Inquisitive Semantics

—the basics—

Jeroen Groenendijk and Floris Roelofsen
(based on joint work with Ivano Ciardelli)

www.illc.uva.nl/inquisitive-semantics

Amsterdam, September 8, 2010

Overview

Basic framework

- Motivation
- Definition and illustration of the semantics
- Some central logical properties

Application

- Attentive *might*

Overview

Basic framework

- Motivation
- Definition and illustration of the semantics
- Some central logical properties

Application

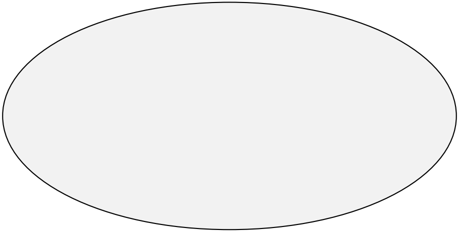
- Attentive *might*

Disclaimer

- Definitions are sometimes simplified for the sake of clarity
- This is all work in progress, there are many open issues, many opportunities to contribute!

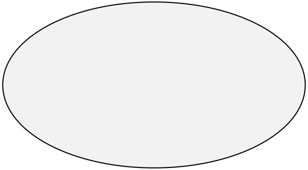
The Traditional Picture

- Meaning = informative content
- Providing information = eliminating possible worlds



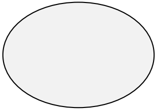
The Traditional Picture

- Meaning = informative content
- Providing information = eliminating possible worlds



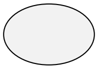
The Traditional Picture

- Meaning = informative content
- Providing information = eliminating possible worlds



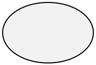
The Traditional Picture

- Meaning = informative content
- Providing information = eliminating possible worlds



The Traditional Picture

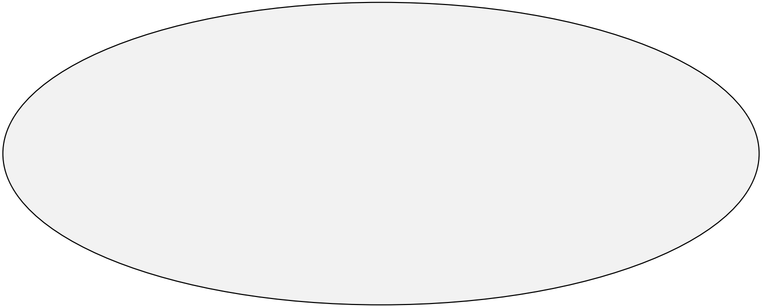
- Meaning = informative content
- Providing information = eliminating possible worlds



- Only captures purely descriptive language use
- Does not reflect the cooperative nature of communication

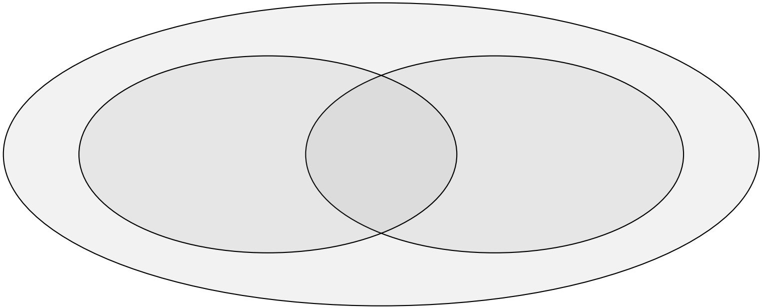
The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive



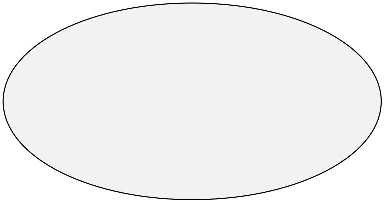
The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive



