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State estimation under quantized measurements: a Sigma-Point Bayesian approach

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Abstract— **Sensors providing only quantized or binary measurements are present in several automation contexts. A remarkable example is the Radio Frequency IDentification technology when only the detection of the tags is used as information for robot localization. In this paper we propose an algorithm which merges some concepts of the Unscented Kalman Filter (UKF) with some aspects of the Particle Filter (PF). The prediction step of the proposed method is like the prediction step of a standard UKF. On the contrary, the correction step of the UKF can not be trivially implemented due to the presence of binary measurements. For this reason a different correction step is proposed here where the sigma-points weights are modified according to their agreement with the measurements, like it is done for particles of a PF. The main advantage of the proposed algorithm with respect to a PF is that much less particles are needed. Moreover, the way to generate particles in the proposed approach is not random but deterministic. A simulative comparison of the proposed approach with respect to a PF and with respect to a Quantized Kalman Filter is reported in the paper.**

I. INTRODUCTION

There are several scenarios where a given quantity is measured only through sensors providing a coarse representation of its actual value. This happens for example when, for some technological/economical reason, a continuous quantity is measured through a set of contiguous binary sensors, like a set of photocells, which can provide a coarse measurement of the position of a given object moving in a unidimensional space (e.g. luggages on a conveyor, people in a corridor, the level of a fluid in a tank, etc.).

A valuable example of this type of situation occurs in binary sensor networks (see e.g. [1], [2]), where the state of the system is measured through a set of binary detectors. As an example in this context, we want to cite the localization of mobile robots through a Radio Frequency IDentification (RFID) system where only the identity of RFID tags positioned in the environment can be detected by a reader installed on the robot: the detection of a certain set of tags tells that the robot is located in a particular region of the environment (see e.g. [3], [4]).

Also the case of quantized measurements resulting from an analog to digital conversion or from a general discretization of the measured quantity belongs to the class of problems addressed in this paper: the continuous quantity to be measured is mapped into a discrete space and the variation of the quantity inside an interval can correspond to the same value of the quantized measurement. In both situations of discretized measurements and of binary detectors, the state space of the system is available through a measurement which can be considered an integer taking values in a finite set: in the case of binary detectors in fact, a measurement is a binary string $s \in \{0,1\}^l$ (with l the number of detectors) and all possible measurements can be always mapped into the set of integers from 1 to 2^l.

The quantized measurements considered in this paper may have quantization regions which are large with respect to typical situations occurring for example in analog to digital conversion. In other words, in the situations mainly addressed in this paper it is common that, despite the state of the system is changing, the output remains constant and, if a discrete time evolution of the system is considered, relatively long sequences of constant measurements can be observed.

The problem of estimating the state of the system under this kind of measurements can be addressed in several ways. One of the most common approaches considered, especially in the context of binary detectors, is the Particle Filter (PF) [5], which has the nice feature that can be easily adapted to deal with the probability model of several kinds of detectors. Several versions of the PF have been developed in the years but usually a satisfactory estimate can be obtained through a PF at the price of a significant computational burden. Notice also that, given the stochastic structure of the filter, if a small number of particles is considered to reduce the computational time, this does not simply provide a minor precision of the method but may often produce an invalid result of the filter.

One alternative to PF could be to face the estimation problem by exploiting the rich literature available on quantized and on constrained Kalman filtering. Most of the existing results for Kalman filtering in presence of quantized measurements (like e.g. [6], [7], [8], [9]) are based on analytical and relatively simple models for the quantization. These methods may be computationally intensive and/or not easily adapted to settings where the quantization regions may have a non trivial shape (like it often is the case when dealing with binary detectors).

