Background and motivation: Yes, google will convert units for you so why bother with this tutorial? 3 reasons:

1. We'll use unit conversion as an exercise to teach how to perform dimensional analysis. Dimensional analysis is extremely useful for deriving the mathematical relationship between a known parameter and an unknown parameter. It will become useful in later tutorials (e.g. deriving scaling relationships and calculus equations).

2. Google doesn't always know about the units you're dealing with. Sometimes units are strange and arbitrary, depending on what you measure, model or analyze. For example, good luck typing the following into google and getting a meaningful answer: [(meters/second)/muscle length]*Newtons*femur length/muscle mass).

3. We often need to calibrate our instruments. Calibration is a unit conversion between what the equipment outputs (usually voltage) and the physical quantity we are measuring. To convert units, we must know the conversion factor (calibration factor). For example if I buy or build a force sensor, I want the sensor output voltage to correspond to the force value. Voltage is converted to Newtons of force via a conversion factor that must be measured.

Note: We won't be using Mathematica yet in this introductory tutorial.

■ 1. Dimensional Analysis by algebra:
Breaking down parameters into their fundamental SI units of mass, length and time (kg, m, s respectively) is crucially important in deriving and proof-reading equations or mathematical expressions. For my equations, I always check that the units match on both sides of the '=' sign. Additionally, I check if the units themselves make physical sense. This will be a recurring theme throughout the following tutorials.

<u>**Examples**</u>
[a] Force = mass x acceleration
Dimensional analysis: mass = kg; acceleration = m/s^2; Therefore:
Force = kg x m/s^2 = Newton 'N'
If it doesn't have the units of kg x m/s^2, it's not force!

[b] A spring is a structure that produces resistive force proportional to its displacement (d):
Spring force = k_s x d; So, given that Spring force units must be (kg x m/s^2), we can figure out what the spring constant (k_s) units must be:

$$N = \frac{kg \times m}{s^2} = k_s \times m$$

Treat the units like numbers and solve for k_s algebraically:

$$kg \times \frac{m}{s^2} = k_s \times m$$

Now, divide by m:

$$kg \times \frac{m}{s^2} \times \frac{1}{m} = k_s = \frac{kg}{s^2}$$

Or, more intuitively, k_s = N/m

■ 2. Dimensional Analysis by the Factor Label Method:
This method multiplies a chain of conversion factors, expressed as fractions, to convert between two parameters.

Examples

[Ex. a] A trivial example: How many meters in 2.8 inches? Google can do this, or you may be able to do it in your head. But let's use the Factor Label Method.

Step 1: Write what we start with on the left and what we end with on the right.
Inches … meters

Step 2: Connect meters and inches via relationships that are easy to find (or that you know already).

1 inch = 2.54 cm
1 meter = 100 cm
Importantly, I think of these conversions as fractions: 1 inch per 2.5 cm OR 2.5 cm per inch. In other words, 1in/2.5cm OR 2.5cm/in.

Step 3: Make a conversion factor with units of what you want to end with (numerator) divided by the units you start with (denominator). In this example, it's inches to meters, so our conversion factor is a quantity that when multiplied by inches gives meters. In other words, the conversion factor has units of meters/inch. To get this, line up your fractions such that the units cancel and you are left with the units that you want. Put the fractions right-side-up or up-side-down as necessary. One way is to start with your first fraction with inch in the denominator.

$$\frac{2.54\ cm}{1\ inch} * \frac{1\ meter}{100\ cm} = \frac{2.54\ \cancel{cm}}{1\ inch} * \frac{1\ meter}{100\ \cancel{cm}} = \frac{0.0254\ meter}{inch}$$

Notice how cm cancels out so we're left with a conversion factor of 0.0254 meters/inch.
Step 4: Multiply your starting unit with your conversion factor

$$2.8\ inchs * \frac{0.0254\ meters}{inch}$$

Inches cancel out, so we're good. Check it in google. Alternatively, you can write the whole chain, starting with 2.8 inches. This is what I usually do:

$$2.8\ \cancel{inches} * \frac{2.54\ \cancel{cm}}{1\ \cancel{inch}} * \frac{1\ meter}{100\ \cancel{cm}} = 0.07112\ meters$$

I ALWAYS write the units explicitly and cross them out. Otherwise, I'm prone to make a mistake.

