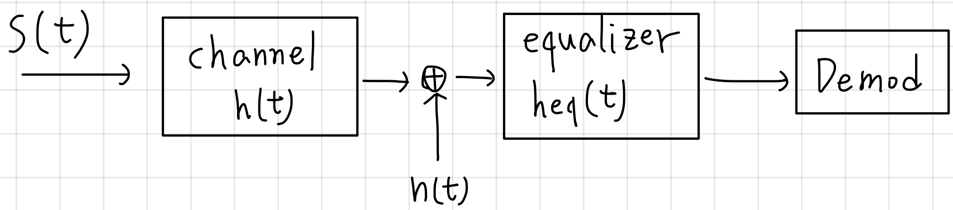
Wireless Communication Ch16

2024 Fall

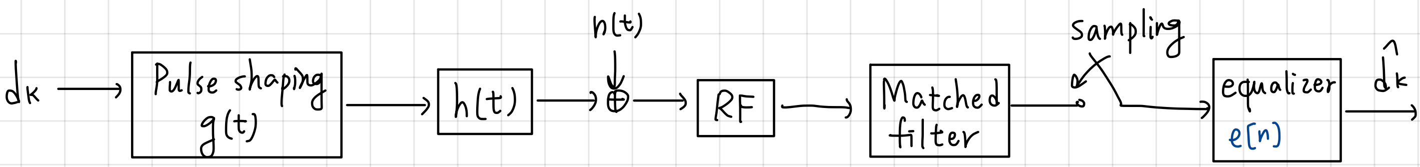
Equalization.

- ISI can degrade the system significantly.
- To mitigate ISI, we use equalization (on time domain)
- From freq domain, equalization can be viewed as a compensation on freq selectivity of channel such that the channel after equalizer is flat.
- For time-domain, we want the overall channel after equalization become a single path channel.
- We assume quasi-static channel.
- let $s(t) = \sum d_i g(t-iT)$



- So, simplest way is to have $H_{eq}(f) = \frac{1}{H(f)}$

  → This is called "zero forcing equalization"



- What we want with ZF ideal is that when there is no noise, we have $\hat{d}_k = d_k$
- We let $f(\hat{t}) = g(t) * h(t) * g_m(t)$, be the effeitive channel.
- $\hat{d}_k = \sum_{i=L}^{L} e_i y[n-i]$

  we want $\hat{d}_k = d_k$, so we design $e_i, \cdots e_0, \cdots e_i$ such $\hat{d}_k - d_k = 0$

$\Rightarrow$ This implies $f[n] * e[n] = \delta[n] \longleftrightarrow E(z) = \frac{1}{F(z)}$

Noise enhancement of ZF equalization

- If we have noise. we have.

$$Y(z) = F(z)\underbrace{D(z)}_{\text{data spectrum}} + \underbrace{N_g(z)}_{\sqrt{N_0}G_m(z)}$$

- If we adopt ZF, we have

$$\hat{D}(z) = \frac{Y(z)}{F(z)} = D(z) + \frac{N_g(z)}{F(z)}$$

- Power of noise : $\frac{N_g(z)}{F(z)} = \frac{N_0 |G_m(z)|^2}{|F(z)|^2} = \frac{N_0 \cancel{|G_m(z)|^2}}{|G(z)|^2 |H(z)|^2 \cancel{|G_m(z)|^2}} = \frac{N_0}{|G(z)|^2 |H(z)|^2}$

- Suppose our pulse shaping function is ideal, we can ignore it.

  $\rightarrow$ Power of noise $= \frac{N_0}{|H(z)|^2}$

- If poor SNR, this implies $|H(f)|^2$ is small .
  Then you have large noise power.

- Thus, noise is enhanced if SNR is low,
  $\rightarrow$ give poor performance.

Note :

$$\begin{cases} \text{If SNR is high , ZF is good} \\ \text{If SNR is low , ZF is NOT a good ideal.} \end{cases}$$

Minimum Mean Square Error (MMSE equalizer)

- Minimum MSE $\rightarrow \min_{e} E\{|\hat{d}_k - d_k|^2\}$

Implimentation

(1) We can directly do MMSE

(2) As the noise is not white, we do the whitening filter first, and them do MMSE.

