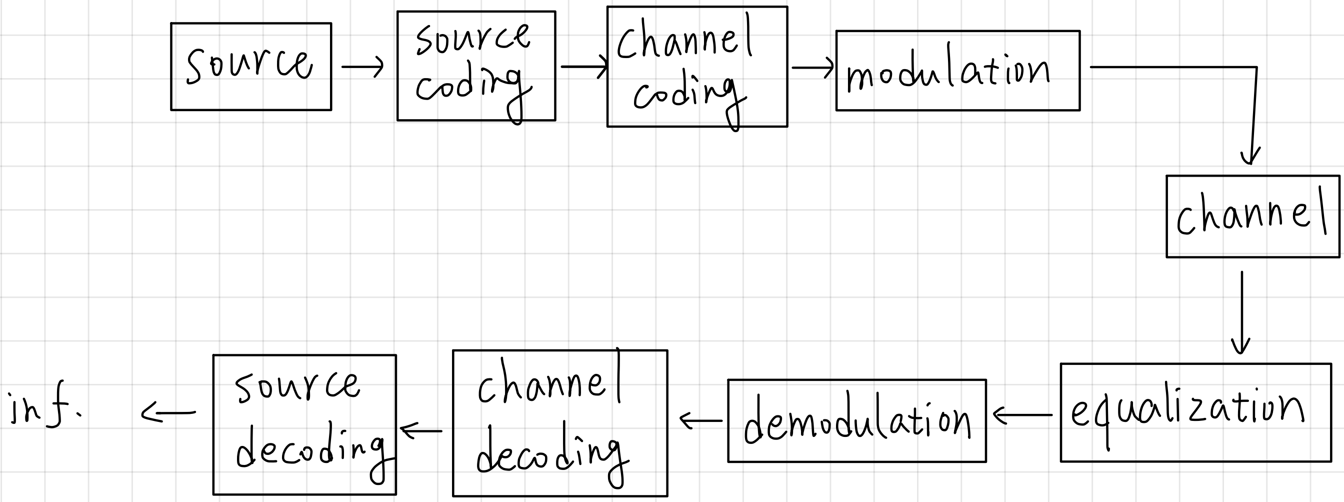
Wireless Communication Ch11

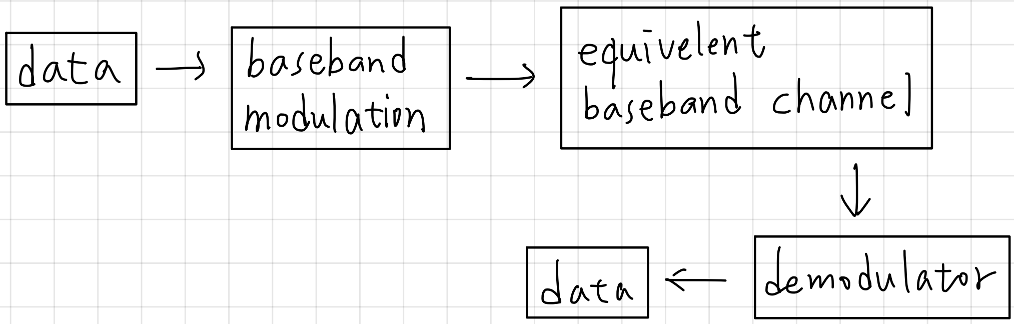
2024 Fall

Ch11

Signal Space description and digital demodulation

Wireless transceiver block diagram.





Basic idea of modulation

- Given K bits and $M=2^k$ distinguishable wave form.
- Map each bit sequence to a waveform at Tx
- At RX. we receive the signal, and try to map it back to the correspounding bit sequence

bandpass signal is $S_{BP}(t) = Re\{S_{LP}(t)\exp(j2\pi f_c t)\}$

$$= S_{LP}^{R}(t)\cos(2\pi f_c t) - S_{LP}^{I}(t)\sin(2\pi f_c t)$$

— $S_{LP}(t) = S_{LP}^{R}(t) + j\,S_{LP}^{I}(t)$

> Note
> $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are almost orthogonal.
> if symbol duration T_s is much larger than $\frac{1}{f_c}$: narrowband

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Pulse Amplitude Modulation (PAM)

$$S_{LP}(t) = \sum_{i=-\infty}^{\infty} C_i\, g(t-nT)$$

- The transmitted signal consist of a series of basic-pulse shape and relvent sequence of symbol to transmit

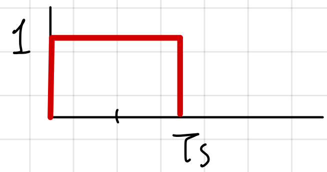
The modulated symbol are C_i, θ_i are independ

— $g(t)$ is pulse shaping function.