There are multiple ways of doing it, but I chose to write the conversions where I'm least likely to make a mistake. For example, I could write 1 cm = 0.3937 inches, but I'm less likely to make a mistake writing 1:2.5 as opposed to 1:0.3937.

[Ex. b] How many degrees in 1.4 radians? The remaining tutorials use radians instead of degrees (it's often most convenient to do so, so get used to it).
A full circle is 2π radians around, so remember the following conversion:

π radians = 180 degrees

We either can make a simple conversion factor of degrees/radian…

$$\frac{180\ degrees}{\pi\ rad} = 57.296\ldots degrees/rad$$

→ 1.4 rad x 57.296 deg/rad = 80.25 degrees.
…OR, we can just multiply everything in a chain (my preference).

$$1.4\,\cancel{rad} * \frac{180\ degrees}{\pi\ \cancel{rad}} = 80.25\ldots\ degrees$$

[Ex. c] How many seconds are in 13.5 fortnights? This is hard to do in my head, so I'll divide large time units into smaller units of time until I get to seconds.

1 fortnight = 14 days
1 day = 24 hours
1 hour = 60 min
1 min = 60 sec

$$13.5\,\cancel{fortnights} * \frac{14\ \cancel{days}}{\cancel{fortnight}} * \frac{24\ \cancel{hours}}{1\ \cancel{day}} * \frac{60\ \cancel{min}}{1\ \cancel{hour}} * \frac{60\ sec}{1\ \cancel{min}} = 1.63296\ \times 10^7\ seconds$$

Note, our conversion factor:

$$conversion\ factor = \frac{14\ days}{fortnight} * \frac{24\ hours}{1\ day} * \frac{60\ min}{1\ hour} * \frac{60\ sec}{1\ min} = \frac{1.2096 \times 10^6\ seconds}{fortnight}$$

Exercises:

1. What are the SI units of pressure (Force/area)?

2. You have a sensor that measures the mechanical power (force x velocity = Watts) of an electric motor. The sensor delivers a voltage output proportional to the power. What would be the SI units of the calibration factor to convert from volts to Watts? I.e. volts x calibration factor = Watts

3. A damper (or dashpot) produces resistive force which is proportional to velocity (meters/s = m/s). For example, a shock absorber is a damper. If you push on a damper slowly, there isn't much resistance. However if you push quickly, the force increases. Squirting fluid out of a syringe has a similar effect. Given that damper Force = Cd x speed, what are the units of the damping coefficient (Cd)?

4. How many degrees in 2.7 radians?

5. How many radians in 104 degrees?

6. How many mm in 6.2 miles?

7. How many square meters in 28.2 cm^2?

8. Let's say you have force plate sensor which delivers increasing voltage corresponding to the amount of force on the plate. For example, when the plate is unloaded, the reading is 0 volts. When there is a force of 5 N on the plate, the sensor reads 2.7 volts. Assuming the sensor is linear, what is the calibration factor? If the plate outputs 7 volts, how many N of force is on the plate?

9. Let's say you're trying to measure the drag on various fish body models. There is a robotic motor which spins a gear which pulls the model fish down a track at a certain translational (linear) speed (see diagram below). Given the following specifications of your motor and the track gearing, derive an equation relating motor voltage output to the approximate linear fish speed in m/s. Use this equation to calculate the maximum model fish speed.

Motor voltage output: 0.07 volts/rpm (i.e. it delivers 0.07 volts if the motor spins at a speed of 0.07 revolutions per minute).
Motor maximum speed: 36,000 rpm
Gear diameter: 3.2 cm