Problem 1, (a)

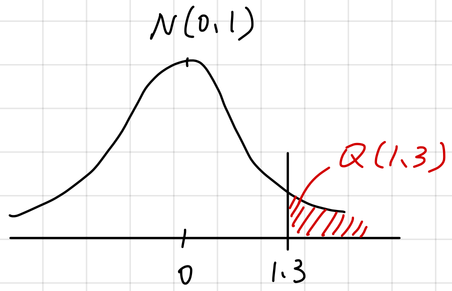
- $f_c = 2\text{GHz}$
- $d_{break} = 15\text{m}$
- pathloss factor $n = 3.7$
- distance between Tx and Rx : 900m
- accommodate a maximum propagation loss 140dB

(i) Calculate the Pathloss

$$PL(d)$$
$$= 20\log_{10}\left(\frac{4\pi d_{break}}{\lambda_c}\right) + 10n\log_{10}\left(\frac{d}{d_{break}}\right)$$
$$= 20\log_{10}\left(\frac{4\pi}{\lambda} 2\times 15\right) + 37\log_{10}(60)$$
$$= 32 + 29.54 + 65.79$$
$$= 127\text{dB}$$

(ii) Total Path loss

$$PL'(d) = PL(d) + X$$
$$= 127 + X$$
$$\text{r.v. } X \sim N(0, 10^2)$$



(iii) Pout with 140dB

$$P_{out} = \Pr\{127 + X \geq 140\} = \Pr\{X \geq 13\}$$
$$= \Pr\left\{\frac{X - \mu_{p,dB}}{\sigma_{p,dB}} \geq \frac{13-0}{10}\right\} = Q(1.3) = 0.0968$$

Ans: 0.0968 #

Problem 1, (b)

- $P_{out} = cdf(\gamma_{min}) = \int_0^{\gamma_{min}} pdf_\gamma(\gamma)\, d\gamma$

$$\boxed{= \int_0^{\infty} \left(1 - \exp\left(-\frac{\pi}{4}\cdot\frac{\gamma_{min}^2}{\bar{\gamma}^2}\right)\right) \frac{20/\ln 10}{\sqrt{2\pi}\,\sigma_F} \cdot \frac{1}{\bar{\gamma}} \cdot \exp\left(-\frac{(20\log_{10}\bar{\gamma} - \mu_{p,dB})^2}{2\sigma_{p,dB}^2}\right) d\bar{\gamma}}$$

- Fading margin $M = 20 = \dfrac{\bar{\gamma}^2}{\gamma_{min}^2} \Rightarrow \gamma_{min} = \sqrt{\dfrac{1}{20}}$

```
>> mu=0;
sigma=10;
C=(20/log(10))/(sqrt(2*pi)*sigma);
r_min=sqrt(1/20);
fun=@(r)C.*exp(-(20.*log10(r)-mu).^2/(2*sigma^2))./r.*(1-exp(-(pi/4)*r_min^2./(r.^2)));
P_out=integral(fun,0,Inf)

P_out =

   0.1543
```

Ans: 15.4% #

Problem 2、

- HW2 P4的做法：將各自的 Fading求出並相加

  $\Rightarrow M_{total,dB} = M_{Rayleigh,dB} + M_{large\text{-}scale,dB} = \underline{21.12\,dB}$

- 此題作法：Compute the fading margin $M_{k.dB}$ for <u>suzuki distribution</u>
- 求 $|M_{total.dB} - M_{k.dB}| = ?$

