Digital Communication

Ch3: Digital Modulation Scheme

2024 Fall

3-2 Memoryless Modulation

M-PAM

Bandpass waveform

Pulse-shaping function

- $S_m(t) = Re\{A_m g(t) e^{j2\pi f_c t}\}$ 等效基頻

$= A_m g(t) \cos(2\pi f_c t)$

Basis

- $\phi_1(t) = \frac{g(t)}{\|g(t)\|}\sqrt{2}\cos(2\pi f_c t)$

vector

- $\vec{S_m} = \frac{A_m}{\sqrt{2}}\|g(t)\|$

Transmit Energy

- $E_m = \frac{1}{2}A_m^2 E_g$

PSK

Bandpass waveform

- $S_m(t) = Re\{g(t) e^{j\theta_m} e^{j2\pi f_c t}\}$

$= g(t)\cos\theta_m \cos(2\pi f_c t) - g(t)\sin\theta_m \sin(2\pi f_c t)$

Basis

- $\phi_1(t) = \frac{g(t)}{\|g(t)\|}\sqrt{2}\cos(2\pi f_c t)$
- $\phi_2(t) = -\frac{g(t)}{\|g(t)\|}\sqrt{2}\sin(2\pi f_c t)$

vector

- $\vec{S_m} = \left[\frac{\|g(t)\|}{\sqrt{2}}\cos(\theta_m), \frac{\|g(t)\|}{\sqrt{2}}\sin(\theta_m)\right]^T$

Transmit Energy

- $E_m = \frac{1}{2}E_g$

優點：Equal Energy

QAM

- 優點：傳送速度提升
- 缺點：錯誤率增加

Bandpass waveform

- $S(t) = A_{m,i}\, g(t)\cos(2\pi f_c t)$

$- A_{m,q}\, g(t)\cdot \sin(2\pi f_c t)$

Basis

- $\phi_1(t) = \frac{\|g(t)\|}{g(t)}\sqrt{2}\cos(2\pi f_c t)$
- $\phi_2(t) = -\frac{\|g(t)\|}{g(t)}\sqrt{2}\sin(2\pi f_c t)$

vector

- $\vec{S_m} = \left[\frac{A_{mi}}{\sqrt{2}}\|g(t)\|, \frac{A_{mq}}{\sqrt{2}}\|g(t)\|\right]^T$

Transmit Energy

- $E_m = \frac{1}{2}E_g(A_{mi}^2 + A_{mq}^2)$

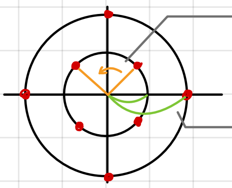
Euclidean Distance

- $\|\vec{S_m} - \vec{S_n}\| = \frac{1}{\sqrt{2}}\|g(t)\|\sqrt{(A_{mi}-A_{ni})^2 + (A_{mq}-A_{nq})^2}$

Minimum Distance

- $\frac{1}{\sqrt{2}}\|g(t)\|\cdot\sqrt{0^2+2^2} = \sqrt{2E_g}$ #

QAM = PAM + PSK



對相位調變

對振幅調變

How to create high-dimensional signal ?

1. Subdivision of Frequency
2. Subdivision of Time
3. Subdivision of both Frequency & Time

Subdivision of Frequency

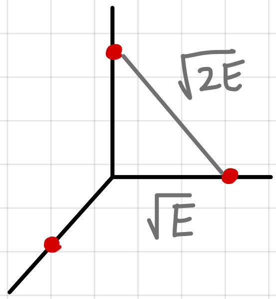
FSK

Bandpass waveform

- $S_m(t)$

$$= Re\left\{\sqrt{\frac{2E}{T}}\, e^{j2\pi(m\Delta f)t} \cdot e^{j2\pi f_c t}\right\}$$

$$= \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + 2\pi m \Delta f t)$$



Basis/vector

- $\phi_m(t) = \frac{1}{\sqrt{E}} S_m(t)$
- $\vec{S_m} = [0, 0 \cdots \sqrt{E} \cdots 0]^T$

- 到原點距離皆相同: $E_m = E$
- 任2點皆等距: $\sqrt{2E}$
- 頻寬大: $B_T = 2W + (M-1)\Delta f$

頻率正交

- 基頻正交: $\Delta f = \frac{1}{T}$
- 旁頻正交: $\Delta f = \frac{1}{2T}$

1. $S_{l,m}(t) = \sqrt{\frac{2E}{T}}\, e^{j2\pi(m\Delta f)t}$; $\|S_{l,m}(t)\| = \sqrt{2E}$

