Wireless Communication Ch6

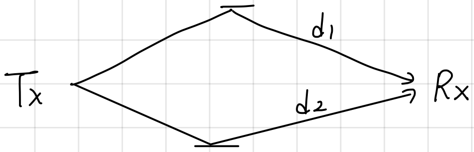
2024 Fall

Wideband channel Characterization.

Midterm 11/14
1320 ~ 1620

- $r = \underline{\underline{h}} \cdot s + n$ with fading distribution.

- Two-path model.



$r(t) = [a_1 s(t-\tau_1) + a_2 s(t-\tau_2)] \, e^{j2\pi f_c t}$

if we consider the equivalent baseband channel model, we have

- $h(t) = a_1' \delta(t-\tau_1) + a_2' \delta(t-\tau_2)$

$a_1' = |a_1| e^{j\phi_1}$

- $H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} \, dt$ $\qquad a_2' = |a_2| e^{j\phi_2}$

$$= \int_{-\infty}^{\infty} [a_1' \delta(t-\tau_1) + a_2' \delta(t-\tau_2)] \, e^{-j2\pi f t} \, dt$$

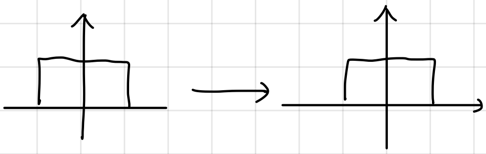
$$= a_1' e^{-j2\pi f \tau_1} + a_2' e^{-j2\pi f \tau_2}$$

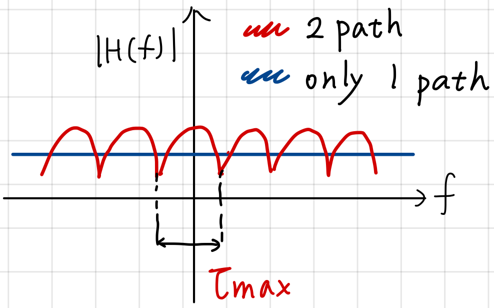
- $|H(f)| = \sqrt{|a_1|^2 + |a_2|^2 + 2|a_1| \cdot |a_2| \cos(2\pi \underline{\underline{f}} \cdot \Delta\tau - \Delta\varphi)}$

$\Delta\tau = \tau_2 - \tau_1$, $\Delta\varphi = \varphi_2 - \varphi_1$ $\qquad$ function of f.

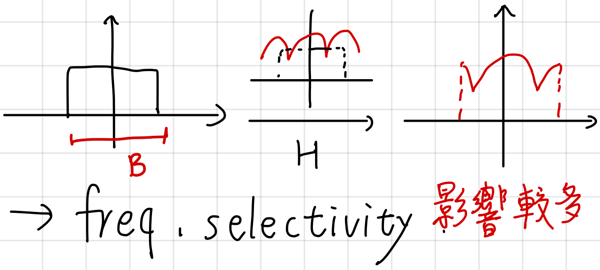
$|H(f)|$ is a function f

- If only one path



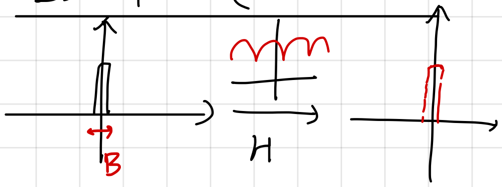


- If 2 path (wideband)



→ freq. selectivity 影響較多

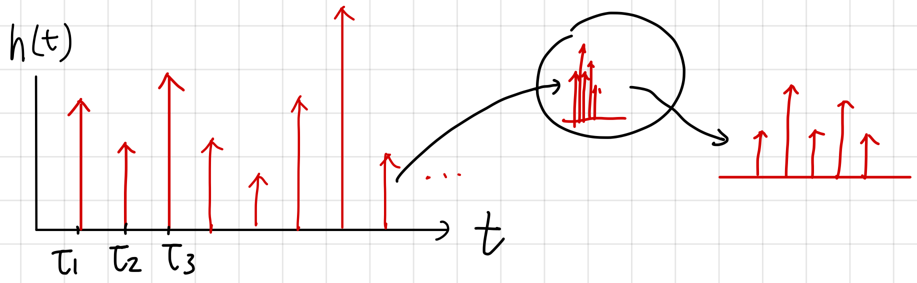
If 2 path (narrowband)



影響較少

General case

$$h(t) = \sum_{i=1}^{N} a_i \, \delta(t - \tau_i) \; ; \; \tau_1 \leq \tau_2 \cdots \tau_N$$



- This is called "delay dispersion".
- This channel is delay dispersive
- Each a_i can still be characterized by fading
  e.g. Rayleigh / Raician or other fading.

