HUCENROTIA LAB

Machine Learning

LABORATORY: Gradient Descent Homework

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Objectives:

- Understand and implement Mini-batch SGD (Algorithm 7.2).
- Extend the algorithm to support Momentum (Algorithm 7.3) and Adam optimization (Algorithm 7.4).
- Apply each optimizer to a binary classification task using the MNIST dataset.
- Evaluate and compare model behavior through accuracy and misclassified samples.
- Practice implementing mathematical update rules directly from textbook equations using NumPy.

Part 1. Instruction

- In this assignment, you will implement **Mini-batch Stochastic Gradient Descent (SGD)** and its extensions using **Algorithms 7.2, 7.3, and 7.4**.
- Your task is to build a **binary classifier** to determine whether an MNIST image matches a specific digit or not (e.g., "Is this a 4 or not?").
- You will implement **three** different methods: **Mini-batch SGD** (Algorithm 7.2), **SGD with Momentum** (Algorithm 7.3) and **Adam Optimizer** (Algorithm 7.4)
- You may write all algorithms in **one file with selectable modes**, or in **three separate files**.
- The code must be implemented **entirely with NumPy**. Do not use external machine learning libraries (e.g., scikit-learn, PyTorch).
- The model should output:
  - Final **accuracy** on the test set.
  - At least **five misclassified test samples**, with true and predicted labels shown.
- Use the **last digit of your student ID** as the `TARGET_DIGIT` for binary classification (e.g., ID ending in 7 → `TARGET_DIGIT = 7`).

Part 2. Arithmetic Instructions.

Algorithm 7.2: Mini-batch stochastic gradient descent

Input: Training set of data points indexed by $n \in \{1, \ldots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Initial weight vector $\mathbf{w}$
Output: Final weight vector $\mathbf{w}$

```
n ← 1
repeat
    w ← w − η∇E_{n:n+B−1}(w) // weight vector update
    n ← n + B
    if n > N then
        shuffle data
        n ← 1
    end if
until convergence
return w
```

Algorithm 7.3: Stochastic gradient descent with momentum

Input: Training set of data points indexed by $n \in \{1, \ldots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Momentum parameter μ
Initial weight vector $\mathbf{w}$
Output: Final weight vector $\mathbf{w}$

```
n ← 1
Δw ← 0
repeat
    Δw ← −η∇E_{n:n+B−1}(w) + μΔw // calculate update term
    w ← w + Δw // weight vector update
    n ← n + B
    if n > N then
        shuffle data
        n ← 1
    end if
until convergence
return w
```

Algorithm 7.4: Adam optimization

Input: Training set of data points indexed by $n \in \{1, \dots, N\}$
Batch size B
Error function per mini-batch $E_{n:n+B-1}(\mathbf{w})$
Learning rate parameter η
Decay parameters β_1 and β_2
Stabilization parameter δ
Output: Final weight vector $\mathbf{w}$

$n \leftarrow 1$
$\mathbf{s} \leftarrow \mathbf{0}$
$\mathbf{r} \leftarrow \mathbf{0}$
repeat
  Choose a mini-batch at random from $\mathcal{D}$
  $\mathbf{g} = -\nabla E_{n:n+B-1}(\mathbf{w})$ `// evaluate gradient vector`
  $\mathbf{s} \leftarrow \beta_1 \mathbf{s} + (1-\beta_1)\mathbf{g}$
  $\mathbf{r} \leftarrow \beta_2 \mathbf{r} + (1-\beta_2)\mathbf{g} \odot \mathbf{g}$ `// element-wise multiply`
  $\widehat{\mathbf{s}} \leftarrow \mathbf{s}/(1-\beta_1^\tau)$ `// bias correction`
  $\widehat{\mathbf{r}} \leftarrow \mathbf{r}/(1-\beta_2^\tau)$ `// bias correction`
  $\Delta\mathbf{w} \leftarrow -\eta \frac{\widehat{\mathbf{s}}}{\sqrt{\widehat{\mathbf{r}}}+\delta}$ `// element-wise operations`
  $\mathbf{w} \leftarrow \mathbf{w} + \Delta\mathbf{w}$ `// weight vector update`
  $n \leftarrow n + B$
  if $n + B > N$ **then**
    shuffle data
    $n \leftarrow 1$
  end if
until convergence
return $\mathbf{w}$

Part 3. Code Template.