2. $\rho_{l\,m,n}$

$$= \frac{\langle S_{l,m}(t), S_{l,n}(t)\rangle}{\|S_{l,m}(t)\| \cdot \|S_{l,n}(t)\|}$$

$$= \frac{1}{T}\int_0^T e^{j2\pi\Delta f(m-n)t}\, dt$$

$$= \mathrm{sinc}\left[\underline{T(m-n)\Delta f}\right] e^{j\pi T(m-n)\Delta f}$$

故基頻正交為 $\Delta f = \frac{1}{T}$

3. $\rho_{m,n}$

$$= Re\{\rho_{l\,m,n}\}$$

$$= \frac{\sin(\pi T(m-n)\Delta f)}{\pi T(m-n)\Delta f} \cos(\pi T(m-n)\Delta f)$$

$$= \mathrm{sinc}\left[\underline{2T(m-n)\Delta f}\right]$$

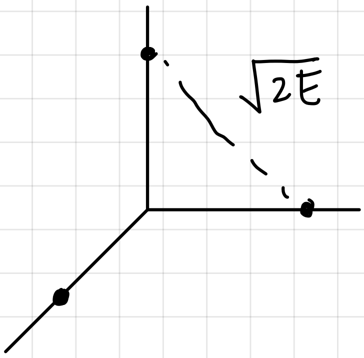
射頻正交為 $\Delta f = \frac{1}{2T}$

Euclidean distance between FSK

$$[S_1\ S_2 \cdots S_M] = \begin{bmatrix} \sqrt{E} & 0 & \cdots & 0 \\ 0 & \sqrt{E} & & \\ & & \ddots & \\ & & & \sqrt{E} \end{bmatrix}$$

$$\Rightarrow \|S_m - S_n\| = \sqrt{2E}$$

- 到原點距離皆相同：$E_m = E$
- 任 2 點皆等距：$\sqrt{2E}$
- 頻寬大：$B_T = 2W + (M-1)\Delta f$

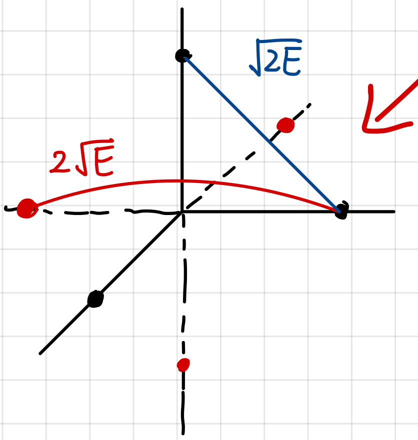


因為浪費頻寬，所以在一個 basis 上傳播 2 個 signal

Biorthogonal multidimensional FSK

$$[S_{-1} \cdots S_{-M}\ S_1 \cdots S_M] = \begin{bmatrix} -\sqrt{E} & \cdots & 0 & \sqrt{E} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & & -\sqrt{E} & 0 & \cdots & \sqrt{E} \end{bmatrix}$$

$$\Rightarrow \|S_m - S_n\| = \begin{cases} \sqrt{2E} & ;\ m \neq n \\ 2\sqrt{E} & ;\ m = -n \\ 0 & ;\ m = n \end{cases}$$

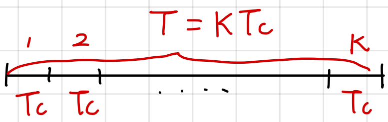


- 到原點的距離仍相同：E
- Crosscorrelation 改變：$\rho_{mn,\ell} = \begin{cases} 0 & ;\ m \neq n \\ -1 & ;\ m = -n \\ 1 & ;\ m = n \end{cases}$ 沒正交
- 傳送速率更佳

Subdivision of Time

$S_m = [\,C_{m,0} \;\; C_{m,1} \; \cdots \; C_{m,K-1}\,]$

where $KT_c = T$



把訊號在 time-domain 切成 K 份傳送

$$\boxed{S_m(t) = \sum_{j=0}^{K} C_{m,j} \cdot g(t - jT_c)}$$

- $C_{m,j} = 1$ represent $+g(t)$ is transmited during time slot j
- $C_{m,j} = -1$ represent $-g(t)$ is transmited during time slot j
- $g(t) = \sqrt{\frac{2E_c}{T_c}} \cos(2\pi f_c t) \;\; ; \; 0 \le t \le T$

