indifferentiation)

The *indifferentiation of a state* s , $s^* = \{\langle i, j \rangle \mid \langle i, i \rangle \in s \text{ & } \langle j, j \rangle \in s\}$.

We will make extensive use of this operation, first of all, in stating the logical notions that characterize relations between sentences, and between

⁶In utility theory indifference and preference relations play an important role as well. The issue whether indifference is taken to be euclidean or not has been lively debated there also.

sentences and states; and secondly in the dialogue management system, when we calculate the effect of subsequent utterances by the participants in the dialogue on the stack of states that constitutes the common ground. Roughly speaking, before we interpret a new utterance, we first ‘reset’ the current state to its indifferentiation.

We are now more than ready to state the semantics for an inquisitive propositional language.

Definition 8 (Inquisitive Update Semantics) Let L_φ be an inquisitive propositional language. The update of a state s with a formula $\varphi \in L_\varphi$, $s[\varphi]$ is recursively defined as follows:

1. $s[p] = \{\langle i, j \rangle \in s \mid i(p) = 1 \& j(p) = 1\}$, for all $p \in \varphi$
2. $s[\neg\varphi] = \{\langle i, j \rangle \in s \mid \langle i, i \rangle \notin s[\varphi] \& \langle j, j \rangle \notin s[\varphi]\}$
3. $s[\varphi \vee \psi] = s[\varphi] \cup s[\psi]$

Before we illustrate the semantics, we define two basic inquisitive semantic notions.

Definition 9 (Inquisitiveness and Informativeness)

1. φ is inquisitive in s iff $\exists i, j \in I: \langle i, j \rangle \in s$, and $\langle i, i \rangle \in s[\varphi] \& \langle j, j \rangle \in s[\varphi]$, but $\langle i, j \rangle \notin s[\varphi]$
2. φ is informative in s iff $\exists i, j \in I: \langle i, i \rangle \in s \& \langle j, j \rangle \in s$, and $\langle i, i \rangle \in s[\varphi]$, but $\langle j, j \rangle \notin s[\varphi]$
3. φ is inquisitive/informative iff φ is inquisitive/informative in ω

A sentence φ is inquisitive in a state s , if a pair of indices $\langle i, j \rangle$ in s is *disconnected* by an update of s with φ , i.e., whereas $\langle i, i \rangle$ and $\langle j, j \rangle$ remain⁷, $\langle i, j \rangle$ is eliminated. This means that $s[\varphi]$ cannot be indifferent. Inquisitiveness of a sentence as such is defined as inquisitiveness in the state of ignorance and indifference.⁸

⁷If $\langle i, j \rangle \in s$, then it also has to be the case that $\langle i, i \rangle \in s$ and $\langle j, j \rangle \in s$, but not vice versa.

⁸Alternatively, inquisitiveness (and informativeness) of a sentence φ as such could be defined as inquisitiveness (informativeness) in *some state* s .

A sentence φ is informative in a state s , if after an update of s with φ some index i is eliminated from the state, and some index j remains. This means that the state $s[\varphi]$ cannot be ignorant, and has to be consistent.

As for the borderline cases of the tautology and the contradiction:

Fact 3 \top and \perp are not inquisitive and not informative.

We semantically characterize questions and assertions in terms of the properties of inquisitiveness and informativeness. A third semantic category are hybrids.

Definition 10 (Questions, Assertions, and Hybrids) Let $\varphi \in L$.

1. φ is a *question* iff φ is not informative.
2. φ is an *assertion* iff φ is not inquisitive.
3. φ is a *hybrid* iff φ is inquisitive and informative.

Fact 3 implies that the borderline cases of contradictions \perp and tautologies \top count as both assertions and questions. This may seem a bit odd, but comes handy in stating Fact 5 and Fact 6 below.

We illustrate the semantics with some pictures, which all concern the update of the state of ignorance and indifference for a language L_\wp , where $\wp = \{p, q\}$, as depicted in Figure 2 above.

Figure 4 illustrates the effect of an update of our example state with the atomic sentence p . Atomic sentences are assertions.

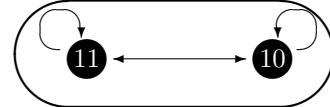
The effect of updating our example state with $p \vee q$ is illustrated in Figure 5. If we update the state with q , and take the union of the state resulting from that with the state that we have seen to be the result of the update with p , we arrive at the state depicted in Figure 5.

The crucial fact about the semantics is the following:

Fact 4 (Hybrid Disjunctions) $p \vee q$ is a hybrid.

There are two overlapping possibilities in $\omega[p \vee q]$.

The effect of the negation of the disjunction $p \vee q$, which is an assertion, is depicted in Figure 6.



(01) (00)

Figure 4: p

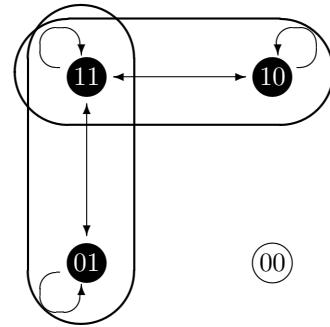


Figure 5: $(p \vee q)$

Figure 7 gives us the result of the double negation of the disjunction $p \vee q$, i.e., of its assertive closure $!(p \vee q)$. The result is an assertion, there is only a single possibility in the picture. Quite generally, the following holds.

Fact 5 (Assertions) For all $\varphi \in L$: $\neg\varphi$, and hence $!\varphi$, is an assertion.

The union of the two states depicted in Figure 6 and Figure 7 results in the state depicted in Figure 8, which corresponds to the yes/no-question $?!(p \vee q)$, i.e. $!(p \vee q) \vee \neg(p \vee q)$.

If we take the union of the states depicted in Figure 5 and Figure 6, we arrive at the state depicted in Figure 9, which corresponds to the alternative question $?(p \vee q)$, i.e. with $(p \vee q) \vee \neg(p \vee q)$.

(11) (10)

(01) (00)

Figure 6: $\neg(p \vee q)$

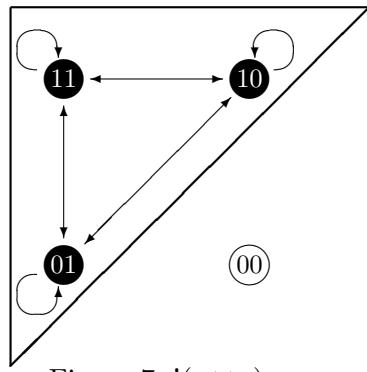


Figure 7: $!(p \vee q)$

Figure 10 illustrates the effect of the update of our example state with the question $?p$, i.e., with $p \vee \neg p$. The following holds quite generally.

Fact 6 (Questions) For all $\varphi \in L$: $? \varphi$ is a question.

If we update the example state with $?q$, and take the union of the outcome of that with the state we depicted in Figure 10, we arrive at the effect of the choice question $?p \vee ?q$.

Equivalence is defined in the obvious way.

Definition 11 (Equivalence) $\varphi \Leftrightarrow \psi$ iff $\forall s: s[\varphi] = s[\psi]$.

In terms of that we can state:

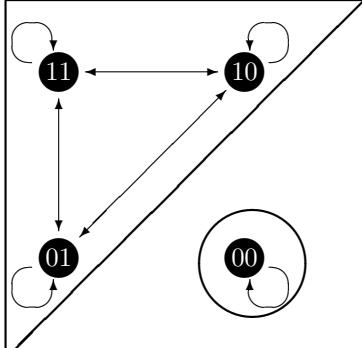


Figure 8: $?!(p \vee q)$

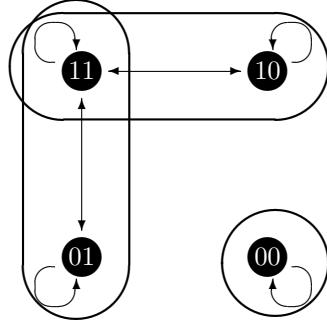


Figure 9: $?(p \vee q)$

Fact 7 (Iteration of ? and \neg)

1. $??\varphi \Leftrightarrow ?\varphi$
2. $!\neg\varphi \Leftrightarrow \neg\varphi$

The second item also means that $!!\varphi \Leftrightarrow !\varphi$. We note one more equivalence:

Fact 8 (Division in Theme and Rheme) For all $\varphi \in L$: $\varphi \Leftrightarrow (?\varphi \wedge !\varphi)$

4 Inquisitive Logic

Although it is perfectly possible to define a notion of entailment along standard lines, we refrain from doing so, since, as is shown in Groenendijk (2008),

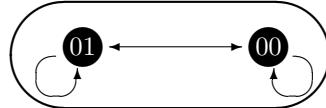
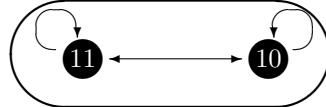


Figure 10: $?p$

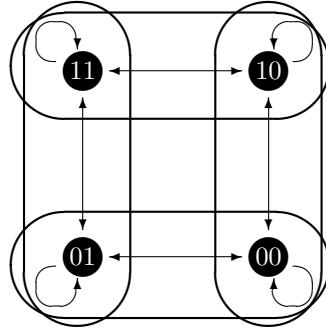


Figure 11: $(?p \vee ?q)$

it does not lead to an appropriate characterization of answerhood and the subquestion relation, which is the minimum that an inquisitive logic is to achieve.

The logical notion that comes with the semantics is the notion of *compliance*, which concerns strict relatedness of utterances to the current issue. This has an informative and an inquisitive side: partial resolution of the current issue, or replacing the current issue by a subissue.

In the pragmatics, compliance will serve as the inquisitive version of the Gricean Maxim of Relation. The inquisitive version of the Maxim of Quantity involves comparison of compliant sentences with respect to informativeness and inquisitiveness. The comparison will be based on a notion of *homogeneity*. Compliant utterances lead to more homogeneous states. The more compliant an utterance is, the more homogeneous is the state it leads to.

We start defining a notion which compares the homogeneity of states.

Definition 12 (Homogeneity)

A state r is at least as homogeneous as a state s , $r \succeq s$ iff

1. $\forall i \in I: \langle i, i \rangle \in r \Rightarrow \langle i, i \rangle \in s$, and
2. $\forall i, j \in I: \langle i, i \rangle \in r \& \langle j, j \rangle \in r \& \langle i, j \rangle \notin r \Rightarrow \langle i, j \rangle \notin s$

The first clause requires for r to be at least as homogeneous as s , that r contains as least as much information as s . The second clause requires that if a pair of indices is disconnected in r , it is to be disconnected in s as well, i.e., r contains at most as much of an issue as s .⁹

The relation of homogeneity partially orders the set of states. The identity relation on the set of indices is the least homogeneous state, the absurd state is the most homogeneous state. We call the least homogeneous state the initial state and denote it by ι .

Definition 13 (Initial State) The *initial state* $\iota = \{\langle i, i \rangle \mid i \in I\}$.

The initial state ι is ignorant, the possibilities in ι correspond one-to-one to the indices in I . See Figure 12 for a picture of the initial state for a language with two propositional variables. Note that the initial state depicted here would be the result of a sequential update of the state of ignorance and indifference as it was depicted in Figure 2, with $?p$ and $?q$.

The indifferentiation of the initial state results in the state of ignorance and indifference.

Fact 9 (Indifferentiation of the Initial state) $\iota^* = \omega$

The state of ignorance and indifference is more homogeneous than the initial state. It holds in general that the indifferentiation of a state is at least as homogeneous as the state as such. As we already mentioned, the initial state

⁹My main motivation for measuring informativeness and inquisitiveness in opposite directions, is that it works for defining answerhood and subquestionhood, and for obtaining interesting quantity implicatures. Independent motivation could come from comparing the notion of homogeneity with the notion of entropy from information theory. But I have not looked into that yet.

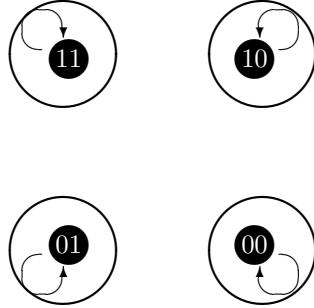


Figure 12: Initial State for $\phi = \{p, q\}$

is the minimal, and the absurd state the maximal element in the homogeneity ordering.

Fact 10 (Homogeneity) $\forall s: \emptyset \succeq s \ \& \ s^* \succeq s \ \& \ s \succeq i$

Adding arbitrary new information to a state always leads to a more homogeneous state, and so does replacing an issue in a state by an arbitrary weaker issue. For example, $\omega[q] \succ \omega[?p]$ and $\omega[?p \vee ?q] \succ \omega[?p]$. But the information that q is not an answer to the question $?p$, and $?p \vee ?q$, though easier to answer, does not constitute a subissue of $?p$. Actually, the latter is an atomic question, that has no proper subissues.

The following notion selects among states r that are at least as homogeneous as the current state s , those in which the issue in s (if there is one) is partially resolved, or replaced by a subissue.

Definition 14 (Relatedness) A state r is related to a state s , $r \propto s$ iff

1. r is at least as homogeneous as s , and
2. every possibility in r is the union of a subset of the set of possibilities in s

By glueing possibilities in s together, we create a weaker subissue in r . But there may also be less of an issue in r than in s , because some possibility in s does not return as (part of) any of the possibilities in r , meaning that in r the issue in s is partially resolved.