A simple approach in the context of quantized measurements, described in [9], is obtained by approximating the quantization error associated with a measurement through a Gaussian noise (i.e., each measurement tells that the real, continuous quantity is the one detected by the sensor plus a Gaussian noise with covariance proportional to the extension

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of the quantization area). This technique is satisfactory if the quantization is fine enough compared with the noise on the non quantized measurement [8], [9], i.e., if the uncertainty on the quantity to be estimated before the measurement is large, in some sense, with respect to the region corresponding to the quantized measurement. This is not always the case. The problem of non fine quantizations, as observed in [6], is that the quantization may introduce correlation among the innovations at different steps of the estimation algorithm, requiring correction techniques (like the ones proposed in [6]). This could be not viable in cases where the regions associated with the quantized measurements have a non trivial shape. This correction, however, does not seem really necessary if the measurements are sparse in the time domain, which would make negligible the correlation among the innovations at different steps. The sparsity of measurements in the time domain can be obtained by applying the correction only when the expected (quantized) measurement differs from the one actually observed. The algorithm corresponding to this idea is the Quantized Extended Kalman Filter (QEKF) described in a RFID robot localization context in [4].

Quantized measurements can be considered also as noisy dynamic constraints on the quantity to be estimated. Techniques taken from the context of constrained Kalman filtering can be applied in this case (see e.g. [10]). Many of these approaches (like, e.g., the modified gain approach of [11]) require an analytical expression for the constraints which is not always simple to derive. When the quantization regions are not too extensive, numerical procedures like a Histogram Filter can be efficiently applied to solve the problem (see e.g. [12], [3]), becoming computational intensive if used with large quantizations areas.

The approach proposed in this paper can be considered somewhat midway between Kalman and Particle filtering. It starts by trying to apply to the problem an Unscented Kalman Filter (UKF) [13]. However, to deal with quantized or binary measurements, the correction step of the UKF is modified by weighting the sigma-points of the UKF as it happens for particles in a PF. A Bayesian approach, approximated through a sigma-points model, is then obtained. The algorithm corresponding to this idea will be referred to as Sigma-Point Bayesian Filter (SPBF) and presents some important features.

First, since the sigma-points are a small set of elements, the computational complexity is much smaller with respect to a PF. In addition, the sigma-points are not randomly generated, unlike the particles of a PF, but are designed according to the deterministic procedure at the base of the unscented transform: in other words we develop a sort of *intelligent* PF which introduces particles with a careful deterministic selection, with the result that only a few particles are needed and it is reduced the probability that particles are generated in not significant positions (which may be a problem, as mentioned, in a PF when a few particles are considered).

Second, with respect to the other approach mentioned above, i.e. the QEKF, which may require a pre-processing of data, the SPBF algorithm does not need any preparation and is then more appropriate in a dynamic changing context (e.g., in the RFID case, if RFID tags are added, removed or fail). In addition, from a simulative point of view, we have observed that at the price of a small increase in the computational burden with respect to QEKF, a much better performance can be obtained.

The paper is organized as follows: in Section II the estimation problem considered in the paper is formally stated. The solution approach proposed in the paper is discussed in Section III. Some simulative results are reported in Section IV and some conclusions are given in Section V.

Notations. The transpose of a vector v is denoted v'. A subvector of v is denoted $v^{i_1:i_2}$ (components from i_1 to i_2). $v = [v_1; v_2]$ defines a vector $v \in \mathbb{R}^{n_1+n_2}$ that piles-up two vectors $v_1 \in \mathbb{R}^{n_1}$ and $v_2 \in \mathbb{R}^{n_2}$ ($v_1 = v^{1:n_1}$ and $v_2 = v^{n_1+1:n_2}$).

II. Problem Formulation

Consider a dynamical system which evolves according to some known discrete time dynamics:

$$x_{t+1} = f(x_t, u_t, \omega_t), \quad t = 0, 1, \ldots \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state vector ($x_0 \sim p(x_0)$), u_t is the (known) control vector and $\omega_t \sim p(\omega_t)$ is a noise vector in $\mathbb{R}^{n_\omega}$. A standard assumption is that the noise sequence ω_t is zero-mean and independent of x_0, so that by the structure (1), ω_t is independent of all x_τ, ($\tau = 0, \ldots, t$).

We assume that the state x_t can be measured through a quantized sensor, which provides an output z_t that takes a finite number of values, say m. Without loss of generality, we assume that $z_t \in \{1, 2, \ldots, m\}$.

