Assignment 3

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Randomised calculation of distances in graph

Time and memory complexity.

1. Space. There are $L = pn$ landmarks, therefore space complexity of computing distances from these landmarks to all other vertices is $O(pn^2)$. Besides that, there are $n - L$ other vertices, for which space complexity is also $O(pn^2)$: pn groups with $\frac{n}{pn}$ members in each of them; $\frac{n}{pn} \cdot (n-L) \approx \frac{n}{p}$. The total capacity is therefore $O(\frac{n}{p} + pn^2) = O(n^{\frac{3}{2}})$ (for $p = \frac{1}{\sqrt{n}}$).
2. Time. For $L = pn$ vertices all m edges need to be passed. Therefore, time complexity is no less than $O(m \cdot n \cdot p) = O(mn^{\frac{1}{2}})$

Optimum value of p. Total amount of memory needed to store distances from L landmarks to all of the other n vertices is (given ball size complexity is $O(\frac{1}{p})$):

$M = \frac{1}{p}n + pn^2$

$M_p^{'} = n^2 - \frac{1}{p^2}n$

It is easy to show that $M_p^{'}$ turns to zero at $p = \frac{1}{\sqrt{n}}$.

Experiment results.