Note the borderline cases. In case r is the absurd state, its single possibility is the empty set, which is a subset of the set of possibilities in s . And the union of all possibilities in s counts as well, which means that the indifferentiation s^* of s counts as related to it. And every state cannot fail to be related to the initial state. So, for relatedness, we get a complete parallel for Fact 10 about homogeneity:

Fact 11 (Relatedness) $\forall s: \emptyset \propto s \ \& \ s^* \propto s \ \& \ s \propto i$

But note also that in case s is indifferent, i.e., if $s = s^*$, only the absurd state and s itself are related to it.

Fact 12 (Indifferent Relatedness) $\forall r, s: r \propto s^* \text{ iff } r = s^* \text{ or } r = \emptyset$

A similar fact holds for homogeneity only in case s is a fully informed state $s = \{\langle i, i \rangle\}$, for some $i \in I$.

Finally, we turn to the logical notion of compliance which evaluates whether a sentence is strictly related to the issue in a state. In terms of that we also define a compliance relation between sentences.¹⁰

Definition 15 (Compliance)

1. φ is *compliant to* s iff $s^*[\varphi] \propto s$
2. φ is *compliant to* ψ iff φ is compliant to $\omega[\psi]$

In determining whether φ is compliant to a state s , we inspect whether the indifferentiation of s updated with φ is related to s . By considering the indifferentiation of s , rather than s itself, we can compare the issue in s with the issue that φ as such gives rise to (if such there is), to see whether it is a subissue of the current issue.¹¹

¹⁰What I call ‘compliance’ now corresponds to what I used to call ‘licensing’ in Groenendijk (1999). I have chosen ‘compliance’ because, unlike ‘licensing’, it clearly suggests that disobedience may not be a bad thing at all.

¹¹It would not do to simply inspect whether φ is not inquisitive in s . For example, $?p \vee ?q$ is not inquisitive in $\omega[?p]$, but $?p \vee ?q$ should not count as a subissue of the current issue $?p$. Under the notion of relatedness it does not: $\omega[?p \vee ?q]$ is not related to $\omega[?p]$, nor the other way around, since the latter is *less* homogeneous than the former.

Being compliant is one thing, being as compliant as you can is yet another. For example, completely resolving the current issue, if you can, is to be preferred over doing so only partially. Comparative compliance prefers *more informative* and *less inquisitive* responses among alternative compliant sentences. It does so by comparing the homogeneity of the states you get by updating with such alternative responses.

Definition 16 (Comparative Compliance)

1. φ is *more compliant to s than ψ* iff
 - (a) φ is compliant to s and ψ is compliant to s , and
 - (b) $s^*[\varphi] \succ s^*[\psi]$
2. φ is *more compliant to χ than ψ* iff φ more compliant to $\omega[\chi]$ than ψ

As we did for compliance, we defined comparative compliance not only relative to a state, but also relative to another sentence.¹² In terms of these notions of compliance we can give an appropriate characterization of answerhood and subquestionhood.¹³

Fact 13 (Answerhood and Subquestionhood)

1. $!\varphi$ is an answer to $?ψ$ iff $!\varphi$ is compliant to $?ψ$
 $!\varphi$ is a complete answer to $?ψ$ iff every answer $!χ$ to $?ψ$ which is more compliant to $?ψ$ than $!\varphi$ is equivalent with \perp
2. $?φ$ is a subquestion of $?ψ$ iff $?φ$ is compliant to $?ψ$
 $?φ$ is a minimal subquestion of $?ψ$ iff every subquestion $?χ$ of $?ψ$ which is more compliant to $?ψ$ than $?φ$ is equivalent with \top

¹²I do so in the definition by considering the state $\omega[\chi]$. Equivalently, I could quantify over all states.

¹³In a partition semantics for questions, such as in Groenendijk (1999), see also ten Cate & Shan (2007), we can define that $?ψ$ is a subquestion of $?φ$ in terms of entailment as $?φ \models ?ψ$, and $!ψ$ being a (partial) answer to $?φ$ as $?φ \models !ψ$. Such definitions do not work when possibilities may overlap. They would, e.g., characterize $?p \vee ?q$ as a subquestion of $?p$, since $?p \models ?p \vee ?q$, and would fail to characterize $p \rightarrow q$ as an answer to $p \rightarrow ?q$, since $p \rightarrow ?q \not\models !(p \rightarrow q)$, but rather the other way around. That compliance does the job nicely, clearly indicates that this is the true logical notion that comes with inquisitive semantics, which brings along that the relation of homogeneity between states, and not the substate relation, is at the basis of the logic. See Groenendijk (2008) for a more detailed discussion of the issue.

I put these characterizations down as a fact, and not as a definition because I want to *claim* that these are appropriate characterizations of these relations from an intuitive point of view. For example, I claim that $?p$ is not a subquestion of $?p \vee ?q$, nor the other way around. Not every complete answer to $?p \vee ?q$ is a partial answer to $?p$. And although it does hold that every complete answer to $?p$ is also a complete answer to $?p \vee ?q$, it should not count as a subquestion, because $?p$ is not easier to answer than $?p \vee ?q$. By the way, $?p \vee ?q$ does have four minimal subquestions. (Minimal subquestions always have only two answers, not necessarily yes/no.)

The following fact indicates that compliance is a very restrictive logical notion.

Fact 14 (Compliance)

1. \perp and \top are compliant to φ
2. φ is compliant to φ
3. if φ is compliant to χ , and χ to ψ , then φ is compliant to ψ
4. Let $\psi \in L_\wp$. φ is compliant to ψ iff $\exists \varphi' \in L_\wp : \varphi \Leftrightarrow \varphi'$ & φ' is compliant to ψ .
5. If φ is compliant to ψ , then $!\varphi$ is compliant to ψ .
6. Apart from what can be obtained from (1)-(5), or what can be obtained from (a)-(d) by using (1)-(5):
 - (a) Only $!\varphi$ is compliant to $!\varphi$
 - (b) Only p and $\neg p$ are compliant to $?p$.
 - (c) Only p and q are compliant to $p \vee q$
 - (d) Only $(\neg)p \vee (\neg)q$ are compliant to $?p \vee ?q$

I don't want to go in these matters in any detail, they are dealt with in Măcarenhas (2008), but it is a rather simple affair to characterize compliance, and comparative compliance, syntactically.

The crux of the matter is that there is an algorithm that puts any sentence into its inquisitive disjunctive normal form. The normal form is a disjunction of which each disjunct is an assertion which corresponds to one of the possibilities that the sentence gives rise to. The normal form directly represents the semantic content in a syntactic way.

You can arrive at the normal form of a compliant sentence by cutting off disjuncts, and by glueing disjuncts together by taking their assertive closure. For one sentence to be more compliant than another means that more disjuncts have been cut off (more informative), or the same disjuncts have been cut off but of the remaining ones more have been glued together (less inquisitive).

5 Dialogue Pragmatics

We consider dialogues with two participants: the *stimulator* and the *responder*. The central dialogue pragmatic notion is that of a *common ground*. We take a common ground to be a state, which, to be worth to be called by that name, has to satisfy the following condition:

Definition 17 (Common Ground) Let c and s and r be states.

c is a *common ground for s and r* iff $s^* \subseteq c^*$ & $r^* \subseteq c^*$.

Only the information in s , r and c plays a role in determining whether c is a common ground for s and r , not the issues.

Issues will be there in the common ground, they will come and go, and play an essential role in steering the process of building on the common ground, but in the end, it is information that matters.

The following fact stresses that only information matters.

Fact 15 (Indifferentiation and the Common Ground) If c is a common ground for s and r , then c^* is a common ground for s and r .

That indifferentiation can not distort a common ground is important in that this operation on the current stage of the common ground will play a role when the stimulator or responder wants to shift the dialogue to another issue. Indifferentiation has the effect of clearing the way for this by setting the old issue temporarily aside.¹⁴

The purpose of inquisitive and informative dialogue is to enhance the common ground by eliminating possibilities from it on the basis of the information that is being exchanged. But in doing so, the stimulator and the

¹⁴To be able to model that an issue is *temporarily* put aside, we will introduce stacks.

responder have to be careful. The most fundamental pragmatic dialogue rule reads as follows:

Definition 18 (First Dialogue Principle) *Maintain a common ground!*

In the next section, we will define *dialogue management rules* that determine the effects of utterances, in combination with reactions upon them, on the common ground. The rules will be such that if they are followed by the stimulator and the responder, a common ground is guaranteed to be maintained.

Eliminating possibilities from the common ground involves updating. Given our semantics, the following holds without exception:¹⁵

Fact 16 (Common Ground and Simultaneous Update) If c is a common ground for s and r , then $c[\varphi]$ is a common ground for $s[\varphi]$ and $r[\varphi]$.

However, it will not always be possible to *accept* a proposed update. It may happen that the update of an informative sentence uttered by the stimulator would lead to an inconsistent state for the responder (or *vice versa*). For a happy continuation of the dialogue, the responder should not let this happen. But then, if she would just not update her state with the utterance of the stimulator, the update of the common ground with it should be cancelled also. Otherwise, there is a serious threat that there is no common ground anymore. This leads to the following pragmatic dialogue rule.

Definition 19 (Second Dialogue Principle) *Keep your state consistent! Publicly announce cancelling a proposed update to maintain consistency.*

The second part of this principle, to publicly announce *cancelling*, is not really an independent rule, but is a natural consequence of the combination of the requirements to maintain a common ground and to keep your state consistent. If the latter would happen ‘silently’, the former would be endangered.

¹⁵ States only contain information *about the world*, not information about information of the dialogue participants. This, together with the fact that the semantics has the update property (for all s and $\varphi: s[\varphi] \subseteq s$) guarantees that simultaneous update preserves a common ground. See Gerbrandy (1999), Chapter. 6.

To make it possible to deal with *proposed* updates of the common ground, and to create room for cancelling them, we will model the common ground as a *stack of states*. Dialogue management will involve the *uptake* of a proposed update, and *absorbing* the effects of publicly announced *cancellation*, *acceptance* or *support* of the update proposal.

Finally, we also put the logical notion of compliance to work as a pragmatic preference rule. It is the inquisitive pragmatic version of the Gricean Maxims of Relation and Quantity:

Definition 20 (Third Dialogue Principle) Other things being equal,
Be as compliant as you can to the current state of the common ground!

The pragmatic notion of comparative compliance is the logical notion we introduced in the previous section. For the responder to try and be compliant means first of all that her utterance should be related to the current state of the common ground, and secondly that her utterance should be as homogeneous to the current state of the common ground as she can possibly make it, i.e., the more informative the better, and the less inquisitive the better.

It is in particular the latter that is new in inquisitive dialogue pragmatics as compared to Gricean pragmatics. And, as we shall see, it will lead to a new dialogue oriented analysis of generalized conversational implicatures pertaining to disjunction. This gives the logical notion of compliance its empirical bite.

The dialogue principles dictate that the stimulator and the responder try to obtain a more homogeneous state of the common ground. We have seen that the least homogeneous state is the initial state ι . Given this, the natural *initial common ground* is the initial state ι , where the following holds:

Fact 17 (Initial Compliance) $\forall s: s \subseteq \iota \Rightarrow \forall \varphi: \varphi$ is compliant to s .

The general aim of a dialogue is eliminating possibilities from ι , where indices and possibilities are interchangeable. We generally do so by focussing on some subissue (of a subissue...). Once a subissue is resolved we are back at a subset of the possibilities in ι , we can focus on a new subissue, etc., etc. That is the general direction that coherent dialogue takes.¹⁶

¹⁶See Roberts (1996), which has much in common with our approach, including to take the initial question under discussion to be The Big Question.

6 Inquisitive Dialogue Management

6.1 Global Architecture

For reasons we have shortly indicated above, to be able to properly manage a dialogue, we model the common ground as a stack of states.

We define stacks in the following way.¹⁷

Definition 21 (Stacks) The set of *stacks* is the smallest set such that:

1. $\langle \rangle$ is a stack
2. If s is a state, and σ is a stack, then $\langle \sigma, s \rangle$ is a stack

Starting from the empty stack $\langle \rangle$, comes handy in defining operations on stacks that percolate down the stack.

Definition 22 (Percolation) Let $[\cdot]$ be an operation on states.

1. $\langle \rangle^\triangleleft[\cdot] = \langle \rangle$
2. $\langle \sigma, s \rangle^\triangleleft[\cdot] = \langle \sigma^\triangleleft[\cdot], s[\cdot] \rangle$

For example, the operation $^\triangleleft[\varphi]$ percolates the update of a state with φ down through all the elements of a stack, until it is halted by $\langle \rangle$.

In our dialogue setting there are two distinctive sets of operations on stacks. The first set of operations concerns the *uptake* of a sentence in the common ground. The second set of operations concerns the *absorption of the reaction* to the result of the uptake.

To start with the latter, we distinguish three kinds of public reactions to the uptake effects of a stimulus, which correspond to three absorption operations on the common ground stack: *to cancel*, *to accept*, and *to support*.

¹⁷Our notion of stacks differs from Kaufmann (2000) in two respects. The first difference is that our recursive definition starts from the empty stack. The second difference is that we push stacks from left to right, instead of from right to left. So, the top of the stack is always on the right hand side.