Step	Procedure

1	#Load Dataset
	`import struct`
	`import numpy as np`
	`import matplotlib.pyplot as plt`
	
	`# ==========Load IDX Files ==========`
	`def load_images(filename):`
	`    with open(filename, 'rb') as f:`
	`        _, num, rows, cols = struct.unpack(">IIII",`
	`f.read(16))`
	`        images=np.frombuffer(f.read(),`
	`dtype=np.uint8)`
	`        images = images[:(len(images)//(rows * cols))`
	`* rows * cols]`
	`        return images.reshape(-1, rows *`
	`cols).astype(np.float32) / 255.0`
	
	`def load_labels(filename):`
	`    with open(filename, 'rb') as f:`
	`        _, num = struct.unpack(">II", f.read(8))`
	`        labels = np.frombuffer(f.read(),`

<table>
<tr><td></td><td><pre>dtype=np.uint8)
        return labels[:num]</pre></td></tr>
<tr><td>2</td><td><pre># ========== 1. Sigmoid Function ==========
def sigmoid(z):
    # TODO: Implement sigmoid function
    pass

====== 2. Mini Batch SGD: Algorithm 7.2 ======
def sgd_minibatch(X, y, eta=0.01, max_iters=10000,
batch_size=64):

    pass

===== 3. Mini Batch SGD with Momentum: Algorithm 7.3 =====
def sgd_minibatch_momentum(X, y, eta=0.01, max_iters=10000,
batch_size=64, momentum=0.9):

    pass

====== 4. Adam Optimizer: Algorithm 7.4 ======
def sgd_Adam(X, y, eta=0.001, max_iters=10000, batch_size=64,
beta1=0.9, beta2=0.999, delta=1e-8):

    pass</pre></td></tr>
<tr><td>3</td><td><pre># ==========Show Misclassified Samples ==========
def show_misclassified(X, true_labels, pred_labels,
max_show=10):
    mis_idx = np.where(true_labels !=
pred_labels)[0][:max_show]
    plt.figure(figsize=(10, 2))
    for i, idx in enumerate(mis_idx):
        plt.subplot(1, len(mis_idx), i + 1)
        plt.imshow(X[idx, 1:].reshape(28, 28), cmap='gray')
        plt.axis('off')
        plt.title(f"T:{true_labels[idx]}
P:{pred_labels[idx]}")
    plt.suptitle("Misclassified Samples")
    plt.show()</pre></td></tr>
<tr><td>4</td><td><pre># ========== 3. Main ==========
if __name__ == "__main__":
    # === Load Data ===
    X_train = load_images("train-images.idx3-ubyte__")
    y_train = load_labels("train-labels.idx1-ubyte__")
    X_test = load_images("t10k-images.idx3-ubyte__")
    y_test = load_labels("t10k-labels.idx1-ubyte__")

    # === Choose binary classification target digit ===
    TARGET_DIGIT = 0   # TODO: Fill in (0 to 9)

    y_train_bin = np.where(y_train == TARGET_DIGIT, 1, 0)
    y_test_bin = np.where(y_test == TARGET_DIGIT, 1, 0)</pre></td></tr>
</table>

	<pre># === Add bias term ===
	    X_train = np.hstack([np.ones((X_train.shape[0], 1)), X_train])
	    X_test = np.hstack([np.ones((X_test.shape[0], 1)), X_test])
	
	    # === Set parameters ===
	
	    # === Train ===
	
	    # === Predict ===
	
	    # === Evaluate ===
	
	    # === Show Misclassified Samples ===</pre>

Grading Assignment & Submission (70% Max)

Implementation(50%):

1. **Correctly** implemented, runs, shows accuracy and sample misclassification of:
    a. **(15%) Mini-batch SGD (Algorithm 7.2)**
    b. **(10%) SGD with momentum (Algorithm 7.3)**
    c. **(5%) SGD with Nesterov momentum (Eq. 7.34)**
    d. **(15%) Adam Optimizer (Algorithm 7.4)**
2. **(5%)** Compare the accuracy and test sample for each algorithm.