$$N_g(z) = \sqrt{N_0}\, G_M(z) \longleftarrow \text{color noise.}$$

we do whitening :

$$y[n] \longrightarrow \boxed{\text{whitening}} \longrightarrow \boxed{\text{Equalizer}} \longrightarrow \hat{d}_k$$

$$d_k \longrightarrow \boxed{\hat{F}(z)} \longrightarrow \underset{h(t)}{\oplus} \longrightarrow \boxed{E(z)} \longrightarrow \hat{d}_k$$

$$\hat{f}[n] = h[n] * g[n]$$

Now, $\hat{d}_n = \sum_{i=-L}^{L} e_i u[n-i] = e^T u$

$$e = [e_{-i}, e_{-i+1} \cdots e_0 \cdots e_i]$$
$$u = [u[n+i] \cdots u[0] \cdots u[n-i]]$$

$$E[|\hat{d}_n - d_n|^2] = E[|e^T u|^2 - 2Re\{u^H e^* d_n\} + |d_n|^2] = J$$

Assume $|d_n|^2 = 1$ without loss of generality.

$\nabla_e J = 0$

$\Rightarrow \nabla_e J = 2e^T E[uu^H] - 2E[u^* d_n]$

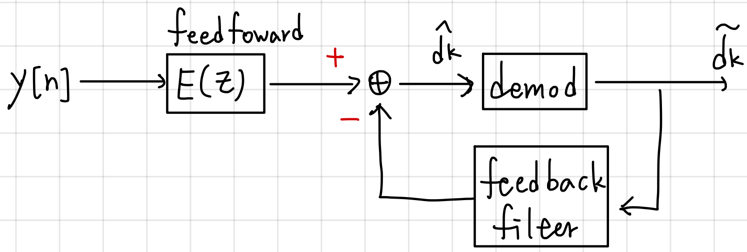
$\Rightarrow e_{opt} = R^{-1}P$, where $R = E[uu^H]$, $P = E[u^* d_n]$

Overall, if we let $L \rightarrow \infty$, them $H_{eq}(z) = \dfrac{1}{F(z) + \frac{N_0}{E[|d_n|^2]}}$ $= \frac{1}{SNR}$

$\Rightarrow$ We see this equalizer is a combination of ZF and noise

- if SNR $\uparrow$, we tend to use ZF
- if SNR $\downarrow$, we tend to not do equalization.

Decision - feedback equalization (DFE)



- Ideal is to use feedback from demodulated data to cancel ISI
- E[z] can be ZF or MMSE
- Issue : If error happen on $\hat{d_k}$, we have even worse performance for the next data symbol
  → error happen again and again.
  → error - propagation.

Maximum Likelihood Sequence Estimation (MLSE)

- $u[n] = \sum_{i=0}^{L} f_i d_{n-i} + n[n]$
  — d_n : transmitted symbol
  — f_i : channel coefficient
  — n : white Gaussian noise

- Assume noise is white with variance σ_n^2
- For a sequence of received data $u[0], u[1] \cdots u[N-1]$
- Joint pdf is :

$$pdf(u|d,f) = \frac{1}{(2\pi\sigma_n^2)^{\frac{N}{2}}} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{n=0}^{N-1}\left|u[n] - \sum_{i=1}^{L} f_i d_{n-i}\right|^2\right)$$

— $u = [u[0] \cdots u[N-1]]^T$
— $d = [d_0 \cdots d_{N-1}]^T$
— $f = [f_0 \cdots f_L]^T$

- The ML solution is to find a vector $\vec{d}$ such that $pdf(u|d,f)$ is maximized

- In other word, we want to minimize

$$\sum_{n=0}^{N-1} \left| u[n] - \sum_{i=0}^{L} f_i d_{n-i} \right|^2$$

- Solution approach

(i) You can try every possible sequence of d and then find the best one

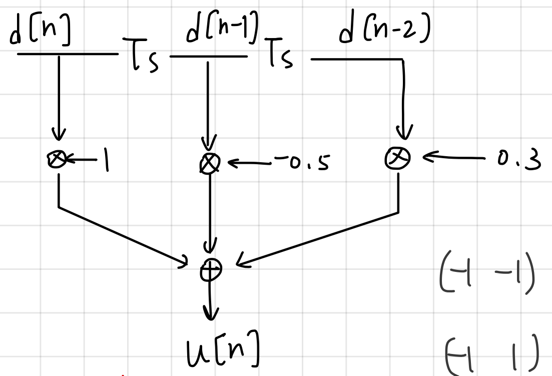
(ii) Viterbi algorithm → still give you the optimum solution.

EX16.3 Viterbi Equalization.