The reason for doing things in this direction is that we (and Kaufmann) define operations on stacks in the update format, i.e., we use a postfix notation for the operations on stacks. Since such operations typically work on the top, or start at the top, of a stack, building up stacks from left to right is more in line with a postfix notation of operations on stacks.

To accept is the default, and signals that the responder can consistently update her state with the stimulus. As we have seen above, in case this is not so, it is essential to publicly announce that cancellation is needed to maintain a common ground.

Support is signalled in case the state of the responder already supported the information provided by the stimulus. Assuming that this also holds for the state of the stimulator, a public announcement of support may result in a definitive *update* of the common ground with the information provided by the stimulus.

The uptake of a sentence φ goes in two steps. The *primary uptake of the semantic content of φ* , and a *secundary uptake of the compliance implicatures of φ* . The primary uptake consists of an operation to *assume φ* , but it is preceded by the operation of *thematizing φ* .

This means that after it has been signalled that to assume φ had to be cancelled, a response may now address the effect of the thematization of φ , which will be an issue, calculated from the current issue in the common ground relative to which φ was uttered, and the *theme of φ* , viz., $?_\varphi$. This allows for critical responses to the stimulus φ .

The secundary uptake *calculates* the compliance implicatures of φ from the effect that to thematize and to assume φ had on the current state of the common ground. The effect of the uptake of implicatures of φ , if such there are, will be a strengthening of what to assume φ has led to, it will *exclude* certain things.

We will distinguish two such exclusion effects, where which of the two will be operative solely depends on whether the stimulus is informative or not in the current state of the common ground.

Since the reactions of to cancel, accept or support, take place after the uptake of φ as a whole, they include a reaction to the implicative part of the uptake of φ .

The overall effect of the uptake of a sentence will always be that the common ground stack is pushed three times, each time putting a new state on top of the stack. The ‘new’ state put on top is not always really new, sometimes it is just a copy of the previous top of the stack. After the uptake as a whole, we clean the stack from such superfluous copies. Absorption of one of the three kinds of reactions we distinguish, will operate on the stack resulting from this cleaning operation.¹⁸

¹⁸One might imagine that the uptake operations themselves would only lead to a push if the result of the operation has an effect. The reason for not doing so, is that the secundary uptake works ‘blindly’ on the results of to thematize and assume. So, even when these have no real effects, we have to be sure of their presence for the calculation of the compliance

So, all in all, with respect to a stimulus sentence φ and a reaction to it, we meet a sequence of a whole bunch of operations on the common ground stack. To summarize:

Structure dialogue management rules

1. *Uptake* of a sentence φ in the common ground
 - (a) *Primary uptake* of the semantic content of φ
 - i. *Thematize* φ relative to current issue in common ground
 - ii. *Assume* φ
 - (b) *Secondary uptake* calculates compliance implicatures
 - i. *Alternative exclusion*, or
 - ii. *Block exclusion*
 - (c) *Cleaning* the uptake.
2. *Absorption* of reaction to the uptake
 - (a) *Cancellation*, or
 - (b) *Acceptance*, or
 - (c) *Support*

6.2 Thematize

We will work our way through defining the rather long list of operations on stacks, and begin with thematizing a sentence.

Definition 23 (Thematize) $\langle \sigma, s \rangle[\varphi]^? = \langle \langle \sigma, s \rangle, s \cup s^*[?\varphi] \rangle$

In thematizing φ in $\langle \sigma, s \rangle$, we *add the theme of* φ , i.e., $?\varphi$, to the current issue in s by taking $s \cup s^*[?\varphi]$.¹⁹

implicatures to work properly.

¹⁹Why do we add $?\varphi$ to the current issue, and not just take $?\varphi$ as such as new current theme? The original motivation comes from conditional questions, which are not represented as such in our language (although they can be expressed). That's why I put this important point in a footnote. Suppose the dialogue has opened with a conditional question $p \rightarrow ?q$. Then $p \rightarrow q$ counts as one of the two answers. So suppose the question is followed by this answer. The theme of $p \rightarrow q$ is the questioned conditional $?(p \rightarrow q)$. Suppose we take this to be the new current issue. That would mean that a critical response

Note that in taking the indifferentiation s^* of s , $s^*[\varphi]$ focusses on the issue behind φ as such. Note also that in case φ is a question, and hence equivalent with $?_\varphi$, $s^*[\varphi] = s^*[_\varphi]$. If the current issue in s can be written as $?_\psi$, then $s = s^*[_\psi]$ and $s \cup s^*[\varphi]$ can be written as $s^*[_\psi \vee ?_\varphi]$. Given this, it is clear that the thematization of a sentence φ in a stack $\langle \sigma, s \rangle$, will put a new state on top of the stack which is homogeneous to s :

Fact 18 (Thematizing and Homogeneity) $\forall s: s \cup s^*[\varphi] \preceq s$.

This need not be the case for relatedness. For example, if the current issue is given by $\omega[?p]$, then both q and $?q$ are not compliant, and the effect of their thematization will lead to a state which is not related to the current state.

$$(1) \quad \omega[?p] \cup \omega[?q] = \omega[?p \vee ?q] \not\propto \omega[?p]$$

Thematization of φ in the initial state, will always have the effect of focussing attention to the theme $?_\varphi$ of φ .

Fact 19 (Initial Thematization)

$$\iota \cup \iota^*[\varphi] = \iota \cup \omega[\varphi] = \omega[\varphi]$$

$$\langle \langle \rangle, \iota \rangle [?_\varphi] = \langle \langle \langle \rangle, \iota \rangle, \omega[\varphi] \rangle$$

Given that any sentence is compliant to the initial state, this fact is a special instance of the following general fact.

Fact 20 (Thematizing and Compliance)

$$\text{If } ?_\varphi \text{ is compliant to } s, \text{ then } s \cup s^*[\varphi] = s^*[\varphi]$$

$$\text{If } ?_\varphi \text{ is compliant to } s, \text{ then } \langle \sigma, s \rangle [\varphi]^? = \langle \langle \sigma, s \rangle, s^*[\varphi] \rangle$$

to the answer $p \rightarrow q$ would address $?(p \rightarrow q)$. But the most natural critical response to $p \rightarrow q$ is not its negation No, $\neg(p \rightarrow q)$, but rather the other answer to the conditional question $p \rightarrow ?q$, i.e.: No, $p \rightarrow \neg q$. This is precisely what adding the theme $?(p \rightarrow q)$ of $p \rightarrow q$ to the original issue $p \rightarrow ?q$ delivers. The disjunction of these two questions is equivalent with the original conditional question. I.e., if the current issue is $p \rightarrow ?q$, then thematization of $p \rightarrow q$ relative to this leaves the current issue as it was.

A question is compliant with a state if it embodies a subissue of the issue in the state. So, thematizing a compliant question means to replace the current issue by a subissue of it.

I will use ten examples to illustrate all the steps in the uptake of a sentence, and the effects of the absorption of the three reactions to sentences we distinguish. Below, I give the effects of thematizing in each of these ten cases.

The examples (1)-(5) concern the uptake of sentences in the initial common ground, where thematization always results in making the theme of the sentences the current issue.

The examples (6)-(10) concern the uptake of sentences relative to a common ground that has already been affected by some earlier moves in the dialogue. Examples (5)-(9) illustrate how the effects of thematizing one and the same sentence p , can differ depending on the current issue in the common ground.

Whereas in (1)-(8) we are dealing with sentences that are compliant to the current state of the common ground, the responses in (9)-(10) are not.²⁰

Ten Examples of Thematizing

- (1) $\langle\langle \rangle, i\rangle [?(p \vee q)]^? = \langle\langle\langle \rangle, i\rangle, \omega[?(p \vee q)]\rangle$
- (2) $\langle\langle \rangle, i\rangle [?!(p \vee q)]^? = \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle$
- (3) $\langle\langle \rangle, i\rangle [p \vee q]^? = \langle\langle\langle \rangle, i\rangle, \omega[?(p \vee q)]\rangle$
- (4) $\langle\langle \rangle, i\rangle [!(p \vee q)]^? = \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle$
- (5) $\langle\langle \rangle, i\rangle [p]^? = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle$
- (6) $\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^? = \langle\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p]\rangle$
- (7) $\langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^? = \langle\langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle$
- (8) $\langle\langle \rangle, i![p \vee q]\rangle [p]^? = \langle\langle\langle \rangle, i[!(p \vee q)]\rangle, \omega[p \vee (\neg p \wedge q)]\rangle$
- (9) $\langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle [p]^? = \langle\langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q) \vee \neg p]\rangle$
- (10) $\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [?q]^? = \langle\langle\langle \rangle, i[?p]\rangle, \omega[?p \vee ?q]\rangle$

²⁰If you want to check my calculations, an easy way to do so is to draw the kind of pictures of states that I used to illustrate the semantics with. A stack of states can be seen as a sequence of such pictures. Since in the examples I use only two propositional variables, the pictures are easy to draw.

The examples (1)-(5) are simple instances of using Fact 19. The only interesting cases are (6)-(10) where we are dealing with responses after a preceding dialogue exchange.

In (6) we find a simple case of a positive answer p to an atomic polar question $?p$. Obviously, since $\omega[?p] \cup \omega[?p] = \omega[?p]$, thematization just puts a copy of the current issue on top of the stack. (Fact 20 applies.)

In case of (7) we are dealing with an utterance of p where the current state of the common ground embodies the hybrid disjunction $p \vee q$. We will see later that this can be the result of acceptance of the hybrid disjunction as such, but also of the alternative question $?(p \vee q)$. Since $\omega[p \vee q] \cup \omega[p] = \omega[p \vee q]$, thematization of p in this situation leaves the current state as it was, and just returns a copy of it on the top of the stack.²¹

In case of (8) we are dealing with an utterance of p where the current state of the common ground just embodies the information that $!(p \vee q)$. Any sentence is compliant in such a situation. (Fact 17 applies.) We can represent $(\iota[!(p \vee q)])^*$ as $\omega[!(p \vee q)]$, and $\omega[!(p \vee q)][?p]$ as $\omega[p \vee (\neg p \wedge q)]$. And the result of thematization $\iota[!(p \vee q)] \cup \omega[p \vee (\neg p \wedge q)] = \omega[p \vee (\neg p \wedge q)]$.

In case of (9) we are dealing with the utterance of a sentence p which is not compliant to the current issue in the common ground, which is the polar yes/no-question $?!(p \vee q)$. The effect of thematization in this situation is $\omega[?(p \vee q)] \cup \omega[?p]$, and can be represented as $\omega[!(p \vee q) \vee \neg p]$.

In case of (10) we are dealing with the utterance of a question $?q$ which is not compliant to the current issue in the common ground, which is the atomic polar question $?p$. The effect of thematization in this situation $\omega[?p] \cup \omega[?q] = \omega[?p \vee ?q]$.

6.3 Primary Uptake: Thematize and Assume

To get a more complete picture, let us move on to the next operation involved in the uptake of a sentence.

Definition 24 (Assume) $\langle \sigma, s \rangle[\varphi]^{\dagger} = \langle \langle \sigma, s \rangle, s[\varphi] \rangle$

We denote the primary uptake, the combination of to thematize and assume, as follows:

Definition 25 (Primary Uptake) $\langle \sigma, s \rangle[\varphi]^{\dagger^1} = \langle \sigma, s \rangle[?]^\dagger[\varphi]^{\dagger}$

²¹This means that a critical response to p after after $p \vee q$, is predicted to be No q rather than No $\neg p$. See also footnote 19.

The effect of the primary uptake of φ results in:

$$\langle \sigma, s \rangle [\varphi]^{\uparrow^1} = \langle \langle \langle \sigma, s \rangle, s \cup s^*[\varphi] \rangle, (s \cup s^*[\varphi])[\varphi] \rangle$$

We can economize this result:

Fact 21 (Primary Uptake) $\langle \sigma, s \rangle [\varphi]^{\uparrow^1} = \langle \langle \langle \sigma, s \rangle, s \cup s^*[\varphi] \rangle, s^*[\varphi] \rangle$

To show that this holds, we have to show that: $(s \cup s^*[\varphi])[\varphi] = s^*[\varphi]$. We have that $(s \cup s^*[\varphi])[\varphi] = (s \cup s^*[\varphi] \cup s^*[\neg\varphi])[\varphi]$. In turn this is the same as $s[\varphi] \cup s^*[\varphi][\varphi] \cup s^*[\neg\varphi][\varphi]$. Since $s^*[\neg\varphi][\varphi] = \emptyset$, and $s^*[\varphi][\varphi] = s^*[\varphi]$, this can be reduced to $s[\varphi] \cup s^*[\varphi]$. Since $s \subseteq s^*$, it also holds that $s[\varphi] \subseteq s^*[\varphi]$. Hence, $(s \cup s^*[\varphi])[\varphi] = s^*[\varphi]$.

Combining this with the previous fact:

Fact 22 (Primary Uptake and Compliance)

If φ is compliant to s , then $\langle \sigma, s \rangle [\varphi]^{\uparrow^1} = \langle \langle \langle \sigma, s \rangle, s^*[\varphi] \rangle, s^*[\varphi] \rangle$

Since any sentence is compliant to the initial state:

Fact 23 (Initial Primary Uptake) $\langle \langle \langle \rangle, i \rangle [\varphi]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[\varphi] \rangle, \omega[\varphi] \rangle$

We consider the primary uptake of our ten examples.