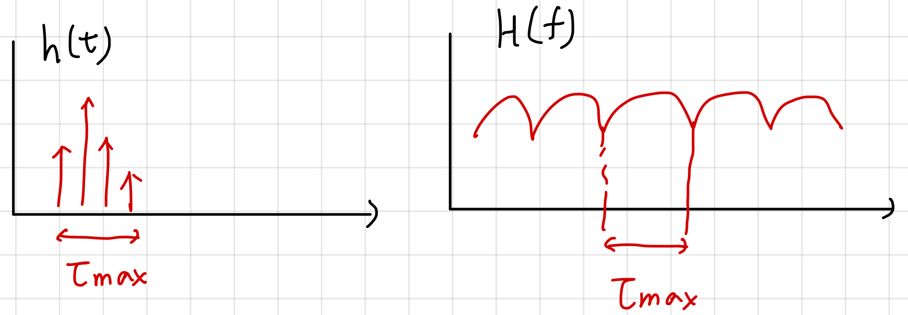
- The maximum excess delay $\tau_{max} = \tau_N - \tau_1$

→ We can mathematatally define narrowband and wideband

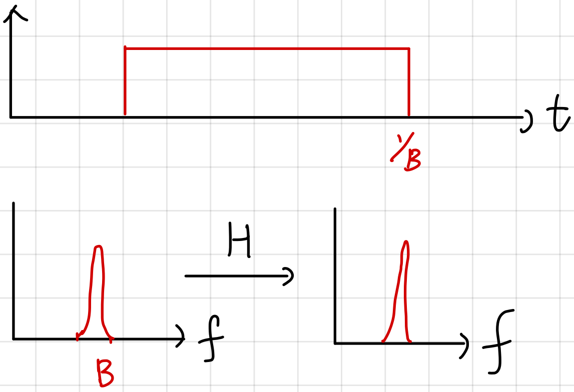
- A system is narrowband if
  the inverse of the system bandwidth B is much larger than τ_{max}
- Otherwise. we have wideband.

根據使用的通道 τ_{max} 決定 narrow or wide

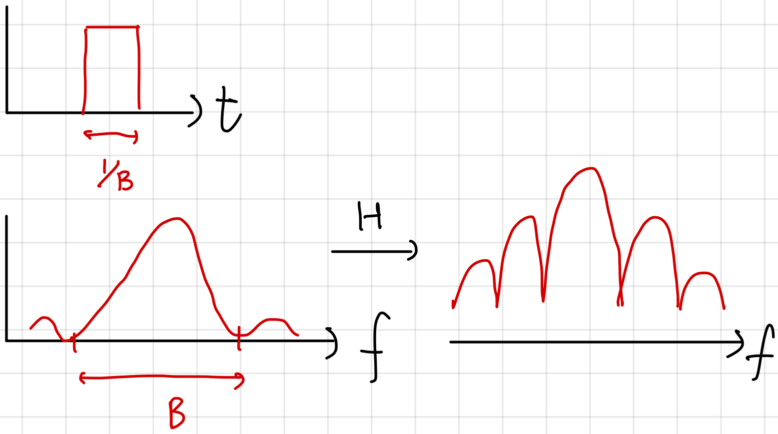
If narrowband, all delay can be viewed as single delay.



If narrow band



If wideband



Characterization of deterministic linear time-variant channel.

- Given channel $h(t,\tau)$, we have

$$r(t) = \int_{-\infty}^{\infty} s(t-\tau)\cdot h(t,\tau)\, d\tau.$$

- To simplify such representation, we commonly assume that the impulse response is varying slowly wrt the time.
- The τ_{max} is much shorter than the time over which the channel change.
- With this assumption. for a given time t, we have $\boxed{h(t,\tau) = h(\tau)}$.

→ This is called quasi-static of a system.