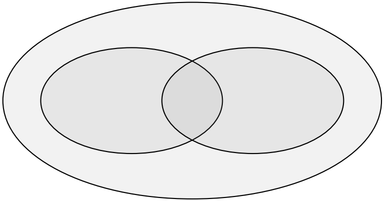
The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive



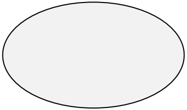
The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive



The Inquisitive Picture

- Propositions as proposals
- A proposal consists of one or more possibilities
- A proposal that consists of several possibilities is inquisitive



A Propositional Language

Basic Ingredients

- Finite set of proposition letters $\mathcal{P}$
- Connectives $\bot$, $\wedge$, $\vee$, $\to$

Abbreviations

- Negation: $\neg\varphi := \varphi \to \bot$
- Classical projection: $!\varphi := \neg\neg\varphi$
- Non-informative projection: $?\varphi := \varphi \vee \neg\varphi$

Semantic Notions

Basic Ingredients

- Index/possible world: function from $\mathcal{P}$ to $\{0, 1\}$
- Possibility: set of indices
- Proposition: set of alternative possibilities

Notation

- $[\varphi]$: the proposition expressed by φ
- $|\varphi|$: the truth-set of φ (set of indices where φ is classically true)

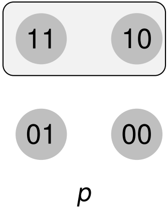
Classical versus Inquisitive

- φ is classical iff $[\varphi]$ contains exactly one possibility
- φ is inquisitive iff $[\varphi]$ contains more than one possibility

Atoms

For any atomic formula φ: $[\varphi] = \{\ |\varphi|\ \}$

Example:

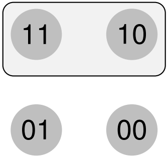


Negation

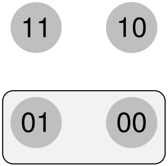
Definition

- $[\neg\varphi] = \{ \overline{\bigcup[\varphi]} \}$
- Take the union of all the possibilities for φ; then take the complement

Example, φ classical:



$[p]$



$[\neg p]$

Negation

Definition

- $[\neg\varphi] = \{\ \overline{\bigcup[\varphi]}\ \}$
- Take the union of all the possibilities for φ; then take the complement

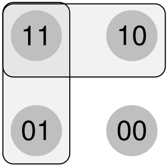
Example, φ inquisitive:

Disjunction

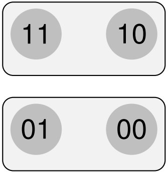
Definition

- $[\varphi \vee \psi] = [\varphi] \cup [\psi]$

Examples:



$p \vee q$



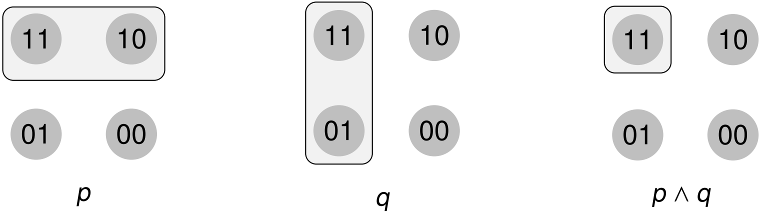
$?p \ \ (:= p \vee \neg p)$

Conjunction

Definition

- $[\varphi \wedge \psi] = \{\alpha \cap \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi]\}$
- Pointwise intersection

Example, φ and ψ classical:

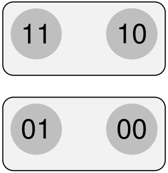


Conjunction

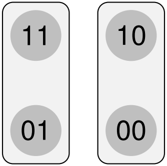
Definition

- $[\varphi \wedge \psi] = \{\alpha \cap \beta \mid \alpha \in [\varphi] \text{ and } \beta \in [\psi]\}$
- Pointwise intersection

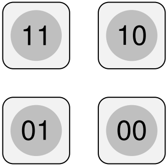
Example, φ and ψ inquisitive:



$?p$



$?q$



$?p \wedge ?q$

Conditionals

Intuition

$$\varphi \rightarrow \psi$$

- Says that if φ is realized in some way, then ψ must also be realized in some way
- Raises the issue of what the exact relation is between the ways in which φ may be realized and the ways in which ψ may be realized

Example

If John goes to London, he will stay with Bill or with Mary

$$p \rightarrow (q \vee r)$$

- Says that if p is realized in some way,
  $q \vee r$ must also be realized in some way

Example

If John goes to London, he will stay with Bill or with Mary

$$p \to (q \lor r)$$

- Says that if p is realized in some way, $q \lor r$ must also be realized in some way
- p can only be realized in one way, but $q \lor r$ can be realized in two ways: by realizing q or by realizing r

Example

If John goes to London, he will stay with Bill or with Mary

$$p \to (q \vee r)$$

- Says that if p is realized in some way,
  $q \vee r$ must also be realized in some way
- p can only be realized in one way, but $q \vee r$ can be realized in two ways: by realizing q or by realizing r
- Thus, $p \to (q \vee r)$ raises the issue
  of whether the realization of p implies the realization of q,
  or whether the realization of p implies the realization of r

Example

If John goes to London, he will stay with Bill or with Mary

$$p \rightarrow (q \vee r)$$

- Says that if p is realized in some way,
  $q \vee r$ must also be realized in some way
- p can only be realized in one way, but $q \vee r$ can be realized in two ways: by realizing q or by realizing r
- Thus, $p \rightarrow (q \vee r)$ raises the issue
  of whether the realization of p implies the realization of q,
  or whether the realization of p implies the realization of r
- The 'ways in which a sentence may be realized' correspond exactly to the possibilities for that sentence

Another way to think about it

Intuition

$$\varphi \rightarrow \psi$$

- Says that there is a certain implicational dependency between the possibilities for φ and the possibilities for ψ
- Raises the issue what this implicational dependency is

Example

If John goes to London, he will stay with Bill or with Mary

$$p \rightarrow (q \vee r)$$

- Two potential implicational dependencies:
  - $p \rightsquigarrow q$
  - $p \rightsquigarrow r$
- The sentence:
  - Says that at least one of these dependencies holds
  - Raises the issue which of them hold exactly

If John goes to London or to Paris, will he fly British Airways?

$$(p \vee q) \rightarrow ?r$$

- Four potential implicational dependencies:
  - $(p \rightsquigarrow r) \ \& \ (q \rightsquigarrow r)$
  - $(p \rightsquigarrow \neg r) \ \& \ (q \rightsquigarrow \neg r)$
  - $(p \rightsquigarrow r) \ \& \ (q \rightsquigarrow \neg r)$
  - $(p \rightsquigarrow \neg r) \ \& \ (q \rightsquigarrow r)$
- The sentence:
  - Says that at least one of these dependencies holds
  - Raises the issue which of them hold exactly

Formalization

- Each possibility for $\varphi \to \psi$ corresponds to a potential implicational dependency between the possibilities for φ and the possibilities for ψ;
- Think of an implicational dependency as a function f mapping every possibility $\alpha \in [\varphi]$ to some possibility $f(\alpha) \in [\psi]$;
- What does it take to establish an implicational dependency f?
- For each $\alpha \in [\varphi]$, we must establish that $\alpha \Rightarrow f(\alpha)$ holds

Formalization

- Each possibility for $\varphi \to \psi$ corresponds to a potential implicational dependency between the possibilities for φ and the possibilities for ψ;
- Think of an implicational dependency as a function f mapping every possibility $\alpha \in [\varphi]$ to some possibility $f(\alpha) \in [\psi]$;
- What does it take to establish an implicational dependency f?
- For each $\alpha \in [\varphi]$, we must establish that $\alpha \Rightarrow f(\alpha)$ holds