The measurement model can be formally described through the conditional probabilities $P\{z_t = j \,|\, x_t = x\}$ of observing at time t each output value $z_t = j \in \{1, \ldots, m\}$ when the state is $x_t = x$. Thus, the description of the quantized sensor is made of a set of m functions

$$P\{z_t = j \,|\, x_t = x\} = c_j(x), \quad x \in \mathbb{R}^n, \; j \in \{1, \ldots, m\}. \tag{2}$$

where $c_j : \mathbb{R}^n \mapsto [0, 1]$ and $\sum_{j=1}^{n} c_j(x) = 1$. Throughout the paper we will use the simplified notation $P(z_t|x_t)$ to denote $c_{z_t}(x_t)$.

Remark 1: In most cases, in dynamic systems the physical quantity to be measured is a state transformation $y_t = h(x_t)$, with $y_t \in \mathbb{R}^q$ (in general $q \leq n$). When y_t is measured by means of coarse low-resolution sensors, the sensor output z_t takes value on a relatively small set of values and can be mapped into a finite interval of integers $\{1, \ldots, m\}$. Thus the sensor model is given by m conditional probabilities

$$P\{z_t = j \,|\, y_t = y\} = \tilde{c}_j(y), \quad y \in \mathbb{R}^q, \quad j \in \{1, \ldots, m\}., \tag{3}$$

$\tilde{c}_j : \mathbb{R}^q \mapsto [0, 1]$ and $\sum_{j=1}^{n} \tilde{c}_j(y) = 1$. Being $y_t = h(x_t)$ (deterministic function) then $P\{z_t = j \,|\, x_t = x\} = P\{z_t = j \,|\, y_t = h(x)\}$, and therefore $c_j(x)$ in (2) can be computed from $\tilde{c}_j(y)$ in (3) as $c_j(x) = \tilde{c}_j\big(h(x)\big)$.

Let $z_{0:t}$ denote the vector of past and current measurements z_τ, $\tau = 0, 1, \ldots, t$. The state estimation problem consists in exploiting the measurements $z_{0:t}$ to compute an estimate $\hat{x}_t$ of the state x_t. If the estimate satisfies some optimality criteria, then it is called *optimal*. Typical optimality criterion are the *minimum Mean Square Error* (MMSE) estimation and the *maximum a posteriori probability* (MAPP) estimation.

The most complete knowledge of the state x_t, given the measurements $z_{0:t}$, is the conditional pdf $p(x_t|z_{0:t})$, from which the conditional mean $x_t^* = \int x_t\, p(x_t|z_{0:t})\, dx_t$ (MMSE estimate) or the MAPP estimate $x_t^* = \arg\max_x p(x|z_{0:t})$ can be computed.

The application of the Bayes formula for the computation of the conditional pdf $p(x_t|z_{0:t})$ requires that the model (1) is transformed into the conditional pdf $p(x_{t+1}|x_t)$, suitably taking into account the known control u_t and the noise pdf $p(\omega_t)$. This step in general can only be implemented numerically through cumbersome computations.

III. Sigma-Point Bayesian Estimation approach

This section presents an easily implementable approximation of the Bayesian estimator to deal with the case of systems with quantized measurements.

A. Exact MMSE Estimation

The Bayesian MMSE estimate of the state x_t, given the measurements $z_{0:t}$, is given by the conditional expectation

$$E\{x_t|z_{0:t}\} = \int_{\mathbb{R}^n} x_t\, p(x_t|z_{0:t})\, dx_t. \tag{4}$$

A measure of the estimation error is given by the error covariance, computed as

$$P_t = \int (x_t - \hat{x}_t)(x_t - \hat{x}_t)'\, p(x_t|z_{0:t})\, dx_t. \tag{5}$$

The a priori information that allows the computation of both (4) and (5) is given by the state-transition model (the conditional pdf $p(x_{t+1}|x_t)$), the sensor model (the conditional probabilities $P(z_{t+1}|x_{t+1})$ in (2)), and the initial state pdf $p(x_0)$.

The recursive computation of the posterior conditional pdf in the integrals (4) and (5) is made of a sequence of prediction and updating steps. Assume that at time $t+1$ we get the new measurement z_{t+1} and the conditional pdf $p(x_t|z_{0:t})$ is known.

