MAT220 manditory 1

Fromsa

February 21, 2018

i)

- **Injective** Let $(a,b),(c,d) \in A \times B$. $f(a,b) = f(c,d) \iff (b,a) = (d,c) \implies (a,b) = (c,d)$. Therefore is f injective.
- **Surjective** Let $(b,a) \in B \times A$. If f is surjective then $\exists (a,b) \in A \times B$ such that $f(a,b) = (b,a)$ and this is the case for f.
- **Bijective** Since f is both injective and surjective, it follows that f is bijective. $\square$

ii) Let $y \in B$ then $\exists (a,b) \in A \times B$ such that $g(a,b) = y$

$$b = y \implies g(a,b) = b$$

Therefore is g surjective $\square$
For g to be surjective A must contain only one element.

iii)

- **Refleksiv** Let $a \in \mathbb{R}$ then $sin(a) = sin(a)$. ✓
- **Symmetric** Let $\{a,b\} \in \mathbb{R}$. If $sin(a) = sin(b)$ then $sin(b) = sin(a)$ ✓
- **Transitiv** Let $\{a,b,c\} \in \mathbb{R}$. If $sin(a) = sin(b)$ and $sin(b) = sin(c)$ then $sin(a) = sin(c)$ ✓

Therefore, $x \sim y$

- The equivalence class of $0, [0] = \{2\pi \cdot k | k \in \mathbb{Z}\}$
- The equivalence class of $\frac{\pi}{2}, [\frac{\pi}{2}] = \{\frac{\pi}{2} + 2\pi \cdot k | k \in \mathbb{Z}\}$

iv)

- **Refleksiv** Let $a \in \mathbb{Q}$ then $a - a = 0 \in \mathbb{Z}$. ✓
- **Symmetric** Let $a, b \in \mathbb{Q}$. If $a - b \in Z$ then $b - a = -(a - b) \in \mathbb{Z}$ ✓
- **Transitiv** Let $a, b, c \in \mathbb{Q}$. If $a - b \in Z$ and $b - c \in \mathbb{Z}$ then $a - c = a - b + b - c = (a - b) + (b - c) \in \mathbb{Z}$ ✓

Therefore, $q \sim p$

- The equivalence class of $\frac{3}{2}, [0] = \{\frac{2k+1}{2} | k \in \mathbb{Z}\}$ same as $[\frac{1}{2}]$
- The equivalence class of $1, [1] = \{k | k \in \mathbb{Z}\}$

v) $2000 = 17 * 117 + 11$ same as $2000 \equiv 11 (mod\, 17)$

vi) If $a|b$ and $a|c$ then by definition, $\exists k_1, k_2 \in \mathbb{Z}$ such that $b = a \cdot k_1$ and $c = a \cdot k_2$.
$b + c = a \cdot k_1 + b \cdot k_2 = a(k_1 + k_2)$
Since $\mathbb{Z}$ is closed under addition, we have that $(k_1 + k_2) \in \mathbb{Z}$
Thus, by definition $a|(b + c)$ $\square$

vii) $gcd(n, n+2) \iff n + 2 = n \cdot 1 + (2) \rightarrow n = 2 \cdot k + 0 \lor n = 2k + (1)$ Where $k \in \mathbb{Z}$
Possible solution are (2) and (1).
$gcd(n, n+3) \iff n + 3 = n \cdot 1 + (3) \rightarrow n = 3k + (2) \lor n = 3k + (1) \lor n = 3 \cdot k + 0$
Where $k \in \mathbb{Z}$
Possible solution are $(3), (2)$ and (1).

viii) Given $a \equiv 1(mod 4)$ and $b \equiv 2(mod 4)$.
Since modulo is closed under addition, we have that $a - b \equiv 1 - 2(mod 4)$ and
Since it is closed under multiplication, we have that $3(a - b) \equiv 3(1 - 2) \equiv -3 \equiv 1(mod 4)$.

ix) _

Addition table for $\mathbb{Z}_6$

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Multiplication table for $\mathbb{Z}_6$

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1