Implementation

- $[\varphi \to \psi] = \{\gamma_f \mid f : [\psi]^{[\varphi]}\}$ where $\gamma_f = \bigcap_{\alpha \in [\varphi]} (\alpha \Rightarrow f(\alpha))$

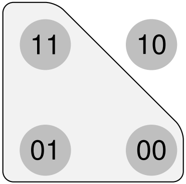
Formalization

- Each possibility for $\varphi \to \psi$ corresponds to a potential implicational dependency between the possibilities for φ and the possibilities for ψ;
- Think of an implicational dependency as a function f mapping every possibility $\alpha \in [\varphi]$ to some possibility $f(\alpha) \in [\psi]$;
- What does it take to establish an implicational dependency f?
- For each $\alpha \in [\varphi]$, we must establish that $\alpha \Rightarrow f(\alpha)$ holds

Implementation

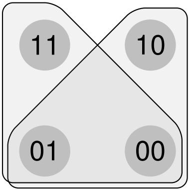
- $[\varphi \to \psi] = \{\gamma_f \mid f : [\psi]^{[\varphi]}\}$ where $\gamma_f = \bigcap_{\alpha \in [\varphi]} (\alpha \Rightarrow f(\alpha))$
- For simplicity, we usually define $\alpha \Rightarrow f(\alpha)$ in terms of material implication: $\overline{\alpha} \cup f(\alpha)$. But any more sophisticated treatment of conditionals could in principle be plugged in here.

Pictures, classical and inquisitive



$p \rightarrow q$

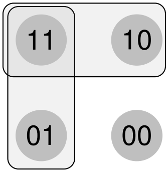
If John goes, Mary will go as well.



$p \rightarrow ?q$

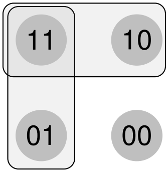
If John goes, will Mary go as well?

Informativeness and Inquisitiveness



- $p \vee q$ is inquisitive: $[p \vee q]$ consists of more than one possibility
- $p \vee q$ is informative: $[p \vee q]$ proposes to eliminate indices

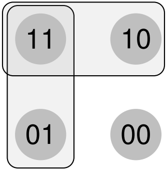
Informativeness and Inquisitiveness



- $p \vee q$ is inquisitive: $[p \vee q]$ consists of more than one possibility
- $p \vee q$ is informative: $[p \vee q]$ proposes to eliminate indices

- $\bigcup[\varphi]$ captures the informative content of φ

Informativeness and Inquisitiveness



- $p \vee q$ is inquisitive: $[p \vee q]$ consists of more than one possibility
- $p \vee q$ is informative: $[p \vee q]$ proposes to eliminate indices

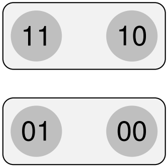
- $\bigcup[\varphi]$ captures the informative content of φ

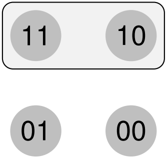
- Fact: for any formula φ, $\bigcup[\varphi] = |\varphi|$

  $\Rightarrow$ classical notion of informative content is preserved.

Questions, Assertions, and Hybrids

- φ is a question iff it is not informative
- φ is an assertion iff it is not inquisitive





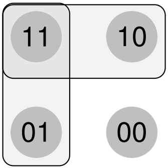
Questions, Assertions, and Hybrids

- φ is a question iff it is not informative
- φ is an assertion iff it is not inquisitive

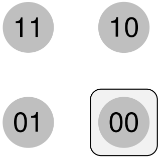
- φ is a hybrid iff it is both informative and inquisitive
- φ is insignificant iff it is neither informative nor inquisitive

Classical closure

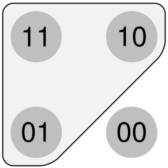
- Double negation always preserves the informative content of a sentence, but removes inquisitiveness