- If quasi-static. we have

$$Y(f) = H(f)\, S(f)$$

$$y(t) = \int_{-\infty}^{\infty} s(t-\tau)\cdot h(\tau)\, d\tau$$

- Let's get back to $h(t,\tau)$ — time-delay domain
  - $H(t,f) = \int_{-\infty}^{\infty} h(t,\tau)\cdot e^{-j2\pi f\tau} d\tau$ — time-freq domain.
  - $S(\nu,\tau) = \int_{-\infty}^{\infty} h(t,\tau)\, e^{-j2\pi \nu\tau} d\tau$ — Dopper-delay domain.

  Corresponds to the Dopper spectrum, we discussed before.

- Finally, we have

$$B(\nu,f) = \int_{-\infty}^{\infty} S(\nu,\tau)\, e^{-j2\pi \nu\tau} d\tau$$

$$= \int_{-\infty}^{\infty} H(t,f)\, e^{-j2\pi ft} dt$$

Dopper-freq domain

$$\begin{tikzcd}
 & \overset{\text{TDM}}{h(t,\tau)} \arrow[dl, shift left] \arrow[dr, shift left] & \\
\text{OFDM}\ H(t,f) \arrow[ur, shift left] \arrow[dr, shift left] & & S(\nu,\tau) \arrow[ul, shift left] \arrow[dl, shift left] \\
 & B(\nu,f) \arrow[ul, shift left] \arrow[ur, shift left] & \text{OTFS}
\end{tikzcd}$$

Stochastic represent of the system.

- We now consider a time-variant stochastic system.
- For the impulse response $h(t,\tau)$, they should be described by using PDFs
- In general, we need the joint pdf for all complex amplitude and phase at every delay τ and time t. → too complicated.
- To deal with this, we choose to look into only the first order and second order statistic.

  → Mean of the random process. Mean : $\overline{y}(t) = E[y(t)]$

  → Auto-correlation of RP. ACF : $R_{yy}(t,t') = E[y(t) \cdot y^*(t')]$

- If we have $\boxed{\text{Gaussian}}$ process, the first order and second order statistic determine the whole process

- Suppose we are given $h(t,\tau)$ with $y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, s(t-\tau)\, d\tau$

$$\rightarrow R_{yy}(t,t') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[s^*(t-\tau)\, s(t'-\tau')]\, E[h^*(t,\tau)\, h(t',\tau')]\, d\tau\, d\tau'$$

$$= \int_{-\infty}^{\infty} R_{ss}(t-\tau, t'-\tau')\, R_{hh}(t,t',\tau,\tau')\, d\tau\, d\tau'$$

- Assume zero mean and s, h are indep.

  we just look at ACF.

Wide sense stastionary assumption.

- ACF NOT depend on t and t', instead it depend on $\boxed{\Delta t = t - t'}$

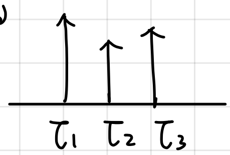
$\rightarrow R_h(t, t', \tau, \tau') = R_h(t, t + \Delta t, \tau, \tau')$

- Interpretation of WSS, the statistical properties of the channel do not change with time.

Uncorrelated scattering assumption

- Contribution from different paths with different delay are uncorrelated.

- This give : $R_h(t, t', \tau, \tau') = P_h(t, t', \tau)\, \delta(\tau - \tau')$



- This is fulfilled when mini-path contribute to a path does not contain any information of other path.

- equivalent : $R_H(t, t', f, f + \Delta f) = R_H(t, t', \Delta f)$

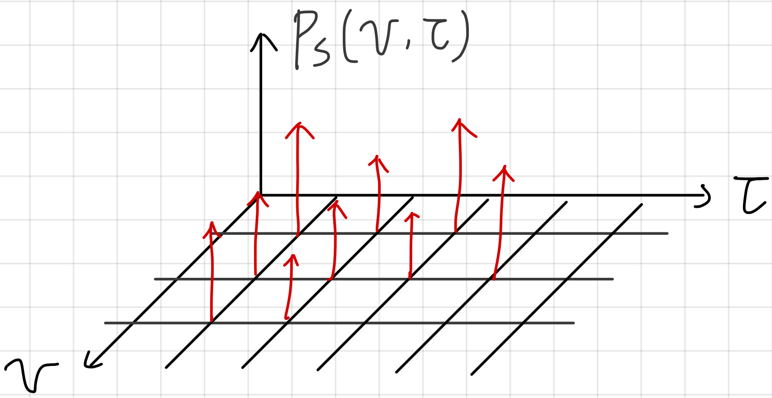
WSSUS assumption

- Different path for different delay are uncorrelated. (US)
- Different Dopper shift are uncorrelated (WSS)

