

## The logic of interrogation

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## Logic and conversation

- ⑥ Standard logic deals with reasoning, entailment
- ⑥ Using standard logic, linguistic semantics deals with phenomena related to entailment
- ⑥ Information exchange more basic use of language than reasoning
- ⑥ Try to make cooperative information exchange a basic notion of logic

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## Linguistic aims

- ⑥ Explain linguistic phenomena using the new logical notions
- ⑥ We will give some illustrations
- ⑥ By-product: a better notion of linguistic answerhood (within a partition semantics of questions)

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## Overview

- ⑥ The game of interrogation
- ⑥ A query language
- ⑥ Semantics for the language
- ⑥ Logical notions to arbitrate the game
- ⑥ Answerhood
- ⑥ Illustration

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## Game of Interrogation

- ⑥ Two players: the interrogator and the witness
- ⑥ The interrogator may only raise issues by asking the witness non-superfluous questions
- ⑥ The witness may only make credible (Quality), non-redundant (Quantity) statements which exclusively address the issues raised by the interrogator (Relation)

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## Logic of interrogation

- ⑥ Define logical notions that arbitrate whether an interrogation proceeds in accordance with the rules
- ⑥ Like standard logic defines the notion of entailment to arbitrate whether an argumentation is in accordance with the rules of valid reasoning

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## Query Language

differs a bit from paper

- ④ Let PL be a language of predicate logic.
- ④ If  $\varphi$  is a sentence of PL, then  $!\varphi$  is a sentence of QL
- ④ If  $\varphi$  is a formula of PL, then  $? \varphi$  is a sentence of QL
- ④ The query operator binds all free variables in  $\varphi$

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## Examples

- ④ Interrogatives ask for the specification of the denotation of an n-place relation ( $n \geq 0$ )
  - ④  $? \exists x P x$
  - ④  $? P x$
  - ④  $? x = a$
  - ④  $? R x y$

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## Proceedings of an interrogation

- ④ Given the strict division of roles, the proceedings of an interrogation can be presented by a sequence of sentences  $\varphi_1; \dots; \varphi_n$  from QL
- ④ We don't have to indicate who said what

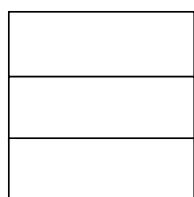
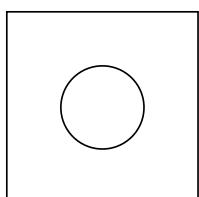
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## Denotational semantics

- ④ Standard truth definition for PL  
 $\| \varphi \|_{w,g} \in \{1,0\}$
- ④ Interpretation for QL
- ④  $\| !\varphi \|_w = \| \varphi \|_{w,g}$
- ④  $\| ?\varphi \|_w = \{v \in W \mid \forall g : \| \varphi \|_{w,g} = \| \varphi \|_{v,g}\}$
- ④ Partition semantics for interrogatives

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## Proposition - Question



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## Example

- ④  $\| ?P x \|_w$  is the set of worlds where the denotation of  $P$  is the same as in  $w$
- ④  $\| ?P x \|_w$  is a proposition which exhaustively specifies which objects have the property  $P$ 
  - ④ So, what you get is the true and complete answer in  $w$

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## Update semantics

- ⑥ In terms of the denotational semantics we define an update semantics for QL
- ⑦ We define the notion  $C[\varphi]$ , the effect of updating a context  $C$  with an indicative or an interrogative sentence  $\varphi$
- ⑧ A context will consist of data (provided by the witness) and issues (raised by the interrogator)

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## Data and Issues

- ⑨ If we would only consider data, a context could be a subset of the set of possible worlds
- ⑩  $C[!\varphi] \subseteq C$
- ⑪ Interrogatives provide no data, they may only raise issues
- ⑫ We model issues by structuring the context

