1(a)

- P_{RX} + Fading margin $\leq P_{TX} + G_{TX} + G_{RX} - PL(d) - L_{TX}$
- Received noise and power :

  $N_0 = -174\,dBm/Hz$, $B = 20\,MHz \Rightarrow 10\log_{10}(2\times10^7) = 73\,dB$, $F = 6\,dB$

  $P_N = -174 + 73 + 6 = -95\,dBm$

  $P_{RX} = -95 + 15 = -80\,dBm$

- Transmit power with pathloss :

  $$PL(d) = \underbrace{20\log_{10}\left(\frac{4\pi}{\lambda_c}\right)}_{32+10} + \underbrace{20\log_{10}(d_{break})}_{20} + \underbrace{n\cdot 10\log_{10}\left(\frac{d}{d_{break}}\right)}_{35} = 97$$

  $$P_{TX} \geq (-80+5) - (5 - 96.5 - 3) = 20\,dBm$$

- <u>Ans: 20 dBm = 100 mW</u> #

1(b)

- P_{RX} + Fading margin $\leq P_{TX} + G_{TX} + G_{RX} - PL(d) - L_{TX}$
- Received noise and power :

  $N_0 = -174\,dBm/Hz$, $B = 1\,GHz \Rightarrow 10\log_{10}10^9 = 90\,dB$, $F = 6\,dB$

  $P_N = -174 + 90 + 6 = -78\,dBm$

  $P_{RX} = -78 + 15 = -63\,dBm$

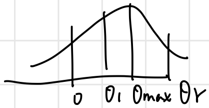
- Transmit power with pathloss :

  $$PL(d) = \underbrace{20\log_{10}\left(\frac{4\pi}{\lambda_c}\right)}_{32+37} + \underbrace{20\log_{10}(d_{break})}_{2\times7} + \underbrace{n\cdot 10\log_{10}\left(\frac{d}{d_{break}}\right)}_{55} = 138$$

  $$P_{TX} \geq (-63+5) - (55 - 138 - 3) = 28$$

- <u>Ans: 28 dBm = 631 mW</u> ※

2. $\theta_r > \theta_{max} > \theta_1 > 0$



Solve $\frac{dG(\theta,\phi)}{d\theta}$ to find maximum

$$\frac{dG(\theta,\phi)}{d\theta} = \left[\frac{d}{d\theta}\sin^n\left(\frac{\theta}{\theta_0}\right)\right]\cos\left(\frac{\theta}{\theta_0}\right) + \sin^n\left(\frac{\theta}{\theta_0}\right)\frac{d}{d\theta}\cos\left(\frac{\theta}{\theta_0}\right)$$

$$= \left(n\sin^{n-1}\left(\frac{\theta}{\theta_0}\right)\cdot\cos\left(\frac{\theta}{\theta_0}\right)\cdot\frac{1}{\theta_0}\right)\cos\left(\frac{\theta}{\theta_0}\right) - \sin^n\left(\frac{\theta}{\theta_0}\right)\frac{1}{\theta_0}\sin\left(\frac{\theta}{\theta_0}\right)$$

$$= \frac{1}{\theta_0}\left[n\cos^2\left(\frac{\theta}{\theta_0}\right) - \sin^2\left(\frac{\theta}{\theta_0}\right)\right]\sin^{n-1}\left(\frac{\theta}{\theta_0}\right) = 0$$

$$\Rightarrow \begin{cases} n\cos^2\left(\frac{\theta}{\theta_0}\right) = \sin^2\left(\frac{\theta}{\theta_0}\right) \\ \sin^{n-1}\left(\frac{\theta}{\theta_0}\right) = 0 \end{cases} \Rightarrow \begin{cases} \sin^2\left(\frac{\theta}{\theta_0}\right) = \frac{n}{n+1} \\ \cos^2\left(\frac{\theta}{\theta_0}\right) = \frac{1}{n+1} \end{cases} \Rightarrow \theta_{max} = \theta_0\tan^{-1}\left(\pm\sqrt{n}\right)$$