<sol>

For Suzuki distribution, with $\mu = 0 \quad \sigma = 5$

$$P_{out} = \int_0^\infty \left(1-\exp\left(-\frac{\pi}{4}\cdot\frac{\gamma_{min}^2}{\bar{\gamma}^2}\right)\right)\frac{20/\ln 10}{\sqrt{2\pi}\,\sigma_F}\cdot\frac{1}{\bar{\gamma}}\cdot\exp\left(-\frac{(20\log_{10}\bar{\gamma}-\mu_{p,dB})^2}{2\sigma_{p,dB}^2}\right)d\bar{\gamma} \le 0.05$$

$$|M_{total} - M_k|_{dB} = |21.12 - 14.45| = 6.67$$

Ans: 6.67 dB #

```
mu = 0;             % dB
sigma = 5;           % dB
P_target = 0.05;
A = 20/log(10)/sqrt(2*pi)/sigma;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 估計r_min介於-10~0db之間時可以滿足 P_out<=P_target=0.05
%r_min_dB = -10;
%r_min = 10^(r_min_dB/10);
%fun = @(r) A*exp(-(20*log10(r)-mu).^2/(2*sigma^2))./r.*(1 - exp(-(pi/4)*r_min^2./(r.^2)));
%P_out = integral(fun,0,Inf);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
LB = 0;
UB = -10;
iter_max = 10;

for i = 1:iter_max
    searchPoint = linspace(LB,UB,11);
    for j = 1:length(searchPoint)-1
        r_min_dB = searchPoint(j+1);
        r_min = 10^(r_min_dB/10);
        fun = @(r) A*exp(-(20*log10(r)-mu).^2/(2*sigma^2))./r.*(1 - exp(-(pi/4)*r_min^2./(r.^2))
        P_out = integral(fun,0,Inf);
        if P_out <= P_target
            UB = searchPoint(j+1);
            LB = searchPoint(j);
            break
        end
    end
end

r_min_dB = LB;
FadingMargin = 2*mu - 2*r_min_dB;
disp(['r_min_dB is ',num2str(r_min_dB)])
disp(['Fading Margin is ',num2str(FadingMargin)])
r_min_dB is -7.2266
Fading Margin is 14.4532
```

Problem 3.

(a)

$$g_m(t) = \begin{cases} \cos\left[2\pi\left(f_c + \frac{b_m}{2\pi} f_{sep}\right)t\right] & ; 0 \le t \le T_s \\ 0 & ; \text{o.w.} \end{cases}$$

$f_m = f_c + \frac{b_m}{2\pi} f_{sep}$ ：訊號以不同的頻率傳送 . Ans: 4-FSK ※

(b) $k = \log_2 4 = 2$. Ans: 2bit ※

(c) Consider $b_m = 1$, $b_n = -1$

$$\langle \cos(2\pi f_c t + f_{sep} t), \cos(2\pi f_c t - f_{sep} t) \rangle = 0$$

$$\int_0^{T_s} \cos(2\pi f_c t + f_{sep} t)\cos(2\pi f_c t - f_{sep} t)\, dt = 0$$

$$\int_0^{T_s} \overset{0}{\cancel{\cos(4\pi f_c t)}} + \cos(2 f_{sep} t)\, dt = 0$$

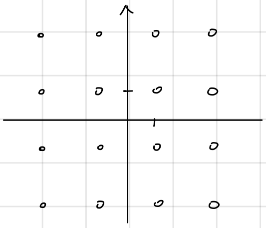
$$\frac{1}{2 f_{sep}} \sin(2 f_{sep} t)\Big|_0^{T_s} = \frac{1}{2 f_{sep}} \sin(2 f_{sep} T_s) = 0$$

$$2 f_{sep} T = k\pi, \quad f_{sep} = \frac{k\pi}{2T}$$

Ans: $f_{sep} = \frac{\pi}{2T}$ #

Problem 4.

