# Digital Communication

## Ch5 Carrier and Symbol Synchronization

2024 Fall

## 5-1. Signal Parameter Estimate.

- Propagation delay $\tau$ between Tx and Rx
- $s(t) = Re[s_l(t)e^{j2\pi f_c t}]$

$\rightarrow r(t)$

$= Re[s_l(t-\tau)e^{j2\pi f_c(t-\tau)} + n_l(t)e^{j2\pi f_c t}]$

$= Re\{[s_l(t-\tau)e^{j\phi} + n_l(t)]e^{j2\pi f_c t}\}$

1. Rx和Tx振盪不同步
2. 時間delay.

- $\phi = -2\pi f_c \tau$
- $\tau$ and $\phi$ are different random variable.
- $\phi \sim U(-\pi, \pi)$

$\Rightarrow r(t) = s_l(t; \phi, \tau) + n(t)$

$\Rightarrow$ Let $\Theta = (\phi, \tau)$ ← 要估計的參數

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因為 $\Theta$ 為 uniform

- MAP estimate

$$\hat{\Theta} = \arg\max_{\Theta} f(\Theta | r, s)$$

$$= \arg\max_{\Theta} f(\Theta) \cdot f(r | \Theta, s)$$

- Assume $\Theta$ are uniform distribution.

$$= \arg\max_{\Theta} \cancel{f(\Theta)} \cdot f(r | \Theta, s)$$ → ML estimate of $\Theta$

- ML estimate

$$f(r | \Theta, s) = \left(\frac{1}{\sqrt{\pi N_0}}\right)^N \exp\left(\frac{-\sum_{i=1}^{N} |r_i - s(\Theta)|^2}{N_0}\right)$$

- $\underbrace{\sum_{i=1}^{N} |r_i - s(\Theta)|^2}_{\text{數位}} = \underbrace{\int |r(t) - s(t;\Theta)|^2 dt}_{\text{類比}}$

$$\Rightarrow \hat{\Theta} = \arg\max_{\Theta} \exp\left(-\frac{1}{N_0} \int |r(t) - s(t;\Theta)|^2 dt\right)$$

- where $\Lambda(\Theta) = \exp\left(-\frac{1}{N_0} \int |r(t) - s(t;\Theta)|^2 dt\right)$

## 5.2-1 ML carrier Phase Estimation

※ Assume $\tau = 0$ and Estimate $\phi$, so $\Theta = \theta$

- $\Lambda(\theta)$

$$= \exp\left(-\frac{1}{N_0}\int |r(t) - s(t;\theta)|^2 dt\right)$$

$$= \exp\left(-\frac{1}{N_0}\int \cancel{r^2(t)} - 2r(t)s(t;\theta) + \cancel{s^2(t;\theta)}\right] dt$$

$$= \exp\left(\frac{2}{N_0}\int r(t)\,s(t;\theta)\,dt\right)$$

- $s(t;\theta) = \mathrm{Re}\left[s_\ell(t)e^{j\theta}e^{j2\pi f_c t}\right]$
- $\|s(t;\theta)\|^2 = \frac{1}{2}\|s_\ell(t)e^{j\theta}\|^2 = \frac{1}{2}\|s_\ell(t)\|^2$

→和 $\theta$ 無關

- $$\boxed{\hat{\phi} = \arg\max_{\theta} \exp\left(\frac{2}{N_0}\int r(t)\,s(t;\theta)\,dt\right)}$$

## Dicision Directed Loops for Estimate $\phi$

- For general modulation,

$$\begin{cases} s_\ell(t) = \sum I_n g(t-nT) \ ; \text{ then} \\ s(t;\theta) = \mathrm{Re}\left[s_\ell(t)e^{j\theta}e^{j2\pi f_c t}\right] \end{cases}$$

- $\Lambda_L(\theta)$

$$= \frac{2}{N_0}\int r(t)\cdot s(t;\theta)\,dt$$

$$= \mathrm{Re}\left[\frac{2}{N_0}\int r(t)\sum I_n g(t-nT)e^{j\theta}e^{j2\pi f_c t}\,dt\right]$$

$$= \boxed{\mathrm{Re}\left[\frac{2}{N_0}\sum I_n y_n e^{j\theta}\right]}$$

$$;\ y_n = \int r(t)g(t-nT)e^{j2\pi f_c t}\,dt$$

$$= \frac{2}{N_0}\mathrm{Re}\left[\frac{2}{N_0}\sum I_n y_n\right]\cos\theta - \frac{2}{N_0}\mathrm{Im}\left[\frac{2}{N_0}\sum I_n y_n\right]\sin\theta$$

- $$\frac{\partial \Lambda_L(\theta)}{\partial \theta} = 0 \rightarrow \hat{\phi} = -\tan^{-1}\left[\frac{\mathrm{Im}\{\sum I_n y_n\}}{\mathrm{Re}\{\sum I_n y_n\}}\right] \#$$

## 5.3 Symbol time estimation

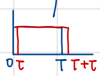
※ Assume $\phi = 0$ and Estimate $\tau$, so $\Theta = \tau$

- $\Lambda(\tau)$

$$= \exp\left(-\frac{1}{N_0}\int |r(t) - s(t;\tau)|^2 dt\right)$$

$$= \exp\left(-\frac{1}{N_0}\int \cancel{r^2(t)} - 2r(t)s(t;\tau) + \cancel{s^2(t;\tau)}\right] dt$$

$$= \exp\left(\frac{2}{N_0}\int r(t)\,s(t;\tau)\,dt\right)$$




若 $\tau \ll T$, 則

$s^2(t;\tau) \approx s(t)$

- $$\Lambda_L(\tau) = \frac{2}{N_0}\int r(t)\,s(t;\tau)\,dt$$

$$\begin{cases} r(t) = s(t;\tau) + z(t) \\ s(t;\tau) = \mathrm{Re}\left\{\sum I_n g(t-nT-\tau)e^{j2\pi f_c(t-\tau)}\right\} \end{cases}$$

- $\Lambda_L(\tau)$

$$= \frac{2}{N_0}\sum I_n \int r(t)g(t-nT-\tau)\cos(2\pi f_c(t-\tau))\,dt$$

$$= \boxed{\frac{2}{N_0}\sum I_n y_n(\tau)}$$

## 5.4 Joint Estimation of Carrier Phase and Symbol Timing.

$$\Lambda(\phi,\tau) = \exp\left(-\frac{1}{N_0}\int |r(t) - s(t;\phi,\tau)|^2 dt\right)$$

- Assume $S_l(t) = \sum I_n g(t-nT) + j\sum Q_n p(t-nT)$

$\rightarrow$ $S_l(t;\phi,t) = \left[\sum I_n g(t-nT-\tau) + j\sum Q_n p(t-nT-\tau)\right] e^{j\phi}$

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|---|---|---|
| ∘ PAM | : | $I_n$ real and $Q_n = 0$ |
| ∘ QAM and PSK | : | $I_n$ complex and $Q_n = 0$ |
| ∘ OQPSK | : | $p(t) = g(t - T/2)$ |

∘ $$\begin{cases} \dfrac{\partial \Lambda_L(\phi,\tau)}{\partial \phi} = 0 \\[2ex] \dfrac{\partial \Lambda_L(\phi,\tau)}{\partial \tau} = 0 \end{cases}$$