(a) $2G_{3dB} = G(\theta_{max})$

$$\rightarrow \sin^n\left(\frac{\theta}{\theta_0}\right)\cos\left(\frac{\theta}{\theta_0}\right) = \frac{1}{2}\left(\frac{n}{n+1}\right)^{\frac{n}{2}}\left(\frac{1}{n+1}\right)^{\frac{1}{2}} = \frac{n^{n/2}}{2(n+1)^{\frac{n+1}{2}}}$$

3dB bandwidth $= \theta_r - \theta_1$ #

(b) $10\,G_{10dB} = G(\theta_{max})$

$$\rightarrow \sin^n\left(\frac{\theta}{\theta_0}\right)\cos\left(\frac{\theta}{\theta_0}\right) = \frac{1}{10}\left(\frac{n}{n+1}\right)^{\frac{n}{2}}\left(\frac{1}{n+1}\right)^{\frac{1}{2}} = \frac{n^{n/2}}{10(n+1)^{\frac{n+1}{2}}}$$

10dB bandwidth $= \theta_r - \theta_1$ #

(c) Maximum directivity $= \dfrac{\text{Power at } \theta_{max}}{\text{Total power}}$

$$= \frac{G(\theta_{max})}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin(\theta)\,d\theta\,d\phi} = \frac{\frac{n^{n/2}}{(n+1)^{\frac{n+1}{2}}}}{\frac{1}{2}\int_0^{\pi}\sin^n\left(\frac{\theta}{\theta_0}\right)\cos\left(\frac{\theta}{\theta_0}\right)\sin(\theta)\,d\theta}$$ #

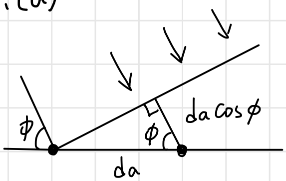
(d) 3dB-bandwidth $= 0.9346$

10dB-bandwidth $= 1.5097$

directivity $= 2.6391$

#

3.(a)



- carrier in complex form is $Ae^{-j2\pi f_c t}$
- $t_k = t_1 + \dfrac{d_a(k-1)\cos\phi}{c} = t_1 + (k-1)\dfrac{d_a}{\lambda_c f_c}\cos\phi$
- $r_k(t) = Ae^{-j2\pi f_c t}\cdot e^{-j\frac{2\pi}{\lambda_c}(k-1)d_a\cos\phi} = Ae^{-j2\pi f_c t}\cdot e^{-j\pi(k-1)\frac{1}{2}}$

$$r = \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_k(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j\pi\frac{1}{2}} \\ \vdots \\ e^{-j\pi(N-1)\frac{1}{2}} \end{bmatrix} Ae^{-j2\pi f_c t} + n \Rightarrow \text{steering vector} = \begin{bmatrix} 1 \\ e^{-j\frac{1}{2}\pi} \\ \vdots \\ e^{-j\frac{\pi}{2}(N-1)} \end{bmatrix} \#$$

3.(b)

Antenna gain:

$$|M_r(\phi)| = \left|\sum_{k=1}^{N} w_k e^{-j\pi(k-1)\frac{1}{2}}\right| = \left|\sum_{k=1}^{N} e^{-j(k-1)(\frac{\pi}{2}-\Delta)}\right| \quad \text{let } w_k = e^{j(k-1)\Delta}$$

(i) $\Delta \neq \frac{\pi}{\sqrt{2}}$, $|M_r(\phi)| = \left|\dfrac{\sin\left[\frac{N}{2}(\Delta-\frac{\pi}{2})\right]}{\sin\left[\frac{1}{2}(\Delta-\frac{\pi}{2})\right]}\right|$

(ii) $\Delta = \frac{\pi}{\sqrt{2}}$, $|M_r(\phi)| = N$ is maximum #

3.(c)

When $\Delta = -\dfrac{2\pi}{d_a} = -\dfrac{4\pi}{\lambda}$

$$\begin{cases} |M_r(\phi)| = \left|\dfrac{\sin\left[\frac{N}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{2})\right]}{\sin\left[\frac{1}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{2})\right]}\right| \\ \text{Noise} \sim N(0, N\sigma_n^2) \end{cases}$$