Prediction step: Chapman-Kolmogorov equation:

$$p(x_{t+1}|z_{0:t}) = \int_{\mathbb{R}^n} p(x_{t+1}|x_t)\, p(x_t|z_{0:t}) dx_t. \tag{6}$$

Updating step: (Bayes formula)

$$p(x_{t+1}|z_{0:t+1}) = \frac{p(z_{t+1}|x_{t+1})\, p(x_{t+1}|z_{0:t})}{p(z_{t+1}|z_{0:t})} \tag{7}$$

where

$$p(z_{t+1}|z_{0:t}) = \int_{\mathbb{R}^n} p(z_{t+1}|x_{t+1})\, p(x_{t+1}|z_{0:t}) dx_{t+1} \tag{8}$$

An accurate numerical computation of the integrals in (4), (6) and (8) is too demanding for real time implementations. The Particle Filter [5], aimed at approximating the integrals through a Monte Carlo approach, in many cases requires a large number of particles to achieve good results.

B. The Sigma-Point Bayesian Filter (SPBF)

Here we propose the use of the *Unscented Transform* (UT, see [13]) to provide a lumped approximation of the prior conditional pdf $p(x_{t+1}|z_{0:t})$ in the prediction step (6), by means of a set of sigma-points. Then, and approximated updating step (7) is computed, providing a lumped sigma-point approximation of the posterior $p(x_{t+1}|z_{0:t+1})$. An estimate $\hat{x}_{t+1}$ of the conditional expectation given in (4) is computed replacing the integral (4) with a weighted summation over the sigma-points. Also an estimate P_{t+1} of the error covariance given by (5) is computed via the sigma-point approximation. Using these lumped approximations, the computation of all integrals in the Bayes estimator is very easy and fast. The resulting formulas for the prediction-step are those of the Unscented Kalman Filter (UKF), while the formulas of the *Particle Filter* (PF) are used in the updating step. Note that the sigma-points in the UT are deterministically chosen, and not extracted from a distribution, like the particles in a PF. On the other hand, the accuracy of the approximation can not be very high, because the number of sigma-points is not large. However, when dealing with a low resolution quantized sensor there is no advantage in the use of a PF with a large number of particles, and the SPBF approach is sufficiently fast and accurate.

According to the UT approach (see [13] for more details), when the noise in the state transition model (1) is not additive, an augmented state x_a must be defined to include the noise vector ω_t, i.e. $x_a = [x; \omega] \in \mathbb{R}^{n_a}$, with $n_a = n + n_\omega$.

The initialization step of the SPBF consists in defining a prediction of $x_{a,0}$ (a priori estimation), denoted $\hat{x}_{a,0}^-$. The noise sequence ω_t is assumed zero-mean, and uncorrelated with x_0, so that $\hat{x}_{a,0}^- = [\hat{x}_0^-; 0]$ ($\hat{x}_0^-$ is the mean-value of x_0) with block diagonal error covariance $P_{a,0}^- = diag\ (\Psi_{x_0}, Q_0)$ (Ψ_{x_0} and Q_0 are the covariances of x_0 and ω_0, respectively). Then, the SPBF iteratively computes the pair $(\hat{x}_{t+1}, P_{t+1})$, suitably processing the pair ($\hat{x}_t$, P_t) at time t, and the (quantized) measurement z_{t+1}. The steps of the SPBF that provide $\hat{x}_{t+1}$ and P_{t+1} are as follows:

Sigma-Point Bayesian Filter (SPBF)

1) define $\hat{x}_{a,t} = [\hat{x}_t; 0]$, $P_{a,t} = diag\ (P_t, Q_t)$ and compute a set of sigma-points $\xi_{i,t}$ and weights w_i, $i = 1, \ldots, 2n_a + 1$ as in [13];
2) transform the sigma-points through the state dynamics (1)

$$\chi_{i,t+1} = f(\xi_{i,t}^{1:n}, u_t, \xi_{i,t}^{n+1:n_a}), \quad i = 1, \ldots, 2n_a + 1. \tag{9}$$