- Suppose a channel tap with $f = [1, -0.5, 0.3]^T$
- Suppose we transmit BPSK and received signal with

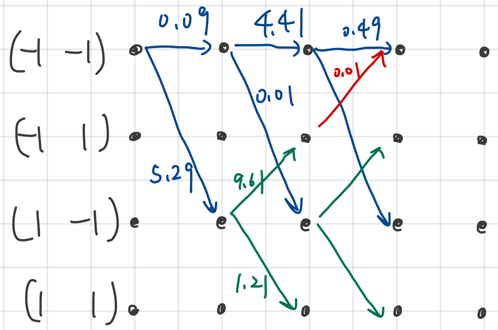
$[-1.1, +1.3, -0.1, +0.1, +1.6] = u$

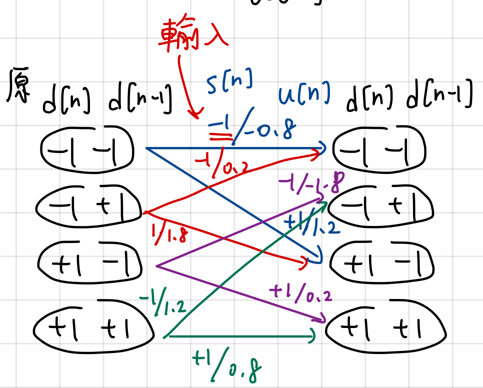
- What is the transmited signal d?



Suppose the starting state is -1, -1

Then, we see the optimal can be closen from the current and the action.





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Capacity and influence of channel coding.

- Capacity is the limit for maximum possible data rate that can be transmitted over wireless channel with negligibly small error rate
- It require both optimum modulation and coding to achieve the capacity.
- Ideal data rate $\triangleq$ capacity.

Capacity in noisy channel

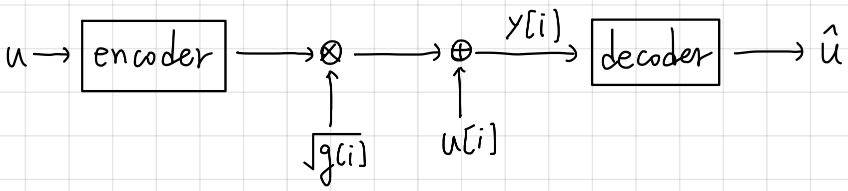
$y = x + n.$

$$\begin{cases} SNR = \dfrac{P}{N_0 B} = \gamma \\ B = \text{bandwidth} \\ P = \text{transmit power.} \end{cases} \rightarrow C = B\log_2(1+SNR)$$

$$\begin{cases} \text{error rate bounded away from } 0, \text{ if } R > C \\ \text{error rate be arbitrurilty small, if } R \le C \end{cases}$$

$\rightarrow$ R: data rate

Capacity in flat fading channel.



$$\begin{cases} x[i] \text{ is the transmitted signal at time } i \\ g[i] \text{ is the channel power gain at time } i \\ u[i] \text{ is the noise at time } i \\ y[i] \text{ is the received signal at time } i \end{cases}$$

$\rightarrow \gamma[i]$ is the SNR at time i

$\rightarrow \gamma[i] = \dfrac{P[i]\,g[i]}{N_0 B}$, $\bar{\gamma} = \frac{1}{T}\sum_{i=0}^{i-1} \gamma[i] = E[\gamma]$

P[i] : transmit power

Case

1. Channel state information (CSI) $g[i]$ is known at the receiver. → received CSI
2. CSI is known at both Tx and Rx → CSIT, perfect CSI
3. CSIT, but only with statistic at Rx

Capacity with CSIR

- Tx does NOT know the CSI. so $P[i] = P[j]$, $i \neq j$

  $P[i] = P[j] = P$, $\forall i, j$

- $\gamma[i] = \frac{P g[i]}{N_0 B}$ → $\gamma[i]$ follow some distribution

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Ergodic capacity

- Snitable is fast fading.

  $$C_{erg} = \lim_{T\to\infty} \frac{1}{T} \sum_{i=0}^{T-1} B\log_2(1+\gamma[i]) = E\left[B\log_2(1+\gamma)\right]$$

  $$= \int_0^\infty B\log_2(1+\gamma) \cdot P_r(\gamma)\, d\gamma$$

  where $P_r(\gamma)$ is the distribution of SNR

- With Jensen's inequation, we see

  $$C_{erg} = E[B\log_2(\gamma)] \underset{\uparrow \text{Jensen's inequation.}}{\leq} B\log(1+E[\gamma]) = B\log(1+\bar{\gamma})$$

  → Fading reduce capacity.