→ WSSUS give

- $R_h(t, t+\Delta t, \tau, \tau') = P_h(\Delta t, \tau)\,\delta(\tau-\tau')$
- $R_H(t, t+\Delta t, f, f+\Delta f) = R_H(\Delta t, \Delta f)$
- $R_S(\nu, \nu', \tau, \tau') = P_S(\nu, \tau)\,\delta(\nu-\nu')\,\delta(\tau-\tau')$
- $R_B(\nu, \nu', f, f+\Delta f) = P_B(\nu, \Delta f)\,\delta(\nu-\nu')$

$P_h(\Delta t, \tau)$: delay cross power spectral density
$R_H(\Delta t, \Delta f)$: time-freq correlation function
$P_S(\nu, \tau)$: scattering function
$P_B(\nu, \Delta f)$: Dopper cross prower spectral density.



It give the clear physical meaning that a bin of a scattering function is determined by a path with a unique angle of arrival (AoA) and delay

give the Dopper shift

Consider

- WSSUS
- quasi-static

→ then, we have a tapped delay line model.

$$h(t,\tau) = \sum_{i=1}^{N} C_i(t)\,\delta(t-t_i)$$

- For a specific t, we have

$$h(t) = \sum_{i=1}^{N} C_i \cdot \delta(t-t_i)\ ;$$

  - $C_i(t)\ \forall i$ are independent.
  - Dopper spectrum determined the change of $C_i\ \forall i$ over time

(考試不考這之前)

Condensed Parameter.

- Using correlation function to describe the channel is still annoying. even if we have WSSUS, there are still two variable.
- An even more condence way is to use a function of variable to characterize the channel.

  → This loses lots of information.

Power Delay Profile (PDP)

— Straight forward way is to get from 2 variable to 1 by integration

— Intergrate the scattering over Dopper domain

→ we have PDP, decay power spectral density.

$$P_h(\tau) = \int_{-\infty}^{\infty} P_s(\upsilon, \tau)\, d\tau$$

— $P_h(\tau)$ indicate how much power density arrive at the RX with a delay between $(\tau, \tau + d\tau)$ irrespective to Dopper shift.

— Although we can do $\int_{-\infty}^{\infty} P_s(\upsilon, \tau)\, d\upsilon$,

a more straight forward way is:

$$P_h(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |h(t, \tau)|^2 dt \text{ (ergdicity)}$$

Dopper Power spectral density

$$P_B(\upsilon) = \int_{-\infty}^{\infty} P_s(\upsilon, \tau)\, d\tau$$

Moments of PDP

- PDP is still a function we want parameter.

(i) time-integrated power (zero moment)

$$P_m = \int_{-\infty}^{\infty} P_h(\tau)\, d\tau$$

→ average power at RX if you transmit at unit power. (other reference can be used)

(ii) Mean delay

$$T_m = \frac{\int_{-\infty}^{\infty} \tau P_h(\tau)\, d\tau}{P_m}$$

(iii) root-mean-square (rms) delay spread.

$$S_\tau = \sqrt{\frac{\int_{-\infty}^{\infty} \tau^2 P_h(\tau)\, d\tau}{P_M} - T_m^2}$$

↳ most important and widely used one

↓ propotional to BER in certain case.

Moment of Dopper spectral.

(i) $P_{B,m} = \int_{-\infty}^{\infty} P_B(\nu)\, d\nu$

(ii) $\nu_m = \dfrac{\int_{-\infty}^{\infty} \nu P_B(\nu)\, d\nu}{P_{B,m}}$ mean Dopper shift.