Ten Examples of Primary Uptake

- (1) $\langle \langle \langle \rangle, i \rangle [?(p \vee q)]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[?(p \vee q)] \rangle, \omega[?(p \vee q)] \rangle$
- (2) $\langle \langle \langle \rangle, i \rangle [?!(p \vee q)]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[?!(p \vee q)] \rangle, \omega[?!(p \vee q)] \rangle$
- (3) $\langle \langle \langle \rangle, i \rangle [p \vee q]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[?(p \vee q)] \rangle, \omega[p \vee q] \rangle$
- (4) $\langle \langle \langle \rangle, i \rangle [!(p \vee q)]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[!?(p \vee q)] \rangle, \omega[!(p \vee q)] \rangle$
- (5) $\langle \langle \langle \rangle, i \rangle [p]^{\uparrow^1} = \langle \langle \langle \langle \rangle, i \rangle, \omega[?p] \rangle, \omega[p] \rangle$
- (6) $\langle \langle \langle \langle \rangle, i \rangle, \omega[?p] \rangle [p]^{\uparrow^1} = \langle \langle \langle \langle \langle \rangle, i \rangle, \omega[?p] \rangle, \omega[?p] \rangle, \omega[p] \rangle$
- (7) $\langle \langle \langle \langle \rangle, i \rangle, \omega[p \vee q] \rangle [p]^{\uparrow^1} = \langle \langle \langle \langle \langle \rangle, i \rangle, \omega[p \vee q] \rangle, \omega[p \vee q] \rangle, \omega[p] \rangle$

- $$(8) \langle\langle \rangle, i[!(p \vee q)]\rangle [p]^{\uparrow^1} = \langle\langle\langle \rangle, i[!(p \vee q)]\rangle, \omega[p \vee (\neg p \wedge q)]\rangle, \omega[p]\rangle$$
- $$(9) \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle [p]^{\uparrow^1} = \langle\langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q) \vee \neg p]\rangle, \omega[p]\rangle$$
- $$(10) \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [?q]^{\uparrow^1} = \langle\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p \vee ?q]\rangle, \omega[?q]\rangle$$

The primary uptakes in (1)-(5) exemplify Fact 23. All of (1)-(10) illustrate Fact 21.

In case of (1) and (2), where we are dealing with compliant questions, the effect of assume after thematize just puts a copy of the latter on the top of the stack. Note that this is not so with the non-compliant question in (10).

It is time to move on to the most interesting part, the secundary uptake of compliance implicatures.

6.4 Implicatures and Suggestions

The primary uptake of a sentence concerns its informative and inquisitive semantic content. It is customary to distinguish on top of what is literally said and implied, things that are suggested or implicated. Gricean pragmatics deals with them as conclusions that can be drawn on the assumption that speakers are cooperative, or compliant to use our terminology.

We will basically follow that pattern, and as is usually done, we concentrate on generalized conversational implicatures, were it will be no surprise that we focus on disjunction.

There will also be rather significant differences in the way in which we approach these pragmatic matters. The first major source for this is that our semantics is rather different from what it usually is, in that issues are just as much a basic ingredient of semantic content as information is. The second source for where our approach is different is that we really model conversations, and suggestions and implicatures are dealt with as coming about in the interplay of the participants in the discourse.

Concerning the differences that stem from our semantics, we already met our notion of comparative Quantity, viz. the logical notion of comparative compliance, that our inquisitive semantics gives rise to. It first of all combines informative quantity with inquisitive quantity, and secondly, although homogeneity prefers more informative utterances over less informative ones, it prefers less inquisitive utterances over more inquisitive ones. This together with the fact that there are hybrid sentences, of which disjunctions are a prime example, will give a whole new twist to quantity implicatures. Comparative compliance will play a key role in our investigations.

It is not uncommon to paraphrase the theoretical notion of an implicature informally as a suggestion, but I make a subtle distinction between the two. The informal definitions run as follows.

Definition 26 (Implicatures and Suggestions)

1. An *implicature* of a sentence φ is a piece of information that is provisionally added to the common ground by a secondary uptake on top of the primary uptake of φ .
2. A *suggestion* of a sentence φ has no immediate effect on the common ground, but forces a compliant responder to φ in a direction where his response ψ has an implicative uptake effect on the common ground.

Just to make sure not to be misunderstood: only implicatures as such, not suggestions, technically play a role in the secondary uptake of a sentence. Suggestions are there, but only when the responder follows a suggestion of the stimulator — by being compliant to the stimulus — will an implicature result. Technically speaking, it is just an implicature of the response as such. The introduction of the two notions stresses the point that implicatures come about in the interplay between stimulator and responder in the dialogue.

The cases we will deal with are the following:²²

- An alternative question $?(p \vee q)$ *implies the exclusion of* $\neg p \wedge \neg q$.
- Both a hybrid disjunction $p \vee q$, and an alternative question $?(p \vee q)$, *suggest the exclusion of* $p \wedge q$.
- A compliant response p to a hybrid disjunction $p \vee q$ or an alternative question $?(p \vee q)$ *implies the exclusion of* $p \wedge q$.

Like any other piece of information provisionally added by the uptake of a sentence, implicatures may be cancelled by the responder. And a responder can refrain from following a suggestion by giving a non-compliant response. Note the following:

²²I take it completely for granted that the reader agrees with me that these exclusion implicatures (or implications) associated with disjunction *are* implicatures (or implications) one wants to account for. As a matter of historical fact, my whole *pragmatic* story came about as a reaction to critique (among others by Maria Aloni) on the outcome of my inquisitive *semantics* for in particular alternative questions. I wanted to underpin my claim that the exclusion effects need not be built into the semantics, but can be accounted for in pragmatics. In essence, this lengthy pragmatic story is a defense of inquisitive semantics.

- *On the responder accepting* the exclusion implicature of an alternative question $?(p \vee q)$, its effect on the common ground becomes the same as that of the hybrid disjunction $p \vee q$.
- *On the responder accepting* the exclusion suggestion of an alternative question $?(p \vee q)$ or a hybrid disjunction $p \vee q$ — compliantly responding with one of the disjuncts p or q — the overall *effect of the dialogue exchange* on the common ground becomes the same as that of a response to an exclusive disjunction.

The source for the exclusion implicature of alternative questions is the following logical fact:

Fact 24 (Source Compliance Implicature Alternative Questions)

1. For any state s : if the alternative question $?(p \vee q)$ is compliant to s , then the polar question $?!(p \vee q)$ is compliant to s as well, and
2. For any state s : $?!(p \vee q)$ is at least as compliant to s as $?(p \vee q)$.

Comparative compliance will invariably prefer the polar question $?!(p \vee q)$ over the alternative question $?(p \vee q)$, because it is *less inquisitive*. The dialogue principle: be as compliant to the common ground as you can, is flouted by the use of an alternative question, *unless we assume* the less inquisitive polar question $?!(p \vee q)$ not really to be inquisitive at all. We can only make sense of that if we assume that the possibility that $\neg p \wedge \neg q$ *apparently does not count*.

The source for the suggestion of exclusiveness of $p \vee q$ and $?(p \vee q)$ is the following logical fact:

Fact 25 (Source Compliance Suggestion Disjunction)

1. $p \wedge q$ is not compliant to $p \vee q$ nor to $?(p \vee q)$, and
2. $p \wedge q$ is more homogeneous to $p \vee q$ and $?(p \vee q)$ than either p or q

The option to compliantly respond to $p \vee q$ or $?(p \vee q)$ with $p \wedge q$ is blocked by relatedness, whereas at the same time $p \wedge q$ would be more homogeneous than either p or q , because $p \wedge q$ is more informative than either p or q .

We can only make sense of that if we assume that the possibility that $p \wedge q$ apparently does not count.

If the responder goes along with that suggestion and responds with one of the disjuncts, say p , she thereby implicates that the *alternative* possibility q is excluded.

This is a piece of Gricean reasoning performed by the author of this paper, using inquisitive logic, and the pragmatic principles based on it. What is to follow is to implement the conclusion of this in defining the contents of the secundary uptake of compliance implicatures in accordance with the outcome of this piece of reasoning.

There is no Gricean reasoning involved for stimulator or responder. By just following the dialogue management rules they implicate and suggest what the logic has told us they implicate and suggest. At the same time, in keeping to these rules, the responder can freely choose to accept, or to cancel, what is implicated; and to be compliant, or not, to what is suggested by utterances of the stimulator.

6.5 Secundary Uptake: Compliance Implicatures

The secundary uptake of compliance implicatures of utterances of sentences splits into two uptake operations: *alternative exclusion*, which only applies to sentences which are informative in the state of the common ground relative to which the sentence is uttered; and *block exclusion*, which applies in the uninformative case. We start with alternative exclusion.

Alternatives can be detected in a stack $\langle\langle\langle\sigma, s\rangle, t\rangle, u\rangle$ that is the result of the primary uptake of a sentence φ in a common ground stack $\langle\sigma, s\rangle$. The definition of alternatives will not mention sentences, and operates purely on the contents of the stack, but you can think of it as determining the alternatives for φ , if such there are.

We will only start searching for alternatives, if φ really addresses the current issue in the common ground, i.e., when φ is compliant to s , which will mean that u is related to s . (We are dealing here with the implicature that comes about by the responder being *compliant to the suggestion of exclusiveness* made by the stimulator.)

We will only put the definition to work in the secundary uptake of φ if φ is informative with respect to the common ground, i.e. when φ provides a partial answer to the current issue, i.e., it selects certain possibilities in t , those which we can find back in u , and discards others, possibilities in t which we do not find back in u . The latter are the alternatives.

Definition 27 (Alternatives) ρ is an *alternative* in $\langle\langle\langle\sigma, s\rangle, t\rangle, u\rangle$ iff

1. $u \propto s$
2. ρ is a possibility in t and ρ is not a possibility in u

Let $s = \omega[p \vee q]$. Let t be the effect of thematizing p in $\omega[p \vee q]$, i.e., $t = \omega[p \vee q]$ as well. Let u be the effect of assuming p , i.e., $u = \omega[p]$. The first condition of the definition is met, p is compliant to $p \vee q$. The possibilities in $t = \omega[p \vee q]$ are the possibility that p : $\{i \in I \mid i(p) = 1\}$ and the possibility that q : $\{i \in I \mid i(q) = 1\}$. The only possibility in $u = \omega[p]$ is the possibility that p . The only alternative in this situation is the possibility that q .

Note that not only the possibility that q counts as an alternative in a situation like the one above, where we were dealing with overlapping possibilities. According to the definition the possibility that q also counts as an alternative after the answer $\neg q$ to the yes/no question $?q$. However, such non-overlapping possibilities will play no role in the exclusion implicature.

The secundary uptake of alternative exclusion is defined as follows:

Definition 28 (Alternative Exclusion)

Let A be the union of the alternatives in $\langle\langle\sigma, s\rangle, t\rangle$.

$$\langle\langle\sigma, s\rangle, t\rangle[\text{EXCLA}] = \langle\langle\langle\sigma, s\rangle, t\rangle, u\rangle, \text{ where } u = \{\langle i, j \rangle \in t \mid i, j \notin A\}$$

Continuing on our example. The new top added to the stack is:

$$\{\langle i, j \rangle \in \omega[p] \mid i, j \notin \{i \in I \mid i(q) = 1\}\} = \omega[p \wedge \neg q]$$

Note that alternative exclusion has no effect when there are no alternatives and the union of the alternatives is the empty set. Neither will alternative exclusion have effect in case there are no overlapping possibilities in s .

It cannot fail to be the case that the result u of alternative exclusion is homogeneous to the old top t , but typically when alternative exclusion has effect, when $u \neq t$, u is *not related* to t . Alternative exclusion has the effect of eliminating *proper parts of possibilities* in t .

We now turn to the second operation, that will take care of the exclusion implicature of alternative questions. First we introduce an auxiliary notion.

Definition 29 (Euclidean Closure) Let s be a state.

The *euclidean closure* s^{\div} of s , is the smallest set s' such that $s \subseteq s'$, and $\forall i, j, k: \langle i, j \rangle \in s' \& \langle i, k \rangle \in s' \Rightarrow \langle j, k \rangle \in s'$

The euclidean closure of a reflexive and symmetric relation turns it into an equivalence relation. It will only have effect on a state s if there are overlapping possibilities in s , and it returns a state where overlapping possibilities are joined together. As a typical example: $\omega[?(p \vee q)]^{\div} = \omega[?!(p \vee q)]$.

We determine the blocks in a state s to be those possibilities in s that we also find in the euclidean closure of s , i.e, the possibilities in s that do not overlap with other possibilities in s . We restrict ourselves to the situation where there are both non-overlapping and overlapping possibilities in s . This is captured by the first clause in the definition below.

Definition 30 (Blocks) ρ is a *block* in s iff

1. $s \neq s^{\div} \& s^{\div} \neq s^*$
2. ρ is a possibility in s and ρ is a possibility s^{\div}

Consider the example where $s = \omega[?(p \vee q)]$. There are three possibilities in $\omega[?(p \vee q)]$: the possibility that p ; the possibility that q and the possibility that $\neg p \wedge \neg q$. The possibilities that p and that q overlap. In the euclidean closure of $\omega[?(p \vee q)]$, which equals $\omega[?!(p \vee q)]$, the union of the possibilities that p and that q forms a single possibility. The possibility that $\neg p \wedge \neg q$ remains untouched, and is the only possibility in $\omega[?(p \vee q)]$ that survives as such in the euclidean closure of $\omega[?(p \vee q)]$.