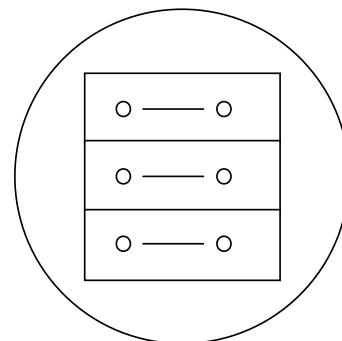
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## Structured contexts

- ⑬ A context  $C$  is a symmetric and transitive relation on the set of possible worlds  $W$
- ⑭ A context  $C$  is an equivalence relation on a subset of  $W$
- ⑮ If two worlds  $w$  and  $v$  are related in  $C$ ,  $\langle w, v \rangle \in C$ , the difference between  $w$  and  $v$  is not an issue
- ⑯ Notation: by  $w \in C$  we mean  $\langle w, w \rangle \in C$

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## Picture of context



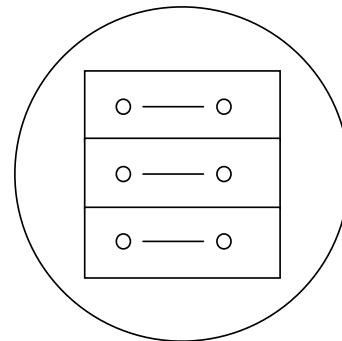
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## Updating contexts

- ⑰  $C[!\varphi] = \{ \langle w, v \rangle \in C \mid ||!|\varphi||_w = ||!|\varphi||_v = 1 \}$
- ⑱  $C[?\varphi] = \{ \langle w, v \rangle \in C \mid ||?|\varphi||_w = ||?|\varphi||_v \}$
- ⑲ For  $\tau = \varphi_1; \dots; \varphi_n$ ,  $C[\tau] = C[\varphi_1] \dots [\varphi_n]$

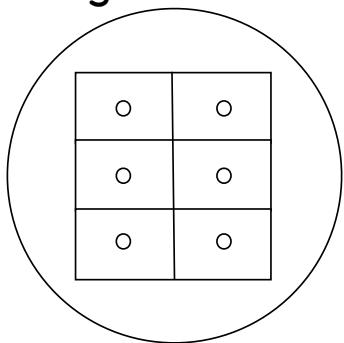
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## Picture of context



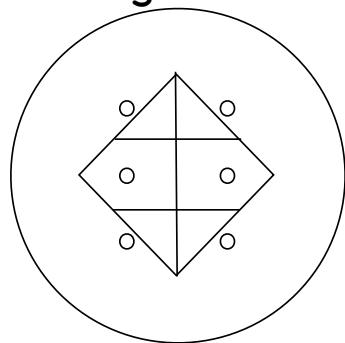
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## Adding an issue



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## Adding data



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## Consistency

- ④  $\varphi$  is consistent with  $\tau$  iff  $\exists C: C[\tau][\varphi] \neq \emptyset$
- ④ Only indicatives can be inconsistent with the context
- ④ Consistency is the logical notion used to arbitrate credibility of the witness
- ④ The witness is judged credible as long as he doesn't contradict himself

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## Informativeness

- ④  $\tau$  entails  $\varphi$  iff  $\forall C: C[\tau] = C[\tau][\varphi]$
- ④  $\varphi$  informative after  $\tau$  iff  $\tau$  does not entail  $\varphi$
- ④ Both indicatives and interrogatives can be uninformative
- ④ Informativeness is the logical notion used to arbitrate whether statements are non-redundant, and questions are not superfluous

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## Examples entailment

- ④  $?P_x$  entails  $?P_a$  and  $? \exists x P_x$
- ④  $! \forall x (P_x \leftrightarrow x=a)$  entails  $?P_x$   
Corresponds to 'complete answerhood' in partition semantics
- ④ Note: allows for over-informative answers
- ④  $? \varphi$  entails  $! \psi$  iff  $! \psi$  is a tautology (or a presupposition of  $? \varphi$ )