(iii) $S_\nu = \sqrt{\dfrac{\int_{-\infty}^{\infty} \nu^2 P_B(\nu)\, d\nu}{P_{B,m}} - \nu_m^2}$: rms Dopper shift

EX 6-1.

$$P_h(\tau) = \delta(\tau - 10\mu s) + 0.3\,\delta(\tau - 17\mu s)$$

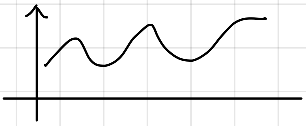
- $P_m = \int_{-\infty}^{\infty} P_h(\tau)\,d\tau = 1.3$

- $T_m = \dfrac{\int_{-\infty}^{\infty} \tau P_h(\tau)\,d\tau}{1.3} = \dfrac{10^{-5} + 0.3 \times 1.7 \times 10^{-5}}{1.3} = 11.6\,\mu s.$

- $S_\tau = \sqrt{\dfrac{\int_{-\infty}^{\infty} \tau^2 P_h(\tau)\,d\tau}{P_m} - T_m^2} = 3\,\mu s.$

Coherent Bandwidth and Coherent time. (6.5.4)

- Coherent bandwidth



→ freq selective channel, different freq components fade differently

→ The correlation between fading on different frequency is smaller if we increase their freq separation.

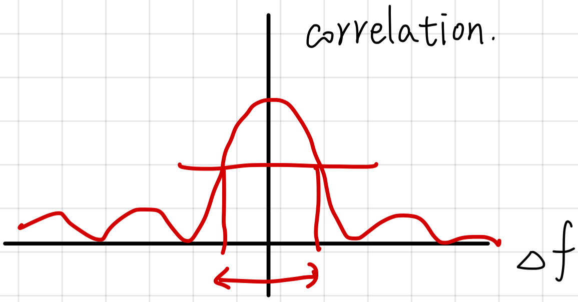
- The coherent bandwidth define the required freq difference such that the correlation coefficient is smaller than a pre-defined threshold (non-coherent/independent)

- Mathematically, we define

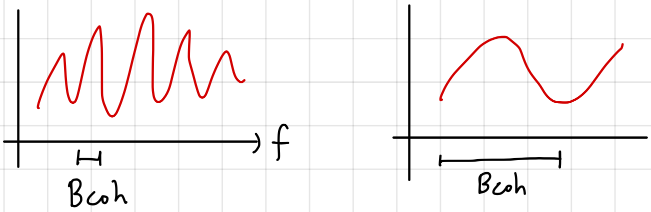
$$B_{coh} = \frac{1}{2}\left[\underset{\Delta f}{\operatorname{argmax}} \left(\frac{|R_H(0,\Delta f)|}{R_H(0,0)} = 0.5 \right) - \underset{\Delta f<0}{\operatorname{argmax}} \left(\frac{|R_H(0,\Delta f)|}{R_H(0,0)} = 0.5 \right) \right]$$

- Define the threshold as 0.5
- usually, this is a complicate function.
- We do approximate from S_τ.

$$B_{coh} \geq \frac{1}{2\pi S_\tau}$$



- Coherent bandwidth measure how fast the fading change from one to another. at freq.



- Coherent time

→ Coherent time measure how fast the channel change from one to another at time.

$$\rightarrow T_{coh} = \frac{1}{2}\left[\underset{\Delta t>0}{\operatorname{argmax}} \left(\frac{|R_H(\Delta t,0)|}{R_H(0,0)} = 0.5 \right) - \underset{\Delta t<0}{\operatorname{argmin}} \left(\frac{|R_H(\Delta t,0)|}{R_H(0,0)} = 0.5 \right) \right]$$

we approximate $T_{coh} = \frac{1}{2\pi S_v}$

Fourier transform relationship

$$\begin{tikzcd}
& P_h(\Delta t, \tau) \arrow[dl, shift right, "F_\tau"'] \arrow[dr, shift left, "F_{\Delta t}"] & \\
R_H(\Delta t, \Delta f) \arrow[ur, shift right, "F_{\Delta f}^{-1}"'] \arrow[dr, shift right, "F_{\Delta t}"'] & & P_s(\nu, \tau) \arrow[ul, shift left, "F_\nu^{-1}"] \arrow[dl, shift left, "F_{\Delta f}^{-1}"] \\
& P_B(\nu, \Delta f) \arrow[ul, shift right, "F_\nu^{-1}"'] \arrow[ur, shift left, "F_\tau"] &
\end{tikzcd}$$

BER under wideband channel without equalization.

- Wideband system has intersymbol interference (ISI) which degrade the BER
- We might use equalization to mitigate ISI
- If we don't, the ISI lead to additional error.