The secundary uptake of block exclusion is defined as follows:

Definition 31 (Block Exclusion) Let B be the union of the blocks in s .

$$\langle \sigma, s \rangle [\text{EXCLB}] = \langle \langle \sigma, s \rangle, t \rangle, \text{ where } t = \{ \langle i, j \rangle \in s \mid i, j \notin B \}$$

Continuing on our example where $s = \omega[?(p \vee q)]$. The new top added to the stack by block exclusion is:

$$\{ \langle i, j \rangle \in \omega[?(p \vee q)] \mid i, j \notin \{ i \in I \mid i(p) = i(q) = 0 \} \} = \omega[p \vee q]$$

It cannot fail to be the case that if t is the result of block exclusion in s , then t is related to s . If block exclusion has any effect, it only eliminates possibilities as a whole.

We are now ready to formulate the secundary uptake of the compliance implicatures as a whole. Every primary uptake of a sentence is *blindly followed* by the following secundary uptake, which operates purely on the stack produced by the primary uptake as such.

Definition 32 (Exclusion)

$$\langle\langle\sigma, s\rangle, t\rangle[\text{EXCL}] = \begin{cases} \langle\langle\sigma, s\rangle, t\rangle[\text{EXCLA}] & \text{if } s^* \neq t^* \\ \langle\langle\sigma, s\rangle, t\rangle[\text{EXCLB}] & \text{if } s^* = t^* \end{cases}$$

In inspecting whether $s^* = t^*$ or $s^* \neq t^*$, the operation detects whether the last sentence was informative in the top of σ or not. If it was — the case of p in response to $p \vee q$ — alternative exclusion is applied, if it was not — the case of $?(p \vee q)$ — block exclusion is applied.

Let us look at our ten examples again. In each case, it is first decided whether block exclusion or alternative exclusion applies. The former is only the case in (1), (2), and (10), where the sentence we are considering is a question.

However, in case of (2) and (10) we are dealing with the yes/no-questions $?!(p \vee q)$ and $?q$, which do not lead to overlapping possibilities, and block exclusion only puts a copy on top of the stack of the issue that was already there as the result of the primary uptake.

In case of the alternative question $?(p \vee q)$, block exclusion has effect, the possibilty that $\neg p \wedge \neg q$ is eliminated, the new top of the stack embodies the hybrid disjunction $p \vee q$.

Ten Examples of Primary Plus Secundary Uptake

- (1) $\langle\langle\langle\langle\langle\langle, i\rangle, ?(p \vee q)\rangle\rangle\rangle\rangle[\text{EXCL}] = \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle\rangle[\text{EXCLB}] =$
 $= \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle$
- (2) $\langle\langle\langle\langle\langle\langle, i\rangle, ?!(p \vee q)\rangle\rangle\rangle\rangle[\text{EXCL}] = \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[?!(p \vee q)]\rangle\rangle[\text{EXCLB}] =$
 $= \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[?!(p \vee q)]\rangle, \omega[?!(p \vee q)]\rangle$
- (3) $\langle\langle\langle\langle\langle\langle, i\rangle, [p \vee q]\rangle\rangle\rangle\rangle[\text{EXCL}] = \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle\langle\langle\langle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle$

- (4) $\langle\langle \rangle, i\rangle [!(p \vee q)]^{\uparrow 1}[\text{EXCL}] = \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q)]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q)]\rangle, \omega[!(p \vee q)]\rangle$
- (5) $\langle\langle \rangle, i\rangle [p]^{\uparrow 1}[\text{EXCL}] = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[p]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[p]\rangle, \omega[p]\rangle$
- (6) $\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^{\uparrow 1}[\text{EXCL}] = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p]\rangle, \omega[p]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p]\rangle, \omega[p]\rangle, \omega[p]\rangle$
- (7) $\langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^{\uparrow 1}[\text{EXCL}] = \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle, \omega[p \wedge \neg q]\rangle$
- (8) $\langle\langle \rangle, i![p \vee q]\rangle [p]^{\uparrow 1}[\text{EXCL}] =$
 $= \langle\langle\langle \rangle, i![p \vee q]\rangle, \omega[p \vee (\neg p \wedge q)]\rangle, \omega[p]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i![p \vee q]\rangle, \omega[p \vee (\neg p \wedge q)]\rangle, \omega[p]\rangle, \omega[p]\rangle$
- (9) $\langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle [p]^{\uparrow 1}[\text{EXCL}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q) \vee \neg p]\rangle, \omega[p]\rangle[\text{EXCLA}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[!(p \vee q) \vee \neg p]\rangle, \omega[p]\rangle, \omega[p]\rangle$
- (10) $\langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [?q]^{\uparrow 1}[\text{EXCL}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p \vee ?q]\rangle, \omega[?q]\rangle[\text{EXCLB}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[?p \vee ?q]\rangle, \omega[?q]\rangle, \omega[?q]\rangle$

In the examples (3)-(9) alternative exclusion applies, but in all cases except for (7), there is no real effect, we just end up with a copy of the effect of to assume the sentence on top of the stack.

Example (9) is a case where the first clause in the definition of alternatives does its work. It requires that the state resulting from assuming the sentence, in this case $\omega[p]$, is related to the result of thematizing, in this case $\omega[!(p \vee q) \vee \neg p]$. Clearly, the former is not related to the latter. Hence, according to the definition of alternatives, there are no alternatives to be found here. Which means that alternative exclusion has no effect beyond pushing the stack with a copy of the result of to assume.²³

²³Note that both possibilities in $\omega[!(p \vee q) \vee \neg p]$ are not possibilities in $\omega[p]$. I.e., without the first clause in the definition, they would have counted as alternatives, and alternative exclusion would have resulted in the absurd state.

Example (7) is the case where p is uttered in a situation where the current state of the common ground embodies the hybrid disjunction $p \vee q$. As we shall see, this situation results after acceptance of the hybrid disjunction, and after acceptance of the alternative question $?(p \vee q)$. In such a situation the alternative that q is excluded, and the new top on the stack in (7) embodies the information that $p \wedge \neg q$.

6.6 Complete Uptake

In all of our ten examples, we find copies of a preceding state in the stack that results from the primary and secondary uptake. We add a final cleaning operation to primary and secondary uptake of a sentence, which removes such superfluous copies from the stack.

Definition 33 (Cleaning)

1. $\langle\langle \rangle, s\rangle[\text{CLEAN}] = \langle\langle \rangle, s\rangle$
2. $\langle\langle \sigma, s\rangle, t\rangle[\text{CLEAN}] = \begin{cases} \langle\langle \sigma, s\rangle[\text{CLEAN}] & \text{if } s = t \\ \langle\langle \sigma, s\rangle[\text{CLEAN}], t\rangle & \text{otherwise} \end{cases}$

Putting everything together, the full uptake of a sentence is as follows.

Definition 34 (Uptake) $\langle\sigma, s\rangle[\varphi]^\uparrow = \langle\sigma, s\rangle[\varphi]^?[\varphi]^\uparrow[\text{EXCL}][\text{CLEAN}]$

Applying the cleaning operation to the outcome of the primary and secondary uptake of our ten examples leads to the following overall uptake results.

Ten Examples of Uptake

- (1) $\langle\langle \rangle, i\rangle[?(p \vee q)]^\uparrow = \langle\langle\langle\langle \rangle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle[\text{CLEAN}] =$
 $= \langle\langle\langle\langle \rangle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle$
- (2) $\langle\langle \rangle, i\rangle[?!(p \vee q)]^\uparrow = \langle\langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle, \omega[?!(p \vee q)]\rangle, \omega[?!(p \vee q)]\rangle[\text{CLEAN}] =$
 $= \langle\langle\langle \rangle, i\rangle, \omega[?!(p \vee q)]\rangle$
- (3) $\langle\langle \rangle, i\rangle [p \vee q]^\uparrow = \langle\langle\langle\langle \rangle, i\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle[\text{CLEAN}] =$

$$\begin{aligned}
&= \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?(p \vee q)] \rangle, \omega[p \vee q] \rangle \\
(4) \quad &\langle \langle \rangle, \iota \rangle [!(p \vee q)]^\uparrow = \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q)] \rangle, \omega[!(p \vee q)] \rangle [\text{CLEAN}] = \\
&= \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q)] \rangle \\
(5) \quad &\langle \langle \rangle, \iota \rangle [p]^\uparrow = \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[p] \rangle, \omega[p] \rangle, \omega[p] \rangle [\text{CLEAN}] = \\
&= \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[p] \rangle \\
(6) \quad &\langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle [p]^\uparrow = \langle \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[?p] \rangle, \omega[p] \rangle, \omega[p] \rangle, \omega[p] \rangle [\text{CLEAN}] \\
&= \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[p] \rangle \\
(7) \quad &\langle \langle \langle \rangle, \iota \rangle, \omega[p \vee q] \rangle [p]^\uparrow = \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[p \vee q] \rangle, \omega[p \vee q] \rangle, \omega[p] \rangle, \omega[p \wedge \neg q] \rangle [\text{CLEAN}] = \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[p \vee q] \rangle, \omega[p] \rangle, \omega[p \wedge \neg q] \rangle \\
(8) \quad &\langle \langle \rangle, \iota[!(p \vee q)] \rangle [p]^\uparrow = \\
&= \langle \langle \langle \langle \langle \rangle, \iota[!(p \vee q)] \rangle, \omega[p \vee (\neg p \wedge q)] \rangle, \omega[p] \rangle, \omega[p] \rangle [\text{CLEAN}] \\
&= \langle \langle \langle \langle \rangle, \iota[!(p \vee q)] \rangle, \omega[p \vee (\neg p \wedge q)] \rangle, \omega[p] \rangle \\
(9) \quad &\langle \langle \langle \rangle, \iota \rangle, \omega[?!(p \vee q)] \rangle [p]^\uparrow = \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q) \vee \neg p] \rangle, \omega[p] \rangle, \omega[p] \rangle [\text{CLEAN}] \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q) \vee \neg p] \rangle, \omega[p] \rangle \\
(10) \quad &\langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle [?q]^\uparrow = \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[?p \vee ?q] \rangle, \omega[?q] \rangle, \omega[?q] \rangle [\text{CLEAN}] = \\
&= \langle \langle \langle \langle \langle \rangle, \iota \rangle, \omega[?p] \rangle, \omega[?p \vee ?q] \rangle, \omega[?q] \rangle
\end{aligned}$$

The one thing to note is that although a primary uptake followed by a secondary uptake may lead to different stacks, the cleaning operation may unify them. This happens in two pairs of our examples.

One pair is (5) and (6). The uptake of the assertion p in the initial common ground has the same effect as the uptake of p in a situation where $?p$ is the current issue.

The other pair is (1) and (3). Cleaning the results in (1), the uptake of the alternative question $?(p \vee q)$ in the initial common ground, and cleaning

the results in (3), the uptake if the hybrid disjunction $p \vee q$ in the initial common ground, are precisely the same.

We now turn to the absorption of reactions of the responder to the complete uptake of a stimulus as they were exemplified above.

6.7 Absorbing Reactions: Cancel, Accept, and Support

As long as the common ground stack and the stacks of the responder and the stimulator simultaneously undergo the uptake operations, the state on top of the common ground stack is a common ground for the states on top of the stacks of the stimulator and the responder. That deals with the first dialogue principle.

The second dialogue principle is to maintain consistency of one's state, and to publicly announce if for this reason a provisional update, an informative step in the uptake, is to be *cancelled*. After that, the responder can relate to the result of thematizing the stimulus, which allows for critical responses.

Even in happier circumstances, not much has been gained yet. The common ground stack has been expanded, but the common ground as such, the bottom of the stack, has remained untouched. Improving on the common ground requires a signal from the responder that she can *accept*, or perhaps even *supports*, the information provided by the stimulus, turning provisional updates into real ones.

We now turn to the operations on stacks that absorb the reactions of cancellation, acceptance and support. For the latter two, we define an auxiliary notion of restricting a state s to the information present in a state t .

Definition 35 (Restriction) $s[t] = \{\langle i, j \rangle \in s \mid \langle i, i \rangle \in t \& \langle j, j \rangle \in t\}$

Restriction eliminates all pairs of indices $\langle i, j \rangle$ in s , when i or j is not present in t . So, we update s with the information present in t .

The following fact tells us that restricting s to $t[\varphi]$ is the same as updating s with the *rHEME !* φ of φ , as long as $t[\varphi]$ is continuous to s . That holds for all states preceding $t[\varphi]$ in a stack.

Fact 26 (Restriction) $s[t[\varphi]] = s[!]\varphi$, if $t[\varphi]^* \subseteq s^*$

The operations to be performed when the need for cancellation, or the possi-

bility for acceptance or support are publicly signalled, are defined as follows. (The definition of support uses the notion of percolation, given in Definition 22, at the very beginning of Section 6.)