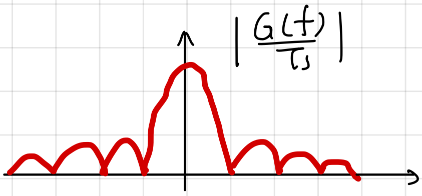
$$\frac{1}{T_s}\int_{-\infty}^{\infty} g(t)\,dt = 1$$

Rectangular pulse

- $g(t) = g_R(t) = \begin{cases} 1 \;;\; 0 \le t \le T_s \\ 0 \;;\; \text{other} \end{cases}$



- $G(f) = G_R(f, T_s) = T_s\,\mathrm{sinc}(\pi f T_s)\, e^{-j\pi f T_s}$



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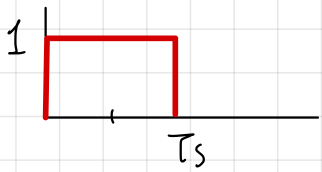
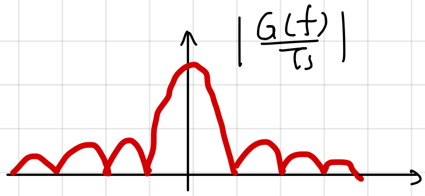




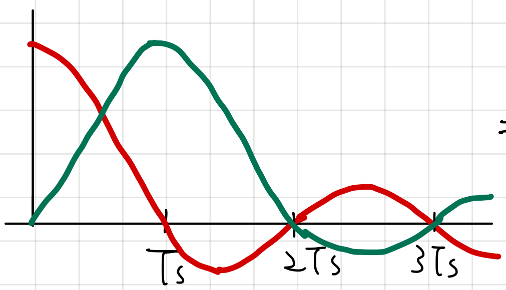
- Nice pulse shape on time-domain
- large sidelobe on freq- domain.

Nyquist Pulse

- To give a lower adj. channel interference, we need to have small sidelobe on freq domain.

- One of the most commonly used class of pulse is Nyquest pulse

Nyquest pulse : $g(nT_s) = \begin{cases} 1 & ; n = 0 \\ 0 & ; n \ne 0 \end{cases}$



$\Rightarrow$ NOT interfere with one another when used transmit different Modulated symb.

Raised-cosine pulse

$$G_{N0}(f,\alpha,T_s)$$

$$= \begin{cases} 1 \; ; & 0 \le |2\pi f| < (1-\alpha)\frac{\pi}{T_s} \\ \frac{1}{2}\left[1 - \sin\left(\frac{T_s}{2\alpha}\left(|2\pi f| - \frac{\pi}{T_s}\right)\right)\right] \; ; & (1-\alpha)\frac{\pi}{T_s} \le |2\pi f| \le (1+\alpha)\frac{\pi}{T_s} \\ 0 \end{cases}$$

The used Raise-cosine Normalization factor

$$\begin{cases} G_N(f,\alpha,T_s) = \frac{T_s}{\sqrt{1-\frac{\alpha}{4}}} G_{N0}(f,\alpha,T_s)\, e^{-j\pi f T_s} \\ h(t) = \begin{cases} \frac{\pi}{4T_s}\,\mathrm{sinc}\left(\frac{1}{2\alpha}\right) & ; t \le \pm \frac{T_s}{2\alpha} \\ \frac{1}{T_s}\,\mathrm{sinc}\left(\frac{t}{T_s}\right)\frac{\cos\left(\frac{\pi\alpha t}{T_s}\right)}{1-\left(\frac{2\alpha t}{T_s}\right)^2} & ; \text{other} \end{cases} \end{cases}$$

- Control α to provide some trade off between time-domain and freq-domain

Root Raise-cosine function.

- In many application, we need "matched filter" at begining RX, to maximum SNR

matched filter.

$$\longrightarrow \boxed{G(f)} \longrightarrow \boxed{H(f)} \longrightarrow \boxed{G^*(f)} \longrightarrow \text{process}$$

- Root-Raise-Cosine

$G_{RN}(f,\alpha,T_s) = \sqrt{G_N(f,\alpha,T_s)} \rightarrow$ to more specific, we need

$$\Rightarrow \boxed{G_{RN}(f)\, G_{RN}^*(f) = G_N(f)}$$

then, Nyquest criterion is still hold with matched filter

Extension Ex 11.1

Suppose we have a 2-user system,

Each transmit their signal to BS with matched filter

Consider 接收端為 $G_N(f)G_N^*(f)$

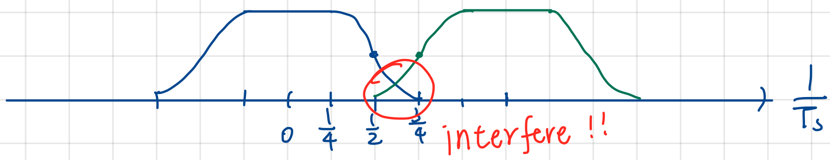
(i) using rasied-cosine

(ii) using root-raised-cosine

(iii) $\alpha = 0.5$

(iv) sparated by $\frac{1.25}{T_s}$

(a) suppose $T_s = 1$, what is SIR ; $\alpha = 0.5$



Raise-cosine

Signal power

$$S_N = \int_{-\frac{0.75}{T_s}}^{\frac{0.75}{T_s}} |G_N(f)G_N^*(f)|^2 df = 0.77$$