(a)



$E_s = \frac{1}{4} d^2 (2+10+10+18) = 10d^2$

$E_b = \frac{1}{4} E_s = \frac{5}{2} d^2$

$d = \sqrt{\frac{2}{5} E_b}$

Ans: $10d^2$ ※

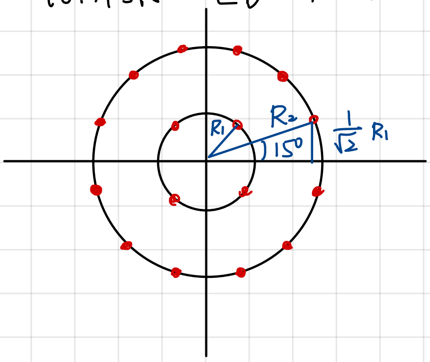
(b)

16QAM和16APSK E_b相同，　$R_2 \sin 15° = \frac{1}{\sqrt{2}} R_1 \rightarrow R_2 = \frac{1}{\sqrt{2} \cdot \sin 15°} R_1 = (1+\sqrt{3}) R_1$

- 16QAM : $E_b = \frac{1}{4} E_b = \frac{5}{2} d^2$
- 16APSK : $E_b = 1.462 R_1^2$

$E_s = \frac{1}{4} R_1^2 \left(1 + 3 (1+\sqrt{3})^2\right) = \frac{13+6\sqrt{3}}{4} R_1^2$

$E_b = \frac{1}{4} E_s = 1.462 R_1^2 = 2.5 d^2$



$\Rightarrow R_1 = 1.307 d$

$d_{min.QAM} = 2d$,

$d_{min.APSK} = \sqrt{2} R_1 = 1.85d$

$\Rightarrow$ 16QAM d_{min} 較大.

Ans:
- 16QAM:
  $S_m(t) = A_{mi}\, g(t) \cos(2\pi f_c t) - A_{mq} \cdot g(t) \sin(2\pi f_c t)$; $A_{mi}, A_{mq} \in \{\pm d, \pm 3d\}$
- 16APSK:
  $$S_m(t) = \begin{cases} R_1 \cos\left(2\pi f_c t + \frac{\pi}{4} + \frac{\pi}{2} k\right) & ; k = 0, 1, 2, 3 \\ (1+\sqrt{3}) R_1 \cos\left(j 2\pi f_c t + \frac{\pi}{4} + \frac{\pi}{6} k\right) & ; k = 0, 1, \cdots 11 \end{cases}$$
- 16QAM has better resistance.
  because 16-QAM d_{min} is larger than 16-APSK ※

Problem 4. (c)

(i) 16QAM : $PAPR = \frac{18d^2}{10d^2} = 1.8$ ※

(ii) 16APSK : $PAPR = \frac{(1+\sqrt{3})^2 R_1^2}{\frac{13+6\sqrt{3}}{4} R_1^2} = 1.276$ ※

Ans: 16-QAM: 1.8
16-APSK : 1.276 ※

Problem 4. (d)

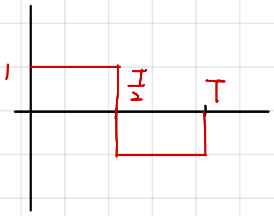
16-QAM 因為有較小的 PAPR.功率分配更平均,因此受到的影響較低.

16-APSK 因為有較大的 PAPR.功率分配不平均,因此受到的影響較大。※

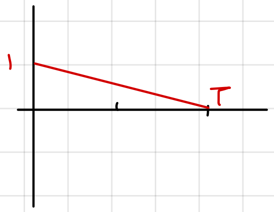
Problem 5. (a)

$T = 2$

$x(t)$



$y(t)$



$$\frac{y}{k} = \frac{b}{k}\varphi_1 + \varphi_2$$

$$y = b\varphi_1 + k\varphi_2$$

$$\cdot\ \phi_1(t) = \frac{x(t)}{\|x(t)\|} \ ; \ \phi_1(t) = \begin{cases} 1/\sqrt{T} & ; 0<t\le \frac{T}{2} \\ -1/\sqrt{T} & ; \frac{T}{2}<t\le T \end{cases}$$

$$\|x(t)\|^2 = \int_0^{\frac{T}{2}} 1\,dt + \int_{\frac{T}{2}}^{T} (-1)^2\,dt = \frac{T}{2} + \frac{T}{2} = T$$