$p \vee q$



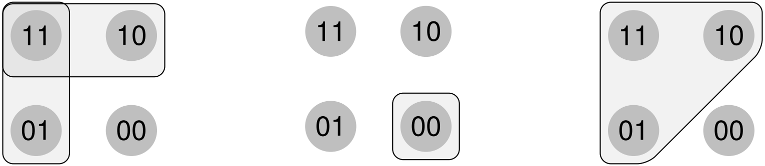
$\neg(p \vee q)$



$\neg\neg(p \vee q)$

Classical closure

- Double negation always preserves the informative content of a sentence, but removes inquisitiveness



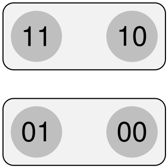
$p \vee q$ $\qquad$ $\neg(p \vee q)$ $\qquad$ $\neg\neg(p \vee q)$

- Therefore, $\neg\neg\varphi$ is abbreviated as $!\varphi$
- $!\varphi$ is called the classical closure of φ

Significance and inquisitiveness

- In a classical setting,
  non-informative sentences are tautologous, i.e., insignificant
- In inquisitive semantics, some classical tautologies come to form a new class of meaningful sentences, namely questions
- Questions are meaningful not because they are informative, but because they are inquisitive

- Example: $?p := p \vee \neg p$



$p \vee \neg p$

Alternative characterization of questions and assertions

Equivalence

- φ and ψ are equivalent iff $[\varphi] = [\psi]$
- Notation: $\varphi \equiv \psi$

Questions and assertions

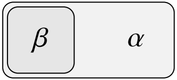
- φ is a question iff $\varphi \equiv ?\varphi$
- φ is an assertion iff $\varphi \equiv !\varphi$

Alternativehood

- If we are only interested in capturing informative and inquisitive content, then we can take a proposition to be a set of alternative possibilities
- That is: no possibility is contained in another
- All possibilities are maximal

Rationale

- Saying that at least one of α and β obtains is just as informative as saying that α obtains



- Asking for information so as to establish at least one of α and β is the same as asking information so as to establish α
- So, as long as we are only interested in capturing informative and inquisitive content, β is redundant

Information, issues, and attention

Attentive content

- However, if we want to capture more than just informative and inquisitive content, then non-maximal possibilities may not be redundant anymore.
- Indeed, the notion of meaning we are exploring is not only suited to capture informative and inquisitive content, but also a sentence's potential to draw attention to certain possibilities

Application

- A novel analysis of *might*

Driving intuition

(1) John might be in London.

(2) John is in London.

(3) Is John in London?

Main contrasts

- (1) differs from (2) in that it does not provide the information that John is in London
- (1) differs from (3) in that it does not request information
- ‘ok’ is an appropriate response to (1), but not to (3)

Main intuition

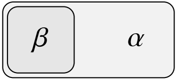
- The semantic contribution of (1) lies in its potential to draw attention to the possibility that John is in London

Attentive content in inquisitive semantics

- The conception of a proposition as a set of possibilities is ideally suited to capture attentive content
- We can simply think of the elements of $[\varphi]$ as the possibilities that φ draws attention to
- At the same time, we can still think of φ as providing and requesting information, just as before
- $\Rightarrow$ informative, inquisitive, and attentive content can all be captured by a single structure

Non-maximal possibilities back aboard

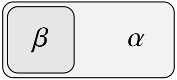
- As long as we are only interested in capturing informative and inquisitive content, non-maximal possibilities are redundant



- But as soon as attentive content becomes of interest as well, non-maximal possibilities cannot be ignored anymore

Non-maximal possibilities back aboard

- As long as we are only interested in capturing informative and inquisitive content, non-maximal possibilities are redundant



- But as soon as attentive content becomes of interest as well, non-maximal possibilities cannot be ignored anymore
- There's no reason we could not draw attention to both α and β
- The only non-maximal possibility that can still be ignored is the empty possibility (it makes no sense to think of any non-contradictory sentence as drawing attention to the empty possibility)