3) use the measure $z_{t+1} \in \{1, \ldots, m\}$ to update the weights

$$\tilde{w}_i = w_i\, P(z_{t+1}|\chi_{i,t+1}) = w_i\, c_{z_{t+1}}(\chi_{i,t+1}), \tag{10}$$

$$w_i^+ = \frac{\tilde{w}_i}{\bar{w}}, \qquad \text{with} \quad \bar{w} = \sum_{i=1}^{2n_a+1} \tilde{w}_i, \tag{11}$$

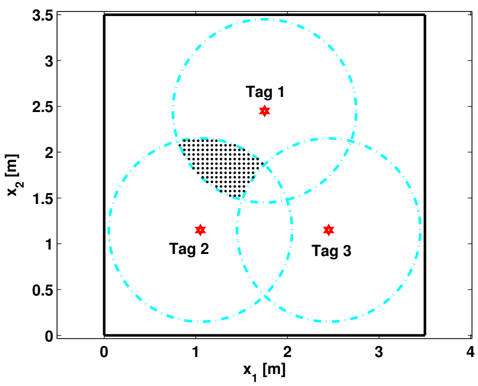

Fig. 1. Cell partition in the case of 3 tags. The dash-dotted circles are the detection areas of the tags. The shaded region is the *cell* where the robot detects the set of tags $\{T_{1,2}\}$

4) compute the updated estimate and covariance

$$\hat{x}_{t+1} = \sum_{i=1}^{2n_a+1} w_i^+ \chi_{i,t+1}, \tag{12}$$

$$P_{t+1} = \sum_{i=1}^{2n_a+1} w_i^+ (\chi_{i,t+1} - \hat{x}_{t+1})(\chi_{i,t+1} - \hat{x}_{t+1})'. \tag{13}$$

Some notes on the SPBF

• The sigma-points generated at step 1) are a lumped approximation $\{(\xi_{i,t}^{1:n}, w_i)\}$ of the posterior $p(x_t|z_{0:t})$;
• Equations (9) of step 2) provide a lumped approximation $\{(\chi_{t+1}, w_i)\}$ of the prior $p(x_{t+1}|z_{0,t})$;
• Equations (10) and (11) of step 3) provide a lumped approximation of equations (7) and (8) (updating step);
• Equations (12) and (13) are a lumped approximations of the integrals (4) and (5);
• The error covariance of the augmented state in step 1) is block diagonal because the noise ω_t is independent of the state estimate $\hat{x}_t$. This happens because the assumptions made on the noise imply that ω_t is independent of present and past measurements $z_{0:t}$, and therefore it is independent of the estimate $\hat{x}_t$, which is a function of $z_{0:t}$.

The equations (10)–(12) are similar to those of a Particle Filter (PF). However, it is important to note that in the SPBF we have no randomly extracted particles but deterministically computed sigma-points. Moreover, it is important to note that although the prediction step uses the unscented transform, the correction step of the standard UKF can not be trivially accomplished due to the quantized nature of the measurements, modeled through conditional probabilities of the type (2).

IV. Simulation Results

The effectiveness of SPBF is evaluated on a robot global localization example taken from [4] by comparing the performance of the proposed approach with the performance produced by a Particle Filter (PF) and the QEKF algorithm adopted in [4].

The robot moves in a 2D environment with coordinates (x_1, x_2) and orientation θ. Let $x = (x_1, x_2, \theta)$ be the state (pose) of the robot. We assume the following discrete time unicycle model:

$$x_{t+1} = \begin{bmatrix} x_{1,t+1} \\ x_{2,t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} x_{1,t} + \frac{v_{R,t}+v_{L,t}}{2}\cos(\theta_t) \\ x_{2,t} + \frac{v_{R,t}+v_{L,t}}{2}\sin(\theta_t) \\ \theta_t + \frac{v_{R,t}-v_{L,t}}{d} \end{bmatrix} = \tilde{f}(x_t, v_t), \tag{14}$$

where d is the distance between the right and left wheels, $v_t = [v_{R,t}\ v_{L,t}]'$ is a vector that collects the distances covered at time step t by the right and left wheels.
We can put this model in the form (1) by considering noisy encoder readings $u = [u_{R,t}\ u_{L,t}]' = v + \omega_t$ on the wheels, where $\omega_t = [\omega_{R,t}\, \omega_{L,t}]'$ is zero-mean Gaussian noise with covariance $\mathrm{diag}\{K_R|v_{R,t}|, K_L|v_{L,t}|,\}$.
Thus, $x_{t+1} = \tilde{f}(x_t, u_t - \omega_t) = f(x_t, u_t, \omega_t)$ as in (1).