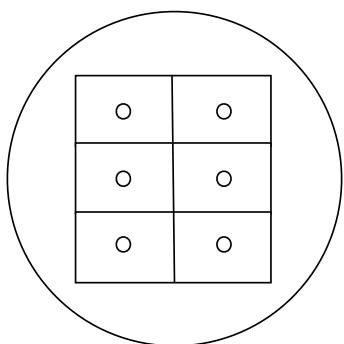
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## Licensing

- ④  $\tau$  licenses  $\varphi$  iff  $\forall C, w, v: <w, v> \in C[\tau] \& w \notin C[\tau][\varphi] \Rightarrow v \notin C[\tau][\varphi]$
- ④ If  $\varphi$  eliminates a world from the context, it should eliminate the whole alternative to which that world belongs
- ④ Licensing is the logical notion used to arbitrate whether the witness exclusively addresses the issues raised by the interrogator

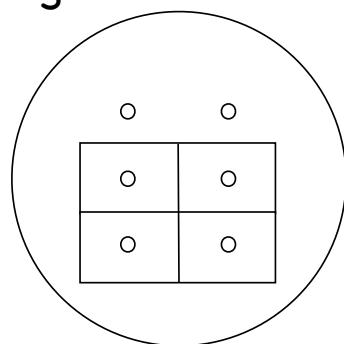
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## Picture of context



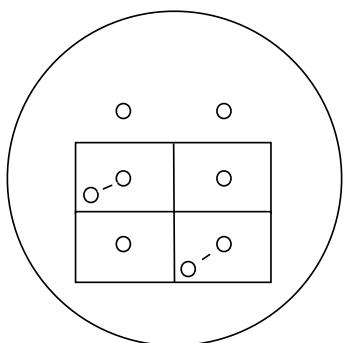
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## Adding relevant data



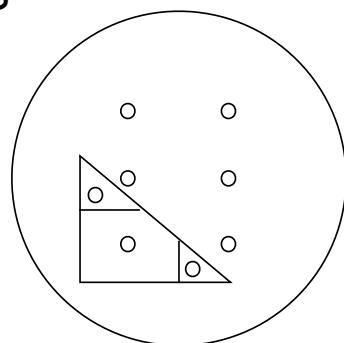
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## Picture of context



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## Being over-informative



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## Remarks on Licensing

- ➊ Licensing is the crucial new logical notion
- ➋ It is typically the formulation of the semantics in update format that gives rise to it
- ➌ The way the notion is defined here is inherently linked to the partition view
- ➍ With overlapping alternatives it does not work anymore

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## Remarks on Licensing

- ➊ Licensing only deals with relatedness of assertions to questions
- ➋ Since questions do not eliminate worlds, questions are always licensed
- ➌ Relatedness of one question to another is rather captured by entailment, which in partition semantics corresponds to the notion of a subquestion
- ➍ Rules of the game prohibit subquestions

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## Fact about Licensing

- ⑥  $\tau$  licenses  $!\varphi$  iff  $\tau$  entails  $?!\varphi$
- ⑥ An indicative is licensed by the context iff the corresponding polar interrogative is part of the issues raised in the context
  - ⑥ Note that this means that from a logical perspective the notion of licensing is superfluous, entailment can do the job

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## Pertinence

- ⑥  $\varphi$  pertinent after  $\tau$  iff  $\varphi$  is consistent with  $\tau$ ;  $\varphi$  is informative after  $\tau$ ; and  $\varphi$  is licensed by  $\tau$
- ⑥ Quality, Quantity and Relation
- ⑥ The logical notion of pertinence arbitrates whether an interrogation is in accordance with the rules of the game

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## Fact about pertinence

- ⑥  $!\varphi$  pertinent after  $\tau$  iff  $!\neg\varphi$  pertinent after  $\tau$
- ⑥  $!\varphi$  pertinent after  $\tau$  iff  $\tau$  entails  $?!\varphi$
- ⑥ Pertinence of an indicative presupposes the corresponding polar question

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## Answerhood

- ⑥  $!y$  is a pertinent answer to  $?!\varphi$  iff  $!y$  is pertinent after  $?!\varphi$
- ⑥ Allows for partial answers, but not for over-informative answers
- ⑥ Let  $!y$  and  $!x$  be pertinent answers to  $?!\varphi$ .  $!y$  is a more informative answer to  $?!\varphi$  than  $!x$  iff  $y$  entails  $x$  (and not vice versa)
- ⑥ Comparing answers nice and easy!