Question(20%):

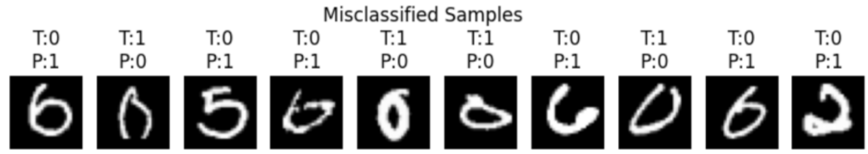
1. (7%) Which optimizer gave you the best test accuracy? Why do you think it performed better than the others?
2. (8%) What is the differences in learning stability, convergence speed, or misclassification types across all algorithm? Please explain with examples or observation from your results.
3. (7%) How did your choice of learning rate, batch size, or momentum affect each optimizer? What values worked best in your experiments?

Submission :

1. Report: Answer all conceptual questions. Include screenshots of your results in the last pages of this PDF File.
2. Code: Submit your complete Python script in either .py or .ipynb format.
3. Upload both your report and code to the E3 system (**Labs4_Homework_Assignment**). Name your files correctly:
    a. Report: StudentID_Lab4_Homework.pdf
    b. Code: StudentID_Lab4_Homework.py or StudentID_Lab4_Homeworkipynb
4. Deadline: Sunday, 21:00 PM
5. Plagiarism is **strictly prohibited**. Submitting copied work from other students will result in penalties.

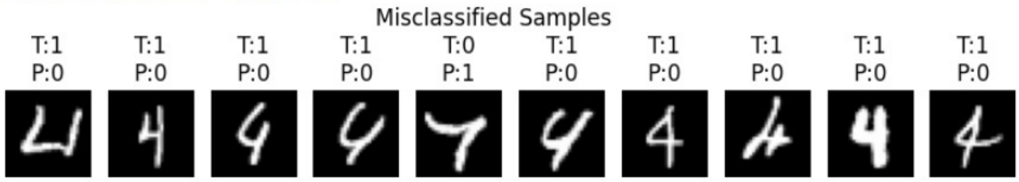
Example Output (Just for reference):

```
[INFO] Header: 60000 images, 28x28
[INFO] Loading 60000 images based on file size
[INFO] Loading 60000 labels based on file size
[INFO] Header: 10000 images, 28x28
[INFO] Loading 10000 images based on file size
[INFO] Loading 10000 labels based on file size

[INFO] Binary classification: '0' vs not-0

Test Accuracy (is 0 or not): 0.9927
```

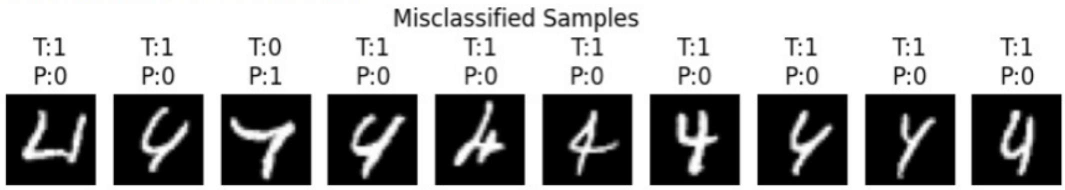


Code Results and Answer:

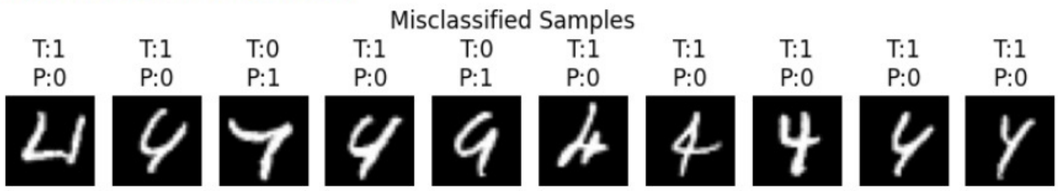
```
Algorithm : Mini-batch SGD
Test Accuracy (is 4 or not) : 0.9722
```