A set of l RFID tags T_h, $h = 1, \ldots, l$, are mounted on the ceiling of the environment, while an RFID reader is mounted on the top of the robot, at a distance d_{rf} from the floor. The positions of the tags are known to the robot. The reading region of each tag has an ellipsoidal shape for a suitable choice of reader's and tag's antennas [4]. The intersection of the ellipsoidal reading region of tag T_h with a plane parallel to the floor at a distance d_{rf} (reader level plane) defines a circle which represents the area where the robot can detect the tag. We can define a boolean variable $s_{h,t}$: $s_{h,t} = 1$ if the robot detects tag T_h at time t; $s_{h,t} = 0$ if the robot does not detect tag T_h. The variables $s_{h,t}$ form a boolean string $[s_{1,t}, \ldots, s_{l,t}]$. This string can be encoded into a natural number which is our quantized measurement $z_t \in \{0, \ldots, m\}$, with $m = 2^l - 1$. Let S_j denote the set of tags T_h that correspond to a given index $j \in \{0, \ldots, m\}$. Let S_0 be the empty set (no tag detected), and $S_m = \{T_1, \ldots, T_l\}$ (all tags detected).

To each set S_j a detection area is associated, that we denote $\mathscr{C}_j$ and call a *cell*. As an example, the shaded area in Fig. 1 is the cell associated to the set of tags $\{T_1, T_2\}$. For instance, $\mathscr{C}_0$, the cell associated to S_0, is a region where no tag is detected, while $\mathscr{C}_m$, the cell associated to S_m, is a region where all tags are detected. Some cells may be empty regions. Note that, in a good layout of the tags, $\mathscr{C}_0$ should be an empty region, while in large workspaces $\mathscr{C}_m$ is likely to be empty. The cells do not intersect and cover all the robot workspace, and therefore form a partition of the workspace.

In a noise-free world, a robot moving inside a cell always detects the same set of tags. In a real (noisy) application, it may happen that a robot inside a cell $\mathscr{C}_j$ detects a different set of tags S_h ($h \neq j$). This depends on several disturbances which may perturb the actual detection range of each tag in each interrogation. To model these disturbances, we consider the probability density function adopted in [4]. In particular, in each interrogation of the tag we consider a perturbed detection range $\tilde{r} = r|1+n_r|$, where r is the nominal detection range and $n_r \sim \mathscr{N}(0, \sigma_r^2)$ is a normal random variable, with standard deviation σ_r, where σ_r is small enough so that the probability of observing a noise $n_r < -1$ is negligible ($r = 80$cm and $\sigma_r = 0.05$ in our simulations). The probability $p_h(x_1, x_2)$ of detecting a tag T_h from a certain position of the

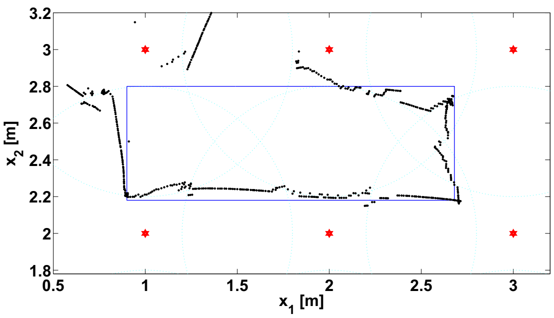

Fig. 2. The path reconstructed through the QEKF

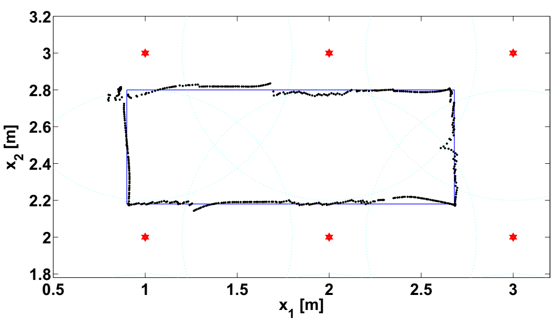

Fig. 3. The path reconstructed through the SPBF

reader (x_1, x_2) can be computed as

$$p_h(x_1,x_2) = 0.5 \cdot erfc\left(\frac{D_h(x_1,x_2)/r - 1}{\sigma_r \sqrt{2}}\right), \tag{15}$$