Interfere power :

$$I_N = \int_{0.5}^{\frac{0.75}{T_s}} |G_N(f) \cdot G_N(f - \tfrac{1.25}{T_s})|^2 df = 0.45 \times 10^{-4}$$

$SNR_N = 42\,dB$

$SNR_{RN} = 35\,dB$

$\Rightarrow$ $SNR_N > SNR_{RN}$

Root Raise cosine :

Signal Power : 0.875

Interfere Power : 2.8×10^{-3}

Signal space diagram

- Present the modulation format using a N-dimensional space as discrete point.
- All modulation format with the same signal representation are equivalent time $\longleftrightarrow$ signal space

Define a signal space

1) Define a set of expansion function that can describe the modulation waveform $\{\phi_n(t)\}_{n=1}^{N}$

2) The set satisfies:

$$\int_0^{T_s} \phi_n(t) \cdot \phi_m^*(t)\, dt = \delta[n-m] = \begin{cases} 1 & ; n=m \\ 0 & ; n \neq m \end{cases}$$

$\rightarrow$ we want orthonormal set.

3) A modulation waveform can be describe as

$$S_m(t) = \sum_{n=1}^{N} S_{m,n}\, \phi_n(t) \quad , \quad S_{m,n} = \int_0^{T_s} S_m(t) \cdot \phi_n^*(t)\, dt$$

$$S_m = [S_{m,1}\ S_{m,2}\ \cdots\ S_{m,N}]^T$$

If we have passband signal. we have

$$\begin{cases} \phi_{BP,1}(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) \\ \phi_{BP,2}(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t) \end{cases}$$

They can converted to have their equivalent baseband represent:

$\phi_1(t) = \sqrt{\frac{1}{T_s}}$ in real domain

$\phi_2(t) = \sqrt{\frac{1}{T_s}}$ in image domain.

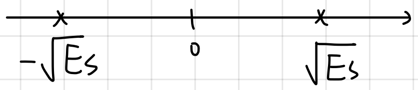
Important modulation format.

◦ BPSK

- $S_{BP}(t) = \sqrt{\frac{2E_s}{T_s}} \cdot P_D \cdot \cos(2\pi f_c t)$

$P_D = \sum_{i=-\infty}^{\infty} b_i \cdot g(t-iT)$; $b_i \in \{\pm 1\}$, $g(t) = g_R(t, T_s)$

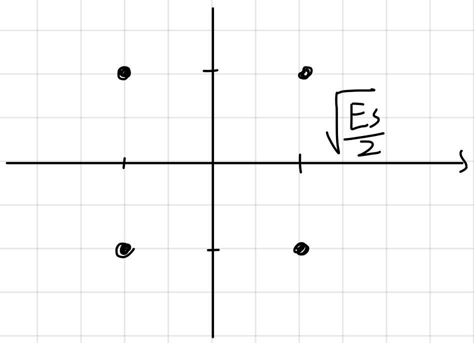
- $S_{LP}(t) = \sqrt{\frac{E_s}{T_s}} \cdot P_D(t) = \sqrt{\frac{E_s}{T_s}} \sum_{i=-\infty}^{\infty} b_i \cdot g(t-iT_s)$



◦ QPSK

- The original bit stream is slipt into 2 sub-stream.
- At each symbol, we transmit 2 bit

$S_{LP}(t) = [P_{1D}(t) + jP_{2D}(t)]\sqrt{\frac{E_s}{2T_s}}$



$E_b = \frac{1}{2}E_s$

- If we have energy symbol : E_s,

the energy per bit is : $\frac{1}{2}E_s$

- To make a fair compairsion between different format

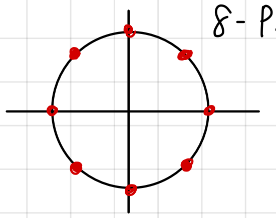
under the same energy per bit, instead of energy per symbol.