$$\cdot\ \varphi_2'(t) = y(t) - \frac{\langle y(t), \phi_1(t)\rangle}{\|\phi_1(t)\|}\phi_1(t) = \begin{cases} \frac{3}{4} - \frac{t}{T} & ; 0<t<\frac{T}{2} \\ \frac{5}{4} - \frac{t}{T} & ; \frac{T}{2}<t<T \end{cases}$$

$$\langle y(t), \varphi_1(t)\rangle = \int_0^{\frac{T}{2}} \left(1-\frac{t}{T}\right)\frac{1}{\sqrt{T}}\,dt - \int_{\frac{T}{2}}^{T} \left(1-\frac{t}{T}\right)\frac{1}{\sqrt{T}}\,dt$$

$$= \frac{1}{\sqrt{T}}\left(t - \frac{t^2}{2T}\right)\Bigg|_0^{\frac{T}{2}} - \frac{1}{\sqrt{T}}\left(t - \frac{t^2}{2T}\right)\Bigg|_{\frac{T}{2}}^{T}$$

$$= \frac{1}{\sqrt{T}}\left\{\frac{T}{2} - \frac{T}{8} - \left[\left(\cancel{T} - \cancel{\frac{T}{2}}\right) - \left(\cancel{\frac{T}{2}} - \frac{T}{8}\right)\right]\right\} = \frac{1}{4}\sqrt{T}$$

$$\|\varphi_2(t)\|^2 = \int_0^{\frac{T}{2}} \Big(\overset{\frac{3}{4}}{\cancel{1-\frac{1}{4}}} - \frac{t}{T}\Big)^2 dt + \int_{\frac{T}{2}}^{T} \Big(\overset{\frac{5}{4}}{\cancel{1+\frac{1}{4}}} - \frac{t}{T}\Big)^2 dt$$

$$\sqrt{\frac{48}{26}}$$

$$= \left(\frac{9}{32} - \frac{3}{16} + \frac{1}{24}\right)T \cdot 2 = \frac{13}{48}T \longrightarrow \|\varphi_2(t)\| = \sqrt{\frac{13}{48}T}$$

Ans: $\varphi_1(t) = \begin{cases} \frac{1}{\sqrt{T}} & , 0<t\le\frac{T}{2} \\ -\frac{1}{\sqrt{T}} & ; \frac{T}{2}<t\le T \end{cases}$; $\varphi_2(t) = \begin{cases} \sqrt{\frac{48}{13T}}\left(\frac{3}{4} - \frac{t}{T}\right) & ; 0<t<\frac{T}{2} \\ \sqrt{\frac{48}{13T}}\left(\frac{5}{4} - \frac{t}{T}\right) & ; \frac{T}{2}<t<T \end{cases}$ #

Problem 5 (b)

(i) $x(t) = \sqrt{T}\,\varphi_1(t) = [\sqrt{T}, 0]$

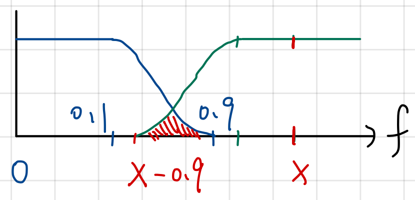
$y(t) = \frac{1}{4}\sqrt{T}\,\varphi_1(t) + \sqrt{\frac{13}{48}T}\,\varphi_2(t) = \left[\frac{1}{4}\sqrt{T}, \sqrt{\frac{13}{48}T}\right]$ #

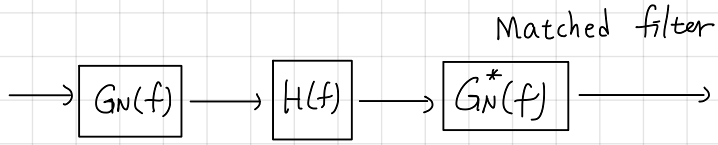
(ii) $\rho_{x,y} = \dfrac{\langle x(t), y(t)\rangle}{\|x(t)\|\cdot\|y(t)\|} = \dfrac{\frac{1}{4}T}{\sqrt{T}\cdot\sqrt{\frac{T}{3}}} = \dfrac{\sqrt{3}}{4}$ #