where $D_h(x_1,x_2) = \sqrt{(x_1 - x_{1,h})^2 + (x_2 - x_{2,h})^2}$ is the distance of the reader from the projection $(x_{1,h}, x_{2,h})$ of tag T_h on the plane at the reader height d_{rf}, and $erfc(\delta) = \frac{2}{\sqrt{\pi}} \int_\delta^\infty e^{-\gamma^2} d\gamma$ is the complement of the error function.

For a given measurement $z_t \in \{0, \ldots, m\}$, S_{z_t} is the set of tags T_h detected. The conditional probability to have the measurement $z_t = j$ for a given state $x = [x_1; x_2; \theta]$ depends only on $[x_1; x_2]$ and can be computed as

$$P\{z_t = j | x_t = x\} = \left(\prod_{T_h \in S_j} p_h(x_1,x_2)\right) \cdot \left(\prod_{T_h \notin S_j} (1 - p_h(x_1,x_2))\right) \tag{16}$$

This function is the sensor model of the type (2) in this application.

In all the simulations of this section, the odometry model of the robot is characterized by the following parameters:

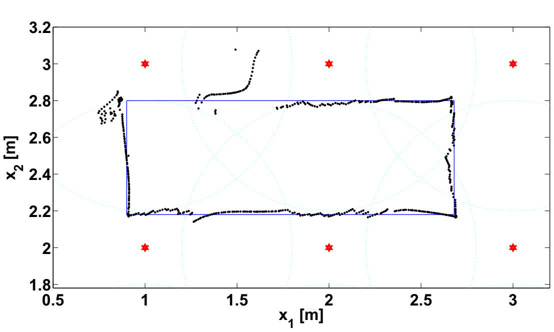

Fig. 4. The path reconstructed through the PF with 1000 particles

TABLE I

A COMPARISON BETWEEN THE DIFFERENT APPROACHES ON THE RECTANGULAR PATH SHOWN IN FIGS. 2, 3 AND 4

Method	J $[cm]$	J_s $[cm]$	T $[s]$
PF100	11.56	7.92	3.39
PF1000	5.92	3.4	33.83
QEKF	9.31	5.52	0.09
SPBF	4.6	3.57	0.5

TABLE II

A COMPARISON BETWEEN THE DIFFERENT APPROACHES ON A RANDOM PATH OF 10000 SIMULATION STEPS

Method	J $[cm]$	J_s $[cm]$	T $[s]$
PF100	5.89	2.82	62.5
PF1000	2.57	1.73	619.5
QEKF	2.56	2.01	1.5
SPBF	1.7	1.07	8.8

$d = 39$ cm, $K_R = K_L = 0.15$ cm. In addition, the RFID measurements refer to a grid of 3×3 RFID tags uniformly distributed on the ceiling of a room 4×4 m^2.

In a first set of simulations, the nominal path considered is the solid thin rectangle reported in Figs. 2, 3 and 4, which is covered clockwise starting from the left-up corner in 560 simulation steps. In these figures the path reconstructed respectively by QEKF, SPBF and PF with 1000 particles is also reported.

To have a quantitative measurement of the performance of the algorithms we introduce a position estimation error

$$e_t := \sqrt{(x_{1,t} - \hat{x}_{1,t})^2 + (x_{2,t} - \hat{x}_{2,t})^2},$$

which is the distance between the actual robot position $[x_{1,t}, x_{2,t}]$ at time step t and its estimate $[\hat{x}_{1,t}, \hat{x}_{2,t}]$. We also consider an average position estimation error:

$$J = \frac{1}{T} \sum_{t=1}^{T} e_t$$

and its steady state counterpart, which will be simply computed as:

$$J_s = \frac{1}{T/2} \sum_{t=T/2+1}^{T} e_t$$

A comparison of the position estimation error e_t corresponding to the approaches in the simulations of Figs. 2, 3 and 4 (and also of a PF with 100 particles) is reported in Fig. 5.

