Wireless Communication

Ch9

2024 Fall

Antenna Directivity and gain

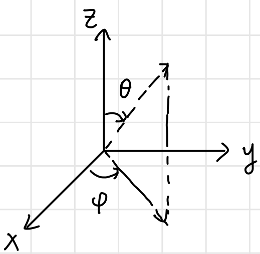
Antenna Directivity

- a measure of how much a transmit antenna concentrates the emitted radiation to a certain direction.
- Defined for far-field

$$D(\Omega) = \frac{\text{Power into unit angle in a direction } \Omega}{\text{Total power into the sphere}}$$

D: directivity pattern Ω: spherical angle

antenna element gain



θ : elevation angle

φ : azimuth angle

- Sometimes, we do normalization, such that

$$\frac{1}{4\pi}\int D(\Omega)\, d\Omega = 1$$

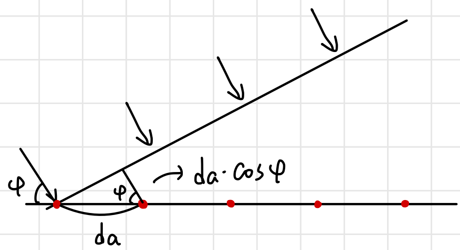
$$\frac{1}{4\pi}\iint D(\theta,\varphi)\sin\theta \, d\theta \, d\varphi = 1$$

- Antenna gain on a direction Ω

$\Rightarrow G(\Omega) = D(\Omega)\,\eta$ → antenna efficiency

Antenna Array gain

- If we have multiple antenna element, we can use them to futher concentrate the energy on a specific direction by construction
- Suppose we have a uniform linear array at RX
- Suppose far-field assumption is valid, and thus we have a planner wave at RX



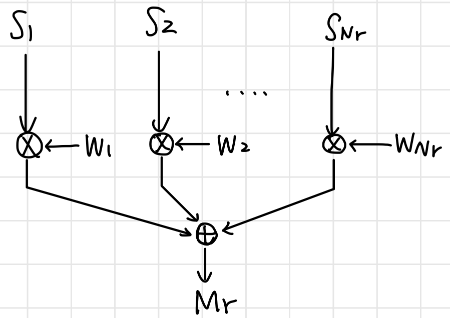
- carrier in complex form is $Ae^{-j2\pi f_c t}$
- Suppose the time signal arrives at element 1 is t_1

  Then, for element k, we know

$$t_k = t_1 + \frac{d_a (k-1) \cos\varphi}{c} = t_1 + (k-1)\frac{d_a}{\lambda_c f_c} \cos\varphi$$

$\Rightarrow$ Signal at element.

$$S_k = Ae^{-j2\pi f_c \left[t_1 + (k-1)\frac{d_a}{\lambda_c f_c}\cos\varphi\right]} = Ae^{-j2\pi f_c t_1} \cdot e^{-j\frac{2\pi}{\lambda_c}(k-1) d_a \cos\varphi}$$



$$\begin{cases} M_r(\varphi) = \left[\sum_{k=1}^{N_r} W_k e^{-j\frac{2\pi}{\lambda_c}(k-1)d_a\cos\varphi}\right] A e^{-j2\pi f_c t_1} \\ |M_r(\varphi)| = A\left|\sum_{k=1}^{N_r} W_k e^{-j\frac{2\pi}{\lambda_c}(k-1)d_a\cos\varphi}\right| \end{cases}$$

- If we let $W_k = e^{j(k-1)\Delta}$; $\alpha = \frac{2\pi}{\lambda_c} d_a \cos\varphi$

$$|M_r(\varphi)| = A\left|\sum_{k=1}^{N_r} e^{-j(\alpha-\Delta)(k-1)}\right|$$

- By letting $\underline{\Delta = \alpha}$, we have

$$|M_r(\varphi)| = A N_r \rightarrow \text{signal power} = \underline{A^2 N_r^2}$$

- Suppose we have noise rv $n_k \sim N(0, \sigma_n^2)$ the noise signal after process is:

$$N = \sum_{k=1}^{N_r} W_k n_k$$

also Gaussian with $\underline{N(0, N_r\sigma_n^2)}$

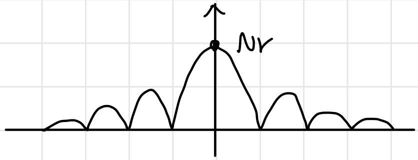
- $$SNR = \frac{A^2 N_r^2}{N_r \sigma_n^2} = N_r \cdot \frac{A^2}{\sigma_n^2}$$

the SNR if only have single antenna

antenna array gain

• If $\Delta \neq \alpha$, we have

$$|M_r(\varphi)| = \left| \frac{\sin\left[\frac{N_r}{2}(\alpha-\Delta)\right]}{\sin\left[\frac{1}{2}(\alpha-\Delta)\right]} \right| \longrightarrow \text{eq } 9.17$$



$D(\Omega)$ $\qquad$ $\frac{A^2}{\sigma_n^2}$

• By consider both antenna element gain & antenna array gain the overall antenna gain is

$$G_{overall}(\Omega) = G_{array}(\Omega) \cdot G_{element}(\Omega)$$

EX.

Suppose $P_t = 1W$. $G_{TX} = G_{RX}$ and the effective antenna area at RX is $A_{ant} = 33cm^2$

What is P_{RX} at $d = 1m$ when using the Friis' law at $f_c = 30MHz$, $300GHz$?

Friis' law : $P_{RX}(d) = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda_c}{4\pi d}\right)^2$

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Hint: Friis' law: $P_{Rx}(d) = P_{Tx} G_{Tx} G_{Rx} \left(\frac{\lambda_c}{4\pi d}\right)^2$

(1) $f_c = 30MHz$

$$G_{RX} = \frac{4\pi}{\lambda_c^2} A_{ant} = \frac{4\pi}{\lambda_c^2} \cdot 33 \times 10^{-4} = G_{TX}$$

$$P_{Rx}(1) = 1 \cdot \left(\frac{4\pi}{\lambda_c^2} 33 \times 10^{-4}\right)^2 \cdot \left(\frac{\lambda_c}{4\pi}\right)^2 = \frac{1}{\lambda_c^2} (33 \times 10^{-4})^2 = 1.1 \times 10^{-7} W$$

(2) $f_c = 300GHz$

$P_{Rx}(1) = 11W$ is unreasonable, so we check its Rayleigh distance

$$d_R = \frac{2 \cdot 33 \times 10^{-4}}{\lambda_c} = 6.6m$$

$1 < 6.6$, is not at far-field, so that model is not valid #