Problem 6.

$$G_N(f,\alpha,T)=\begin{cases}1 & ;\ 0<|f|<\frac{1-\alpha}{2T}\\ \frac{1}{2}\left(1-\sin\left(\frac{T}{2\alpha}\left(|2\pi f|-\frac{\pi}{T}\right)\right)\right) & ;\ \frac{1-\alpha}{2T}<|f|<\frac{1+\alpha}{2T}\\ 0\end{cases}$$

$\alpha=0.8,\ T=1$





(a)

◦計算接收功率. $S_N=\int_{-0.9}^{0.9}\left|G_N(f,0.8,1)\cdot G_N^*(f,0.8,1)\right|^2 df=0.6375$

◦計算干擾功率: $SIR=\frac{S_N}{I_N}\geq 25 \rightarrow I_N\leq\frac{0.6375}{316.227}=2.016\times10^{-3}$

$$I_N=\int_{X-0.9}^{0.9}\frac{1}{16}\left[1-\sin\left(\frac{5}{8}\left(|2\pi f|-\pi\right)\right)\right]^2\left[1-\sin\left(\frac{5}{8}\left(|2\pi(f-x)|-\pi\right)\right)\right]^2 df\leq 2.016\times10^{-3}$$

Ans: $X\geq 1.2037$ Hz , s.t SIR $\geq$ 25dB ※

(b)

user 2 move toward the BS → d = 100m

◦原本的 PL(d) : $38\log_{10}\left(\frac{200}{5}\right)=60.88$ dB

◦移動後的 PL(d) : $38\log_{10}\left(\frac{100}{5}\right)=49.44$ dB

⇒ received signal power of user2 increase 11.44 dB

Ans: $SIR_1=25-11.44=13.56$
$SIR_2=25+11.44=36.44$ ※

(c)

$SIR_1-11.44\geq 25$, $SIR_1\geq 36.44$

$\frac{S_{N1}}{I_N}\geq 10^{3.644}\Rightarrow I_N\leq\frac{0.6375}{4405}=1.447\times10^{-4}$ Repeat (a) by matlab

Ans: $X\geq 1.3667$ Hz #

(d)

Increase user 1 P_{TX} by 11.44 dB or

Decrease user 2 P_{TX} by 11.44 dB

```
T = 1;
alpha = 0.8;
th1 = (1 - alpha) / 2;
th2 = (1 + alpha) / 2;
SIR = 25; % dB

GN = @(f) 1 * (abs(f) <= th1) + 0.5 * (1 - sin((abs(2 * pi * f) - pi) / (2 * alpha))) .* (abs(f) > th1) .* (abs(f) <= th2);
fun = @(f) GN(f) .^ 4;
SN = integral(fun, -th2, th2);
disp(['signal power = ', num2str(SN)])
IN = SN / 10^(SIR / 10);
disp(['maximum interference power = ', num2str(IN)])

% assume the separate x >= 1
iter_max = 5;     % search accuracy
x = 1;            % initial point
UB = 2;           % initial search boundary

for iter = 1:iter_max
    sp = x : 10^(-iter) : UB;  % set up search points
    for step = 1 : length(sp) - 1
        fun = @(f) (0.25 * (1 - sin(5 / 8 * (2 * pi * f - pi))) .* (1 - sin(5 / 8 * (2 * pi * (f - sp(1 + step))) - pi))) .^ 2;
        FUN = integral(fun, sp(1 + step) - 0.9, 0.9);
        if FUN <= IN
            x = sp(step);
            UB = sp(step + 1);
            break
        end
    end
end

disp(['minimum separation = ', num2str(x)])
signal power = 0.6375
maximum interference power = 0.002016
minimum separation = 1.5057
```