Inquisitive, informative, and attentive sentences

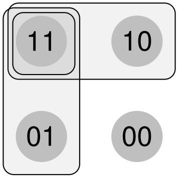
Definitions

- φ is informative iff it proposes to eliminate indices, i.e., $|\varphi| \neq \omega$
- φ is inquisitive iff $[\varphi]$ contains at least two maximal possibilities
- φ is attentive iff $[\varphi]$ contains a non-maximal possibility

Example

- $p \vee q \vee (p \wedge q)$ (*p* or *q* or both)

  informative, inquisitive, and attentive

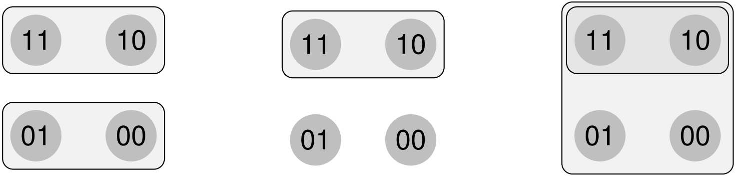


Questions, Assertions, and Conjectures

Definitions

- φ is a question iff it is neither informative nor attentive
- φ is an assertion iff it is neither inquisitive nor attentive
- φ is a conjecture iff it is neither informative nor inquisitive

Examples

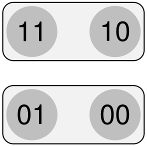


$?p$ $\quad$ p $\quad$ $\top \vee p$

Insignificance

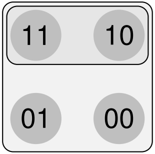
- In the classical setting, any sentence that is non-informative is a tautology, i.e., insignificant

- In inquisitive semantics, many classical tautologies come to form a new class of meaningfull sentences, namely questions



- However, in the 'restricted' setting, any sentence that is neither informative nor inquisitive is still insignificant

- In the unrestricted setting, many of these 'inquisitive tautologies' come to form another class of meaningfull sentences, namely conjectures



Might

Intuition

- $\Diamond p$ draws attention to the possibility that p, without providing or requesting any information

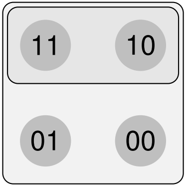
  More generally:

- $\Diamond\varphi$ draws attention to all the possibilities for φ, without providing or requesting information

Implementation

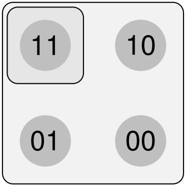
- Define $\Diamond\varphi$ as an abbreviation of $\top \vee \varphi$

Illustrations



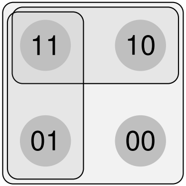
$\Diamond p$

It might be rainy



$\Diamond(p \wedge q)$

It might be
rainy and windy



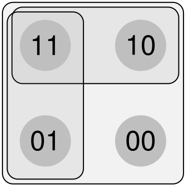
$\Diamond(p \vee q)$

It might be
rainy or windy

Might and conjectures

Every *might* sentence is a conjecture

- $\Diamond\varphi$ is never informative
- $\Diamond\varphi$ is never inquisitive
- So $\Diamond\varphi$ is always a conjecture



$\Diamond(p \vee q)$

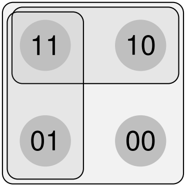
Every conjecture can be expressed by a *might* sentence

- φ is a conjecture if and only if $\varphi \equiv \Diamond\varphi$

Might and conjectures

Every *might* sentence is a conjecture

- $\Diamond\varphi$ is never informative
- $\Diamond\varphi$ is never inquisitive
- So $\Diamond\varphi$ is always a conjecture



$\Diamond(p \vee q)$

Every conjecture can be expressed by a *might* sentence

- φ is a conjecture if and only if $\varphi \equiv \Diamond\varphi$
- φ is a question if and only if $\varphi \equiv ?\varphi$
- φ is an assertion if and only if $\varphi \equiv !\varphi$

Closure properties of conjectures

For any φ and ψ:

- $\Diamond\varphi$ is a conjecture;
- if φ and ψ are conjectures, then so is $\varphi \wedge \psi$;
- if at least one of φ and ψ is a conjecture, so is $\varphi \vee \psi$;
- if ψ is a conjecture, then so is $\varphi \rightarrow \psi$.