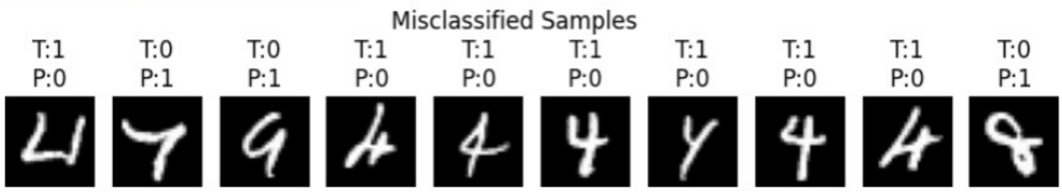

```
Algorithm : SGD with momentum
Test Accuracy (is 4 or not) : 0.9802
```




```
Algorithm : SGD with Nesterov momentum
Test Accuracy (is 4 or not) : 0.9792
```




```
Algorithm : Adam Optimizer
Test Accuracy (is 4 or not) : 0.9827
```



1. (7%) Which optimizer gave you the best test accuracy? Why do you think it performed better than the others?

According to the evaluation results:

Optimizer	Test Accuracy
Mini-batch SGD	0.9722
SGD with Momentum	0.9802
SGD with Nesterov Momentum	0.9792
Adam Optimizer	**0.9827** ✅

The best test accuracy was achieved by **Adam Optimizer (0.9827).** I believe Adam outperformed the other optimizers for the following reasons:

- Adam combines **momentum** (first-order gradient) and **RMSprop** (second-order moment estimation), enabling it to adaptively adjust the learning rate and converge faster and more reliably.
- On challenging examples from the MNIST dataset—particularly distorted or slanted instances of the digit "4"—Adam can leverage historical gradient information to better fine-tune the update step, helping avoid over-adjustment or under-adjustment.
- Due to its built-in adaptive mechanism, Adam is less sensitive to hyperparameters like batch size or learning rate, which contributes to its overall stability and strong performance.

2. (8%) What is the differences in learning stability, convergence speed, or misclassification types across all algorithms? Please explain with examples or observation from your results.

🔹 Learning Stability & Convergence Speed

- **Mini-batch SGD**: Slowest convergence among all methods; the update direction is easily affected by noise. Frequently misclassifies digits that look similar to 4, such as 9 or 7.
- **Momentum**: More stable than plain SGD. It suppresses oscillations and accelerates convergence, resulting in better accuracy and more consistent updates.
- **Nesterov Momentum**: Provides a "look-ahead" correction ability, achieving slightly faster convergence. However, in this task, its final accuracy is slightly lower than standard momentum, possibly because the advantage of lookahead is limited in binary classification.
- **Adam**: Fastest and most stable. Training requires little to no manual tuning of parameters, and convergence is smooth throughout.

🔹 Misclassification Analysis

From the visualized misclassified samples:

- All methods misclassify digits **"that look like 4"**, especially distorted versions of 9 or 7.
- **Adam** misclassified the fewest samples, and mostly only when the digit shapes were extremely distorted or ambiguous.
- **Momentum** and **Nesterov** produced similar types of misclassification, indicating that they behave similarly in handling gradient dynamics.
- **SGD** showed a more scattered pattern in misclassification, suggesting that its decision boundary is less sharp and well-defined.

3. (7%) How did your choice of learning rate, batch size, or momentum affect each optimizer? What values worked best in your experiments?

◆ Learning Rate (η)

- For **SGD-based optimizers**, the best performance was achieved with **η = 0.01**. A higher rate (e.g., 0.1) caused severe oscillations, while a smaller rate (e.g., 0.001) slowed down convergence significantly.
- For **Adam, η = 0.001** worked best. A larger learning rate would interfere with Adam's internal adaptive learning mechanism.

◆ Batch Size

- A **batch size of 64** achieved a good trade-off between gradient noise and stability.
- Smaller batches (e.g., 32) introduced more randomness into gradient updates. This may help escape local minima, but it also reduced accuracy slightly.
- Larger batches (e.g., 128) slowed convergence and increased the risk of overfitting.

◆ Momentum

- For **Momentum** and **Nesterov**, using **μ = 0.9** resulted in the best performance.
- When tested with **μ = 0.5**, convergence was much slower and frequently got stuck.
- When tested with **μ = 0.99**, overshooting or unstable acceleration occurred.