- Transmit energy : $E_m = KE_c$
- Crosscorrelation coefficient (if differ by only one component $C_{m,j}$)

$$\rho_{m,n} = \frac{\langle S_m, S_n \rangle}{\|S_m\| \cdot \|S_n\|} = \frac{(K-1)E_c - E_c}{KE_c}$$

$$= 1 - \frac{2}{K}$$

因為 $K-1$ 個皆相同 : $(K-1)E_c$
只有 1 個不同 : $-E_c$

- Minimum Euclidean distance (if differ by only one component $C_{m,j}$)

$$\min \| S_m - S_n \| = \min \sqrt{\|S_m\|^2 + \|S_n\|^2 - 2\langle S_m, S_n \rangle}$$

$$= \sqrt{KE_c + KE_c - 2(K-2)E_c}$$

$$= 2\sqrt{E_c}$$

- Upper bound on "the number of channel symbol"
  - Hadamard signal

$$H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{bmatrix} \; ; \; H_0 = [1]$$

→ 雖然有 2^n 種組合方式，但為了 orthogonal，故只選 n 種 symbol 作為傳送設計方式為 Hadamand signal . H_n

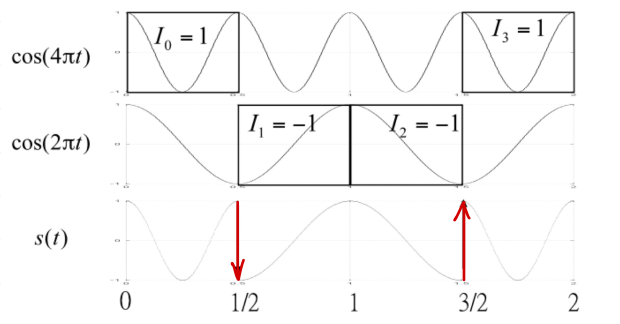
3-3 Signaling scheme with memory

- 傳統 FSK signal 問題.

- $S_m(t)=\sqrt{\frac{2E}{T}}\cos(2\pi f_c t+2\pi(m\Delta f)t)$

Ex.

$$S(t)=\begin{cases}\cos(2\pi t) & ; I_n=1,\ m=0\\ \cos(4\pi t) & ; I_n=-1,\ m=1\end{cases}$$



在不同 oscillator 切換時會有高頻的產生 (不連續)

- CPFSK.

  - $\phi(t;I_n)=4\pi f_d T\int_{-\infty}^{t}d(\tau)\,d\tau$

• 解決方式 CPFSK

• phase change (the derivative of phase)

$$d(t)=\sum I_n\, g(t-nT)$$

• Baseband of CPFSK:

phase (the integration of phase change)
為了讓相位連續

$$\boxed{S_\ell(t)=\sqrt{\frac{2E}{T}}\exp\left(j\left[4\pi T f_d\int_{-\infty}^{t} d(\tau)\,d\tau+\phi_0\right]\right)}$$

- f_d : peak frequency deviation
- ϕ_0 : initial phase

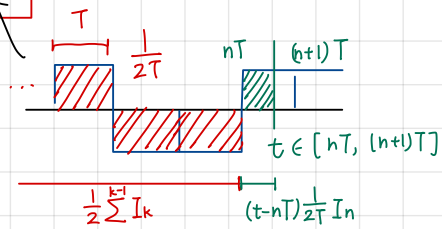
• Bandpass of CPFSK: $S_\ell(t)=\sqrt{\frac{2E}{T}}\,e^{j\phi(t;I)}$

where

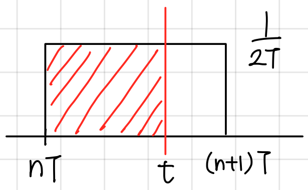
$$\phi(t;\{I_n\}) = 4\pi T f_d \int_{-\infty}^{t}\left[\sum_k I_k\, g(\tau-kT)\,d\tau\right]d\tau$$

$$= \boxed{2\pi T f_d\sum_{k=-\infty}^{n-1} I_k + 2\pi f_d (t-nT) I_n} \;;\; t\in[nT,(n+1)T]$$

$$= \underbrace{\theta_n}_{\text{memory}} + 2\pi h\cdot I_n\cdot q(t-nT)$$



- $h = 2f_d T$: modulation index
- $\theta_n = \pi h \sum^{n-1} I_k$: memory
- $q(t) = \begin{cases} 0 & ; t<0 \\ \frac{t}{2T} & ; 0<t<T \\ \frac{1}{2} & ; t\geq T \end{cases} \Rightarrow q(t-nT) = \begin{cases} 0 & ; t<nT \\ \frac{t-nT}{2T} & ; nT<t<(n+1)T \\ \frac{1}{2} & ; t\geq (n+1)T \end{cases}$