The values of J and J_s computed as an average of 100 independent simulation runs (each run with a proper odometry noise (ω_R, ω_L) and RFID noise n_r), considering the rectangular path reported in Figs. 2, 3 and 4, are given in Table I, together with the total computational time required by each approach to estimate all the robot path in each simulation.

In Table II we report the average of 100 independent simulation runs executed considering a random path of

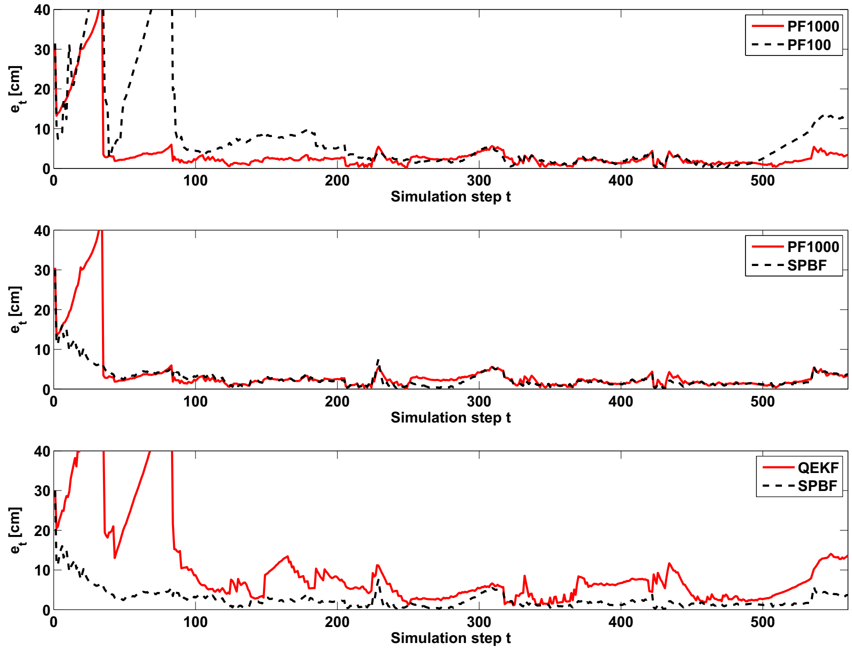

Fig. 5. A comparison of the position estimation error of the three approaches corresponding to the simulations reported in Figs. 2, 3 and 4

10000 simulations steps: according to this random path, the robot moves at constant speed on straight lines and stops performing a turn when the distance from the walls in the direction of motion becomes less than 100 cm.

From all these figures and the tables it can be observed how the proposed approach provides a satisfactory performance also with respect to the PF with 1000 particles which requires a computational effort about two orders of magnitude larger than the one required by SPBF.

V. Conclusions and Future Works

A. Conclusions

In this paper a Sigma-Point Bayesian Filter is proposed to deal with estimation problems characterized by quantized measurements. The proposed algorithm is derived from the Unscented Kalman Filter by modifying the correction step, which is executed according to a numerical approximation of the Bayes update as in Particle Filters. The simulative performance of the proposed filter is comparable with the performance of a PF even if a much smaller computational load is needed. The main advantage of SPBF w.r.t. a PF is that particles of SPBF are generated through a deterministic procedure (the one introduced in the standard UKF to approximate a Gaussian density), with the consequence that a small number of entities (the sigma-points) is enough for a proper characterization of the posterior of the state of the system. A PF usually requires a much larger number of particles to obtain similar performances.

B. Future Works

A limit of SPBF, like it also happens in the case of a standard UKF, is that a unimodal approximation is considered to describe the posterior of the state of the system. In the case of multi-modal posteriors, a PF would be more effective. The design of a multi-hypothesis SPBF could be explored to face the problem.

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