Definition 36 (Cancellation, Acceptance, and Support)

1. $\langle\langle\sigma, s\rangle, t\rangle[\perp] = \begin{cases} \langle\sigma, s\rangle & \text{if } s \text{ is not indifferent} \\ \langle\sigma, s\rangle[\perp] & \text{otherwise} \end{cases}$
2. $\langle\langle\sigma, s\rangle, t\rangle[\diamond] = \begin{cases} \langle\langle\sigma, s\rangle, t\rangle & \text{if } s = s[t] \\ \langle\sigma, s[t]\rangle & \text{if } s \neq s[t] \text{ and } s[t] \text{ is not indifferent} \\ \langle\sigma, s[t]\rangle[\diamond] & \text{otherwise} \end{cases}$
3. $\langle\langle\sigma, s\rangle, t\rangle[\top] = \begin{cases} \langle\langle\sigma, s\rangle, t\rangle & \text{if } s = s[t] \\ \langle\sigma^\triangleleft[t], s[t]\rangle & \text{if } s \neq s[t] \text{ and } s[t] \text{ is not indifferent} \\ \langle\sigma, s[t]\rangle[\top] & \text{otherwise} \end{cases}$

All three operations deconstruct a stack, popping a state from the stack in case of cancellation, and pulling information down the stack in case of acceptance and support, and the operations keep on doing so until they meet a state in which there remains an issue. Even after that, support keeps percolating information all the way down the stack.

In case of acceptance and support, there is also a ‘check’ at the beginning (the first clause in their definition), whether the reaction concerns a sentence which was informative in the state of the common ground relative to which the sentence was uttered. Restriction of s to the informative content of t will typically have no effect in case compared to s , t embodies no new information, i.e., if t just embodies a change in current issue as compared to s . It has to be a different issue, otherwise the cleaning operation would have eliminated one of the two.

Note that even if the sentence uttered was a question, because of the complicity implicature of block exclusion, it can have an informative effect on the common ground.

So, after one of these operation has been performed, there is always an issue to relate to for the next move in the dialogue, until we meet the unlikely situation where all possible issues have been resolved.

Let us look at our ten examples again. In the first line the result of the uptake of the sentence in the common ground as we found it above is given. In the next three lines you find the results of cancelling, acceptance, and

support, where in most cases the latter two have the same effect. This time, we will discuss our ten examples one by one.

Ten Examples of Uptake plus Absorption

The first example concerns an initial alternative question.

- (1) $\langle\langle\rangle, \iota\rangle [?(p \vee q)]^\uparrow = \langle\langle\langle\rangle, \iota\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle$
- $\langle\langle\rangle, \iota\rangle [?(p \vee q)]^\uparrow[\perp] = \langle\langle\langle\rangle, \iota\rangle, \omega[?(p \vee q)]\rangle$
- $\langle\langle\rangle, \iota\rangle [?(p \vee q)]^\uparrow[\Diamond] = \langle\langle\langle\rangle, \iota\rangle, \omega[?(p \vee q)][!(p \vee q)]\rangle = \langle\langle\langle\rangle, \iota\rangle, \omega[p \vee q]\rangle$
- $\langle\langle\rangle, \iota\rangle [?(p \vee q)]^\uparrow[\top] = \langle\langle\langle\rangle, \iota![p \vee q]\rangle, \omega[p \vee q]\rangle$

Although semantically alternative questions are not informative, pragmatically they function like hybrid disjunctions: they are semantically inquisitive, and informative by implicature. Hence, there can be reason to not be able or willing to accept the pragmatic informative content, and to publicly signal cancelling. The effect of that is that the alternative question as such is the current issue, and a response with $\neg p \wedge \neg q$ is now compliant.

In the case of acceptance, the default case, the implicated information that $!(p \vee q)$ is pulled one level down the stack, where it leads to the upgrade of the current issue from the alternative question to a hybrid disjunction, and only informative responses with p or q are now compliant. Note that this brings us to the situation that was the starting point of example (7), where as we saw, the compliant response with p implicates that $\neg q$. That is why we said that an alternative question suggests exclusive disjunction.

The difference between acceptance and support is that the implicated information is percolated down to the bottom of the stack, the common ground as such. This means that it only still counts as a common ground if the state of both responder and stimulator (are made to) support the implicated information.

In the case of acceptance there are still remote possibilities to move the implicature out of the way after all, and to happily continue the dialogue. In the case of support this is no longer possible, one should call upon *revision* of the common ground as such to restart the dialogue.

The second example concerns an initial disjunctive yes/no-question.

- (2) $\langle\langle\rangle, \iota\rangle [?!(p \vee q)]^\uparrow = \langle\langle\langle\rangle, \iota\rangle, \omega[?!(p \vee q)]\rangle$
- $\langle\langle\rangle, \iota\rangle [?!(p \vee q)]^\uparrow[\perp] = \langle\langle\rangle, \iota\rangle$

$$\begin{aligned} \langle\langle\rangle,\iota\rangle [?!(p \vee q)]^\uparrow[\diamond] &= \langle\langle\langle\rangle,\iota\rangle,\omega[?!(p \vee q)]\rangle = \\ &= \langle\langle\rangle,\iota\rangle [?!(p \vee q)]^\uparrow[\top] \end{aligned}$$

In this case cancellation has the more drastic effect of completely ignoring the issue, bringing us back to the initial situation, as if nothing had happened. Both acceptance and support have no effect at all. The positive and negative answer are compliant responses.

Note that the situation where acceptance of the question brings us, was the starting point of example (9), there we considered a non-compliant answer to this yes/no-question.

The third example concerns an initial hybrid disjunction:

$$\begin{aligned} (3) \quad \langle\langle\rangle,\iota\rangle [p \vee q]^\uparrow &= \langle\langle\langle\langle\rangle,\iota\rangle,\omega[?(p \vee q)]\rangle,\omega[p \vee q]\rangle \\ \langle\langle\rangle,\iota\rangle [p \vee q]^\uparrow[\perp] &= \langle\langle\langle\rangle,\iota\rangle,\omega[?(p \vee q)]\rangle \\ \langle\langle\rangle,\iota\rangle [p \vee q]^\uparrow[\diamond] &= \langle\langle\langle\rangle,\iota\rangle,\omega[p \vee q]\rangle \\ \langle\langle\rangle,\iota\rangle [p \vee q]^\uparrow[\top] &= \langle\langle\langle\rangle,\iota![p \vee q]\rangle,\omega[p \vee q]\rangle \end{aligned}$$

Since the full uptake of this hybrid disjunction is the same as that of the alternative question in (1), the three reactions we distinguish cannot fail to have the same effect as well.

Only now, in case of cancellation, the source of the information that the responder is apparently not able or willing to accept, belongs to the semantic content of the sentence as such. The response with $\neg p \wedge \neg q$, which is compliant after cancellation, denies the disjunction as such, not an implicature. But, from the pragmatic point of view, this makes little difference. If we don't protest against the implicature of the alternative question, the information it concerns gets the same status as the corresponding implication of the disjunction. And similarly in case of support.

The fourth example concerns an initial disjunctive assertion.

$$\begin{aligned} (4) \quad \langle\langle\rangle,\iota\rangle [!(p \vee q)]^\uparrow &= \langle\langle\langle\langle\rangle,\iota\rangle,\omega[?!(p \vee q)]\rangle,\omega[!(p \vee q)]\rangle \\ \langle\langle\rangle,\iota\rangle [!(p \vee q)]^\uparrow[\perp] &= \langle\langle\langle\rangle,\iota\rangle,\omega[?!(p \vee q)]\rangle \\ \langle\langle\rangle,\iota\rangle [!(p \vee q)]^\uparrow[\diamond] &= \langle\langle\rangle,\iota![p \vee q]\rangle = \\ &= \langle\langle\rangle,\iota\rangle [!(p \vee q)]^\uparrow[\top] \end{aligned}$$

In this case we are dealing with an initial plain assertion. After cancellation, denying or doubting the information asserted are compliant moves in the dialogue. If the initial assertion is accepted, the information it contains is pulled all the way down to the bottom of the stack, since the effect of $\omega[?!(p \vee q)][!p \vee q] = \omega[!(p \vee q)]$, which is indifferent. So the operation of acceptance moves down to the next stage, which is the initial state, which then gets updated with the information. And, hence, the effect is the same as that of absorbing support.

So, also in case of acceptance, to maintain a common ground, the responder must be prepared to really update her own state with the information provided. If she would not be willing to do so, she should cancel and express her doubt.²⁴

The effect of accepting an initial assertion is that we are back at what has remained of the initial big issue, making it possible to respond with, or let the stimulator continue with, any sentence, as long as it is not in open conflict with the assertion that has just been accepted. Example (8) started from this situation.

The fifth example concerns a simple case of an initial assertion.

$$\begin{aligned}
(5) \quad & \langle \langle \rangle, i \rangle [p]^{\uparrow} = \langle \langle \langle \rangle, i \rangle, \omega[?p] \rangle, \omega[p] \rangle \\
& \langle \langle \rangle, i \rangle [p]^{\uparrow}[\perp] = \langle \langle \langle \rangle, i \rangle, \omega[?p] \rangle \\
& \langle \langle \rangle, i \rangle [p]^{\uparrow}[\diamond] = \langle \langle \rangle, i[p] \rangle = \\
& = \langle \langle \rangle, i \rangle [p]^{\uparrow}[\top]
\end{aligned}$$

There is little to discuss about this simple example, it follows the pattern of the previous one, it is another instance of an initial assertion. The example is there to illustrate the differences between the next four examples, which all concern the uptake of the same atomic assertion p , but in different situations.

The sixth example concerns an assertion, where the current issue is the corresponding yes/no-question.

²⁴Elaborations of the system could enable other types of moves, which can keep the information in a hypothetical way. For example, a response of the form *Then* φ , which may point at a certain ‘difficulty’ with updating with the information provided by a stimulus. Perhaps, on hearing what consequences an update with her utterance has relative to the information of the responder, the stimulator may think twice about whether she wants to uphold her assertion. Perhaps φ embodies information that she would not be happy with either.

$$\begin{aligned}
(6) \quad & \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^{\uparrow} = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle, \omega[p]\rangle \\
& \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^{\uparrow}[\perp] = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle \\
& \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^{\uparrow}[\diamond] = \langle\langle \rangle, i[p]\rangle = \\
& = \langle\langle\langle \rangle, i\rangle, \omega[?p]\rangle [p]^{\uparrow}[\top]
\end{aligned}$$

In this example, the assertion p is the response to the apparently accepted corresponding yes/no-question. Cancelling brings us back to this question, and makes the opposite answer $\neg p$ compliant. Accept and support have the same effect, and the outcome is the same as in the previous example where p was an initial assertion.

The seventh example concerns an assertion, where the current state of the common ground embodies a hybrid disjunction.

$$\begin{aligned}
(7) \quad & \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^{\uparrow} = \langle\langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle, \omega[p \wedge \neg q]\rangle \\
& \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^{\uparrow}[\perp] = \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle \\
& \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^{\uparrow}[\diamond] = \langle\langle \rangle, i[p \wedge \neg q]\rangle = \\
& = \langle\langle\langle \rangle, i\rangle, \omega[p \vee q]\rangle [p]^{\uparrow}[\top]
\end{aligned}$$

As we saw above, the situation in which p is asserted is the one that results from accepting the alternative question $?(p \vee q)$ or the hybrid disjunction $p \vee q$. And p implicates the exclusion of the other alternative q .

Cancellation keeps popping the stack until by doing so it meets a state which is not indifferent, which is the state which embodies the hybrid disjunction. After this, the responder can come up with alternative response q , which, since it will implicate $\neg p$, also denies the effect of the uptake of p .²⁵

Acceptance and support have the same effect of leading to the update of the initial common ground state with $p \wedge \neg q$. So, the combined effect of a stimulus which is the alternative question $?(p \vee q)$, or the hybrid disjunction $p \vee q$, and a compliant response with p , exposes the effect of exclusive disjunction. But it is not disjunction as such that is exclusive, it is the choice of one of the disjuncts that excludes the other one. Stimulator and responder together bring about the exclusion effects.

The eighth example concerns an assertion, where the current issue in the common ground is a disjunctive yes/no-question.

²⁵See footnote 21.

$$\begin{aligned}
(8) \quad & \langle\langle \rangle, \iota[!(p \vee q)] \rangle [p]^\uparrow = \langle\langle\langle \rangle, \iota[!(p \vee q)] \rangle, \omega[p \vee (\neg p \wedge q)] \rangle, \omega[p] \rangle \\
& \langle\langle \rangle, \iota[!(p \vee q)] \rangle [p]^\uparrow[\perp] = \langle\langle\langle \rangle, \iota[!(p \vee q)] \rangle, \omega[p \vee (\neg p \wedge q)] \rangle \\
& \langle\langle \rangle, \iota[!(p \vee q)] \rangle [p]^\uparrow[\diamond] = \langle\langle \rangle, \iota[p] \rangle = \\
& = \langle\langle \rangle, \iota[!(p \vee q)] \rangle [p]^\uparrow[\top]
\end{aligned}$$

In this example p is asserted in the situation that results from the acceptance of the assertive disjunction $!p \vee q$, example (4) discussed above. Note, first of all, that p is just as compliant in this case as after a hybrid disjunction or alternative question.

