Digital Signal Processing

Ch6 Structure for Discrete-Time System

2024 Fall

Linear constant coefficient difference equation

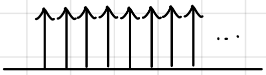
- $\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m] \longrightarrow \sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{m=0}^{M} b_m z^{-m} X(z)$

- system function : $H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{M} b_m z^{-m}}{\sum_{k=0}^{N} a_k z^{-k}} = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1-\sum_{k=1}^{N} a_k z^{-k}}$

- $y[n] = a_1 y[n-1] + \cdots + a_N y[n-N] + b_0 x[n] + \cdots + b_M x[n-M]$

FIR (Rectangular Pulse)

- $h[n] = \frac{1}{M} \sum_{k=0}^{M-1} \delta[n-k]$



- $H(e^{j\omega}) = \frac{1}{M} \sum_{n=0}^{M-1} z^{-n}$

$\rightarrow y[n] = \frac{1}{M} \left(x[n] + x[n-1] \cdots x[n-M+1] \right)$

Rational IIR (Causal exponential)

- $h[n] = a^n u[n]$
- $H(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}$

$\rightarrow y[n] = x[n] + a x[n] + a^2 x[n+2] \cdots = a y[n-1] + x[n]$

Non-Rational IIR (Ideal Lowpass Filter)

- $h[n] = \frac{\sin \omega_c n}{\pi n}$
- $H(e^{j\omega}) = \begin{cases} 1 \ ; & |\omega| < \omega_c \\ 0 \ ; & \text{o.w} \end{cases}$

$H(z) \neq \frac{\sum b_m z^{-m}}{\sum a_k z^{-k}} \Rightarrow \sum a_k y[n-k] = \sum b_m x[n-m]$

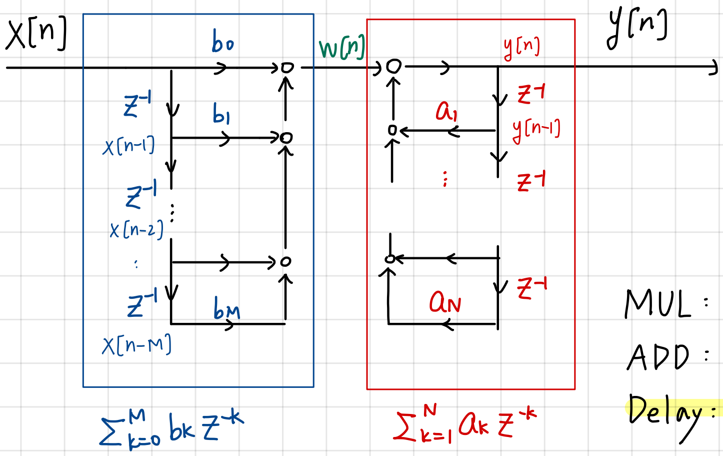
- $H(z)$ is NOT rational $\rightarrow$ cannot be represented as Linear const coeff difference equation
- NO finite cost
- output cannot be computed recursively.

Structure for Rational System

- Direct form $\begin{cases} 1 \\ 2 \end{cases}$
- Cascade form
- Parallel form

$\boxed{\text{Direct form 1}}$ (IIR)

- $y[n] = a_1 y[n-1] + \cdots + a_N y[n-N] + b_0 x[n] + \cdots + b_M x[n-M]$
- $H(z) = \dfrac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$

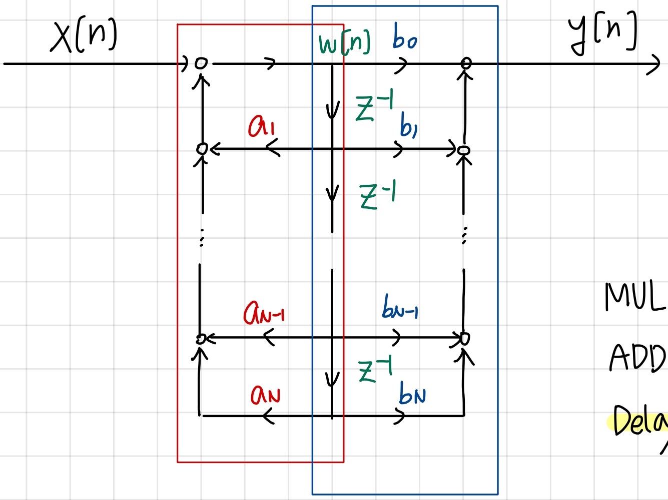


MUL : $M+N+1$

ADD : $M+N$

Delay : $M+N$

$\boxed{\text{Direct form 2}}$ (IIR)