$\rightarrow$ Array gain $= \dfrac{|M_r(\phi)|^2}{N} = \dfrac{\text{signal power gain}}{\text{noise power gain}}$

$$= \frac{1}{N}\left|\frac{\sin\left[\frac{N}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{2})\right]}{\sin\left[\frac{1}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{2})\right]}\right|^2 \#$$

3.(d)

When $\Delta = -\left[\dfrac{2\pi}{d_a} + \dfrac{\pi}{N}\right] = \dfrac{-4\pi}{\lambda} - \dfrac{\pi}{N}$

$$\begin{cases} |M_r(\phi)| = \left|\dfrac{\sin\left[\frac{N}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{N}-\frac{\pi}{2})\right]}{\sin\left[\frac{1}{2}(\frac{-4\pi}{\lambda}-\frac{\pi}{N}-\frac{\pi}{2})\right]}\right| \\ \text{Noise} \sim N(0, N\sigma_n^2) \end{cases}$$

$\rightarrow$ Array gain $= \dfrac{|M_r(\phi)|^2}{N} = \dfrac{\text{signal power gain}}{\text{noise power gain}}$

$$= \frac{1}{N}\left|\frac{\sin\left[\frac{N}{2}(-\frac{4\pi}{\lambda}-\frac{\pi}{N}-\frac{\pi}{2})\right]}{\sin\left[\frac{1}{2}(-\frac{4\pi}{\lambda}-\frac{\pi}{N}-\frac{\pi}{2})\right]}\right|^2 \#$$

4(a)

- P_{RX} + Fading margin $\leq P_{TX} + G_{TX} + G_{RX} - PL(d) - L_{TX}$

- Received noise and power :

$$N_0 = -174\,dBm/Hz \;;\; B = 1\,GHz \quad F = 7$$
$$P_N = N_0B = -174 + 10\log_{10}10^9 + 7 = -77\,dBm$$
$$P_{RX} = -77 + 20 = -57\,dBm$$

- Transmit power with pathloss :

$$PL(d) = \underbrace{20\log_{10}\left(\frac{4\pi}{\lambda_c}\right)}_{\substack{32+20\log_{10}38 \\ 31.6}} + \underbrace{20\log_{10}(d_{break})}_{\substack{+20\log_{10}20 \\ 20+6=26}} + \underbrace{n\cdot 10\log_{10}\left(\frac{d}{d_{break}}\right)}_{+\,4.1\cdot 7} = 118.3$$

$$G_{TX} \geq (-57+13) - (45 - 5 - 118.3 - 4) = 38.3\,dB \approx 6760.83 \;\#$$

4(b)

$$\text{Antenna gain} = \left|\sum_{k=1}^{N} W_k e^{-j\frac{2\pi}{\lambda}(k-1)d\cos\phi}\right| = \left|\sum_{k=1}^{N} \frac{1}{\sqrt{N}} e^{-j\pi(k-1)/\sqrt{2}}\right|$$

$$= \frac{1}{\sqrt{N}}\left|\frac{\sin\left(\frac{\sqrt{2}}{2}\pi N\right)}{\sin\left(\frac{\sqrt{2}}{2}\pi\right)}\right| \text{ (amplitude gain)}$$

$$G_{TX}\text{: power gain at Tx} = \frac{1}{N}\left|\frac{\sin\left(\frac{\sqrt{2}}{2}\pi N\right)}{\sin\left(\frac{\sqrt{2}}{2}\pi\right)}\right| \geq 6760.83$$, N has no solution

---

4(c)

let $W_k = \frac{1}{\sqrt{N}} e^{j\theta_k}$; $k = 1, 2, \cdots N$

$$\text{Antenna gain} = \left|\sum_{k=1}^{N} \frac{1}{\sqrt{N}} e^{j\theta_k} e^{-j\pi(k-1)/\sqrt{2}}\right| = \left|\sum \frac{1}{\sqrt{N}} e^{j[\theta_k - \frac{1}{\sqrt{2}}(k-1)\pi]}\right|$$

When $\theta_k = \frac{\pi}{\sqrt{2}}(k-1)$; Antenna gain $= \frac{1}{\sqrt{N}} N = \sqrt{N}$

G_{TX} = power gain at Tx $= (\sqrt{N})^2 = N \geq 6760.83$

$N = 6761 \;\#$