But the effects are different. In case of acceptance and support, the initial common ground gets just updated with p as such, as in the examples (5) and (6). So, the exclusion effect is lacking. After accepting the response with p to $!p \vee q$, the dialogue can be happily and compliantly continued with q . Something that was possible in (7) only in case of first cancelling p .

So, the basic difference between assertive and hybrid disjunction is that the latter has, and the former does not have, exclusion effects in the dialogue after a response with one of the disjuncts.

Cancellation in this case just pops the assertion p from the top of the stack, and brings us to the state which embodies the hybrid disjunction $p \vee (\neg p \wedge q)$, where now $\neg p \wedge q$, is compliant. But note that the assertion of $\neg p$ would be sufficient to express this in the situation at hand, where the information that $!p \vee q$ is already accepted.

The ninth example concerns an assertion which is not compliant to the current issue in the common ground.

$$\begin{aligned}
(9) \quad & \langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle [p]^\uparrow = \langle\langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q) \vee \neg p] \rangle, \omega[p] \rangle \\
& \langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle [p]^\uparrow[\perp] = \langle\langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle, \omega[!(p \vee q) \vee \neg p] \rangle \\
& \langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle [p]^\uparrow[\diamond] = \langle\langle \rangle, \iota[p] \rangle = \\
& = \langle\langle\langle \rangle, \iota, \omega[?!(p \vee q)] \rangle [p]^\uparrow[\top]
\end{aligned}$$

In this case we are dealing with a non-compliant assertion of p in the situation where the yes/no-question $!p \vee q$ is accepted as the current issue. Still, despite non-compliance, the effect of accepting or supporting p in this case are the same as in the compliant situations we met in (5), (6), and (8). The information p provides is added to the common ground.

Cancellation in this case also just pops the assertion p from the top of the stack, but now brings us to the state which embodies the *question* $!(p \vee q) \vee \neg p$ as the current issue. Both the negation $\neg p$ of the response, but also a rejoinder with the answer $!(p \vee q)$, which is weaker than the proposed answer p , are now compliant. So, this rejoinder just tells us that p ‘goes too far’, without excluding p .

The tenth and last example concerns a question which is not compliant to the current issue in the common ground.

$$\begin{aligned}
(10) \quad & \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle [?q]^{\uparrow} = \langle \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle, \omega[p \vee ?q] \rangle, \omega[?q] \rangle \\
& \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle [?q]^{\uparrow}[\perp] = \langle \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle, \omega[p \vee ?q] \rangle \\
& \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle [?q]^{\uparrow}[\diamond] = \langle \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle, \omega[p \vee ?q] \rangle, \omega[?q] \rangle = \\
& = \langle \langle \langle \rangle, i \rangle, \omega[p] \rangle [?q]^{\uparrow}[\top]
\end{aligned}$$

This example also deals with a non-compliant response, the question $?q$ which is not a subquestion of the current issue $?p$. Although with compliant questions the effect of to assume and to thematize are identical, this is not the case here. To assume leads to turning the new question $?q$ into the current issue. Compliant responses now, are answers to this question.

Of course, from the fact that $?q$ is non-compliant *locally* in the dialogue, we need not conclude that responder is non-cooperative. It is easy to imagine a situation where she has no direct answer to $?p$, but where she has the information that if q then p . Unfortunately, she does not know whether q . Perhaps the stimulator does.

So, it is very well imaginable that *globally* the responder is compliant. And in the fortunate circumstances that the stimulator gives the answer q to her question, she can return to the original question p , with the indirect answer *Then p*.²⁶ Which might, or might not, be acceptable to the stimulator. But if it is, her original question $?p$ is resolved.

Most likely, the state of the stimulator will not already have supported the indirect answer, because given that she could provide a positive answer

²⁶This is another story, but the *Then* in front of the response is rather essential. It signals that the responder draws the conclusion that p on the basis of combining her own information and information provided by the stimulator. Combining information from different sources is a tricky affair. Natural language is aware of that, and forces to signal a warning that this has happened. That is the discourse function of *Then* in the response. (See Groenendijk, Stokhof & Veltman (1997)). To model this formally, we could add to our stack approach a dialogue version of the *conclude*-operation, as it is given in Kaufmann (2000), in dealing with the use of stacks to model modal subordination.

to $?q$, this would have meant that $?p$ was not inquisitive in her own state, which would have made it a strange thing to ask.

Cancellation was basically motivated to signal that the sentence uttered was inconsistent with the state of the responder, which motivates popping the common ground stack in order to make sure that a common ground is maintained, but the operation of cancellation as defined above is not restricted to this type of situation. As we see in example (10), it also allows for the cancellation of an issue, in this case $?q$, moving to the next issue ‘below’ it in the stack, in this case the weaker question $?p \vee ?q$. It makes sense to cancel $?q$ and move to $?p \vee ?q$, in case you lack an answer to q .

In the situation as sketched, where $?p$ was her own question, to which presumably she has no answer, and where she cancelled the counter question $?q$ that came in response, presumably because she has no answer to that one either, the stimulator will be unable to provide one of the four complete answer to $?p \vee ?q$. But there are many partial answers to this disjunction of questions, one of which she may be able to provide on the basis of her information, such as $\neg q \vee p$. (I.e., $q \rightarrow p$, if the language contained implication).

If the state of the responder can accept this as a new piece of information, it could very well be possible for her to now fully answer what has remained of the disjunction of questions $?p \vee ?q$, with *Then p*, thereby also resolving the original question $?p$ posed by the stimulator. The responder would be in the right position to do this if her state supports the information $p \vee q$.

This concludes the discussion of our ten examples. I end the discussion of the dialogue management with two remarks.

First, I do not want to dwell upon this right now, but it is an interesting issue precisely in which ways the three reactions we distinguished are signalled in natural language dialogues. For cancelling an informative uptake, *No* would be a good candidate. But in the case of cancelling a question we just discussed, something like *Well* might be a way to do it. Also a non-compliant response such as $p \wedge q$ to $?p \vee q$, or not accepting its implicature that $\neg p \wedge \neg q$ is excluded, are naturally preceded by *Well*, and followed or preceded by *Actually*. One way to signal support is to start with *Yeah*. Acceptance being the default case, there seems no real need for special linguistic tools. Just the absence of *Yeah* and *No*, in combination with a happy continuation of the dialogue may suffice.

Finally, it is important to note that it is not essential in order to maintain a common ground that the current state of the stimulator actually fully

supported the information, implicatures and suggestions, signalled by her own utterance. The only thing that matters in order to maintain a common ground, is that *after* the responder has signalled that she accepts them, upon which the common ground is inevitably made to accept them, the stimulator performs the same operation signalled by the responder on her own stack as well.

This is particularly relevant for implicatures and suggestions made by the stimulator, which certainly need not concern information strongly supported by her state. But they are pieces of information that the stimulator should be willing to accept once the responder has signalled that she is willing to do so.

Implicatures and suggestions become effective in the common ground by interaction between stimulator and responder. This also means that there is no reason for the dialogue not to proceed happily when suggestions and implicatures are not accepted. It need be no signal of really conflicting information.

This gives a different look upon the Gricean Maxim of Quality. In our dialogue pragmatics, it can be a ‘softer’ notion, then it is usually taken to be, and guarding Quality is much more of a ‘collective responsibility’ for stimulator and responder than a personal responsibility for each of them separately. And, in the end, Quality need not be stated as a separate rule (we didn’t), it more or less follows from the first dialogue principle: maintain a common ground. Although it may go unnoticed, a lie ruins the common ground. Perhaps this is the logical explanation for why compulsive liars tend to come to firmly believe in their own lies.

7 Loose End

The loose end is formed by explicitly non-exclusive disjunctions and alternative questions, like $p \vee q \vee (p \wedge q)$ and $?(p \vee q \vee (\neg p \wedge \neg q))$. They are semantically fully equivalent to their non-exclusive brethren. One could use Manner to deal with them. They obviously involve more processing effort. Hence, there must be a reason for asking this extra effort, etc.

One can take this to invite an alternative interpretation where, e.g., $p \vee q \vee (p \wedge q)$ is taken to mean $\text{ONLY}(p) \vee \text{ONLY}(q) \vee (p \wedge q)$. The good news is that our notion of alternative exclusion gives us the tools we need to implement the required interpretation of **ONLY**.

A more radical solution, not requiring to actually introduce **ONLY** as an operator in the language, is to declare that disjunction is pragmatically

ambiguous between two ways of turning a relation on states into a euclidean relation. One is standard euclidean closure where you extend the relation by adding pairs $\langle j, k \rangle$ in case $\langle i, j \rangle$ and $\langle i, k \rangle$, but not $\langle j, k \rangle$. Leading from $p \vee q$ to $!(p \vee q)$.

The other way might be called euclidean disclosure (or perhaps the other way around), where you remove pairs $\langle j, k \rangle$ in case $\langle i, j \rangle$ and $\langle i, k \rangle$, but not $\langle j, k \rangle$. Leading from $p \vee q$ to $(p \wedge \neg q) \vee (q \wedge \neg p)$.

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All these operations on stacks will always preserve continuity of the stack. The inquisitive dialogue pragmatics prefers homogeneous stacks. These two properties are defined using the notions of the relations of continuity and homogeneity between states, defined above.

Definition 37 (Continuous and Homogeneous Stacks)

We stipulate: $\langle\langle \cdot, \cdot \rangle, s \rangle$ is homogeneous (and hence continuous)

1. $\langle\langle \sigma, s \rangle, t \rangle$ is *continuous* iff $\langle \sigma, s \rangle$ is continuous and $t^* \subseteq s^*$.
2. $\langle\langle \sigma, s \rangle, t \rangle$ is *homogeneous* iff $\langle \sigma, s \rangle$ is homogeneous and $t \succeq s$.

What is to be expected, and maybe even required, is that if $\langle \sigma, s \rangle$ is homogeneous, and φ is homogeneous to s , then the effect of the uptake of φ in the stack, followed by one of the three absorption operations, preserves homogeneity of the stack.

Inquisitive pragmatics not only prefers homogeneity, but also relatedness, and hence the combination of the two: compliance. Relatedness of a stack is defined using the notion of relatedness between states. Compliance of stacks combines relatedness and homogeneity.

Definition 38 (Related and Compliant Stacks)

We stipulate: $\langle\langle \cdot, \cdot \rangle, s \rangle$ is related (and hence compliant).

1. $\langle\langle \sigma, s \rangle, t \rangle$ is *related* iff $\langle \sigma, s \rangle$ is related and $t \propto s$.
2. σ is *compliant* iff σ is homogeneous and σ is related

Again, what is to be expected, and maybe even required, is that if $\langle \sigma, s \rangle$ is related, and φ is related to s , then the effect of the uptake of φ in the stack, followed by one of the three absorption operations, preserves relatedness of the stack.

Although in the subsections to follow, in which we define the operations on stacks, we will pay some attention to issues pertaining to the preservation of homogeneity and relatedness of a stack, we postpone a fuller discussion to the next main section of the paper.

Although thematization always preserves homogeneity, the primary uptake of a sentence does not. Our example (10) is a case in point.

$$(10) \quad \langle\langle\langle \ , i\rangle, \omega[?p]\rangle \ [?q]^{\uparrow\uparrow^1} = \langle\langle\langle\langle \ , i\rangle, \omega[?p]\rangle, \omega[?p \vee ?q]\rangle, \omega[?q]\rangle$$

Since $\omega[?q] \not\geq \omega[?p \vee ?q]$, the stack we obtain from $\langle\langle\langle\langle\langle\langle, i\rangle, \omega[?p]\rangle [?q]^{\uparrow 1}$ is not homogeneous.

We will prove in the next main section that what does hold in general is:

Fact 27 (Homogeneity of Primary Uptake)

If $\langle \sigma, s \rangle$ is homogeneous, and φ is homogeneous to s , then $\langle \sigma, s \rangle[\varphi]^{ \uparrow^1}$ is homogeneous.

A Preservation of Homogeneity and Relatedness

This section is still under construction, and concerns a meta-logical issue which I take to be important in establishing whether the dialogue management rules are logically correct. Another issue is of course whether they are empirically interesting. That I leave to others to decide.

I have the logical intuition (it is no more than that) that what determines the correctness of the system of dialogue management rules, is whether they preserve the properties of homogeneity, relatedness, and compliance of stacks, that were defined above.

By preservation I mean that if the stack we start from has one of these properties, and the sentence uttered stands in the corresponding relation to the current state of the stack relative to which the sentence was uttered, then the stack that results from performing the uptake of the sentence followed by either one of the three absorption operations, has the property in question as well.

One might expect that the latter should not necessarily be immediately taken into account, that we could study the issue separately for the uptake operations and the absorption operations. But apart from the fact that I *know* already that the full uptake in isolation will *not* always preserve relatedness, whereas it is likely that in combination with the absorption operations this can be overcome, it is also in line with the whole story that we should look at the combination of the two.

The bottom line of the story is that the full interpretation of dialogue moves is established by the *interaction* of the participants in the dialogue, and, of course, the reactions of cancellation, acceptance and support to the uptake of the sentence are an integral part of this interactive process. So, we shouldn't be too surprised that in particular relatedness of the common ground stack is cooperatively established.