MUL : $M+N+1$

ADD : $M+N$

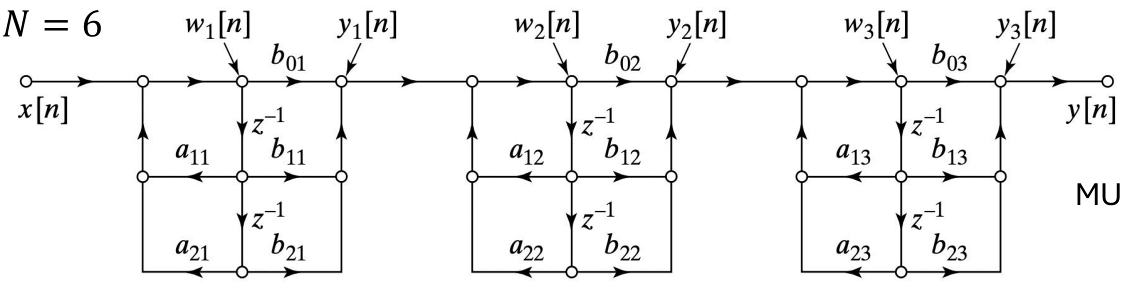
Delay : $\max(N, M)$

Cascade form (IIR)

串聯 N_s 組 2^{nd} order，也可串聯 1^{st} order

$$H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 - a_{1k}z^{-1} - a_{2k}z^{-2}}$$

- Ex: $N = 6$



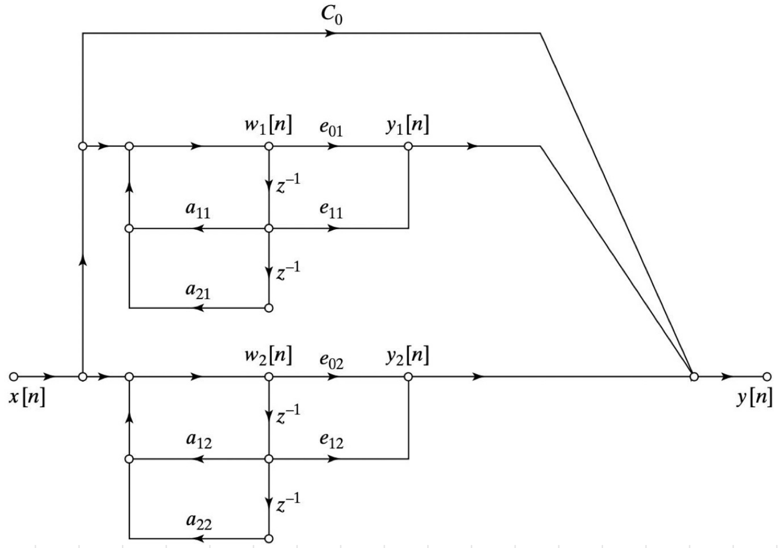
MUL: $5N/2$

- H(z) is express as a product of a number of 2^{nd} or 1^{st} order section.

Parallel form (IIR)

用部份分式，拆成 1^{st} order 和 2^{nd} order 的和.

$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=1}^{N_s} \frac{e_{0k} + e_{1k}z^{-1}}{1 - a_{1k}z^{-1} - a_{2k}z^{-2}}$$



- FIR system do NOT have parallel form

→ we can NOT do partial fraction expansion when denominator is constant

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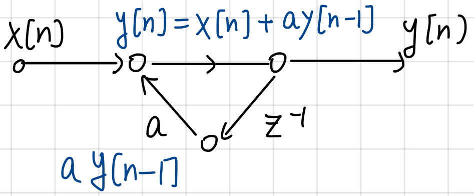
Feedback

- closed path that begin and end at same node.
- Feedback loop is necessary for a system to be IIR.