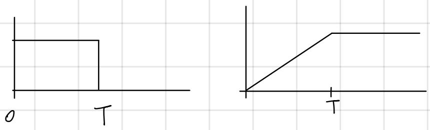


CPM (Continuous Phase Modulation)

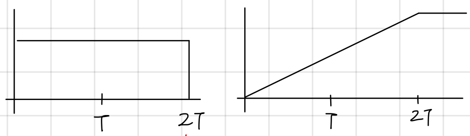
phase微分後沒有脈衝

- CPM is generalization of CPFSK
  - $\phi(t;I_n) = 2\pi \sum^{n} h_k \cdot I_k \cdot q(t-kT)$
    - $\{I_n\}$ is the sequence of PAM symbol in $\{\pm 1, \pm 3 \cdots \pm(M-1)\}$
    - h_k is modulation index, if varies with k, it is called "multi-h CPM"
    - $\begin{cases} \text{If } g(t)=0 \text{ for } t \geq T,\ S_\ell(t) \text{ is called full-response CPM} \\ \text{Otherwise, it is called partial-response CPM} \end{cases}$

Full-response



Partial-response



影響前後訊號
→ 頻譜效率更好

MSK (Minimum Shift Keying)

- MSK is a special case of CPFSK with

$$\boxed{h_k = 2f_d T = \frac{1}{2}\ ,\ g(t) = \frac{1}{2T} \text{ for } 0<t<T\ ,\ I_n \in \{\pm 1\}}$$

- $\phi(t;I_n) = 2\pi \sum^{n} I_k h_k q(t-kT)$

$$= \frac{\pi}{2}\sum^{n-1} I_k + \pi I_n \underbrace{q(t-nT)}_{\frac{t-nT}{2T}}$$

$$= \quad \theta_n \quad + \pi I_n \left(\frac{t-nT}{2T}\right)$$

- $S_{MSK}(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \phi(t;I_n)\right)$

$$= \sqrt{\frac{2E}{T}} \cos\left(2\pi\left(f_c + \frac{I_n}{4T}\right)t - \frac{n\pi I_n}{2} + \theta_0\right)$$

  - Since $I_n \in \{\pm 1\}$, $S_{MSK}(t)$ has two freq component

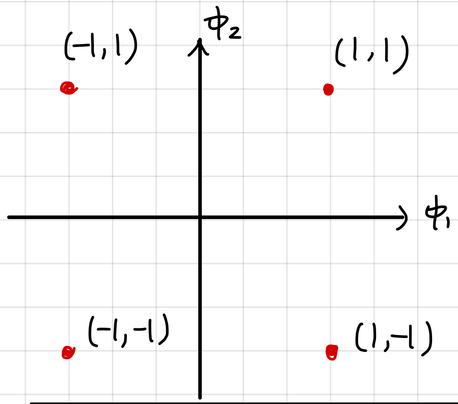
$$\left.\begin{aligned} f_1 &= f_c + \frac{1}{4T} \\ f_2 &= f_c - \frac{1}{4T} \end{aligned}\right\} \Rightarrow \Delta f = \frac{1}{2T}$$

FSK射頻正交時，最小頻率差為$\Delta f = \frac{1}{2T}$

故稱為 MSK

OQPSK

· 傳統 QPSK 的問題：



ex：

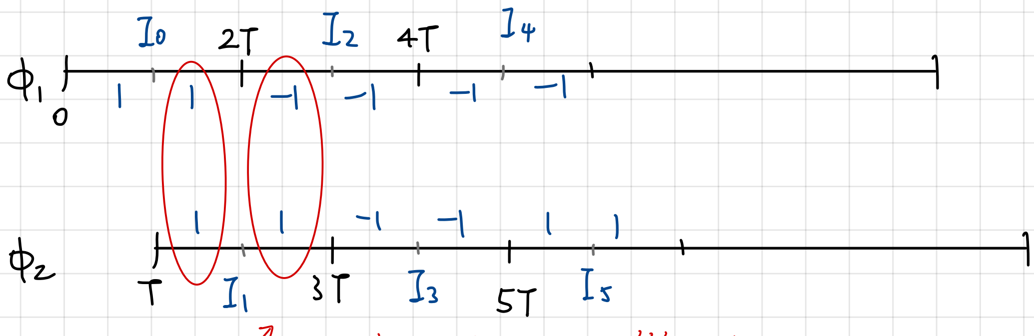
$(I_0\, I_1) = (+1\ +1)$

$(I_2\, I_3) = (-1\ -1)$ 180°

$(I_4\, I_5) = (-1\ +1)$

$$\boxed{S_{QPSK}(t) = \sum I_{2n}\, g(t-2nT)\cos(2\pi f_c t) - \sum I_{2n+1}\, g(t-2nT)\sin(2\pi f_c t)}$$