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## Examples answers

- ⑥ Pertinent answers to  $?P_x$
- ⑥  $!P_a$
- ⑥  $!\neg P_a$
- ⑥  $!(P_a \wedge P_b)$
- ⑥  $!\forall x P_x$
- ⑥  $!\forall x(P_x \leftrightarrow x=a)$

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## Illustration

Alf rescued Bea. And No-one else.

Ambiguous:

Rab;  $\neg \exists x(Rxb \wedge x \neq a)$

Rab;  $\neg \exists x(Rax \wedge x \neq b)$

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## Illustration

(Who rescued Bea?)

Alf rescued Bea. And No-one else

Ambiguity resolved:

Rab;  $\neg \exists x(Rxb \wedge x \neq a)$

Rab;  $\neg \exists x(Rax \wedge x \neq b)$

Explanation:

Not licensed after ?Rxb; Rab

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## Illustration

(Whom did Alf rescue?)

Alf rescued Bea. And No-one else

Ambiguity resolved:

Rab;  $\neg \exists x(Rxb \wedge x \neq a)$

Rab;  $\neg \exists x(Rax \wedge x \neq b)$

Explanation:

Not licensed after ?Rax; Rab

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## Illustration

(Whom did Alf rescue?)

Alf rescued Bea. And, actually, no-one else

Ambiguity returns:

Rab;  $\neg \exists x(Rxb \wedge x \neq a)$

Rab;  $\neg \exists x(Rax \wedge x \neq b)$

Presupposition of addressing existing issue is cancelled

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## Presupposing an issue

Alf rescued Bea

Alf rescued Bea

presupposes

presupposes

Did Alf rescue Bea?

Who rescued Bea?

preserved under  
negation

Alf did not rescue Bea

presupposes

Who rescued Bea?

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## Only

Who rescued Bea? Only Alf rescued Bea.

?Rxb; Rab  $\wedge \neg \exists x(Rxb \wedge x \neq a)$

Who rescued Bea? \*Alf rescued only Bea

?Rxb; Rab  $\wedge \neg \exists x(Rax \wedge x \neq b)$

Not a pertinent answer

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## A remaining issue?

Did someone rescue Bea?

Yes. Alf rescued Bea.

Is this equally correct if the 'Yes' is missing?

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## Conclusion

- ⑥ Enriching the notion of meaning to embody both information and issues opens a new perspective on dealing with pragmatic issues in rather standard logical terms
- ⑥ The notion of licensing embodies a very strict logical notion of relatedness to the context, but the illustrations suggest that such a strict notion is linguistically relevant

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## Looking ahead

- ⑥ On all levels, the system is rather restricted
- ⑥ The game is very limited and artificial
- ⑥ Even as a first order query language the language is poor as compared to natural language
- ⑥ The idea that a new perspective on the notion of meaning is at stake does not really play a role

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## Data and issues

- ⑥ In our language providing data and raising issues is divided over two different categories of sentences
- ⑥ It might be interesting to look at hybrid cases, where e.g. an indicative sentence (implicitly) raises an issue as well
  - ⑥ Someone came to visit me yesterday
    - ⑥ Who was it?

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## Language

- ⑥ Things that could be added:
  - ⑥ Questions as subformulas
  - ⑥ Conditional questions
  - ⑥ Which questions
    - ⑥ What happens to the partition view?

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## Game

- ⑥ Turn the game into a more realistic dialogue game, where really exchange of information plays a role
- ⑥ Extend relatedness/licensing to questions as well
- ⑥ Allow for critical moves in the game: denial, doubt

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