Ex) IIR

$$H(z) = \frac{1}{1-az^{-1}}$$

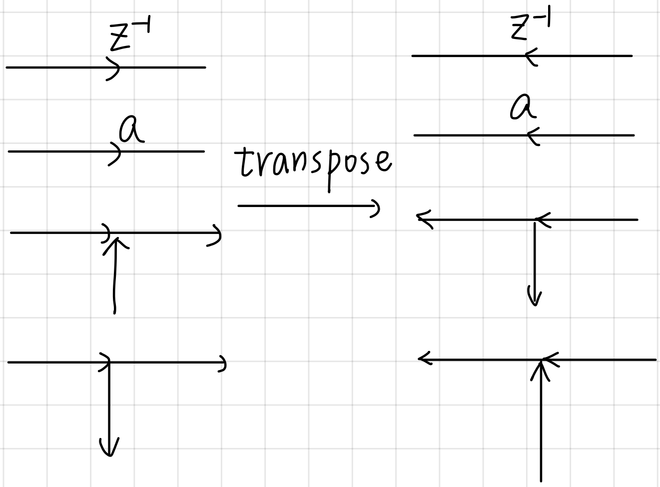


Ex) FIR

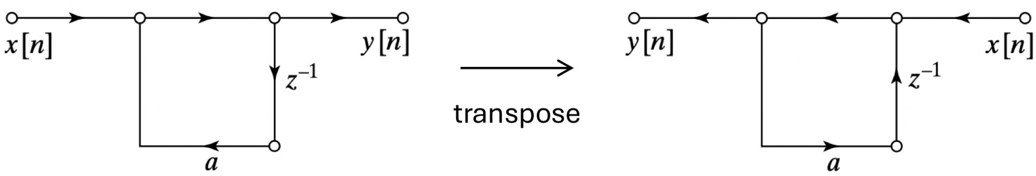
$$H(z) = \frac{1-a^2z^{-2}}{1-az^{-1}} = 1+az^{-1}$$

Transposed Form

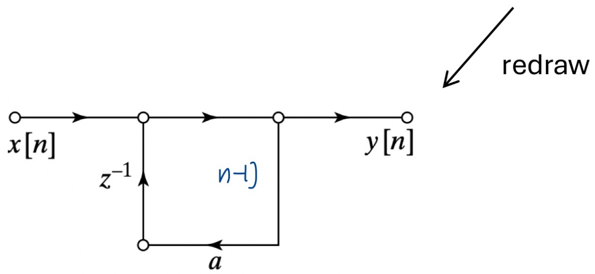
1. Reversing all the branches
2. Reversing input and output



Ex:



$$H(z) = \frac{1}{1-az^{-1}}$$



Discrete-Time network

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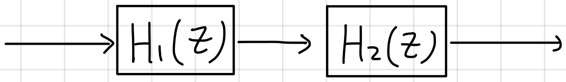
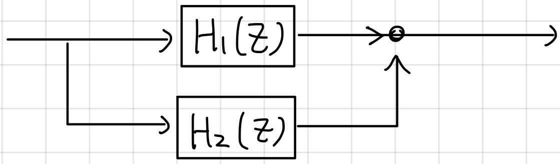
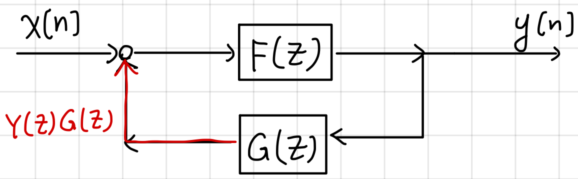
∘ **Cascade**: $H(z) = H_1(z) \cdot H_2(z)$



∘ **Parallel**: $H(z) = H_1(z) + H_2(z)$

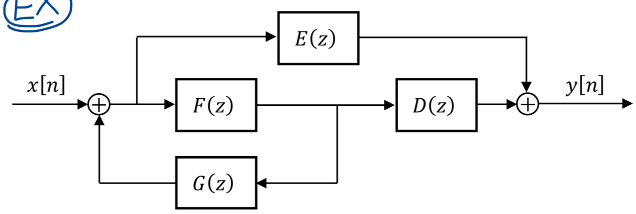


∘ **Feedback**: $H(z) = \dfrac{F(z)}{1 - F(z)G(z)}$



$\rightarrow [X(z) + Y(z)G(z)]\, F(z) = Y(z)$

EX



$$H(z) = \frac{F(z)}{1 - F(z)G(z)} D(z) + \frac{F(z)}{1 - F(z)G(z)} \cdot \frac{E(z)}{F(z)} = \frac{F(z)D(z) + E(z)}{1 - F(z)G(z)}$$