- If we have delay spread.
  - $\overline{BER}_{Delay} = K_1 \left(\frac{S_\tau}{T_s}\right)^2$
  - K_1 some constant related to your modulation approach.
  - $S_\tau \uparrow \quad \overline{BER}_{Delay} \uparrow$
  - $T_s \uparrow \quad \overline{BER}_{Delay} \downarrow$

- If we have Doppler spread.

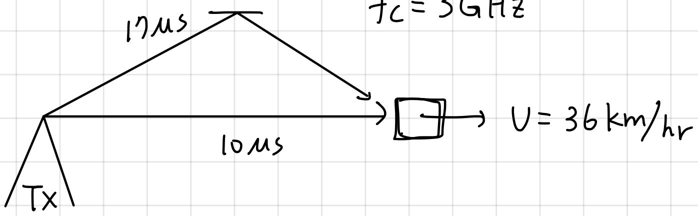
  $\overline{BER}_{Doppler} = K_2 (\nu_{max} T_s)^2$
  - K_2 is some constant related to modulation scheme
  - $\nu_{max} \uparrow \quad \overline{BER}_{Doppler} \uparrow$
  - $T_s \downarrow \quad \overline{BER}_{Doppler} \downarrow$

- Trade off exist between $\overline{BER}_{Doppler}$ & $\overline{BER}_{Delay}$ when deciding T_s

$\begin{cases} \text{To reduce the impact of Doppler, you want a wideband system,} \\ \text{To reduce the impact of Delay spread, you want narrowband system} \end{cases}$

- $\overline{BER}_{total} \leq \overline{BER}_{noise} + \overline{BER}_{Delay} + \overline{BER}_{Doppler}$

Ex 期末.



- Suppose $K_1 = 4$, $K_2 = \frac{1}{2}$
- $P_h(\tau) = \bar{\Omega}\left[\delta(\tau - 10\mu s) + 0.3\,\delta(\tau - 17\mu s)\right]$

; $S_\tau = 3\mu s = 3\times10^{-6}\,s$

; $\nu_{max} = \frac{V}{C} f_c = \frac{36000}{3600} \times \frac{3\times10^9}{3\times10^8} = 100Hz$

$$\begin{cases} \overline{BER}_{delay} = K_1\left(\frac{S_\tau}{T_s}\right)^2 = 4\left(\frac{3\cdot10^{-6}}{T_s}\right)^2 \\ \overline{BER}_{Doppler} = K_2(\nu_{max} T_s)^2 = \frac{1}{2}(100 T_s)^2 \end{cases}$$

(a) If SNR = 20dB. BPSK, Rayleigh.

Q: $\overline{BER}_{total}$

A: $\overline{BER}_{total} = \overline{BER}_{noise} + \overline{BER}_{delay} + \overline{BER}_{Doppler}$.

- $\overline{BER}_{noise} = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right] \approx \frac{1}{4\bar{\gamma}} = 2.5\times10^{-3}$

$\rightarrow \overline{BER}_{total} = 2.5\times10^{-3} + 4\left(\frac{3\cdot10^{-6}}{10^{-4}}\right)^2 + \frac{1}{2}(100\cdot10^{-4})^2 = \underline{6.15\times10^{-3}}$ #

(b) If we want to minimize $\overline{BER}_{total}$. by changing T_s, What T_s should be ?

$\rightarrow$ focus on $\overline{BER}_{delay}$ and $\overline{BER}_{Doppler}$

$$T_s^* = \arg\min_{T_s}\left[4\left(\frac{3\times10^{-6}}{T_s}\right)^2 + \frac{1}{2}(100 T_s)^2\right] \rightarrow \frac{d\,\overline{BER}(T_s)}{d\,T_s} = 0$$

$\rightarrow 4\times9\times10^{-12}\times(-2)T_s^{-3} + 10^4\,T_s = 0$

$T_s^4 = 72\times10^{-16}$ $\quad T_s = 2.9\times10^{-4}\,s$

$\rightarrow \overline{BER}_{delay} = \overline{BER}_{Doppler} = 4.2\times10^{-4}$

$\rightarrow \overline{BER}_{total} = 2.5\times10^{-3} + 4.2\times10^{-4}\times2$

$= 3.34\times10^{-3} < 6.15\times10^{-3}$