· OQPSK



相臨時間只有改變一個 bit，

⇒ 不會有 180° 的情況

$$\boxed{S_{OQPSK}(t) = \sum I_{2n}\, g(t-2nT)\cos(2\pi f_c t) - \sum I_{2n+1}\, g(t-(2n+1)T)\sin(2\pi f_c t)}$$

3.4 Power Spectrum of Digital Modulated signal.

- 隨機數位波
- $S_\ell(u,t) = \sum I_n(u) \cdot g(t-nT)$
  - $I_n(u)$: random sequence
  - $g(t)$: pulse shaping function

$$\begin{cases} E[S_\ell(t)] = \mu_I \sum g(t-nT) & \text{mean is time-varying} \\ R_{s\ell}(t+\tau, t) = R_{s\ell}(t+\tau+kT, t+kT) & \text{autocorrelation with period } T \end{cases}$$

$\rightarrow$ cyclostationary process : Mean and autocorrelation are periodic

Time-average autocorrelation function

- $R_{s\ell}(\tau) = \frac{1}{T} \sum_m R_I(m) R_g(\tau - mT)$
  - $R_I(m) = E[I_k(u) I_n^*(u)]$
  - $R_g(\tau) = \int_{-\infty}^{\infty} g(t+\tau) g^*(t)\, dt$

Average Power Spectrum Density

- $\boxed{\overline{S_{s\ell}}(f) = \frac{1}{T} S_I(f) |G(f)|^2}$

EX.1

input information is real and mutually uncorrelated

> **Poisson sum formula:**
> $$\sum e^{-j2\pi f m T} = \frac{1}{T}\sum \delta\left(f-\frac{m}{T}\right)$$

$$R_I(m)=\begin{cases}\sigma_I^2+\mu_I^2\ ; & m=0\\ \mu_I^2\ ; & m\neq 0\end{cases}$$ ← 直流成份

Hence, $S_I(f)=\sigma_I^2+\mu_I^2\boxed{\sum_{m=-\infty}^{\infty} e^{-j2\pi f m T}} = \sigma_I^2+\mu_I^2\boxed{\frac{1}{T}\sum\delta\left(f-\frac{m}{T}\right)}$

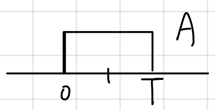
and
$$\boxed{\overline{S_{se}}(f)=\underbrace{\frac{\sigma_I^2}{T}|G(f)|^2}_{\text{continuous}}+\underbrace{\frac{\mu_I^2}{T^2}\sum\left|G\left(\frac{m}{T}\right)\right|^2\delta\left(f-\frac{m}{T}\right)}_{\text{discrete}}}$$

- 1. discrete term vanish, when input information has zero mean: $\mu_I=0$
- 2. With zero mean input information, the average power spectrum density $\overline{S_{se}}(f)$ is determined by $|G(f)|^2$

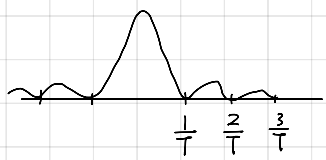
$\Rightarrow \overline{S_{se}}(f)=\frac{\sigma_I^2}{T}|G(f)|^2$

Ex1-1. Average Power Spectrum Density for rectangular pulse

- $g(t)=A[u(t)-u(t-T)]$



- $G(f)=AT\,\text{sinc}(fT)\,e^{-j\pi f T}$; $|G(f)|^2=A^2T^2\,\text{sinc}^2(fT)$

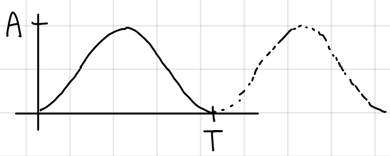


Hense
- $S_I(f)=\sigma_I^2+\mu_I^2\frac{1}{T}\sum\delta\left(f-\frac{m}{T}\right)$
- $\overline{S_{se}}(f)=\frac{1}{T}S_I(f)|G(f)|^2$