EX. Flat fading as follow

$$\sqrt{g} = \begin{cases} 0.05 & , \text{ w.p } 0.1 \\ 0.5 & , \text{ w.p } 0.5 \\ 1 & , \text{ w.p } 0.4 \end{cases}$$

Capacity with outage prob.

- suitable for slow fading.
- We denote γ_{min} as the minimum SNR. to have for a transmission.

  $\rightarrow C_{min} = B\log_2(1+\gamma_{min})$: minimum capacity to have for channel.

$$\begin{cases} C(\gamma) < C_{min} \Rightarrow \text{error rate} \rightarrow 1 \\ C(\gamma) > C_{min} \Rightarrow \text{error rate} \rightarrow 0 \end{cases}$$

$\rightarrow$ effective rate $R = (1-P_{out})\, B\log_2(1+\gamma_{min})$

- You can do rate adaptation such that you can maximized R.
- If you have CSIT, you can do rate adaptation.

Capacity with both CSIR and CSIT.

- Both Tx and Rx have CSI
- Suppose we can allocate resource to multiple channel from $i = 1, 2, \cdots, I$.
- We want allocate different power on different $g[i], \forall i$, but remain the same average power.

$$\rightarrow C = \sum_{i=1}^{T} C[i] = \sum_{i=1}^{T} B\log_2\left(1+\frac{P[i]\,g[i]}{N_0 B}\right)$$

$$\rightarrow \max_{P[0]\cdots P[T-1]} C[i] \;,\; \text{s.t. } \frac{1}{T}\sum_{i=0}^{T-1} P[i] \le \bar{P}, \quad P[i] \ge 0,\ \forall i$$

$\longrightarrow$ We use "water filling alg" to solve.

Water-filling (Lagrange multiplier)

- $J(P) = \sum_{i=0}^{T-1} B\log_2\left(1+\frac{P[i]\,g[i]}{N_0B}\right) - \lambda\left[\frac{1}{T}\sum_{i=0}^{T-1} P[i] - \bar{P}\right]$

$$\rightarrow \frac{dJ(P)}{dP[i]} = \frac{B}{\ln 2}\,\frac{1}{1+\frac{P[i]\,g[i]}{N_0B}} - \frac{\lambda}{T} = 0$$

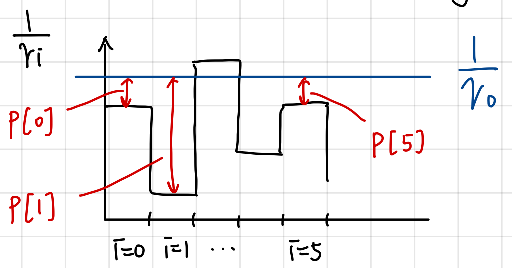
$$\rightarrow P[i] = \frac{1}{\lambda}\frac{TB}{\ln 2} - \frac{N_0B}{g[i]},\ \forall i$$

$$P(i) = \left[\frac{1}{\lambda}\frac{TB}{\ln 2} - \frac{N_0B}{g[i]}\right]^+ \ ;\ [a]^+ = \max(a, 0)$$

$\rightarrow$ Final solution

$$\begin{cases} P^*[i] = \left[\frac{1}{\lambda}\frac{TB}{\ln 2} - \frac{N_0B}{g[i]}\right]^+,\ \forall i \\ \text{such that } \sum_{i=0}^{T-1} P^*[i] = \bar{P} \end{cases}$$

let $\frac{1}{\lambda}\frac{TB}{\ln 2} = \frac{1}{\gamma_0}$ and $\frac{N_0B}{g[i]} = \frac{1}{\gamma_i}$



BER with channel coding. (error-correction code)

- Suppose we have (n,k) code, where the input is k bits and the output is n bit, $k \le n$.

$$k \text{ bit} \rightarrow \boxed{\text{encoder}} \longrightarrow n \text{ bit}.$$

- code rate $= \frac{k}{n}$
- Spectral efficiency $= R\frac{k}{n}$.

For Example.

最多可更正 2bit，超出即為 error.

If we have $(7,3)$ code that can help receiver at most 2 bit then the prob. of having error become:

- error rate $= 1 - P_r(\text{no bit error}) - P_r(1 \text{ bit error}) - P_r(2 \text{ bit error})$
- error rate $= 1-(1-P_e)^7 - 7(1-P_e)^6 - C^7_2(1-P_e)^5$