Examples

(4)	John might be in London.	$\Diamond p$
(5)	John might be in London and Bill in Paris.	$\Diamond p \wedge \Diamond q$
(6)	John is in London, or he might be in Paris.	$p \vee \Diamond q$
(7)	If John is in London, Bill might be in Paris.	$p \rightarrow \Diamond q$

Might meets disjunction and conjunction

Zimmermann's observation (NALS 2000)

- The following are all equivalent:

(8) John might be in London or in Paris. $\Diamond(p \vee q)$

(9) John might be in London
or he might be in Paris. $\Diamond p \vee \Diamond q$

(10) John might be in London
and he might be in Paris. $\Diamond p \wedge \Diamond q$

Might meets disjunction and conjunction

Important note

- *Might* behaves differently in this respect from clear-cut epistemic modals
- The following are not equivalent:

(11) It is consistent with my beliefs that John is in London or it is consistent with my beliefs that he is in Paris.

(12) It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

- Problematic if *might* is analyzed as an epistemic modal

Might meets disjunction and conjunction

Further observation

- For the equivalence to go through, it is crucial that John cannot be both in London and in Paris at the same time

Szabolcsi's scenario

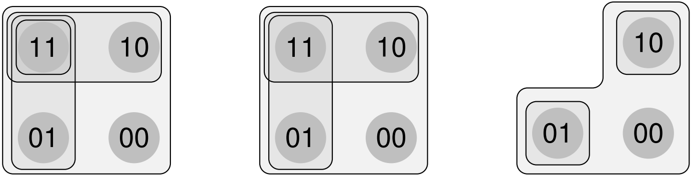
- We need an English-French translator, i.e., someone who speaks *both* languages. In that context, (15) is perceived as a useful recommendation, while (13) and (14) are not.

(13) John might speak English or French. $\Diamond(p \vee q)$

(14) John might speak English or he might speak French. $\Diamond p \vee \Diamond q$

(15) John might speak English and he might speak French. $\Diamond p \wedge \Diamond q$

Might meets disjunction and conjunction



(a) $\Diamond p \wedge \Diamond q$

(b) $\Diamond p \vee \Diamond q$
$\equiv \Diamond(p \vee q)$

(c) $\Diamond p \wedge \Diamond q$
$\equiv \Diamond p \vee \Diamond q$
$\equiv \Diamond(p \vee q)$

- Whenever the disjuncts are mutually exclusive, as in (c), all three formulas are equivalent
- If the disjuncts are not mutually exclusive, then $\Diamond p \wedge \Diamond q$ differs from the other two in that it draws attention to the possibility that p and q both hold.
- This is what makes $\Diamond p \wedge \Diamond q$ a useful recommendation in Szabolcsi's scenario

Might meets negation

Basic observation

Standard sentential negation never takes scope over *might*

(16) John might not be in London. $\# (\neg > \Diamond)$

Important note

Might $\neq$ 'it is consistent with my information that'

(17) It is not consistent with my information
that John is in London. $\checkmark(\neg > \Diamond)$

Explanation

$\neg\Diamond\varphi$ is always a contradiction

Similar, but more complex effects in conditionals (discussed later)

What's next?

- Support-based definition of the semantics
- Inquisitive entailment
- Inquisitive logic, link with intuitionistic logic
- Pragmatics
- First-order case
- ...

What's next?

- Support-based definition of the semantics
- Inquisitive entailment
- Inquisitive logic, link with intuitionistic logic
- Pragmatics
- First-order case
- ...

These slides will be posted on the course website:

www.illc.uva.nl/inquisitive-semantics