One more remark before we turn to the logical stuff. We concentrate here on the preservation of homogeneity, relatedness and compliance. This may give the impression as if to preserve these properties of the common ground stack is essential to a happy dialogue as such. That is certainly not the case. Compliance is pragmatically preferred, hence the possibility to derive compliance implicatures, but everything works equally fine when for whatever good reason non-compliance is called upon. That is one of the virtues of the system as a whole.

At the heart of the motivation for the whole stack approach is to smoothly allow for critical moves in the dialogue, denying or doubting what has been said. In turn, the ultimate motivation for that is to maintain a common ground.²⁷

Now the logical stuff. From Fact 15, $\forall s: s \cup s^*[\varphi] \preceq s$, it follows immediately that no matter what:

Fact 28 (Thematization Preserves Homogeneity)

If $\langle \sigma, s \rangle$ is homogeneous, then $\langle \sigma, s \rangle [\varphi]^?$ is homogeneous

We have seen in the examples we gave that relatedness is not guaranteed to obtain automatically. However, one would expect the following to hold. [It has been shown in the meantime by Sophia Arnoult that this conjecture is false!]

Question 1 (Thematization and Relatedness)

If $\langle \sigma, s \rangle$ is related and φ is related to s , then $\langle \sigma, s \rangle [\varphi]^?$ is related

A structurally similar fact, but concerning the homogeneity of primary uptake, we already noted when we introduced the notion of primary uptake.

Fact 29 (Homogeneity of Primary Uptake)

²⁷Actually, but that is another story, enforcing compliance on others, as parents tend to do with their children, not to speak of what absolute regimes enforce on their subjects, is predicted by the theory to be a bad idea. It does not achieve what it intends to achieve: that authority determines a common ground. By forbidding critique it achieves the opposite: the lack of a common ground, where the next step is punishment or worse, if authority detects it, as the means to restore the dictated common ground.

If $\langle \sigma, s \rangle$ is homogeneous, and φ is homogeneous to s , then $\langle \sigma, s \rangle[\varphi]^{\uparrow^1}$ is homogeneous.

The proof runs as follows. We have seen that the following holds.

$$\langle \sigma, s \rangle[\varphi]^{\uparrow^1} = \langle \langle \langle \sigma, s \rangle, s \cup s^*[\varphi] \rangle, s^*[\varphi] \rangle$$

Given that $\langle \sigma, s \rangle$ is homogeneous, we know from the definition of homogeneity of stacks, that $\langle \sigma, s \rangle[\varphi]^{\uparrow^1}$ is homogeneous if $s \cup s^*[\varphi] \succeq s$ and $s^*[\varphi] \succeq s \cup s^*[\varphi]$. The former corresponds to homogeneity of thematization, which we have already seen to hold. Only the latter remains to be shown, on the assumption that φ is homogeneous to s , i.e., that $s[\varphi] \succeq s$.

Continuity, i.e., that $(s^*[\varphi])^* \subseteq (s \cup s^*[\varphi])^*$, certainly holds. What remains to be shown is:

$$\forall i, j: \langle i, i \rangle \in s^*[\varphi] \& \langle j, j \rangle \in s^*[\varphi] \& \langle i, j \rangle \notin s^*[\varphi] \Rightarrow \langle i, j \rangle \notin s \cup s^*[\varphi]$$

Since $s \cup s^*[\varphi] = s \cup s^*[\varphi] \cup s^*[\neg\varphi]$, we can split this in three cases.

$$\forall i, j: \langle i, i \rangle \in s^*[\varphi] \& \langle j, j \rangle \in s^*[\varphi] \& \langle i, j \rangle \notin s^*[\varphi] \Rightarrow \langle i, j \rangle \notin s^*[\varphi] \quad (\text{a})$$

This is trivially true.

$$\forall i, j: \langle i, i \rangle \in s^*[\varphi] \& \langle j, j \rangle \in s^*[\varphi] \& \langle i, j \rangle \notin s^*[\varphi] \Rightarrow \langle i, j \rangle \notin s^*[\neg\varphi] \quad (\text{b})$$

This is simple as well, since if $\langle i, i \rangle \in s^*[\varphi] \& \langle j, j \rangle \in s^*[\varphi]$, then $\langle i, i \rangle \notin s^*[\neg\varphi] \& \langle j, j \rangle \notin s^*[\neg\varphi]$, and hence $\langle i, j \rangle \notin s^*[\neg\varphi]$.

What remains to be shown is that (c) holds, on the assumption that $s[\varphi] \succeq s$.

$$\forall i, j: \langle i, i \rangle \in s^*[\varphi] \& \langle j, j \rangle \in s^*[\varphi] \& \langle i, j \rangle \notin s^*[\varphi] \Rightarrow \langle i, j \rangle \notin s \quad (\text{c})$$

The assumption that $s[\varphi] \succeq s$ guarantees the following:

$$\forall i, j: \langle i, i \rangle \in s[\varphi] \& \langle j, j \rangle \in s[\varphi] \& \langle i, j \rangle \notin s[\varphi] \Rightarrow \langle i, j \rangle \notin s$$

Since $\langle i, i \rangle \in s[\varphi]$ iff $\langle i, i \rangle \in s^*[\varphi]$, this boils down to showing that:

$$\forall i, j: \text{if } \langle i, j \rangle \notin s[\varphi] \Rightarrow \langle i, j \rangle \notin s, \text{ then } \langle i, j \rangle \notin s^*[\varphi] \Rightarrow \langle i, j \rangle \notin s$$

This immediately follows from the fact that $s \subseteq s[\varphi] \subseteq s^*[\varphi]$. End of proof.

Of course, the next issue is whether we can obtain the same preservation result with respect to relatedness. On top of the first question, assuming a positive answer to it [but the answer is negative, see above], we would have to answer the second one:

Question 2 (Primary Uptake Preserves Relatedness)

If $\langle \sigma, s \rangle$ is related and φ is related to s , then $\langle \sigma, s \rangle [\varphi]^\uparrow$ is related

Probably the first thing to address, because it seems the easiest bit, is the following. Is this the case?

$$\forall s: s^*[\varphi] \propto s \cup s^*[\varphi]$$

The conjecture would be that whereas thematization always preserves homogeneity, and to assume preserves homogeneity in case the sentence in question is homogeneous to the state on top of the stack, relative to which the sentence is uttered, we get the mirror picture for relatedness: after thematizing φ to assume φ is always related, but relatedness of thematizing requires that the thematized sentence as such is related to the state on top of the stack, relative to which the sentence is uttered.

Of course, a positive answer to question 1 and question 2, will answer the question whether if φ is compliant to s , and $\langle \sigma, s \rangle$ is compliant, the primary uptake of φ in $\langle \sigma, s \rangle$ is guaranteed to preserve compliance of the stack.

Let's turn to the secundary uptake. We made the following remark about alternative exclusion:

- It cannot fail to be the case that the result u of alternative exclusion is homogeneous to the old top t , but typically when alternative exclusion has effect, when $u \neq t$, u is *not related* to t . Alternative exclusion has the effect of eliminating *proper parts of possibilities* in t .

This is clear enough. If the utterance of a sentence involves alternative exclusion the secundary uptake is bound to preserve homogeneity, but is equally bound *not* to preserve relatedness. So, as far as the full uptake of a sentence is concerned, compliance preservation will not hold.

But here the story gets interesting, because if we look beyond the uptake of a sentence, and consider the absorption of the reactions to the sentence as well, then it is pretty clear (but remains to be proved) that all three operations of cancel, accept and support, will *restore relatedness* if it were broken by the uptake of alternative exclusion.

The situation is much easier for block exclusion:

- It cannot fail to be the case that if t is the result of block exclusion in s , then t is both homogeneous and related to s . If block exclusion has any effect, it eliminates a whole possibility.

About the last operation that finishes off the full uptake, cleaning a stack, it is obvious that it preserves relatedness and homogeneity.

So, all in all, concerning the full uptake of a sentence we seem to have established the following:

Fact 30 (Homogeneity of Uptake)

If $\langle\sigma, s\rangle$ is homogeneous, and φ is homogeneous to s , then $\langle\sigma, s\rangle[\varphi]^\uparrow$ is homogeneous.

I trust (but it remains to be proved [See above!]) that this will not be disturbed if we add an absorption operation. But the real interesting and open case is captured by the following question:

Question 3 (Relatedness of Uptake)

If $\langle\sigma, s\rangle$ is related, and φ is related to s , then $\langle\sigma, s\rangle[\varphi]^\uparrow[\perp]$, $\langle\sigma, s\rangle[\varphi]^\uparrow[\Diamond]$, and $\langle\sigma, s\rangle[\varphi]^\uparrow[\top]$ are related as well.

B Dialogue Management Rules

In this appendix I have collected all the dialogue management rules.

Definition 1 (Stacks) The set of *stacks* is the smallest set such that:

1. $\langle \rangle$ is a stack
2. If s is a state, and σ is a stack, then $\langle \sigma, s \rangle$ is a stack

Definition 2 (Percolation) Let $[.]$ be an operation on states.

1. $\langle \rangle^\triangleleft[.] = \langle \rangle$
2. $\langle \sigma, s \rangle^\triangleleft[.] = \langle \sigma^\triangleleft[.], s[.] \rangle$

Definition 3 (Thematize) $\langle \sigma, s \rangle[\varphi]^? = \langle \langle \sigma, s \rangle, s \cup s^*[? \varphi] \rangle$

Definition 4 (Assume) $\langle \sigma, s \rangle[\varphi]^\uparrow = \langle \langle \sigma, s \rangle, s[\varphi] \rangle$

Definition 5 (Alternatives) ρ is an *alternative* in $\langle \langle \langle \sigma, s \rangle, t \rangle, u \rangle$ iff

1. $u \propto s$ & $u \succeq s$
2. ρ is a possibility in t and ρ is not a possibility in u

Definition 6 (Alternative Exclusion)

Let A be the union of the alternatives in $\langle \langle \sigma, s \rangle, t \rangle$.

$$\langle \langle \sigma, s \rangle, t \rangle[\text{EXCLA}] = \langle \langle \langle \sigma, s \rangle, t \rangle, \{ \langle i, j \rangle \in t \mid i, j \notin A \} \rangle$$

Definition 7 (Euclidean Closure) Let s be a state.

The *euclidean closure* s^\div of s , is the smallest set s' such that $s \subseteq s'$, and $\forall i, j, k: \langle i, j \rangle \in s' \& \langle i, k \rangle \in s' \Rightarrow \langle j, k \rangle \in s'$

Definition 8 (Blocks) ρ is a *block* in s iff

1. $s \neq s^\div \wedge s^\div \neq s^*$
2. ρ is a possibility in s and ρ is a possibility s^\div

Definition 9 (Block Exclusion) Let B be the union of the blocks in s .

$$\langle\langle\sigma,s\rangle[\text{EXCLB}] = \langle\langle\sigma,s\rangle, \{\langle i,j \rangle \in s \mid i,j \notin B\}\rangle$$

Definition 10 (Exclusion)

$$\langle\langle\sigma,s\rangle,t\rangle[\text{EXCL}] = \begin{cases} \langle\langle\sigma,s\rangle,t\rangle[\text{EXCLA}] & \text{if } s^* \neq t^* \\ \langle\langle\sigma,s\rangle,t\rangle[\text{EXCLB}] & \text{if } s^* = t^* \end{cases}$$

Definition 11 (Cleaning)

1. $\langle\langle\ ,s\rangle[\text{CLEAN}] = \langle\langle\ ,s\rangle$
2. $\langle\langle\sigma,s\rangle,t\rangle[\text{CLEAN}] = \begin{cases} \langle\sigma,s\rangle[\text{CLEAN}] & \text{if } s = t \\ \langle\langle\sigma,s\rangle[\text{CLEAN}],t\rangle & \text{otherwise} \end{cases}$

Definition 12 (Uptake) $\langle\sigma,s\rangle[\varphi]^\uparrow = \langle\sigma,s\rangle[\varphi]^?[\varphi]^\uparrow[\text{EXCL}][\text{CLEAN}]$

Definition 13 (Restriction) $s[t] = \{\langle i,j \rangle \in s \mid \langle i,i \rangle \in t \wedge \langle j,j \rangle \in t\}$

Definition 14 (Cancellation, Acceptance, and Support)

1. $\langle\langle\sigma,s\rangle,t\rangle[\perp] = \begin{cases} \langle\sigma,s\rangle & \text{if } s \text{ is not indifferent} \\ \langle\sigma,s\rangle[\perp] & \text{otherwise} \end{cases}$
2. $\langle\langle\sigma,s\rangle,t\rangle[\diamond] = \begin{cases} \langle\langle\sigma,s\rangle,t\rangle & \text{if } s = s[t] \\ \langle\sigma,s[t]\rangle & \text{if } s \neq s[t] \text{ and } s[t] \text{ is not indifferent} \\ \langle\sigma,s[t]\rangle[\diamond] & \text{otherwise} \end{cases}$
3. $\langle\langle\sigma,s\rangle,t\rangle[\top] = \begin{cases} \langle\langle\sigma,s\rangle,t\rangle & \text{if } s = s[t] \\ \langle\sigma^\triangleleft[t],s[t]\rangle & \text{if } s \neq s[t] \text{ and } s[t] \text{ is not indifferent} \\ \langle\sigma,s[t]\rangle[\top] & \text{otherwise} \end{cases}$