◦ M-PSK

$$S_{LP}(t) = \sqrt{\frac{2E_s}{T_s}} \exp\left(j \frac{2\pi}{M}(m-1) \right) \; ; \; m = 1, 2 \cdots M$$

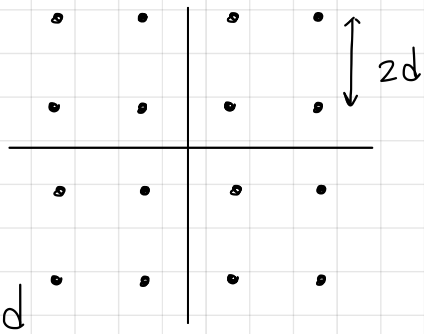
$$= \left[\cos\left(\frac{2\pi}{M}(m-1) \right), \sin\left(\frac{2\pi}{M}(m-1) \right) \right] \sqrt{E_s}$$

8-PSK



◦ High order QAM

16-QAM



- To transmit more bit per symbol.
- multiple amplitude and phase level.

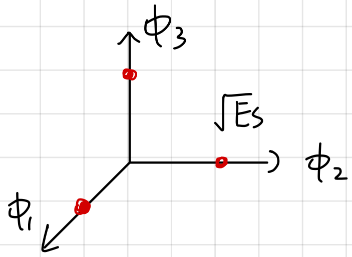
→ M-QAM

$$S_{LP}(t) = (2m_1 + 1 - \sqrt{M})d + j(2m_2 - 1 - \sqrt{M})d$$

$$\leftrightarrow \left[2m_1 + 1 - \sqrt{M}, \; 2m_2 - 1 - \sqrt{M} \right] d$$

$$m_1, m_2 = [1, 2, \cdots \sqrt{M}]$$

◦ BFSK



◦ In FSK, different modulation symbol are transmit using different freq.

BFSK : $\tilde{f}_c = f_c \pm f_{mod}$

$$\left.\begin{aligned} \phi_{BP,1}(t) &= \sqrt{\frac{2}{T_s}} \cos(2\pi (\underline{f_c + f_{mod}}) t) \\ \phi_{BP,2}(t) &= \sqrt{\frac{2}{T_s}} \cos(2\pi (\underline{f_c - f_{mod}}) t) \end{aligned}\right\} \text{they are orthogonal.}$$

- Differential - BPSK.
- Non coherent BPSK. Non coherent BFSK.
- On-OFF modulation (visiable light communication)

Example 11.2 (力爭上游)

Suppose we have a 16-QAM with distance $2d$

What is the $\overline{E_s}$ (average energy) in terms of d for 16 QAM

(Assume every symbol is transmit with equal probability)

- $\overline{E_s} = \frac{1}{4} d^2 \left[(1+1)+(1+9)+(1+9)+(9+9)\right] = 10 d^2 \Rightarrow d = \sqrt{\frac{E_s}{10}}$

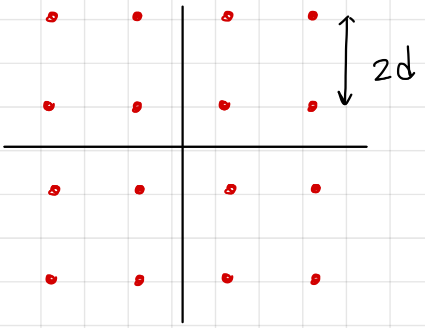
- Following this example. we compair 16QAM with 16-PSK, Suppose $E_b = 10$

a) Find $E_s = E_b \cdot 4 = 40$

b) Expression 16-QAM, 16-PSK.

c) compair d_{min} and Peak Average Power Ratio (PAPR)

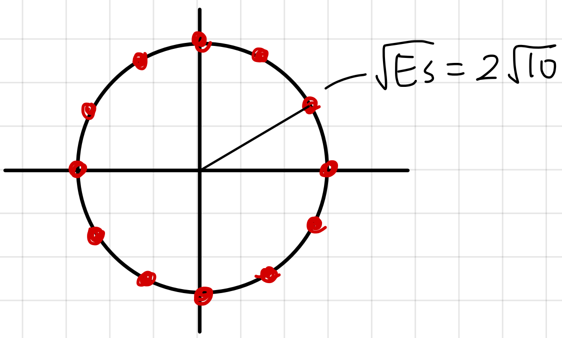
16-QAM



$d = \sqrt{\frac{E_s}{10}} = 2$

- $d_{min} = 4$
- $PAPR = \frac{9d^2 + 9d^2}{10d^2} = 1.8$

16-PSK



$\left[2\sqrt{10}\cos\left(\frac{2\pi}{16}(m-1)\right),\ 2\sqrt{10}\sin\left(\frac{2\pi}{16}(m-1)\right)\right]$

- $d_{min} = 2\sqrt{E_s}\sin\left(\frac{\pi}{16}\right) \approx 2.46$
- $PAPR = 1$