$=\sigma_I^2A^2T\,\text{sinc}^2(fT)+\mu_I^2A^2\delta(f)$ $\propto f^{-2}$

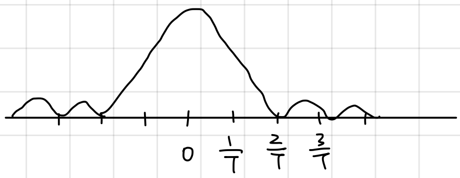
Ex1-2. Average Power Spectrum Density for raised cosine pulse

- $g(t)=\frac{A}{2}\left[1+\cos\left(\frac{2\pi}{T}\left(t-\frac{T}{2}\right)\right)\right](u(t)-u(t-T))$



- $G(f)=\frac{AT}{2}\,\text{sinc}(fT)\,\frac{1}{1-f^2T^2}\,e^{-j\pi f T}$

$|G(f)|^2=\frac{A^2T^2}{4}\,\text{sinc}^2(fT)\,\frac{1}{1-2f^2T^2+f^4T^4}$



Hense
- $S_I(f)=\sigma_I^2+\mu_I^2\frac{1}{T}\sum\delta\left(f-\frac{m}{T}\right)$
- $\overline{S_{se}}(f)=\frac{1}{T}S_I(f)|G(f)|^2$ $\propto f^{-6}$

When $\{I_n\}$ is correlated : $I_n = b_n + b_{n-1}$

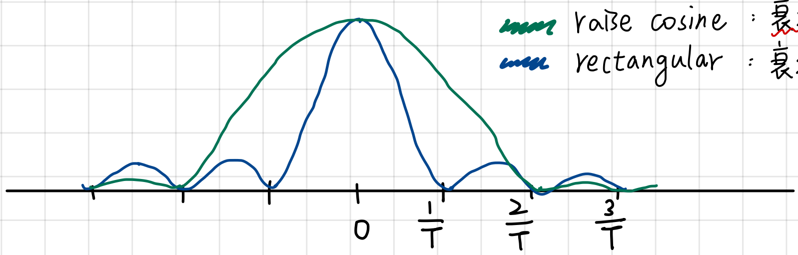
- $R_I(m) = \begin{cases} 2 \ ; & m=0 \\ 1 \ ; & m=\pm 1 \\ 0 \ ; & \text{other} \end{cases}$ $\quad |G(f)|^2 = A^2 T^2 \operatorname{sinc}^2(fT)$
- $S_I(f) = 2 + e^{j2\pi fT} + e^{-j2\pi fT} = 2(1+\cos(2\pi fT)) = 4\cos^2(\pi fT)$
- $\overline{S_{se}}(f) = \frac{1}{T} S_I(f) \cdot |G(f)|^2 = \frac{4}{T}\cos^2(\pi fT)|G(f)|^2$

觀念彙總:

(i) 同為 uncorrelated, rectangular 和 raised cosine 的差異

raise cosine : 衰減快,但 mainlobe 寬 ($\propto f^{-6}$)

rectangular : 衰減慢,但 mainlobe 窄 ($\propto f^{-2}$)



→ The smoother pulse shaping (raise cosine) ⇒ the greater bandwidth efficiency

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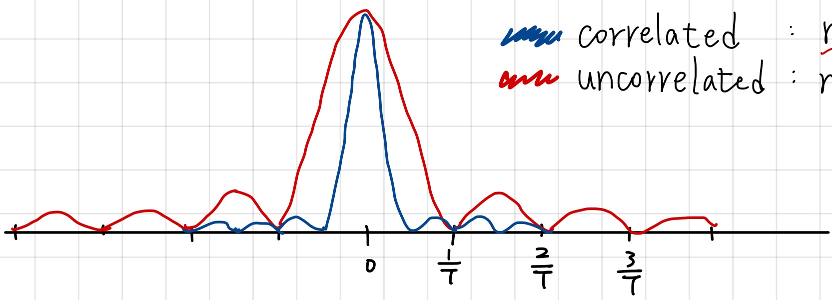
(ii) 同為 rectangular, correlated 和 uncorrelated 的差異

correlated : mainlobe 窄 且, 衰減快

uncorrelated : mainlobe 寬 且 衰減慢



(iii)

- Modulation index h :

  The lower modulation index h
  → The higher bandwidth efficiency ← MSK 的 h = 0.5

- Pulse shape g(t) :

  The smoother g(t)
  → The greater bandwidth efficiency ← raise cosine result in higher bandwidth efficiency than rectangular